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OPTIMAL PARAMETERS OF LEADER DEVELOPMENT IN LIGHTNING

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Abstract

The dependences between the different parameters of a leader in lightning theoretically are obtained.The physical mechanism of the instability leading to the formation of the streamer zone is supposed.The instability has the wave nature and is caused by the self-influence effects of the space charge.Using a stability condition of the leader propagation a dependence between the current across a leader head and the its velocity of moving is obtained.The dependence of the streamer zone length on the gap length is obtained.It is shown that the streamer zone length is saturated with the increasing of the gap length.A comparison of the obtained dependences with the experimental data is resulted.

1 Introduction

A consideration of leader discharge propagation is based on the investigation of charged particles moving in electric field. A particle moves in the potential created by all others charges.The calculation of that self-consistent potential is a difficult mathematical problem.Therefore it is important to construct an alternative approach for the analysis of such self-consistent problem. Particularly,an approach similar to the one used nonlinear wave theory for analysis of the self-influence effects in a nonlinear media turns out to be effective.At such approach a leader process is described by the nonlinear wave equations of evolution type, which from the continuity equations for the particles and Poisson equation for the electric field may be obtained.Note,that the nonlinear wave equation being constructed for the complex wave function $\Psi(x,y,z,t)$,determined from the condition $\rho = |\Psi|^2$,where ρ is the charge density.Such approach allows to understand the physical picture of many effects in leader process,in particurlar,the mechanism of instability,leading to the streamer corona formation.

In this paper a qualitative consideration of physical processes in a leader discharge caused by the self-influence effects is suggested.

2 Streamer zone formation

Leader process begins from the formation of streamer corona,consisting of multitude individual streamers.It is known that the streamers formation is caused by the avalanche-streamer transition.However a series of pecularrieties of streamer have not a physical explanation so far.In particular the physical picture of the keeping of stremer head radius along the all its trajectory is not known.It is necessary to solve the two-dimensional equations for explanation of this effect:

$$\frac{\partial n}{\partial t} = \frac{1}{e}\,\mathrm{div}\,j + \alpha v n + w, \qquad (1)$$

$$\frac{\partial p}{\partial t} = \alpha v n + w, \qquad (2)$$

$$\mathrm{div}\,E = \frac{4\pi e}{\varepsilon}(p-n), \qquad (3)$$

where $j = en\mu E + eD\nabla n$ is the current density, n and p are number density of electrons and positive ions,respectively, α is the ionization coefficient, v is the drift velocity, D is the diffusion coefficient, w is the charges produced due to the gas photoionization per volume and time.

A quasiclassical approach for the analysis of this system of equations allows to elucidate the physical picture of streamer radius preservation.Coulomb repulsive interaction between the charges,leading to the decrease of particles number and the ionization in the potential created by the same charges,which lead to the increase of charge number are the basic processes keeping the streamer radius. A next qualitative analysis may be presented. The growth of number of charges in some region at the time dt is determined by the expression

$$dQ_+ = \nu_i Q\, dt, \qquad (4)$$

where $\nu_i = \alpha \cdot v$ is the ionization coefficient. The decrease of the particles at the same time dt from this region because of Coulomb interaction is equal to

$$dQ_- = 4\pi\rho r^2 dr = 4\pi\rho r^2 v\, dt, \qquad (5)$$

where ρ is the mean density of charges, $v = \mu E_r = \mu Q/4\pi\varepsilon_0 r^2$, μ is the mobility of the electrons, E_r is the electric field at the streamer head with radius r . A stationary propagation of streamer take place at the fulfilment of equality

$$dQ_+ = dQ_- \qquad (6)$$

Hence it follows that the mean density of charges and the radius of streamer are equal

$$\rho \simeq \alpha \varepsilon_0 E_0, \qquad (7)$$

where $E_0 \simeq E_r$ is the external electric field,

$$r_0 \simeq \frac{3v}{\nu_i} = \frac{3}{\alpha}, \qquad (8)$$

i.e. is determined by only the gas properties.

