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IMPROVED FINITE-ELEMENT METHODS FOR ROTORCRAFT STRUCTURES

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ARO 6.1 Review

IMPROVED FINITE-ELEMENT METHODS FOR ROTORCRAFT STRUCTURES

Howard E. Hinnant

Aerostructures Directorate
U.S. Army Aviation Systems Command

April 23, 1991

This report documents a presentation given at Langley Research Center on April 23, 1991. The purpose of the presentation was to give an overview of the past year's research to the Army Research Organization represented by Dr. Gary Anderson. The research is directed at improving finite element methods for rotorcraft airframes. The main portion of the presentation covers the development of a modification to the finite element method which eliminates interelement discontinuities.

Outline

- Objective
- Recent Developments in Finite Elements
  - Geometric Entities
  - Inter-Element Continuity
  - Dependent Rotational Degrees of Freedom
  - Current Status
- Georgia Tech Grant
- Adaptive Numerical Integration
- Summary

Howard E. Hinnant

There are three basic areas which have been the focus of this research during the past year. The majority of this talk is about recent developments in finite elements. Next, there is a brief discussion about our grant with Dewey Hodges at Georgia Tech. Finally, a new numerical integration technique, which was developed last year, is presented.

Objective

Dynamic analysis of rotorcraft airframes has much potential for improvement. The objective of this research is to obtain that improvement by developing better analyses and better modeling techniques.

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The overall objective of this research is to improve the capability of dynamic finite element analyses. This can be done by improving the analysis methods or by improving the modeling techniques used in today's analyses. This research project investigates both possibilities, however, here we focus on improved methods as opposed to better modeling techniques.

FEAT

Finite Element Advanced Technology

FEAT is a research finite element code under development. It's purpose is to provide an environment in which new finite element methodologies and modeling techniques can be experimented with.

Last year I presented a p-version, tapered beam element for FEAT. This year the scope has been extended to include shell and brick elements as well. However, to successfully incorporate different kinds of elements in the same analysis, the fundamentals of the analysis method itself needed some reworking.

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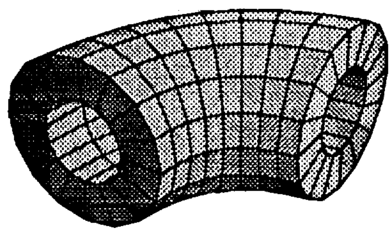
The following slides depict some of the problems which motivated me to modify the methodology of finite elements.

ARO 6.1 Review

Typical Problems In Current Finite Element Analyses



It can take many elements to model complex geometries.

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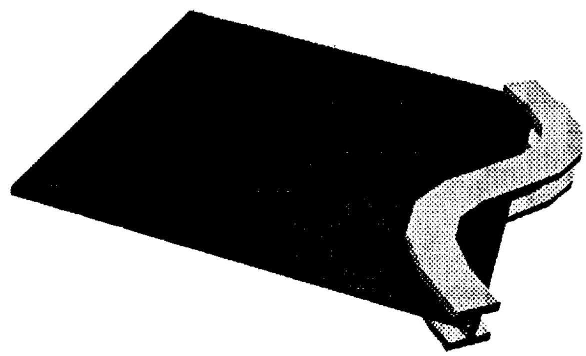
Sometimes the geometry of a model is complex enough that mesh subdivision (the number of elements) is controlled by geometric considerations, instead of by convergence considerations. For example, in modeling a curved pipe in a conventional h-version code, one is probably not concerned that the model does not have enough elements (degrees of freedom), because many elements are needed just to capture the geometry. A more ideal situation would be to specify only enough elements (degrees of freedom) to ensure convergence, and still model the geometry exactly.

Typical Problems In Current Finite Element Analyses (continued)



Discontinuities between different types of elements. (Plate - Beam example shown)

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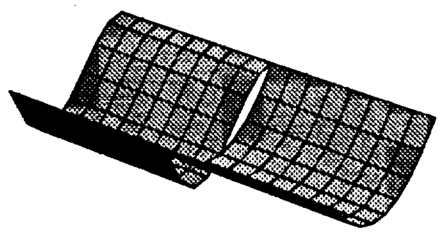
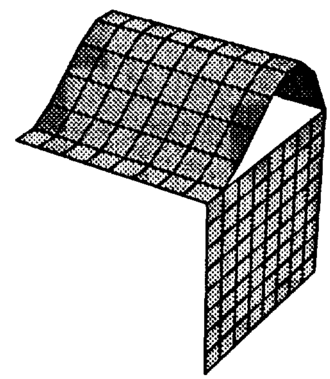
Many finite element models use different types of elements in the same analysis. A prime example is the rotorcraft fuselage. This model usually consists of beams for stringers and plates for skin. The problem is that continuity is not maintained between beams and plates except at the grid points. The same problem exists when mixing beams and bricks, and when mixing plates and bricks. The importance of the discontinuity is not known, but the only way to analyze the effects of it are to get rid of the discontinuities and see if the results improve.

Typical Problems In Current Finite Element Analyses (continued)

Discontinuities between high- and low-order elements.



Discontinuities between non-coplanar plate elements.



The solution to these problems lies in using what I refer to as *geometric entities.*

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Discontinuities also can exist between adjacent high- and low-order elements. Transition elements are sometimes used to combat this problem. This requires that the user change his mesh (geometry) at the point where the order changes, regardless of what the physical geometry is doing. It would be best, however, if the user could just specify order based on engineering judgement, and let the program adjust for interelement continuity.

Plates typically have cubic out-of-plane displacements but maintain only linear displacement fields for in-plane displacements. When two plates are connected in an out-of-plane fashion, as shown, the linear displacement field of the vertical plate can not keep up with the cubic field of the horizontal plate. This creates a gap between the plates during deformation.

A new finite element methodology using geometric entities has been developed which completely solves the interelement discontinuity problem, and which greatly aids in the modeling of complex geometry.

