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## Re-Ionization and Decaying Dark Matter

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Gunn-Peterson tests suggest that the Universe was re-ionized after the standard recombination epoch. We present a systematic treatment of the ionization process by deriving the Boltzmann equations appropriate to this regime. A compact solution for the photon spectrum is found in terms of the ionization ratio. These equations are then solved numerically for the Decaying Dark Matter scenario, wherein neutrinos with mass of order 30 eV radiatively decay producing photons which ionize the intergalactic medium. We find that the neutrino mass and lifetime are severely constrained by Gunn-Peterson tests, observations of the diffuse photon spectrum in the ultraviolet regime, and the Hubble parameter.

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## 1. Introduction

The standard model of the early Universe predicts that free electrons and protons combined to form hydrogen atoms at a redshift of order 1000. After this “recombination” epoch, almost all matter is predicted to be in the form of neutral hydrogen. The Gunn-Peterson [1] test is a way of examining this prediction. In particular, the absence of a dip in the spectra of distant quasars on the blue side of the Lyman alpha line is convincing evidence that there is very *little* neutral hydrogen in the intergalactic medium. To get a feel for the conflict between the prediction and the observations, consider the limit imposed by Steidel and Sargent [2] on the intergalactic hydrogen density,  $n_H$ :

$$n_H < 8.4 \times 10^{-12} h \text{ cm}^{-3} \quad (1.1)$$

at a redshift  $z = 2.64$ . Here  $h$  is the Hubble constant today in units of  $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  and lies between .4 and 1. The standard model prediction [3] for the hydrogen density at this redshift is roughly  $7 \times 10^{-6} \text{ cm}^{-3}$ , violating the Steidel-Sargent limit by six orders of magnitude.

It appears then that there is a diffuse spectrum of ionizing radiation permeating the Universe [4]. What is the source of this radiation? Some years ago Sciama [5] proposed that neutrinos may be both the dark matter and the source of ionizing radiation. Neutrinos are particularly attractive candidates to serve this dual role because cosmology predicts that there are many background neutrinos; they exist in numbers comparable to the background photons. If the mass of one of the species of neutrinos is in the range  $25 - 100 \text{ eV}$ , then they can be the dark matter [6]. Further, if the lifetime for the process

$$\nu_{\text{Heavy}} \rightarrow \nu_{\text{Light}} + \gamma \quad (1.2)$$

is short enough – but longer than the age of the Universe, so most of the heavy neutrinos are still around to be the dark matter – then the photons coming from neutrino decays can fully ionize the intergalactic medium [7].

In this paper, we will analyze the Decaying Dark Matter (DDM) scenario and see what predictions it makes. In particular, for any given values of the neutrino mass and lifetime, we will calculate the photon spectrum today as a result of all past neutrino decays, and we will find the predicted value of the neutral hydrogen density. To do this, in Section 3 we derive the Boltzmann equations which govern the photon spectrum and the ionization ratio. It will be shown that the ionization ratio depends on the electron temperature, so

we will derive the Boltzmann equation for this quantity as well. Our Boltzmann treatment complements and – to some extent – extends previous work on the subject [8] [9].

The possibility of radiatively decaying particles has been raised in many different cosmological contexts [10] [11] [12]. In all of these cases, it is useful to have an expression for the photon spectrum as a function of redshift. In Section 4, we solve the Boltzmann equation for the photon spectrum in terms of the ionization ratio. Our discussion is kept general so it can be applied to different decaying particle scenarios and also to other proposed explanations of the Gunn-Peterson tests.

In section 5, we present the results of a numerical integration of the Boltzmann equations. There are two requirements which together bound the neutrino lifetime. The lifetime must be short enough so that almost all hydrogen atoms are ionized. If the lifetime is too short, though, we would have observed some of the photons from the neutrino decays in the diffuse UV background. These complementary bounds constrain the neutrino lifetime to be of order  $10^{23} - 10^{24}$  seconds. Besides the quantitative results, our most striking prediction is that the photon spectrum today will be sharply cut off at long wavelengths.

First, though, for the purpose of orientation, several general comments are presented in the next section.

