S7-61
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p 10

N92-11735

MJ700801

# Nested Ocean Models: Work In Progress

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**Abstract**

This paper details the ongoing work of combining three existing software programs into a nested grid oceanography model. The HYPER domain decomposition program [8], the SPEM ocean modeling program [5] and a Quasi-Geostrophy model written in England are being combined into a general ocean modeling facility. This facility will be used to test the viability and the capability of two-way nested grids in the North Atlantic.

## 1 Introduction

We are beginning work on a basin wide coarse grid overlaid with finer grids that follow major mesoscale and dynamic features in the North Atlantic basin. The grid management will be handled by the HYPER domain decomposition program [8]. We will consider several combinations of solution methods to be used including nesting a primitive equation fine mesh solution method within another primitive equation coarser grid solution, and a primitive equation fine mesh solution within a coarser quasi-geostrophic model solution.

It is well known that to refine the entire coarse mesh in space for ocean circulation modeling would be inefficient; it would require large amounts of memory and waste processor time in quasi-geostrophic regions. In short, refining the entire coarse mesh is overkill. For explicit time-evolution solution methods of primitive equations the advancement must also be severely refined in time to account for the gravity wave stability constraint. This results in an excessive number of time steps. Alternatively, an implicit solution on a fully refined mesh results in a very large matrix problem.

We are attacking two areas of fundamental ocean modeling directly. The efficiency of the boundary conditions between quasi-geostrophic and primitive equation models should be advanced based on the insight acquired from our hierarchical approach to the nesting experiments. The second fundamental area is ocean modeling in general. Nested basin/regional grids are a new concept for ocean applications, and in this respect oceanographic modeling lags behind atmospheric and aerodynamic modeling. But the success of domain decomposition in these more advanced modeling areas provides encouragement that our research efforts are timely, central and well directed towards new, successful applications.

## 2 The Navier-Stokes Equations on a Rotating Sphere

To examine the Navier-Stokes equations on a rotating sphere in a rotating reference frame $R$, let $I$ denote an inertial reference frame, and let $r$ be a radius of that sphere. Then

$$(r_t)|_I = (r_t)|_R + \Omega \times r$$

where $r_t = u$ is velocity and the second term on the right-hand side of the equation is the motion a non-rotating observer would see because of the rotation of the sphere. Then

$$(u_I)_t|_I = (u_R)_t|_R + 2\Omega \times u_R + \Omega \times (\Omega \times r) + \Omega_t \times r.$$

On the right-hand side of this equation the second term is the Coriolis acceleration, the third term is the centripetal acceleration, and the fourth term is the acceleration resulting from any changes in the rotation speed.

For geophysical applications here on Earth this last acceleration term is discarded except for *very* long time scales, and centripetal acceleration can be expressed as a potential,

$$\Omega \times (\Omega \times r) = -\nabla \Theta_c,$$

that then can be added to the gravitational force potential to net a new geophysical force potential.

The total or material derivative of a scalar quantity is the same in both reference frames. Hence the form of the conservation of mass and the thermodynamic equations remains the same.

To estimate the frictional forces $F$, we could assume a Newtonian fluid with a symmetric Navier-Stokes internal pressure tensor. But this molecular dissipative strength would have an unknown relationship to the dissipative strength of a given mesoscale ocean phenomena. In general, a qualitative description of the transfer of energy and momentum between scales of interest, and not these smaller molecular scales, are parameterized based on known qualitative ocean behavior.

# 3 Governing Equations

Coupling hydrodynamics and thermodynamics, consider an adiabatic, inviscid fluid. It can be described by conservation of momentum, a continuity equation, and an energy equation coupled with an equation of state. That is,

$$\frac{d}{dt}u + \alpha \nabla P + F = 0$$

$$\frac{d}{dt}\alpha - \alpha \nabla \cdot u = 0$$

$$\frac{d}{dt}P + P\gamma \nabla \cdot u = 0$$

where $u \in R^3$ is the velocity, $\alpha$ is the specific volume, $P$ is the pressure, and $\gamma$ is the ratio of specific heats. Here $F$ is the Coriolis and gravity forces, and $\frac{d}{dt} = \frac{\partial}{\partial t} + u \cdot \nabla$ is the total time derivative.