Definitions

Geometric Entity

- **What it is:**
  - It Defines Geometry.
  - It may possess degrees of freedom.
  - It may be composed of other geometric entities.
- **What it isn't:**
  - It does not specify mass or stiffness relationships among degrees of freedom.
- **Examples:**
  - Point
  - Line
  - Surface
  - Volume

Finite Element

- **What it is:**
  - It specifies mass & stiffness relationships among degrees of freedom.
  - It is associated with a geometric entity.
- **What it isn't:**
  - It does not possess degrees of freedom.
  - It does not have a geometry of it's own.
- **Examples:**
  - Rigid Mass
  - Beam
  - Shell
  - Brick

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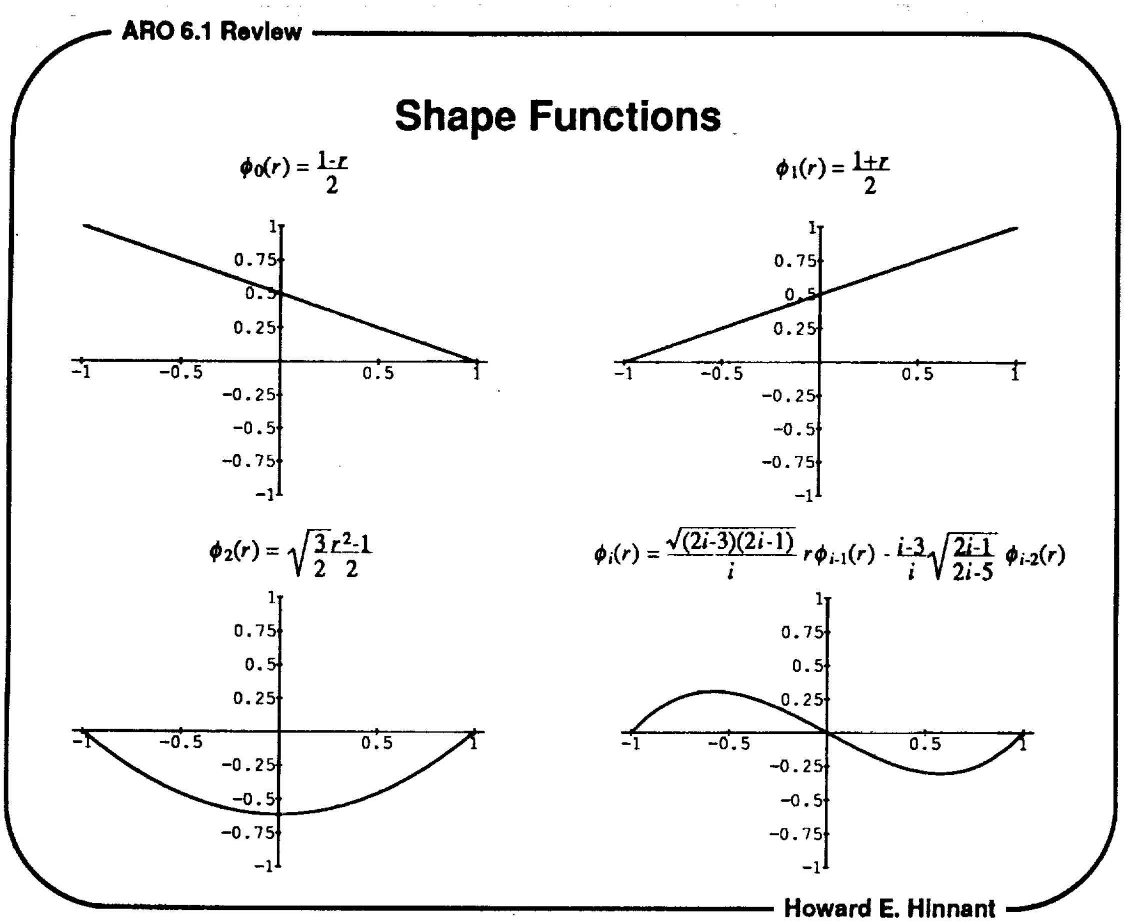
A new class of mathematical objects has been identified for use in finite element analyses. This class is referred to as geometric entities. The grid point is the first example of a geometric entity, and has been in use since the first finite element program. Here, the idea of the grid point has been generalized to include lines, surfaces, and volumes. This generalization is necessary and advantageous when working with bricks, shells, and beams in the same analysis.

It is important to make a clear distinction between geometric entities and finite elements. Each of these mathematical objects is important and has specific duties and responsibilities during an analysis. A geometric entity's first responsibility is to define a geometry (location and shape). Speaking mathematically, this would be a transformation from a local, curvilinear coordinate system to a global, orthonormal coordinate system. The geometric entity may also possess degrees of freedom, allowing it to translate and/or rotate during deformation. A higher dimensional entity (line, surface, or volume) is composed of lower-dimensional entities. A geometric entity does not specify any mass or stiffness relationship among the degrees of freedom.

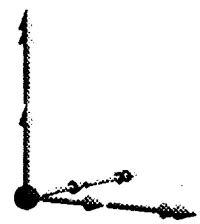
A finite element's only responsibility is to provide mass and stiffness relationships among the degrees of freedom. It is associated with a geometric entity and assumes that geometry. A brick is associated with a single volume instead of 8 points as would be done in a traditional finite element program. A shell is associated with a surface, a beam with a line, and a rigid mass with a point. Finite elements do not contain any degrees of freedom. This is contrary to every other p-version finite element code.



Before discussing the geometric entities in more detail, the shape functions need to be defined. Shape functions are polynomials used for interpolation. They interpolate geometry, translational deformation and rotational deformation. The same shape functions are used for all of these purposes, giving this analysis an isoparametric flavor. However, this is not truly an isoparametric analysis because the same number of shape functions need not be used for geometry and deformation. The shape functions are symbolized by the Greek letter ϕ, and a subscript which indicates the order of the polynomial. The exception to this rule is the zeroth shape function. It is actually linear instead of a constant. The first two shape functions are the typical C0-type shape functions. The higher-order ones are integrals of Legendre polynomials and are the same as used by Szabo and Babuska. The shape functions are zero at the end points, and their first derivatives form an orthonormal set. The orthogonality produces numerical stability in the element matrices even at high orders.

Geometric Entities
Point

The point is the simplest of the geometric entities. It consists only of a spatial location, and from 0 to 6 degrees of freedom.



$x^l_{point} = \hat{x}^l$ **3 Spatial Coordinates (Defines Geometry)**

$u^l_{point} = \hat{u}^l$ **3 Degrees of freedom (translation)**

$\theta^l_{point} = \hat{\theta}^l$ **3 Degrees of freedom (rotation)**

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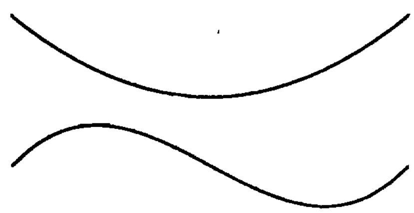
The point is the basis for all geometric entities. Of course, the idea of a point has been used in finite elements since the beginning. The point defined here is identical to NASTRAN's grid point and to the points of virtually every other program. It defines a location with three coordinates, and it has up to six degrees of freedom (three translations, three rotations). The point can be extrapolated from zero dimensions into one, two, and three dimensions. This process forms the set of mathematical objects referred to as geometric entities.

