N92-10746

HORIZONTAL STRESS IN PLANETARY LITHOSPHERES FROM VERTICAL PROCESSES

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Introduction. Understanding the state of stress in a lithosphere is of fundamental importance for planetary geophysics. It is intimately linked to the processes which form and modify tectonic features on the surface and reflects the behavior of the planet's interior, providing a valuable constraint for the difficult problem of determining interior structure and processes. On the Earth, most of the stress in the lithosphere can be related to lateral tectonics associated with plate dynamics; however, the tectonics on many extra–terrestrial bodies (such as the Moon, Mars, and most of the outer–planet satellites) appears to be primarily vertical in nature, and the horizontal stresses induced by vertical motions and loads are expected to dominate the deformation of their lithospheres [e.g., 1–5]. We are concerned here only with changes in the state of stress induced by processes such as sedimentary and volcanic deposition, erosional denudation, and changes in the thermal gradient that induce uplift or subsidence. This analysis is important both for evaluating stresses for specific regions in which the vertical stress history can be estimated, as well as for applying the proper loading conditions to global stress models. It is also of interest for providing a reference state of stress for interpreting stress measurements in the crust of the Earth (e.g., [6]). Most of the previous work on this subject has been directed toward the latter problem.

All references to "lithosphere" in this abstract should be understood to refer to the elastic lithosphere, that layer which deforms elastically or brittlely when subjected to stresses on geological time scales.

Boundary Conditions. A great deal of confusion exists in the literature about the effects of vertical changes in the lithosphere on its horizontal state of stress (σ_h). Much of this confusion can be traced to an uncertainty in the type of lateral boundary condition applied to the lithospheric column. Generally, a lateral constraint condition has been assumed [e.g., 7–9] in which the horizontal displacement is assumed to vanish due to the "resistance" of the surrounding rock. In that case changes in the vertical stress results in an additional horizontal stress due to a combination of (a) lateral stress accommodation ("Poisson stress"), (b) isostatic subsidence or uplift on a sphere, and (c) thermal re–equilibration, resulting in a horizontal stress given by [9]

$$\sigma_h = \rho_c g \Delta h \, \frac{1}{1-\nu}\left(\nu + \frac{E}{\rho_m g R} + \frac{\alpha E}{\rho_c g}\frac{dT}{dz}\right) \quad (1)$$

where ν is Poisson's ratio, Δh is the thickness of crustal material of density ρ_c added, g is the acceleration of gravity, ρ_m is the density of the material beneath the crust, E is Young's modulus, R is the radius of the planet, α is the linear coefficient of thermal expansion, and dT/dz is the thermal gradient.

McGarr [10] pointed out fundamental logical inconsistencies in the lateral constraint assumption, and argued on both theoretical and observational grounds that this situation is "thoroughly improbable". In its place, he advocated a fixed stress boundary condition, in which the state of stress in the region outside the area involved in the vertical changes is unaffected. No quantitative justification for this assumption was presented by McGarr [10], but its intuitive appeal can be confirmed in the following way: Consider the lithosphere to be an elastic plate overlying an inviscid substrate. Let the edges be fixed at a distance X from the region undergoing vertical deformation (i.e., lateral constraint at a distance). If the vertical processes result in a horizontal displacement Δx at one end of the plate, the resulting strain in the plate will be $\Delta x/X$ and the change in the horizontal stress will be proportional to that strain. It is obvious that if X is sufficiently large, then the change in stress outside the loaded region will essentially vanish for any induced finite horizontal displacement.

In order to calculate stresses in the loaded region, McGarr [10] required that horizontal forces balance and solved for the resulting stresses. This is analogous to a horizontal version of an isostatic calculation [11]. He further assumed that: (a) changes in the overburden thickness are equivalent to changes in the elastic lithosphere thickness; (b) changes in the temperature of the lithosphere affect the lithosphere thickness through thermal expansion alone; and (c) thermal erosion and underplating of the lithosphere can be modelled as thickness changes alone, independent of other thermal effects. His result can be written:

$$\sigma_h = \rho_c g \Delta h \left\{ -\frac{H}{2\Delta h}\left(\frac{\Delta h}{H+\Delta h} + \frac{\alpha \Delta T}{1+\alpha\Delta T}\right) + \frac{\rho_c}{\rho_m}\left[\frac{H+\frac{\Delta h}{2}}{H+\Delta h} - \frac{\frac{2H}{\Delta h}\alpha\Delta T\left(1+2\alpha\Delta T\right)}{\left(1+\alpha\Delta T\right)\left(1+3\alpha\Delta T\right)^2}\right]\right\} \quad (2)$$

where H is the thickness of the lithosphere and $\Delta T = \Delta h \, dT/dz$ is the average temperature change of the lithosphere.