## 2. General Considerations

With very little work, one can get a feel for the range of neutrino masses and lifetimes necessary for the DDM scenario to work. First, we note that cosmological neutrinos with a mass in the desired range are very non-relativistic today. Therefore, the energy of the photon emitted in a decay is  $E_\nu/2 = m_\nu/2$  [as long as the emitted neutrino is much less massive than the decaying one]. This must be greater than the energy required to ionize a hydrogen atom,  $\epsilon_0 \equiv 13.6$  eV, so the first constraint is on the mass of the decaying neutrino:

$$m_\nu > 27.2 \text{ eV}. \quad (2.1)$$

A second constraint emerges when we note that if the energy density of neutrinos is enough to set  $\rho/\rho_{\text{critical}} \equiv \Omega = 1$ , then there is a relationship between the Hubble constant and the neutrino mass. This follows since

$$\Omega_\nu h^2 = \frac{m_\nu}{91.5 \text{ eV}}. \quad (2.2)$$

If we now require  $\Omega_\nu + \Omega_{\text{baryon}} = 1$ , then we can express  $h$  as a function of  $m_\nu$ :

$$h(m_\nu) = \left[ \frac{m_\nu}{91.5 \text{ eV}} + .0125 \right]^{1/2}. \quad (2.3)$$

Here – and throughout – we have set  $\Omega_{\text{baryon}} h^2 = .0125$ , the favored value emerging from cosmic nucleosynthesis calculations [3]. We note that since  $m_\nu$  must be greater than  $2\epsilon_0 = 27.2 \text{ eV}$  for the emitted photons to be able to ionize hydrogen,  $h$  must be greater than 0.55. Since we are assuming a matter-dominated Universe with  $\Omega = 1$ ,  $h$  must be less than 0.65 in order for the age of the Universe to be greater than  $10^{10}$  years [13]. This places an upper bound on  $m_\nu$ :

$$m_\nu < 37.5 \text{ eV}. \quad (2.4)$$

A final constraint comes from requiring that all the hydrogen atoms in the intergalactic medium have been ionized. If every photon produced ionizes a hydrogen atom, then – if decaying neutrinos are to be solely responsible for the ionization – the number of neutrinos which have decayed by now must be greater than the number of hydrogen atoms. But the number density of neutrinos that have decayed by now is just  $(t_0/\tau)n_\nu(t_0)$ , where  $t_0$  is the present age of the Universe;  $\tau$  is the neutrino lifetime; and  $n_\nu(t_0)$  is the cosmological number density of one species of neutrinos, roughly a third the number density of background photons. As a first estimate, let us assume that the number density of hydrogen [ionized or not] in the intergalactic medium is of order the diffuse baryon number density,  $4 \times 10^{-10}$  times the number density of background photons. By requiring the number density of decayed neutrinos to be greater than the hydrogen number density, we arrive at the third constraint, this one on the neutrino lifetime:

$$t_0 < \tau < 10^9 t_0 \quad (2.5)$$

where the lower limit follows from the requirement that most of the heavy neutrinos have not yet decayed. We will see in the next few sections that the lower limit is much larger than  $t_0$  due to ultraviolet constraints [14] and the upper limit is much smaller than  $10^9 t_0$  due to the effects of recombination. The favored regime which emerges –  $10^{23} - 10^{24}$  seconds – is extremely interesting. The standard model of particle physics predicts [15] a lifetime much longer than this for a 30 eV neutrino, so convincing evidence of this scenario would require drastic modifications of the standard model [16] [17].

How good is the assumption that the number density of diffuse hydrogen today is of order the cosmological baryon number density? The answer to this question depends

upon how much and when baryons clumped together. One might imagine that all of the baryons clumped together (into pop III objects) shortly after recombination. Such a scenario would explain the null Gunn-Peterson result but is rather unlikely as a model of structure formation. It is more likely that in the era in which quasars formed (the epoch the Gunn-Peterson test probes) virtually all the baryonic matter is still diffuse. Subsequently more clumping, though how much is unknown, took place. In order to allow for this uncertainty we shall introduce a clumping factor  $f$  such that

$$n_B^{\text{diffuse}} \equiv f n_B^{\text{cosmological}} \quad (2.6)$$

where  $f \leq 1$ . It is fairly safe to assume that for the oldest quasars  $f$  is of order unity, while today it could be up to three orders of magnitude lower [18].