The equations for the point are listed here. The x equation defines the geometry. The superscript l runs from 1 to 3 and indicates the three orthogonal axes ($x1$, $x2$, and $x3$). The equation merely states the undeformed location of the point in space. The u equation represents the translational deformation. Again, there are three components denoted by the superscript l. The rotational degrees of freedom are represented by the θ equation. Note that the form of the three equations is the same. This fact will remain true with all geometric entities.

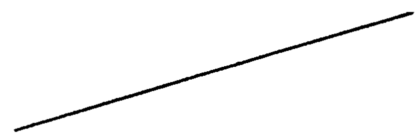
Once a point is defined, a rigid mass (finite element) can be associated with it. So far, this is no different than conventional finite elements.

Geometric Entities
Line

A line consists of 2 points plus optional "curvature points" which can curve the line in 3-D space. The line may possess 0 or more degrees of freedom in translation and rotation.



The first two line degrees of freedom. Line degrees of freedom don't influence the associated points.



Influence of point deformation on a line.

$$x^l_{line}(r^1) = \phi_0(r^1)x^l_{point\,1} + \phi_1(r^1)x^l_{point\,2} + \sum_{i=2}^{P_{xr}} \phi_i(r^1)\hat{x}^l_i \longleftarrow$$ **Curvature points.**

$$u^l_{line}(r^1) = \phi_0(r^1)u^l_{point\,1} + \phi_1(r^1)u^l_{point\,2} + \sum_{i=2}^{P_{ur}} \phi_i(r^1)\hat{u}^l_i \longleftarrow$$ **Line translational degrees of freedom.**

$$\theta^l_{line}(r^1) = \phi_0(r^1)\theta^l_{point\,1} + \phi_1(r^1)\theta^l_{point\,2} + \sum_{i=2}^{P_{\theta r}} \phi_i(r^1)\hat{\theta}^l_i \longleftarrow$$ **Line rotational degrees of freedom.**

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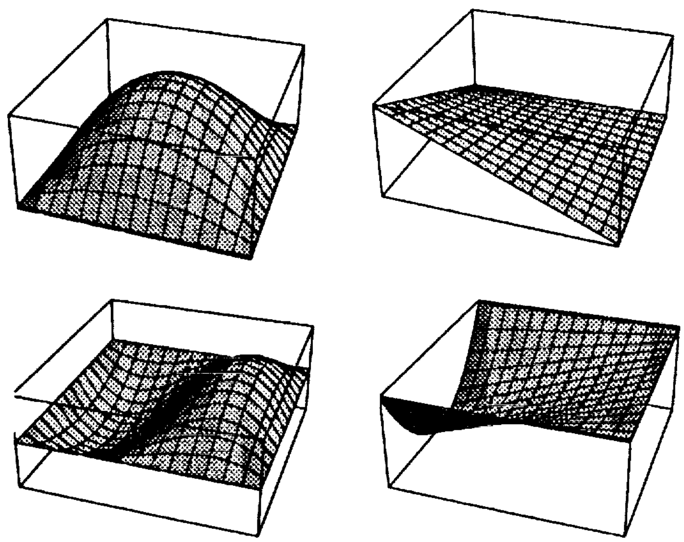
To create the line, the zero dimensional point is generalized to one dimension. The line is made up of two points which serve to mark the beginning and ending of the line. The line may be straight, or the user may define intermediate "curvature points" through which the line must pass.

The line may possess degrees of freedom. These degrees of freedom are distinct from the degrees of freedom defined by the line's points. If the line has no degrees of freedom, then the line will deflect in a linear fashion according to the line's end points. If the line has degrees of freedom, then it's middle can move without affecting the motion of the end points. The user can specify as many degrees of freedom for a line as computer memory allows.

The first two terms in all of the equations represent the linear behavior of the line between two points. In the geometry equation, a series of curvature points can be added to give the line a generally curved shape. In the deformation equations, these extra terms represent degrees of freedom. Notice that there does not need to be the same number of curvature points as there are translational degrees of freedom or rotational degrees of freedom. However, for any one equation there are the same number of higher-order terms for each value of the superscript l. This is an extremely important property. In traditional terms, this means that a line's axial deformation will be the same order as it's bending deformation. This is necessary so that the higher-order degrees of freedom can be transformed to global coordinates and maintain displacement continuity.

Once a line has been defined, a beam finite element can be associated with it. This is different than traditional programs where a beam is associated with two points. One advantage here is that the beam will assume the geometry of the line, no matter how curvy.

Geometric Entities
Surface



Two surface degrees of freedom.

Influence of point deformation on a surface.

Influence of line deformation on a surface.

$$x^l_{surface}(r^1,r^2) = \phi_0(r^1)x^l_{line\ 1}(r^2) + \phi_1(r^1)x^l_{line\ 2}(r^2) + \phi_0(r^2)x^l_{line\ 3}(r^1) + \phi_1(r^2)x^l_{line\ 4}(r^1)$$
$$- \phi_0(r^2)\phi_0(r^1)x^l_{line\ 1}(-1) - \phi_0(r^2)\phi_1(r^1)x^l_{line\ 2}(-1) - \phi_1(r^2)\phi_0(r^1)x^l_{line\ 1}(1) - \phi_1(r^2)\phi_1(r^1)x^l_{line\ 2}(1)$$
$$+ \sum_{j=2}^{P_{su}} \sum_{i=2}^{P_{sv}} \phi_j(r^2)\phi_i(r^1)\hat{x}^l_{ji}$$