Typical boundary conditions for a real lithosphere almost certainly lie between the two end members outlined above, and are more likely nearer the fixed stress case. Thus these two end members can be used to place bounds on the magnitude of deviatoric stresses induced in the lithosphere by vertical processes. However, the published expressions for both situations [9,10] are incorrect, due to mistaken assumptions in their derivations. In this investigation I have derived the horizontal stress relations for addition or removal of overburden after correcting these errors.

<u>Constant Stress Boundary Condition.</u> The assumptions (a)–(c) given above for the constant stress condition are inconsistent with current understanding of the relationship between temperature and the structure of the lithosphere [e.g., 12]. For a given geologic material, the effective thickness of the elastic lithosphere is determined by the depth to a critical isotherm, which is in turn a function only of the thermal gradient. The addition or removal of overburden will not result in any change to the total lithosphere thickness once thermal re–equilibration has occurred. However, the lithosphere is, in general, composed of two major layers: a crustal layer and an elastic mantle layer, both of which overlie ductile mantle. Changes (e.g., erosion, deposition) near the surface involve material with the density of crustal rocks whereas the compensating changes at the base of the lithosphere (due to vertical migration of the critical isotherm) involve the mantle, which has a higher density. Thus the integrated vertical stress will decrease and the buoyancy of the column will increase. (Note that if the base of the elastic lithosphere is within the crust, there is no net change in the the mechanical state after addition or removal of material at the surface.)

With these modifications to the lithospheric model, the derivation of the horizontal stress can be carried out using horizontal force balance methods, giving

$$\sigma_h = -\rho_c g \Delta h \left(1 - \frac{\rho_c}{\rho_m}\right) \frac{H_c + \frac{\rho_c}{\rho_m} H_m - \frac{1}{2}\Delta h \left(\frac{\rho_m}{\rho_c} - 1\right)}{H_c + H_m} \quad (3)$$

where H_c and H_m are the thicknesses of the crust and mantle portions of the lithosphere, respectively. Using nominal estimates for material parameters ($\alpha = 10^{-5}$/K°, dT/dz = 20°/km, ρ_c = 2.6 Mg/m^3, ρ_m = 3.3 Mg/m^3, H = 30 km, $H_c = H_m$ = 15, Δh = 5 km), I find that the deviatoric horizontal stress is about –0.2 times the overburden stress ($\sigma_v = \rho_c g \Delta h$), whereas that predicted by equation (2) is about +0.3 σ_v. Thus there is about a 50% correction in the stress difference ($\sigma_h - \sigma_v$).

Horizontal Constraint Boundary Condition. An implicit assumption in previous derivations of the horizontal stress for this case is that the three contributions to the stress are independent, and can be computed separately and added together (see, e.g., Haxby and Turcotte [9]). Such is not the case, however. For example, the horizontal expansion induced by vertical compression acts to help support the lithosphere in a spherical geometry ("arch support"); this reduces the vertical displacement and hence the compression due to subsidence on a sphere (which depends only on the displacement).

I have derived an expression for the horizontal stress in a thick spherical lithosphere (after Love [13]):

$$\sigma_h = (3\lambda + 2\mu)\, A + \frac{2\mu}{r^3} B \tag{4}$$

where

$$A = \frac{-\rho_c g \Delta h}{3\lambda + 2\mu}\left[1 - \left(\frac{R - H}{R}\right)^3 \frac{1 - \omega'}{1 + \omega}\right]^{-1}$$

$$B = \frac{\rho_c g \Delta h R^3}{4\mu}\left[1 - \left(\frac{R}{R - H}\right)^3 \frac{1 + \omega}{1 - \omega'}\right]^{-1}$$

and where λ and μ are Lame's constants, $\omega = \rho_m g(R - H)/4\mu$ is a stress parameter and $\omega' = \omega \cdot 4\mu/(3\lambda + 2\mu)$.

The new result agrees with equation (1) in the limiting cases $\omega >> 1$ (Poisson stress dominates) and $\omega << 1$ (displacement stress dominates). But for values of the elastic parameters and ω appropriate for the Earth (e.g., using the previous parameter values and $\lambda = \mu = 5\times10^{10}$ Pa), the horizontal stress from equation (1) (omitting the thermal term) is $+0.5\ \sigma_v$, whereas the corrected stress from equation (4) is about $+1.2\ \sigma_v$. Thus the stress difference is of the opposite sign and about 40% larger in magnitude.

Thermal expansion effects for the lateral constraint case, along with expressions appropriate to thermal uplift or subsidence, are the subject of future work. It will then be possible to re–evaluate a large number of problems involving horizontal stress effects from vertical tectonics on planetary lithospheres.

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