### 3. Evolution Equations

In this section we derive the Boltzmann equations which govern the photon spectrum and the ionization ratio [19]. In a flat FRW Universe, the distribution function  $f(k, t)$  of a particle species  $\psi$  evolves [20] according to the following equation:

$$\left\{ \frac{\partial}{\partial t} - \frac{\dot{R}}{R} k \frac{\partial}{\partial k} \right\} f_\psi(k, t) = C_\psi(k, t) \quad (3.1)$$

where  $R(t)$  is the scale factor of the Universe. The term on the left hand side is the generalized Liouville operator acting upon  $f_\psi$ , which takes into account the cosmological expansion. The term on the right side, represents all the physical interactions involving the particle. For example, for scatterings  $\psi + a \leftrightarrow b + c$ , the collision term is

$$\begin{aligned} C_\psi(k, t) = & \frac{-1}{2E_\psi(k)} \int \frac{d^3 p_a}{(2\pi)^3 2E_a(p_a)} \int \frac{d^3 p_b}{(2\pi)^3 2E_b(p_b)} \int \frac{d^3 p_c}{(2\pi)^3 2E_c(p_c)} \\ & \times (2\pi)^4 \delta^4(k + p_a - p_b - p_c) \left[ |M|^2_{\psi+a \rightarrow b+c} f_\psi(k) f_a(p_a) \right. \\ & \left. - |M|^2_{b+c \rightarrow a+\psi} f_b(p_b) f_c(p_c) \right]. \end{aligned} \quad (3.2)$$

Here, and throughout,  $E_i(q) \equiv \sqrt{q^2 + m_i^2}$  for any particle  $i$ . The matrix elements squared include summations over the spin states of particles  $a, b$  and  $c$  but not of  $\psi$ . Note that we have not included Pauli (Bose) suppression (enhancement) factors, as they are negligible.

We shall first consider the distribution function of the photons,  $f_\gamma(k, t)$ . The aim of this paper is to elucidate the most important physics of the reionization epoch, so accordingly only the dominant terms will be included on the right hand side of the Boltzmann equation. For the photons this includes the decay process

$$\nu_H(q) \rightarrow \nu_L(p)\gamma(k); \quad (3.3)$$

the heavy neutrino decaying into a light neutrino [assumed massless] and a photon. In analogy to Eq. (3.2), we have

$$C_{\text{decay}} = \frac{1}{2k} \int \frac{d^3p}{(2\pi)^3 2p} \int \frac{d^3q}{(2\pi)^3 2E_\nu(q)} F_\nu(q, t) |M|^2 (2\pi)^4 \delta^4(q - p - k) \quad (3.4)$$

where  $F_\nu(q, t)$  is the distribution function of the massive neutrinos and  $|M|^2$  is the square of the amplitude for radiative decay, summed over massive and light neutrino spin states. The inverse process is unimportant at the temperatures of interest, far below the heavy neutrino mass.

We know that  $F_\nu(q)$  constrains  $q$  to be of order the temperature; since we are interested in temperatures much less than  $m_\nu$ , we have

$$\delta[E(q) - |\vec{q} - \vec{k}| - k] \approx \frac{1}{2} \delta(k - m_\nu/2). \quad (3.5)$$

Using this simplification and the fact that

$$2 \int \frac{d^3q}{(2\pi)^3} F_\nu(q, t) = n_\nu(t) \quad (3.6)$$

where  $n_\nu(t)$  is the number density of the heavy  $\nu$  and  $\bar{\nu}$  [hence the factor of 2], we find

$$C_{\text{decay}} = \frac{n_\nu(t) \pi^2}{\tau k_0^2} \delta(k - k_0). \quad (3.7)$$

Here  $k_0$  is the energy of the decay-produced photon.

$$k_0 \equiv \frac{m_\nu}{2}, \quad (3.8)$$

and  $\tau$ , the lifetime of the decaying neutrino, is given by  $\tau = 16\pi m_\nu / |M|^2$ .