And similarly for u and θ

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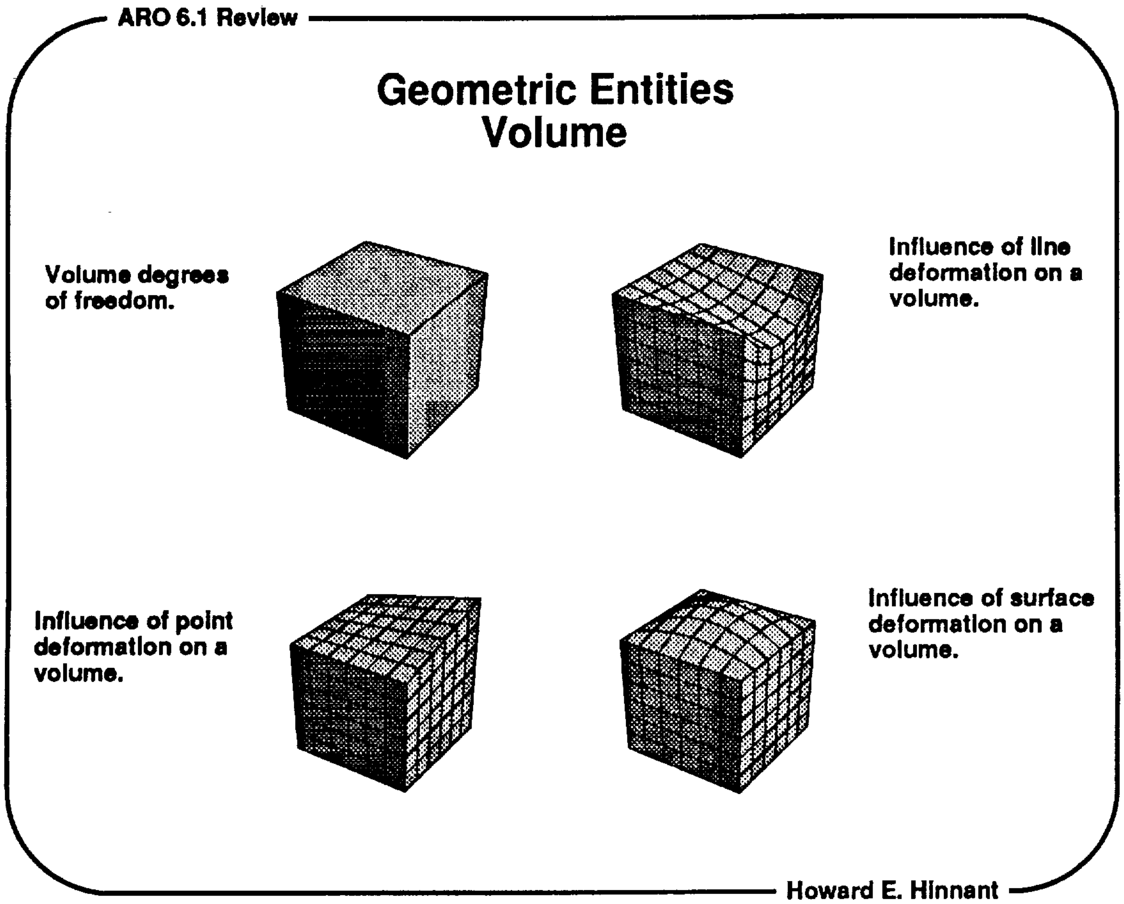
Extrapolating the line into two dimensions gives rise to the surface. The surface consists of four lines. Each of those lines may be curved or straight. The surface will warp linearly to meet each line. In addition, the user may specify curvature points through which the interior of the surface must pass.

The surface's degrees of freedom will only effect it's interior. Two low-order degrees of freedom are pictured above. The higher-order degrees of freedom get more "bubbly" but still go to zero at the surface's boundaries. This slide also shows the effects on a surface when its lines and points deflect. The surface will warp itself to follow the deflection of the points and lines, even if it has no degrees of freedom of it's own.

Only the geometry equation is shown because of lack of space. But the form is the same for the deformation equations. Note that the equation depends only on the four lines explicitly. The points which comprise those lines do not appear. This modularity helps to protect the integrity of the surface throughout the analysis. Also note that the number of higher-order terms in the r_1 and r_2 directions need not be the same. For example, u_1 may be a quadratic in r_1 and cubic in r_2. However, u_2 and u_3 have the same order in r_1 and r_2 as does u_1 so that displacement continuity can be maintained during transformations.

Once a surface is defined, a shell element can be associated with it. The shell will have the curvature of the surface and will deform according to the degrees of freedom defined by the surface and it's sub-geometric entities. Since a surface consists of four lines, it is important to make the distinction between lines and beams. It would not be appropriate to define the boundaries of a surface (or shell element) with four beam elements.



The 3-dimensional geometric entity is called the volume. It is made up of six surfaces, each of which may be curved. In addition, the user may specify curvature points through which the interior of the volume must pass.

The degrees of freedom of a volume only affect its interior and thus are not visible in a plot. This does not mean that they are not important, though. The user may specify any number of degrees of freedom for the volume in each of the three coordinates independently (r_1, r_2, r_3). One should exercise some constraint when raising the number of degrees of freedom in all directions because the number of unknowns rises very rapidly (as a cubic). In many applications, high order may only be needed in one direction. For example, a cantilevered beam being modeled by a brick element: the volume may need only linear or quadratic functions in the transverse coordinates, but cubic or quartic in the axial coordinate. The user has the freedom to do this.

A volume does not need degrees of freedom to deform. As shown above (and consistent with lines and surfaces) a volume will linearly warp to conform to the deformation of it's surfaces, lines, and points.

Once a volume is defined, a brick finite element can be associated with it. The brick will adopt the curved shape of the volume, no matter how convoluted. The user can therefore create quite complex geometries using lines, surfaces, and volumes, and use the same beams, shells, and bricks to model the complex geometries as one would use to model simple linear geometries.

Geometric Entities
Volume (continued)