Note that at the $r<r_0$ $dQ_+<dQ_-$ and the conductivity σ is insufficient in order to oust the field from this region in consequence of Maxwell relaxation at the time $t \sim 1/\nu_i$. Therefore the radius r grows to value r_0, i.e. when the Maxwell relaxation time $\tau_m = 1/4\pi\sigma$ is compared with the conductivity growth time $t \sim 1/\nu_i$ .Later on the growth of streamer radius is ceased because of the quickly ousting of field from the streamer head forward. It is noted that such picture occures in the electronegative gases,where the influence of processes in a channel is not essential.In the electropositive gases the radius of the streamer depends also on the conductivity of a channel and must be greater than the $r_0 \simeq 3/\alpha$. A charge of the head of streamer may be eva-

luated as follows:

$$q_0 = \frac{4\pi}{3}\rho r_0^3 = \frac{4\pi}{3}\alpha \varepsilon_0 E_0 \left(\frac{3}{\alpha}\right)^3 \approx 10^2 \frac{\varepsilon_0 E_0}{\alpha^2} \qquad (9)$$

At the $E_0 = 24$ kV/cm and $\alpha = 10^2 cm^{-1}$ in air we obtaine, that the $q_0 = 2.4 \cdot 10^{-12} C$ or $N_e = 10^7$. In SF_6 for the development of streamer the field $E_0 = 89$ kV/cm is necessary. Then the $\alpha = 0.75 \cdot 10^3 cm^{-1}$ and the streamer charge is equal to $q_0 \approx 1.6 \cdot 10^{-12} C$ ($N_e \approx 10^7$). A process in the streamer channel influence on its development only up to distance $\ell \sim v_{str}/\nu_a$, where ν_a is the attachment coefficient ($\nu_a \sim 10^{-7}$ sec in air). At further removing from the electrode a streamer head lose the conductive connection with its, but the necessary intensification of field on the head is ensured because of its polarization in the external electric field.

A such mechanism allows to explain the dependence of streamer velocity on the value of external electric field. So, from the system (1-3) may be obtained

$$v_{str} = \frac{\partial n/\partial t}{\partial n/\partial x} = v_0 + \alpha v_0 r_0 + w, \qquad (10)$$

where $\alpha(E) = A\exp(-B/E)$, E is the electric field, A and B are constant, $v_0 = v_{min} \sim \sim r_0/\tau_M \sim \mu E_0$.
From here one can see that the streamer velocity has the threshold character of dependence on the external electric field. This leads to the quickly stopping of streamer at the decrease of field lower the critical value E_{cr}.

In the nonuniform gaps the critical field is reached only near by electrode. However in the long gaps only the numerous streamers are propagated, which form the streamer zone. Elucidation of physical mechanism of instability leading to the formation of streamer zone is of interest. When the charge density reaches the critical value, the breaking into the threads (streamers) is occuring, i.e. the analogy with the breaking of light beam or acoustics wave in nonlinear media is exist. Note that the instability leading to the formation of streamer zone has the wave nature and is not connected with the temperature instability. The role of critical power in this case the critical charge density in the leader head plays, and besides the number of streamers is equal to $N_{str} = Q/q_0$ at the inculcate into the gap of charge Q.

3 Physical picture of leader propagation

Characteristic peculiarity of propagation both positive and negative leaders is the essential influence of the space charge of the streamer zone. Formation of a new leader head and its moving is caused owing to the self-influence processes and the ability of streamers to propagate in the region of the weekly field.

3.1 Pinching effect in the leader front

As the mechanism of a pinch usually the low-temperature overheating instability is suggested [1]. However the time of pinching in this case is determined by the thermal processes. A next physical picture of the pinch not connected with the thermal processes may be suggested. Because of the nonuniform distribution of the electric field at the leader front the distribution of charges created in this field turn out to be also nonuniformity, i.e. the nonuniform distribution of conductivity $\sigma(r,x) = e\mu n(r,x)$ is formed. The axis region of the head has the greater conductivity. Therefore the electric field is ousted from there forward quickly than the from periphery regions. The ousting time of field is determined by the Maxwell relaxation time of charges $\tau_M \sim 1/4\pi\sigma$. Thus the cross electric field is created, pinching the charges into the axis region and leading to the pinch of the head. The velocity of pinching is determined by the degree of nonuniformity of conductivity and Maxwell relaxation time τ_M.