## 3.1 Primitive Equations

The hydrostatic approximation neglects vertical acceleration in the vertical equation of motion;

$$\frac{\partial P}{\partial z} = \frac{-g}{\alpha}.$$

The Boussinesq approximation replaces density with a zeroth order mean density everywhere except where multiplied by gravity. The combined hydrostatic and Boussinesq approximations are used to formulate a set of reduced equations known as the primitive equations.

These equations are as follows:

$$\frac{d}{dt}u_H + \alpha\nabla_H P + F_H = 0$$

$$\alpha\frac{\partial P}{\partial z} + g = 0$$

$$\frac{d}{dt}\alpha - \alpha\nabla\cdot u = 0$$

$$\frac{d}{dt}P + P\gamma\nabla\cdot u = 0$$

where $u_H = (u_1, u_2)^t$, $u = (u_H, w)^t$,$\nabla_H = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})^t$.

## 3.2 Geostrophy

Geostrophy, or geopotential flow, retains only the balance between the Coriolis force and the potential field. Geostrophy:

$$fv = g\eta_x$$

$$fu = -g\eta_y$$

is the first approximation in an asymptotic expansion of the primitive equations. Here $\eta$ is the variation of the sea surface height, a measure of pressure.

## 3.3 Quasi-Geostrophy

Large-scale ocean movement is typically quasi-geostrophic. Asymptotically, quasi-geostrophic motion has time scales not smaller than the advective time scale. It is geostrophic to lowest order, yet retains dynamics. Velocity fields can change, but they do so in continuous geostrophic balance with pressure. Hence there are temporal derivatives retained. The quasi-geostrophic equations are

$$u_t - fv = -g\eta_x$$

$$v_t + fu = -g\eta_y$$

$$\eta_t + (uH)_x + (vH)_y = 0.$$

Here $H + \eta$ is the mean height, $H$, plus the variation of thickness $\eta$.

## 4 Related Research

Spall and Holland in unpublished work nested a primitive equation model within a quasi-geostrophic model. They found that the quasi-geostrophic boundary conditions seriously dampened the primitive equation physics close to the boundary, reducing them to essentially quasi-geostrophic physics in the boundary region. Reducing or eliminating this boundary layer is the principle focus of our current efforts.

Thompson and Schmitz [7] varied the damping time scale on outflow boundary conditions for a model of the gulf stream. They found that the outflow dynamics and hence the location of the Gulf stream are significantly impacted by the outflow boundary conditions. With such strong impact, the possibility of numerical artifacts in regional models due to boundary conditions seems large. The lack of existence of well-posed boundary conditions for primative equations complicates this problem because no comparisons with true boundary conditions can be made even as approximations [7]. See the article by Dr. Blake in this proceedings for more detailed information.

This leads to the importance of getting the boundary conditions *physically* correct. It is known that the subcharacteristics of the Euler equations, upon which the primitive equations are based, can have combined inflow and outflow characteristics at both advective inflow and outflow boundaries, dependent on the sound speed. Hence the dynamics of the refined regions typically has an affect on the surrounding flow fields. Using one-way boundary conditions for inflow and one-way for outflow is not, in general, sufficient. There must be a stronger interaction between the dynamics of the coarse and the refined meshes.

To strengthen the interaction Spall and Holland [9] added a direct, but averaged, insertion of the streamfunction field (generated using the refined primitive equation depth-averaged horizontal velocity values on the refined mesh) onto the coarser quasi-geostrophic solution. They allow the refined mesh to dictate the regional external flow component.

Our hypothesis is that the external flow is insufficient. The strong dynamics in the gulf stream are forced by internal instabilities. We are currently testing both baroclinic and barotropic nudging for all prognostic variables to establish a stronger relationship between the regional dynamics and the coarse mesh solution behavior. This will be done while maintaining Spall and Hollands quasi-geostrophic to primitive equation boundary conditions [9], which are barotropic, to see if we can improve upon their results. If we are successful, we will experiment with a quasi-geostrophic coarse mesh solution and try to find more comprehensive two-way communication techniques that reduce or eliminate the boundary layer formation found in the previous work.

We are currently using the barotropic modon defined in [9], and a baroclinic vortex problem.