Another important process affecting the photon distribution is photoionization of hydrogen –  $\gamma(k)H(k') \rightarrow e(p)P(p')$  – and the inverse process, recombination –  $e(p)P(p') \rightarrow \gamma(k)H(k')$ . These lead to a collision term

$$C_{\text{ion/rec}} = -\frac{1}{2k} \int \frac{d^3 p'}{(2\pi)^3 2E_P(p')} \int \frac{d^3 p}{(2\pi)^3 2E_e(p)} \int \frac{d^3 k'}{(2\pi)^3 2E_H(k')} \quad (3.9)$$

$$\times |M|_{H\gamma \rightarrow eP}^2 (2\pi)^4 \delta^4(p + p' - k - k') [f_\gamma(k)g_H(k') - g_e(p)g_P(p')]$$

where  $g_H(k')$ ,  $g_e$ ,  $g_P$  are the distribution functions of hydrogen atoms, electrons and protons, and  $M$  is the amplitude for the process in which a hydrogen atom in the ground state gets ionized. There is no contribution to the photon spectrum due to ionization from excited states of hydrogen, since at these low temperatures, all excited states that might be produced immediately drop down to the ground state: only the  $n = 1$  state is occupied. [Appendix A treats this question quantitatively.] All the integrals in Eq. (3.9) can be carried out assuming thermal equilibrium amongst free electrons, protons, and hydrogen atoms. Since the rate for elastic scattering – the process which bring the species into thermal equilibrium – is much greater than the expansion rate, thermal equilibrium is a good assumption. Consider first the term in Eq. (3.9) with  $f_\gamma g_H$ . The integrals over  $p$  and  $p'$  can be rewritten in terms of the cross section for photoionization, and the integral over  $k'$  can be rewritten in terms of  $n_H$  yielding,

$$C_{\text{ionization}} = -n_H \sigma_{H\gamma \rightarrow eP} f_\gamma(k, t) \Theta(k - \epsilon_0). \quad (3.10)$$

Now consider the term in Eq. (3.9) with  $g_e g_P$ . Due to thermal equilibrium, each of these distribution functions are Maxwellian with a common temperature  $T_e$ :

$$g_e(p) = \frac{n_e}{2} \left( \frac{2\pi}{m_e T_e} \right)^{3/2} e^{-p^2/2m_e T_e} ; \quad g_P(p') = n_e \left( \frac{2\pi}{M T_e} \right)^{3/2} e^{-p'^2/2M T_e} \quad (3.11)$$

where  $M$  is the proton mass and the factor of 2 in the denominator for  $g_e$  accounts for the two spin states of electrons [nuclear spin states play no role, since those in  $g_P$  cancel those in  $g_H$ ]. Note that the proton distribution function,  $g_P$ , is proportional to  $n_P$ , which we have set equal to  $n_e$  since there are an equal number of free electrons and protons. [Any difference between the two is due to the small fraction of helium atoms, which we neglect.] These expressions for the distribution functions together with energy conservation can be used to do the integrals in Eq. (3.9). Specifically we write

$$g_e(p)g_P(p') = \frac{n_e^2}{2} \left( \frac{2\pi}{m_e T_e} \right)^{3/2} \left( \frac{2\pi}{M T_e} \right)^{3/2} e^{-[k - \epsilon_0 + k'^2/2M]/T_e}$$

$$= \frac{n_e^2}{2} \left( \frac{2\pi}{m_e T_e} \right)^{3/2} \left( \frac{2\pi}{M T_e} \right)^{3/2} e^{-[k - \epsilon_0]/T_e} \frac{2g_H(k')}{n_H} \left( \frac{M T_e}{2\pi} \right)^{3/2} \quad (3.12)$$



where the first equality follows from energy conservation and the second from the fact that  $g_H$  is also assumed Maxwellian. We can now proceed as before, integrating over  $p, p'$ , and  $k'$  to find

$$C_{\text{recombination}} = \left( \frac{2\pi}{m_e T_e} \right)^{3/2} n_e^2 \sigma_{H\gamma \rightarrow eP} e^{-[k-\epsilon_0]/T_e}. \quad (3.13)$$

Physically we see that at low temperatures the photons produced in recombination will have energies very close to  $\epsilon_0$ , as expected.