3.2 Plasma clots formation

It is known [1], that at the front of the streamer zone of negative leader the plasma clots are formed, from which in the opposite direction the positive volume leader is propagated. A physical mechanism of the plasma clot formation is not clear. Lower the physical mechanism of plasma clot formation is suggested. It is known that the streamers starting out of the leader tip are connected with the leader head galvanically up to the distance approximately of few centimeters. A maximum length is determined by the disintegration time of the plasma in the old parts of the streamer channel $\ell_{max} = v_s/\nu$, $\nu = 1/\tau$ is the electron detachment frequency. This time in air is equal to $\tau \sim 10^{-7}$ sec. A further propagation of the streamers take place at the absence of the galvanic connection with the leader tip. The losses of the energy are compensated at the expense of the external electric field energy. The plasma formations with the length of approximately 1 cm are polarizated in the electric field. A force acting on the dipoles in the nonuniform electric field equal to

$$\bar{F} = \bar{p}\nabla \cdot \bar{E},$$

where $p = q\ell$ is the dipole moment, $\bar{E}$ is the electric field.
Hence it follows that the plasma dipoles draw in the strong field region, i.e. the focusing of dipoles take place. Note that the formation of plasma clot not depends on the polarity of leader and take place also in a positive leader.

3.3 Stepped leader propagation mechanism

A continuous or stepped propagation of a leader to be take place in the dependence on the polarity and the humidity. A negative leader propagates only in the stepped form. A positive leader may to propagate both continuous and stepped forms.

Two forms of the stepped propagation of positive leader may be suggested. The first of these is connected with the feeding difficulties of the leader channel, and the second with the formation of plasma clot at the front of streamer zone, analogically to the negative leader. In the first case the time pause between the steps or the flashing of the leader channel is not connected with the length of the streamer zone and the velocity of the leader and not has a periodic character. In the second case the pause time between the steps by the velocity of the leader and the length of streamer zone is determined. This it seems leads to the decrease of the time interval between the steps when the leader approaches to the ground. A schematic picture of stepped propagation of a leader is presented in fig.1.

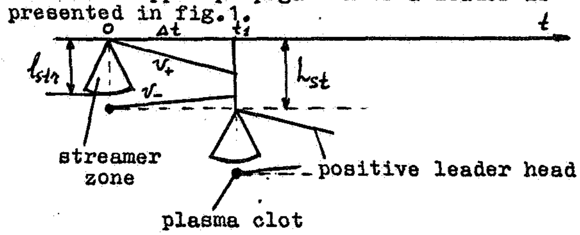

Fig. 1

It is seen from fig.1 that the pause time between the steps depends on the velocities of the positive v_+ and negative v_- leaders and the streamer zone length l_{str}. Effective velocity of the stepped leader propagation or the mean leader velocity is determined as the

$$v_{eff} = \frac{H}{t} = \frac{N l_{st}}{N \Delta t} \approx \frac{l_{str}}{\Delta t}, \qquad (11)$$

where H is the gap length, t is the full time of leader propagation, N is the number of steps.
As it follows from (11) at the $\Delta t \approx$ const the effective velocity of stepped leader grows with the increasing of streamer zone length.

4 Optimal parameters of leader

A stability propagation of a leader is possible at the establishment of balance between the processes in the channel, leader head and streamer zone (Fig.2). In particular, a

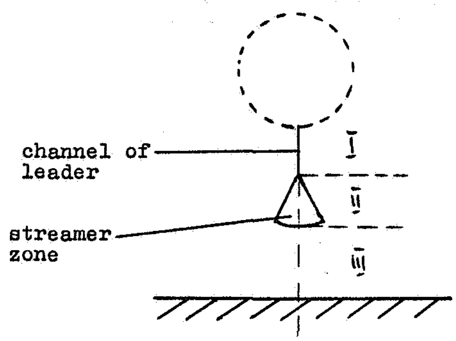

Fig.2

full current flowing in each these regions must be the same. In a channel the current is determined in the main by the conduction current

$$i_c = \sigma E_c \pi r_c^2, \qquad (12)$$

where r_c is the channel radius, σ is the conductivity.
In a streamer zone the displacement current caused by the moving of charges flows

$$i_s = q \cdot v_l, \qquad (13)$$

where q is the charge per unit length, v_l is the leader velocity.
Finally at the front of streamer zone the "clean" displacement current, not connected with the charges moving takes place

$$i_j = \varepsilon_0 \frac{\partial E}{\partial t} \cdot S, \qquad (14)$$

where S is the area of streamer zone front. Note that $q \sim \varepsilon_0 E_s l_s$ and $S \sim l_s^2$, where E_s is the electric field in the streamer zone, l_s is the streamer zone length.
Hence it follows the correlation

$$j_h \cdot \pi \cdot r_h^2 \sim \varepsilon_0 E_s l_s v_l \sim \varepsilon_0 \frac{\partial E}{\partial z} \cdot v_l \cdot l_s^2, \qquad (15)$$

where $\partial E/\partial z$ is the derivative of electric field along the propagation direction of leader at the streamer zone front, j_h is the current density in the leader head, r_h is the radius of leader head.
Note that the electric field in the streamer zone is kept along the all its length [2].
A pinch of leader head takes place at the reaching of critical power $W_{cr} = j E_h = \sigma_h \cdot E_h^2$, where $\sigma_h = \mu \cdot \rho_{cr}$ = const. Since $E_h \sim r_h$, then $j_h \sim r_h$. Then from (15) we obtain the correlation $r_h^3 \sim l_s \cdot v_l$.