We are working with flat topography at first, so that we can combine either fixed or sigma (stretched) vertical coordinate models.

# 5 Initial Boundary Value Problems

For modeling purposes these equations must, of course, be viewed as initial boundary-value problems (IBVP). Computational models use IBVP for hindcasts, nowcasts, and forecasts as well as for physical studies that examine phenomena of interest such as energy cascades, eddy shedding, and coastal upwelling. There are two prevalent boundary conditions used in ocean models: rigid walls and open boundary conditions. The trick, of course, is to find open boundary conditions that are well posed and yet not overconstrained. Because one cannot simultaneously diagonalize the coefficient matrices for the multidimensional advective terms of the Navier-Stokes equations, this is often a time-consuming guessing game.

To define open boundary conditions in general, define a system of equations

$$Lu = F$$

with given initial conditions

$$u(x : t = 0) = u_0(x)$$

and characteristic boundary conditions

$$u^1 = Su^2 + g$$

where $u = (u^1, u^2)^t$ are the prognostic variables, and $S$ is a generalized reflection operator [4].

But for reduced equations there can also be modeling constraints such as hydrostatic balance or incompressibility. The modeling constraints assumed in order to reduce the equations (that is, the asymptotic balances chosen) must be enforced on the initial conditions and the boundary conditions to avoid introducing inappropriate length and time scales.

Many obvious well-posed boundary conditions are overspecified, which leads to the formation of boundary layers within which the solution adjusts to the additional information.

## 5.1 Open Boundary Conditions

The major issue to address is boundary conditions. Oliger and Sundstrom in [4] detail some boundary treatment for geophysical problems, and show that point-wise local boundary conditions for the primitive equations are not well-posed: the regional open boundary problem is open-ended. It is not known, however, whether non-local boundary conditions, such as those generated with a domain decomposition method where boundary conditions that are derived from a larger domain are or are not well-posed. We may have fewer problems with open boundary conditions at a two-sided boundary. The quasi-geostrophic boundary layer within the nested primitive equation model in the unpublished work of Spall and Holland indicate that open boundary conditions on a nested grid is a major problem that must be addressed before two-way nesting will be successful.

Our approach has been to work from the primitive equation homogeneous model backwards to the quasi-geostrophic model, assessing the differences in the information transmitted across boundaries, to derive better open boundary conditions for the simplier quasi-geostrophic model.

For the heterogeneous boundary conditions between the primitive equation and quasi-geostrophic regions the quasi-geostrophic boundaries need to evolve as if there was primitive equation physics in the region surrounding the refined domain. To insure this we are monitoring $\frac{\partial \rho}{\partial t}$ on the boundary, where quasi-geostrophic physics, which is less vertically diverse, is statically stable, and comparing the evolving quasi-geostrophic boundary against a fine mesh primitive equation global solution. This is one of our measures of error that is physically based.

# 6 HYPER

The HYPER program, described in Perkins [5], looks at domain decomposition as a tool to combine grids for computational efficiency and for model flexibility. It currently can locate where refined grids should be placed based on asymptotic and-or physical criteria and it initializes the refined grids using local uniform mesh refinement.

Our current work is a static domain decomposition; we are running experiments on the influence of the internal boundaries on the flow pattern, and are interested in the flow through that boundary. These results are currently being prepared and will appear in a later report.

The goal is to follow different asymptotic regimes within the ocean basin that are identifiable as distinct physical regions of the ocean. The problem is that, once you're inside a reduced physics region, such as a quasi-geostrophic region where there is no ageostrophic flow, there may be no way for the model to evolve the complete physics you hope to recover by using the refined meshes. For example, in the gulf stream region meanders pinch off to form eddies. Many aspects of the physics contribute to this pinching off. In such cases the reduced quasi-geostrophic model will not reproduce the ageostrophic behavior in the initial conditions of the refined mesh, and hence will miss some of the time dependent interactions that contribute to the dynamically significant event of ring shedding. There will be no ageostrophic signals in the initial conditions to interpolate onto the refined mesh. In such a situation, the data used to help initialize the model may make up for some of the missing physics, but the time scales may be off.