While there are other processes which in principle should be included in the Boltzmann equation, such as Compton scattering and Bremsstrahlung [21], since the rates of these processes are much slower, they can be neglected. Thus the photon equation is

$$\begin{aligned} \left\{ \frac{\partial}{\partial t} - \frac{\dot{R}}{R} k \frac{\partial}{\partial k} \right\} f_\gamma(k, t) &= \frac{n_\nu(t) \pi^2}{\tau k_0^2} \delta(k - k_0) \\ &+ \left( \frac{2\pi}{m_e T_e} \right)^{3/2} n_e^2 \sigma_{H\gamma \rightarrow eP} e^{-[k-\epsilon_0]/T_e} - n_H \sigma_{H\gamma \rightarrow eP} f_\gamma(k, t) \Theta(k - \epsilon_0). \end{aligned} \quad (3.14)$$

We must now derive an equation for the neutral hydrogen density. Following the principles above, we can write the non-integrated equation as

$$\begin{aligned} &\left\{ \frac{\partial}{\partial t} - \frac{\dot{R}}{R} k' \frac{\partial}{\partial k'} \right\} g_H(k', t) \\ &= \frac{1}{2E_H(k')} \int \frac{d^3 p}{(2\pi)^3 2E_e(p)} \int \frac{d^3 p'}{(2\pi)^3 2E_P(p')} \int \frac{d^3 k}{(2\pi)^3 2k} (2\pi)^4 \delta^4(p + p' - k - k') \\ &\times \left[ |M|^2_{eP \rightarrow H\gamma} g_e(p) g_P(p') - |M|^2_{H\gamma \rightarrow eP} f_\gamma(k) g_H(k') \right] \end{aligned} \quad (3.15)$$

where we have explicitly distinguished the two amplitudes for reasons to be made clear shortly. Summing over hydrogen spin states and integrating over hydrogen momenta we find

$$\begin{aligned} R^{-3} \frac{d}{dt} (n_H R^3) &= 2n_P \int \frac{d^3 p}{(2\pi)^3} g_e(p) (\sigma_{eP \rightarrow H\gamma} v) \\ &- 2n_H \int \frac{d^3 k}{(2\pi)^3} f_\gamma(k) \sigma_{H\gamma \rightarrow eP}. \end{aligned} \quad (3.16)$$

The recombination cross section  $\sigma_{eP \rightarrow H\gamma}$  must include capture into all possible states of hydrogen, since at low electron temperatures, direct recombination to excited states gives an important contribution to the recombination rate. Since the lifetime of excited states (even metastables) is much shorter than the time scale of ionization,  $\sigma_{H\gamma \rightarrow eP}$  need only include ionization from the ground state.

To simplify the equation further we define the neutral hydrogen ratio

$$r \equiv \frac{n_H}{n_B} \quad (3.17)$$

where  $n_B$  is the number density of baryons in the intergalactic medium. We are neglecting the small fraction of helium atoms, so  $n_e = n_p = n_B(1 - r)$ . Plugging in the relevant cross sections for ionization and recombination [22] and performing the integrations, we have

$$\frac{dr}{dt} = n_B(\sigma v)_R(\epsilon_0/T_e)^{1/2}(0.43 + 1/2 \ln \frac{\epsilon_0}{T_e})(1 - r)^2 - \tilde{n}_\gamma \sigma_I r \quad (3.18)$$

where

$$\begin{aligned} \tilde{n}_\gamma \sigma_I &\equiv 2 \int \frac{d^3 k}{(2\pi)^3} f_\gamma(k) \sigma_{H\gamma \rightarrow eP} \\ &= \sigma_I \frac{\epsilon_0^3}{\pi^2} \int_{\epsilon_0}^{\infty} \frac{dk}{k} f_\gamma(k). \end{aligned} \quad (3.19)$$

Here  $\sigma_I = 6.2 \times 10^{-18} \text{ cm}^2$  is the ionization cross section at threshold, and we have taken  $\sigma_{H\gamma \rightarrow eP}$  to fall off as  $k^{-3}$ . The recombination cross section is  $(\sigma v)_R = 5.2 \times 10^{-14} \text{ cm}^3 \text{ sec}^{-1}$ .

Since the recombination rate depends upon the electron temperature, we must derive an equation for it. We begin with the Boltzmann equation for the electron distribution  $g_e(p, t)$ . There are several relevant interactions which must be included on the right hand side of the Boltzmann equation. First, there are elastic processes: Coulomb scattering with other free electron; Coulomb scattering with free protons; and elastic scattering off hydrogen atoms. We also must include a term representing recombination and its inverse, photoionization. Finally, a free electron can scatter off the background photons and lose energy, so we must include a term for Compton scattering. Thus the equation for the electron distribution function is

$$\left\{ \frac{\partial}{\partial t} - \frac{\dot{R}}{R} p \frac{\partial}{\partial p} \right\} g_e(p, t) = C_{\text{elastic}}^{eP} + C_{\text{elastic}}^{ee} + C_{\text{elastic}}^{eH} + C_{\text{ion/rec}} + C_{\text{Compton}}. \quad (3.20)$$