$$
\begin{aligned}
x^l_{volume}(r^1,r^2,r^3) = {} & \phi_0(r^1)x^l_{surface1}(r^2,r^3) + \phi_1(r^1)x^l_{surface2}(r^2,r^3) \\
& + \phi_0(r^2)x^l_{surface3}(r^1,r^3) + \phi_1(r^2)x^l_{surface4}(r^1,r^3) \\
& + \phi_0(r^3)x^l_{surface5}(r^1,r^2) + \phi_1(r^3)x^l_{surface6}(r^1,r^2) \\
& - \phi_0(r^2)\phi_0(r^1)x^l_{surface1}(-1,r^3) - \phi_0(r^2)\phi_1(r^1)x^l_{surface2}(-1,r^3) \\
& - \phi_1(r^2)\phi_0(r^1)x^l_{surface1}(1,r^3) - \phi_1(r^2)\phi_1(r^1)x^l_{surface2}(1,r^3) \\
& - \phi_0(r^3)\phi_0(r^1)x^l_{surface1}(r^2,-1) - \phi_0(r^3)\phi_1(r^1)x^l_{surface2}(r^2,-1) \\
& - \phi_1(r^3)\phi_0(r^1)x^l_{surface1}(r^2,1) - \phi_1(r^3)\phi_1(r^1)x^l_{surface2}(r^2,1) \\
& - \phi_0(r^3)\phi_0(r^2)x^l_{surface3}(r^1,-1) - \phi_0(r^3)\phi_1(r^2)x^l_{surface4}(r^1,-1) \\
& - \phi_1(r^3)\phi_0(r^2)x^l_{surface3}(r^1,1) - \phi_1(r^3)\phi_1(r^2)x^l_{surface4}(r^1,1) \\
& + \phi_0(r^3)\phi_0(r^2)\phi_0(r^1)x^l_{surface1}(-1,-1) + \phi_0(r^3)\phi_0(r^2)\phi_1(r^1)x^l_{surface2}(-1,-1) \\
& + \phi_0(r^3)\phi_1(r^2)\phi_0(r^1)x^l_{surface1}(1,-1) + \phi_0(r^3)\phi_1(r^2)\phi_1(r^1)x^l_{surface2}(1,-1) \\
& + \phi_1(r^3)\phi_0(r^2)\phi_0(r^1)x^l_{surface1}(-1,1) + \phi_1(r^3)\phi_0(r^2)\phi_1(r^1)x^l_{surface2}(-1,1) \\
& + \phi_1(r^3)\phi_1(r^2)\phi_0(r^1)x^l_{surface1}(1,1) + \phi_1(r^3)\phi_1(r^2)\phi_1(r^1)x^l_{surface2}(1,1) \\
& + \sum_{k=2}^{P_{xt}} \sum_{j=2}^{P_{xs}} \sum_{i=2}^{P_{xr}} \phi_k(r^3)\phi_j(r^2)\phi_i(r^1)\hat{x}^l_{kji}
\end{aligned}
$$

And similarly for u and θ

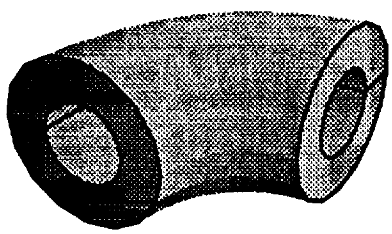
Howard E. Hinnant

The geometry equation for the volume is shown here for completeness. The deformation equations are not shown because of space limitations. The volume is dependent only on six surfaces and not any lines or points. Most of the equation is just to do the linear interpolation among the surfaces. The higher-order terms are written very compactly in the last term of the equation.

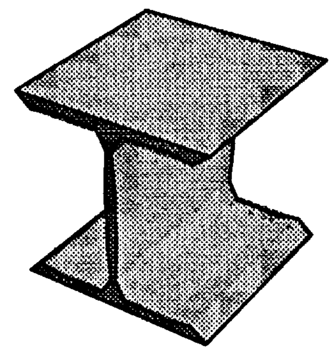
ARO 6.1 Review

Advanced Geometry

Lines, surfaces, and volumes can be shaped to model complex geometries. The user simply specifies points through which the geometry passes, and FEAT constructs a smooth interpolation.



Volume mapped into a curved pipe.



Volume mapped into an I - beam.

A single brick element can model either of these complex geometries.

Howard E. Hinnant

Two examples of what can be done by curving lines and surfaces are shown above. Both of these shapes started out as cubes, but by curving the lines and surfaces appropriately, a curved pipe and an I-beam were created. A single brick element can be used to model either of these geometries. There is no thin-wall assumption on the pipe and no beam theory associated with the I-beam. These analyses would result in the 3-D elasticity solution. The pipe could have as few as 12 degrees of freedom and the I-beam as few as 24. However, with so few degrees of freedom, accuracy would be compromised. To add degrees of freedom (and get accurate answers) only the order of the shape functions needs to be modified; the existing geometry does not.

Still, this analysis is not limited to brick elements. If the thin-wall assumption for the pipe can be made, then it could be modeled with a shell associated with a curved surface (instead of the volume). A straight line would be the appropriate geometry to model the I-beam with a beam element.

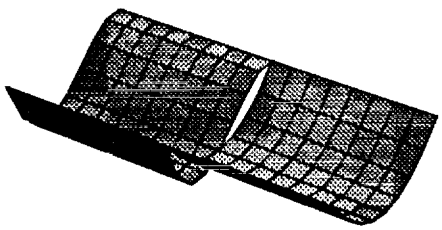
The analysis of anisotropic materials is also included. The material coordinate system is an orthonormal one which follows the curvature of the geometry but which is rotated from the curvilinear system by a user specified amount.

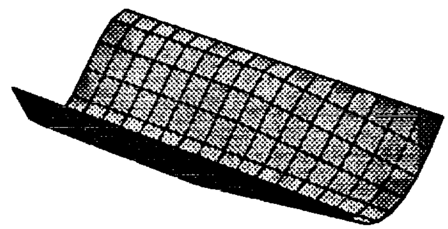
Fancy geometry is possible without geometric entities. However, using the geometric entities makes modeling complex geometry much easier. For example, for the I-beam above, only the 4 vertical lines of the cube needed to be curved. The surfaces and volume of the I-beam curved automatically to follow the lines. For the pipe, the 4 vertical lines of the cube were curved into circles to make a cylinder. Then the 4 horizontal lines were curved into a quarter circle. At no point was it necessary to deal with the entire volume at once.

Maintaining Inter-Element Continuity While Varying Shape Function Order

Problem:

Part of the model needs high-order shape functions while another part requires only low-order shape functions.





Traditional programs:

Discontinuities exist between high-order elements and low-order elements.

FEAT:

The use of geometric entities allows a smooth continuous transition from high order to low order.