4.1 The velocity of leader

The velocity of leader analogically to the velocity of streamers is determined by the effective ionization coefficient α_{eff} before the head:

$$v_l = v_0 + \alpha_{eff} \cdot v_0 \cdot r_h \qquad (16)$$

It is seen from (16), that at the $\alpha_{eff} r_h \ll 1$ the velocity of leader is constant. Therefore $l_s \sim r_h^3$. The current in the head $i \sim r_h^3 \sim l_s \sim \varphi_l$, where $\varphi_l \sim E_s \cdot l_s$ is the potential of leader head. At the $\alpha_{eff} r_h \gg 1$ $v \sim \alpha_{eff} \cdot v_0 \cdot r_h$ and the streamer zone length is proportional to the square of head radius $l_s \sim r_h^2$ (l_s/r_h^2 =const).
For the current in the leader head we obtain

$$i_l \sim r_h^3 \sim l_s^{3/2} \sim v_l^3 \qquad (17)$$

Hence it follows that the current grows with the increase of potential of leader head as

$$i_l \sim \varphi_l^{3/2}$$

and the velocity

$$v_l \sim i^{1/3} \sim \varphi_l^{1/2}$$

The velocity dependence on the current flowing across the leader head is presented in fig.3.

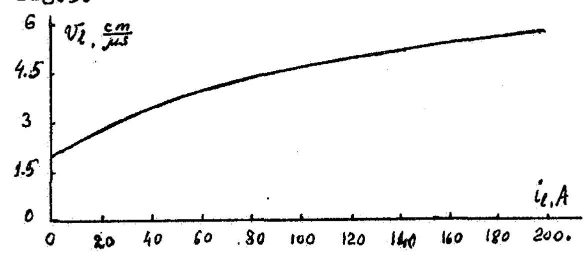

Fig.3

4.2 The streamer zone length

From (15) the equation for the determining of the dependence of streamer zone length length on the gap length may be obtained:

$$\varepsilon_0 \cdot \frac{\partial E}{\partial z}(l_s, H) \cdot v_l \cdot l_s^2 = q \cdot v_l \qquad (18)$$

It is known that the electric field intensity is determined by the equation

$$E = \text{grad}\, \varphi$$

where $\varphi = \varphi_1 + \varphi_2$ is the sum of the potential φ_1 created by the electrode and the potential φ_2 created by the space charge of the streamer zone.
The potential φ is the solution of Poisson equation $\Delta\varphi = 4\pi\rho$ and may be obtained from the integral equation

$$\varphi = \iint \frac{\sigma\, dS}{\varepsilon_0 R} + \iiint \frac{\rho(r)\, dV}{\varepsilon_0 R} \qquad (19)$$

where $dS = R_0^2 \cdot \sin\theta \cdot d\theta \cdot d\alpha$, R is the radius of electrode, $\sigma = \varepsilon_0 E_0 / 2\pi$ is the surface charge density, E_0 is the electric field on the electrode surface, R is the distance between the element of charge and the point of observing, $dV = r^2 dr \sin\theta \cdot d\theta \cdot d\alpha$, θ is the angle of integration.
The distribution of the charge density along the radius of streamer zone may be determined on the known electric field from the equation

$$\text{div}\, E = 4\pi\rho \qquad (20)$$

Since the electric field intensity in the streamer zone is not changed along the all its length [3] then from (20) we obtaine

$$\rho(r) = \frac{\rho_0 \cdot r_0}{r}, \qquad (21)$$

where $E_h = \rho_0 \cdot r_0 / r_0$ is the electric field near the leader head. Integrals (19) may are calculated analytically. Calculating the derivative $\partial E/\partial x$ and substituting its to the equation (18) the equation for determining of the dependence $l_{str} = f(H)$ may be obtained. Fig. 4 shows the calculated value of streamer zone length l_{str} as a function of gap length H.