# 7 Domain Decomposition

Domain decomposition allows the mesh to evolve with the solution. It has been applied to elliptic and hyperbolic equations for several years. The interested reader can refer to Chan, et al., [2] for an extensive bibliography on elliptic and hyperbolic domain decomposition methods.

Formally we describe the domain decomposition of a discrete coarse mesh, $\Omega^1$, on the computational mesh $\Omega$, for a fixed time $t = n\Delta t$, by letting $(p(t) - 1)$ be the number of refined subdomains used at time $t$ and $\{\Omega^k\}_{k=2}^{p(t)}$ be those subdomains:

$$\bigcup_{k=1}^{p(t)} \Omega^k(t) \subset \Omega \,.$$

After the domain is decomposed, local uniform spatial mesh refinement, as developed by Berger [1], is applied to the new subdomains. The time step may also be refined on these regions.

The sequencing for one coarse time step of magnitude $\Delta_c t$ from time $t^n$ to time $t^n + \Delta_c t$, where $n$ indexes the discrete time steps on the coarse mesh, is presented next. Let the temporal refinement ratio from the coarse mesh to the refined meshes be $r$, and notate this $r\Delta_f t = \Delta_c t$, so that a subscript "$c$" informs us that we are on the coarse mesh, and a subscript "$f$" informs us that we are discussing one of the refined meshes. The domain decomposition algorithm follows:

Domain Decomposition Algorithm

```
Advance coarse mesh
Mark points with significant mesoscale and ageostrophic potential
Cluster these points into refined meshes
DO r times
    Solve equations on refined meshes
ENDDO
Nudge refined values onto coarse mesh
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When all of the refined meshes have been advanced $r$ refined time steps to the next coarse time step, their values at time $t^{n+1}$ are passed to the coarse mesh where a nudging technique modifies the coarse advanced values and produces an aggregate solution on the coarse mesh. Let $F^A$, $F^1$, and $\{F^k\}_{k=2}^{p(t)}$ represent the discrete operators for the aggregate solution on the original discretized mesh $\Omega^1$, the solution on the coarse mesh $\Omega^1$, and the separate solutions on each of the refined subdomains $\{\Omega^k\}_{k=2}^{p(t)}$, respectively. Then the aggregate solution on the coarse discretized mesh is given by

$$F^A(\Omega^1) = \mathcal{C}\left[\{F^k(\Omega^k)\}_{k=2}^{p(t)}, F^1(\Omega^1)\right] \tag{7.1}$$

where the operator $\mathcal{C}$ is a nudging technique that may vary between experiments.

We use domain decomposition as a tool to combine the explicit coarse mesh solution method with the refined mesh solution methods to satisfy our varying numerical requirements in a computationally efficient way. We use a two-level refinement scheme consisting of one coarse mesh and a set of overlaid refined meshes, where the coarse mesh adequately represents quasi-geostrophic behavior, while the refined meshes adequately resolves the more physically complete primitive equations.

Refined to coarse mesh communication (feedback) can take the form of value averaging, as in Berger [1] and as in Spall and Holland [6].

We are using a nudging data assimilation technique for the initial experiment and we do not include explicit conservation enforcement.

Our domain decomposition work focuses on two-way interactive nested grid communications and the development of good internal boundary conditions. We are particularly interested in examining heterogeneous open boundary conditions between different asymptotic regions. Because the major geophysical equations are not known to be well-posed as an initial value boundary problem, in general, this issue becomes important.

# 8 Initial Conditions

Our long range plans are to build a basin wide grid, and overlay it with refined grids about regions of ageostrophic dynamic regions. The refined grids will then follow mesoscale or planetary scale dynamical features.

But our current work is much less ambitious. We have constructed a box model and are using Spall and Hollands [12] barotropic modon and baroclinic vortex problems to examine the viability and desirability of different communication schemes between the coarse and refined meshes.

A barotropic modon is a coherent, concentric streamfunction. The barotropic flow is the primary mode of a quasi-geostrophic equation formulated as a Sturm-Liouville problem (all other modes of the Sturm-Liouville problem are referred to as baroclinic). It has an analytic solution, it is quasi-geostrophic, and uses an infinite beta plane approximation. The result is a coherent depth independent (barotropic) structure that moves at constant speed.

The baroclinic vortex has no analytic solution, and is defined using a Gaussian pressure distribution with maximum geostrophic velocity of $100cm/sec$. The initial velocity fields are calculated to be in geostrophic balance with the prescribed Gaussian pressure field.

Our experiment follows a hierarchical approach. Beginning with a homogeneous domain decomposition we use a full, coarse primitive equation model and keep track of the flow across the "future" internal boundaries. Then we introduce the nested grid into the same problem and analyze any changes in the boundary information flow. This is used as our error due to boundary conditions only. This error is measured both in root mean squared error and in phase error. Small shifts of mesoscale features are not always bad compared to changes in dynamics within those mesoscale features.

Once we complete our homogeneous studies we will move to a heterogeneous domain decomposition with a quasi-geostrophic coarse grid overlaid with a primitive equation refined grid. Again we will compare the flow across the internal boundaries. Then we will add a feedback loop that uses the nudging data assimilation technique from the refined mesh to the coarse mesh and nudge to the true boundary information. This way we can measure the improvement due to the nudging feedback loop.

The quasi-geostrophic coarser model will be forced by the nudging from the refined primitive equation model. The unforced quasi-geostrophic model is statically stability, but the primitive equation is not. The forced behavior perturbs the reduced equation dynamics, so that a time series of its behavior in the refined region will not be statically stability due to the feedback interaction. But with known density changes at the boundaries from our true solution, we can measure how well the reduced dynamics are being influenced by the regional models with their more complete physical models. Another metric is the apparent ageostrophic time series behavior in the quasi-geostrophic coarser model. After nudging, the ageostrophic adjustment to the quasi-geostrophic coarse mesh is calculated, and a time series of this difference is the ageostrophic forcing of the quasi-geostrophic model. Where this difference is small, there is no need to maintain a refined mesh, so this metric can be used to eliminate refined meshes that are no longer needed, but it can not help us locate where refined meshes should be placed.

## 8.1 Semi-Spectral Primitive Equation Model (SPEM)

The primitive equation SPEM model of Haidvogel et. al. [3] has prognostic variables for horizontal velocity, $u$ and $v$, and temperature $t$, and salinity $s$. It uses the hydrostatic and Boussinesq approximation. The resulting equations are advanced on an scattered Arakawa "C" grid in the horizontal, while the vertical is spectral, with Chebyshev modes. It has a rigid lid approximation at the surface (no variations or "waves" in sea surface height).

# 9 Current Summary

The computational demands of fully three-dimensional global ocean modeling seem to require a nested heterogeneous adaptive grid solution. However, the implementation difficulties are robust. The need for physically realistic open boundary conditions is already well documented, mostly a result of a "grand challenge" issued several years ago. Our experience indicates that an equally pressing need is to provide modeling-consistent asymptotically nested initial conditions for each new nested grid.

The scientific aspects of the work are focused on the boundary condition formulation and on the two-way grid communication mechanisms under development.

# References


[1] M.J. Berger, *Adaptive Mesh Refinement For Hyperbolic Partial Differential Equations*, Ph.D. Dissertation, Stanford University, 1982.

[2] Chan, T.F., R. Glowinski, J. Periaux, and O.B. Widlund, eds, *Domain Decomposition Methods*, SIAM, 1989.

[3] Haidvogel, Wilkin, and Young, *A semi-spectral primitive equation ocean circulation model using vertical sigma and orthogonal curvi-linear horizontal coordinates*, JCP, 1989, in press.

[4] J. Oliger and A. Sundstrom, *Theoretical and practical aspects of some initial boundary value problems in fluid dynamics*, SIAM J. Appl. Math. **35**, no. 3 (1978) 419–446.

[5] Perkins, A.L.,*Parallel Heterogeneous Mesh Refinement For Multidimensional Convection-Diffusion Equations Using An Euler-Lagrange Method*, Ph.D. Dissertation, UCRL-53950, 1989.

[6] Spall, M.A., and W.R. Holland, *A Nested Primitive Equation Model For Oceanic Applications*, to appear in J. Phys. Ocean.

[7] Thompson, J.D., and W.J. Schmitz, *A Limited-Area Model of the Gulf Stream: Design, Initial Experiments, and Model-Data Intercomparison*, J. Phys. Ocean., Vol. 19, No. 6, June 1989.