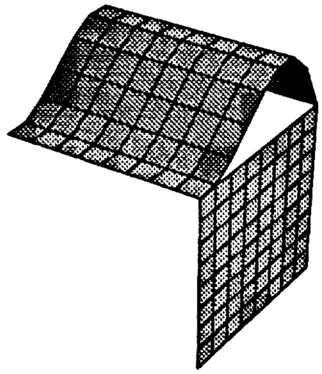
Howard E. Hinnant

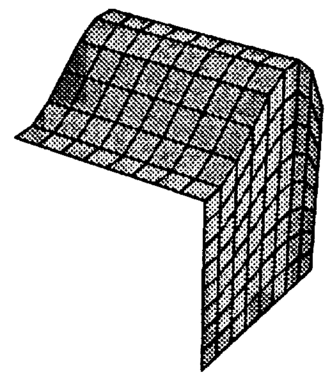
The use of geometric entities makes interelement continuity over a wide range of conditions and elements completely automatic. For example, sometimes part of the model needs high-order shape functions to capture the physics, while other parts of the same model are hardly moving. One could simply specify high-order shape functions for the entire model, but this would waste a large amount of computational time. Using high-order shape functions over an entire model would be analogous in h-version codes to putting in a very fine mesh everywhere, when it is needed only in a small, specific area.

Traditional p-version codes incorporate the higher-order degrees of freedom into the elements themselves. This can lead to displacement discontinuities between adjacent high- and low-order elements. The same thing also can happen when different order elements are used in a h-version code. In FEAT, the elements don't have degrees of freedom; the geometry does. Furthermore, due to the construction of the shape functions, one can place a hundredth order surface right next to a linear surface and maintain displacement continuity. This is because adjacent surfaces share the same line, and that line can only be in one place at a time. Surfaces are used as an example here because they're easy to visualize, but all geometric entities maintain interelement continuity.

ARO 6.1 Review

The Non-Coplanar Plate Problem





Traditional programs:

Plate elements have cubic out-of-plane deformation, but only linear in-plane deformation. This can cause displacement discontinuities between non-coplanar plates.

FEAT:

Any two adjacent surfaces share the same line between them. This eliminates displacement discontinuities.

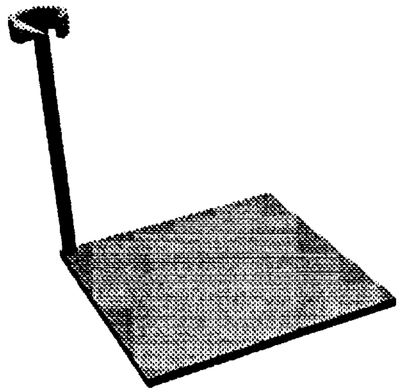
Howard E. Hinnant

In traditional finite element programs, plate elements have cubic shape functions for the out-of-plane bending, and a linear displacement field in-plane. When two plates are attached out-of-plane with respect to each other, this can cause discontinuities, even though both elements are identical.

It is impossible for a surface to deflect as a cubic out-of-plane while being restricted to linear in-plane displacement. The slides concerning geometric entities (pg. 10-14) explain that displacements along the three axes are always of the same order. This is why that restriction is applied. Therefore, when using geometric entities, non-coplanar plates will maintain continuity.

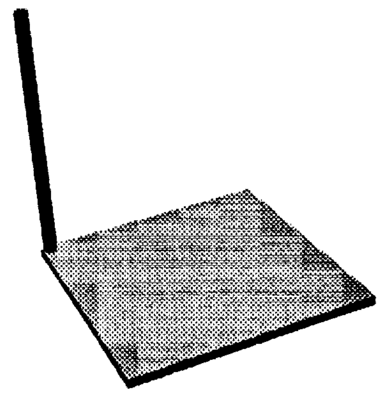
ARO 6.1 Review

Dependent Rotational Degrees of Freedom



Traditional programs:

Out-of-plane rotation for plate element has zero stiffness. A beam attached as shown will spin like a top.



FEAT:

Plate drilling degree of freedom is constrained in an elastically consistent manner. Beam torque will transfer to plate.

Howard E. Hinnant

Traditional plate elements contain another annoying problem. Due to consistent elastic assumptions when creating a plate element, there is no degree of freedom for the out-of-plane (drilling) rotation. This creates problems in the non-coplanar plate situation, or when plates are mixed with other types of elements. For example: If a beam is connected to a plate as shown above in NASTRAN, and a torque is applied to the beam, the beam will spin like a top. The plate will not resist the beam at all. NASTRAN will let the user "zero out" the out-of-plane rotation (effectively giving the plate infinite stiffness in that area), but one can intuitively see that either of these extremes gives the wrong result.

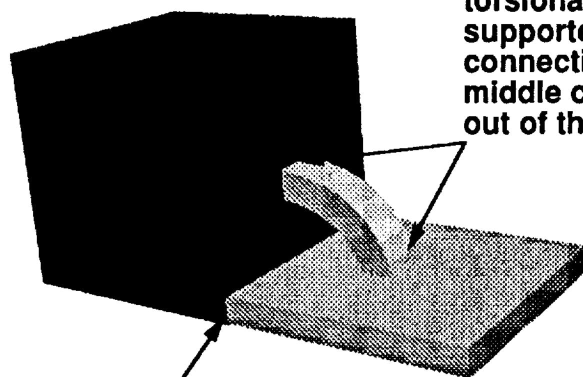
The answer lies in constraining the out-of-plane rotation to the in-plane translations of the plate in an elastically consistent manner. This process is termed "dependent rotational degrees of freedom". This is not a completely new idea, but it has never been implemented in a major finite element code.

Because this is implemented as a constraint equation, the strain energy of the plate is not affected. Therefore the integrity of plate theory is not compromised.

ARO 6.1 Review

Dependent Rotational Degrees of Freedom (continued)

Not only is the beam torsional coupling not supported, but connecting to the middle of an element is out of the question.