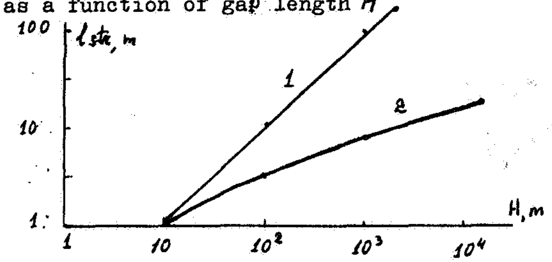

Fig. 4

It is found that when the influence of electrode is not take into accounted the streamer zone length grows linearly to the gap length (curve 1). The streamer zone length is saturated when the influence of electrode is take into accounted (curve 2).

5 Fractal nature of lightning

It is known that even at the identical external conditions (gap geometry, atmosphere conditions, applied voltage) the characteristics of discharge behave accidentally. In particular the trajectory of lightning represents something crooked line changing from case to case. However it may be showed that the channel dynamics is described by the determinic equations, i.e. the chance picture of trajectory is determined by the internal properties of leader, but not the external chance influences. A series of quantitative characteristics may be introduced which allow to differ the one picture of discharge from other. These are fractal dimensions. So, the channel length of lightning L measured by the put on the sections with the length ε depend on the minimal length L_{min} by the degree manner

$$L(\varepsilon) = \varepsilon \left(\frac{L_{min}}{\varepsilon}\right)^D,$$

where D is the fractal dimension, changing in the interval $1 < D < 2$. Fractal dimension D change at the changing of characteristic length of straight sections of channel and its orientation angle. In one's turn this characteristic length is connected with the streamer zone length that determines the parameters of the electromagnetic radiation of lightning. Therefore the amplitude-frequency characteristics of lightning radiation also possess by the fractal nature. This property may be used for the re-establishment of the channel parameters on the characteristics of the electromagnetic radiation.

A growth of streamer zone of leader may be described also on the basis of the growth law of branching physical system, possessing by the fractal properties. The fractal dimension D of the streamer zone may be determined by means of calculation the number of streamers branches (or the streamers heads), keeping in the sphere with radius R at different R :

$$N(R) = \int_0^R \rho(r) \cdot r^{d-1} \cdot dr = R^D, \qquad (22)$$

where $\rho(r)$ is the density distribution of streamers head, d is the space dimension. Hence it follows that the charge density distribution satisfies the law

$$\rho(r) \sim r^{D-d} \qquad (23)$$

It is known that the electric field intensity in the streamer zone is keeped along the its length [2]. From the Poisson equation $div\,E = 4\pi\rho$ we obtaine, that this take place at the $\rho(r) \sim r^{-1}$, i.e. at the $D = 2$ in three-dimensional space. Therefore the streamer zone represent the fractal structure with the dimension $D = 2$.

6 Discussion and conclusion

The dependences obtained above may be used at the calculating of leader parameters in lightning. Using the relations (17) we can evaluate the streamer zone length l_{sk} or the leader head potential U_l of lightning. It is known [3] that the streamer zone length is equal to $l_{sk} \approx 1$m and $U_l \approx 500$ kV in the laboratory gaps at the leader current $i_l \approx 1$ A . A characteristic current of leader in lightning is equal to $i_l \approx 100$ A [4]. Hence we obtain that the characteristic streamer zone length is equal to $l_{sk} \approx 20$ m, and the potential of leader head $U_{sk} \approx 10$ MV, that agrees with the experimental observations.

In table 1 some values of current in leader i_l, radius of leader r_l, potential of leader head U_l, velocity of leader v_l, streamer zone length l_{sk} for different gap length H are presented. At the current in leader $i_l = 1$ A the values of parameters are presented from the laboratory experiments in long air gap.

Table 1

i_l, A	r_l, mm	U_l, MV	v_l cm/μs	l_{sk}, m
1	1	0.5	2	1
10	1.1	2.4	2.2	4.8
100	2.3	10.6	4.6	21.2

It is seen from table that the calculated values agree with the experimental data obtained for lightning. Adduced relations between the parameters are related to the leader stage of discharge. However these determine also the characteristics of discharge in the return stroke stage. So, streamer zone length determines a duration of return stroke current, connected with the neutralization of space charge around the channel. As was shown above, streamer zone length is saturated at the growth of gap length. This explains the slow change of the duration of return stroke current from case to case. Note that streamer zone length determines also the amplitude value of return stroke current.

The obtained correlations may be used at the determining of such parameters of lightning as the potential of cloud, current and space charge neutralized by return stroke on the characteristics its electromagnetic radiation.

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