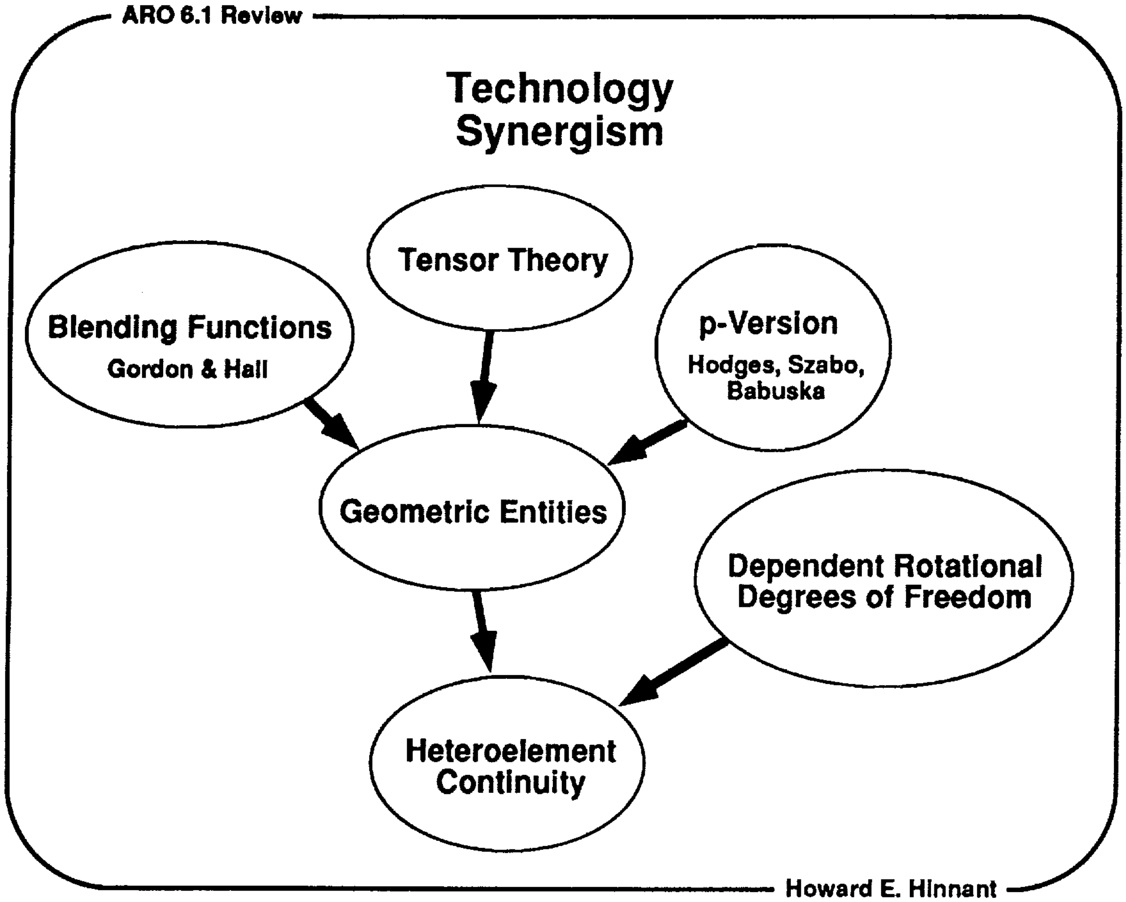
Traditional programs are pinned at the brick - plate interface.

FEAT fixes all of these problems, providing complete inter-element continuity (in translation and rotation) even across different types of elements (heteroelement continuity).

Howard E. Hinnant

Dependent rotational degrees of freedom are not limited to plates. They can be applied to bricks as well. Without them, rotational discontinuity would exist between bricks, plates, and beams. As an extra convenience, dependent degrees of freedom can be placed anywhere on an element, not just at a corner. This allows, for example, a beam element to connect to the middle of a brick or a plate. As the ends of the beam rotate, the brick and plate elements will react in an elastically consistent manner.

With geometric entities and dependent rotational degrees of freedom combined, adjacent elements of different types will maintain continuity in translation and rotation. This can be referred to as heteroelement continuity. Airframes are always modeled with beams for stringers and plates for the skin. Not one of the analyses for any rotorcraft has ever maintained continuity between the skin and the stringers. It simply isn't possible with a NASTRAN style analysis. Admittedly, the effect of the discontinuity is an unknown at this time. But it is only with geometric entities and dependent rotational degrees of freedom that we will ever learn if it is an important effect or not.



This research represents the collaboration of several technologies. Geometric entities are only possible by combining tensor theory with the work done by Gordon and Hall in the early seventies. Blending functions were originally developed to handle automated grid generation for structural analysis. However, blending function theory amounts to nothing more than a generalized coordinate transformation. Tensor theory was created to handle general coordinate transformations. By combining the two, the geometry and the strains and stresses can be transformed in the same manner. Bringing the idea of p-version into geometric entities allows for a consistent treatment of the undeformed and the deformed state of the structure.

Combining the ideas of geometric entities with dependent rotational degrees of freedom makes possible interelement continuity across all elements in both translation and rotation. Advanced beam and plate theories may be incorporated into this analysis. If the theories can be written in a C0-continuous fashion, then there will be no trouble bringing them into FEAT.

ARO 6.1 Review

Current Status

- **FEAT currently consists of 6000 lines of code.**
- **No analysis capability yet.**
- **Brick element should be running in a few months.**
- **Beam and shell elements will follow.**

Howard E. Hinnant

FEAT currently consists of about 6000 lines of code. However, most of this is user interface, data base interface, and mathematical utilities. So far there is no analysis capability. The brick element will be implemented first. This might be completed in a few months. The beam and shell will follow.

ARO 6.1 Review

Georgia Tech Grant

Modeling of Composite Beams and Plates for Static and Dynamic Analysis

Principal Investigator:

Dewey H. Hodges

- Developing methodologies for rigorous determination of plate properties for anisotropic, nonhomogeneous structures.
- Developing methodologies for space-time beam finite elements to be used in dynamic analyses.
- This work will directly impact studies done with FEAT.
- Progress is very encouraging.
- Currently in second year out of three-year grant.

Howard E. Hinnant

Other work includes the monitoring of a Georgia Tech. grant entitled "Modeling of Composite Beams and Plates for Static and Dynamic Analysis". The principal investigator is Dewey Hodges, and he is studying plates and beams together as thin bodies, treating the theories similarly as much as practical. Dr. Hodges is working on a rigorous determination of anisotropic, nonhomogeneous plate properties.

Dr. Hodges is also looking into space-time beam finite elements. These elements discretize both space and time to get a displacement field as a function of time. The space-time beam can be visualized as a plate element with one dimension being space and the other dimension being time.

This work could have a major impact on how beam and plate elements are implemented in FEAT.

ARO 6.1 Review

Integration

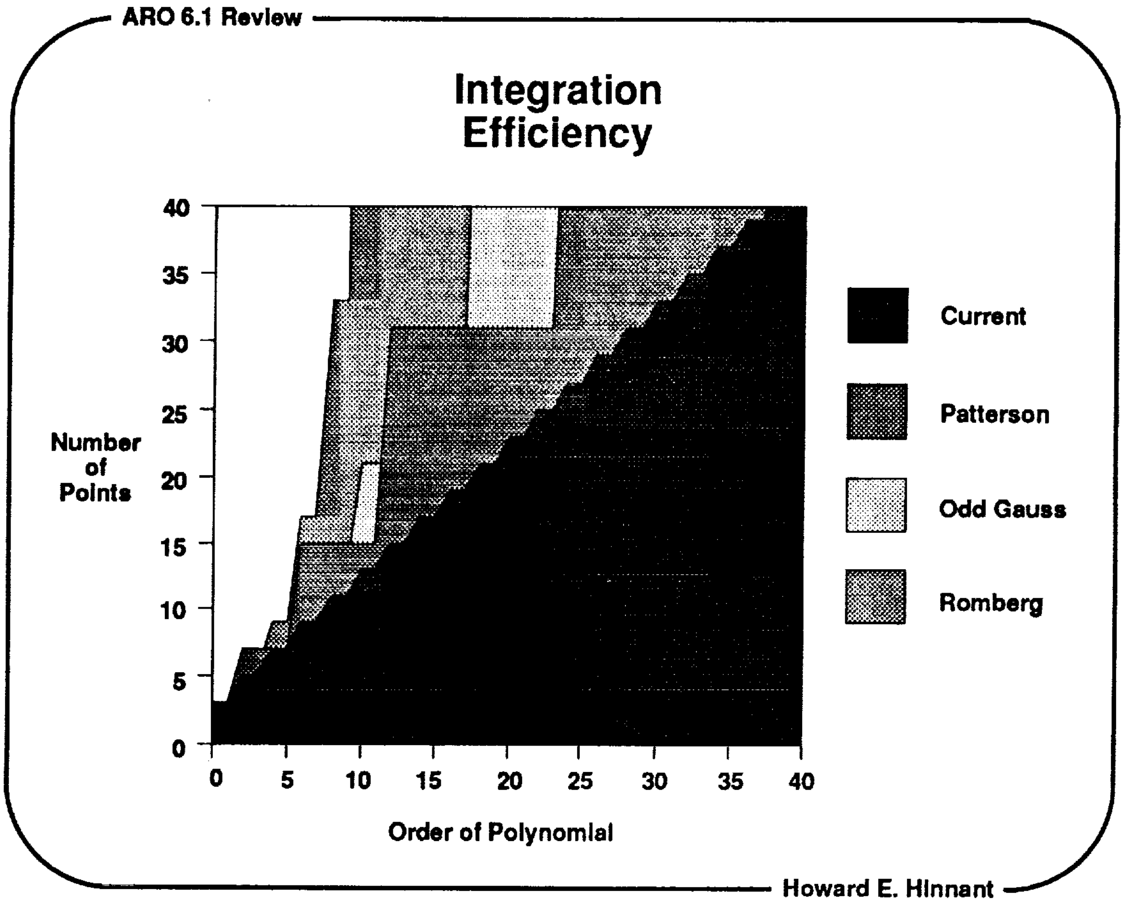
- A new adaptive numerical integration technique has been developed.
- Adaptivity is important due to the vast range of integration orders needed in finite elements.
- It appears to be more efficient than any other common adaptive method.
- More testing is needed to determine it's true value against the more exotic methods.

Howard E. Hinnant

The final area which this research has been involved in over the past year is numerical integration. In the formation of the element mass and stiffness matrices of a finite element program, it is often necessary to resort to numerical integration. If not handled properly, this step can become unreasonably expensive, or introduce inaccuracies into the analysis. To further complicate matters, each member of the element matrices needing integration may require a different integration rule to achieve accuracy. If integrating in more than one dimension (shells and bricks) the same matrix member may require different integration orders for the different dimensions. These problems suggest that an adaptive scheme which would automatically modify it's behavior for each matrix member and each dimension would be an ideal solution.

Adaptive schemes are not new. Romberg integration is probably the oldest and best known adaptive integration technique. The problem with Romberg is that it is outrageously expensive, so much so that it's cheaper to use Gauss formulas of increasing order until accuracy is reached.

A new adaptive technique has been developed that is more efficient than Romberg, more efficient than using a series of Gauss formulas, and more efficient than Patterson integration (The Optimum Addition Of Quadrature Points, 1967). This technique is conceptually very similar to Romberg integration. Instead of requiring the doubling of the number of function evaluations for each increase in accuracy, however, this technique requires only two function evaluations for each increase in accuracy. As FEAT becomes operational, it will provide a testbed for the proving of this new integration technique against other techniques.



This chart shows the relative efficiencies of a few adaptive integration methods. Integration methods are often stated in terms of what order of polynomial can be exactly integrated and with what number of function evaluations. The vertical axis shows the number of function evaluations, and the horizontal axis represents the order of the polynomial which can be exactly integrated. A perfect method would be a horizontal line at the bottom of the chart indicating exact integration of any polynomial with no function evaluations. Romberg is the most expensive method shown here. Romberg requires the doubling of points every time the polynomial order is raised by two.

Odd Gauss is a method that starts with a 1 point Gauss formula, then tries a 3 point and checks to see if the answer has changed. If so a 5 point formula is used, then 7, and so on until convergence is reached. It is remarkable that Odd Gauss is more efficient than Romberg, despite the fact that Odd Gauss throws away all function evaluations (except the center one) every time a new formula is tried. In contrast, Romberg uses all function evaluations at every step.

Patterson, an adaptation of Odd Gauss, seeks to use all of the old function evaluations at every step. Patterson integration adds integration points to an existing formula in an optimum manner. This would work very well except for one snag. To achieve optimum placement, the number of points must be doubled at every step. This brings the efficiency of Patterson down to just barely better than Odd Gauss.

The current integration scheme is clearly more efficient than the other techniques shown here. It requires only two function evaluations for every increase in accuracy. It also uses the information of each point at every step. The implementation is very similar to Romberg.

Summary

- A new finite element methodology (geometric entities) has been theorized. Implementation and validation are underway.
  - This new methodology promises interelement continuity across all elements.
  - This new methodology is being implemented as an anisotropic, curvilinear, p-version, beam, shell, and brick finite element program.
- Other activities include:
  - A pertinent grant with Dr. Hodges at Georgia Tech. concerning the modeling of composite beams and plates.
  - The development of a new adaptive numerical integration technique with improved efficiency.

This slide summarizes the points covered in this presentation.

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16. Abstract

This report documents a presentation given at Langley Research Center on April 23, 1991. The purpose of the presentation was to give an overview of the past year's research to the Army Research Organization represented by Dr. Gary Anderson. The research is directed at improving finite-element methods for rotorcraft airframes. The main portion of the presentation covers the development of a modification to the finite element method which eliminates interelement discontinuities.

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