We now outline a simple recipe for dealing with the elastic collision terms. (i) Multiply this equation by the kinetic energy  $p^2/2m_e$  and integrate over phase space: number of spin states  $\times d^3 p / (2\pi)^3$ . (ii) Do the same for all particles with which the electron interacts elastically. (iii) Add all these equations. Then the integrals over the elastic terms all vanish

due to energy conservation. As an example, consider electron -proton elastic scattering. The sum of these – after the relevant multiplications and integrations is –

$$\begin{aligned}
& \int \frac{d^3 p}{(2\pi)^3} (p^2/2m_e) C_{\text{elastic}}^{eP} + \int \frac{d^3 p'}{(2\pi)^3} (p'^2/2M) C_{\text{elastic}}^{Pe} \\
&= \int \frac{d^3 p}{(2\pi)^3 2E_e(p)} \int \frac{d^3 p'}{(2\pi)^3 2E_P(p')} \int \frac{d^3 q}{(2\pi)^3 2E_e(q)} \int \frac{d^3 q'}{(2\pi)^3 2E_P(q')} \quad (3.21) \\
&\times (2\pi)^4 \delta^4(p + p' - q - q') \sum_{\text{spins}} |M|^2 \\
&\times [p^2/2m_e + p'^2/2M][g_e(q)g_P(q') - g_e(p)g_P(p')].
\end{aligned}$$

But, by energy conservation, the last line here is

$$(q^2/2m_e + q'^2/2M)g_e(q)g_P(q') - (p^2/2m_e + p'^2/2M)g_e(p)g_P(p'). \quad (3.22)$$

This changes sign under the interchange of dummy variables ( $q \leftrightarrow q'; p \leftrightarrow p'$ ) while the integration measure is invariant under such a change. Therefore the sum in Eq. (3.21) vanishes.

The above prescription throws away some of the information contained in the non-integrated equation. However, we can compensate for this by assuming that the distributions are Maxwellian. Then the integrated equation determines the one parameter in the Maxwellian distribution: the temperature  $T_e$ . Specifically, it is straightforward to show that the left hand side of the sum of the integrated Boltzmann equations is

$$R^{-5} \frac{d}{dt} \left\{ \frac{3}{2} R^5 T_e (n_H + n_e - n_P) \right\}. \quad (3.23)$$

We can write the densities in terms of  $r$  and  $n_B$  and then use the fact that  $n_B R^3$  is constant to rewrite this as

$$\frac{3}{2} n_B R^{-2} \frac{d}{dt} \{ R^2 T_e (2 - r) \} = \frac{3}{2} n_B (2 - r) \left\{ \dot{T}_e + 2 \frac{\dot{R}}{R} T_e - \frac{\dot{r}}{2 - r} T_e \right\}. \quad (3.24)$$

If the right hand side of the Boltzmann equation were zero – that is, in the absence of any interactions – and if  $\dot{r}$  were 0 then we find that  $T_e$  falls as  $R^{-2}$ , a well-known result. Another good check on Eq. (3.24) is to ignore expansion and interactions. Then as the neutral hydrogen fraction increases ( $\dot{r} > 0$ ),  $T_e$  increases as well. This reflects the fact that the temperature is shared by the 3 species: electrons, protons, and hydrogen atoms. If one

hydrogen atom is added, both an electron and a proton must be subtracted, so the total number of degrees of freedom has dropped by one. This leads to a rise in temperature.

It remains to find the right hand side of the Boltzmann equation; that is, to integrate the remaining collision terms in Eq. (3.20) and its proton and hydrogen counterparts. First consider the recombination/ionization terms. Schematically they sum to

$$\sum_{\text{momenta, spins}} |M|^2 \{f_\gamma(k)g_H(k') - g_e(p)g_P(p')\} [(p^2/2m_e) + (p'^2/2M) - (k'^2/2M)]. \quad (3.25)$$

By energy conservation, the energies in square brackets are just equal to  $k - \epsilon_0$ . Thus, when integrating over  $f_\gamma g_H$ , the  $p$  and  $p'$  integrals can be done easily. When integrating over  $g_e g_P$ , we can simply set the energies equal to the electron kinetic energy, since the heavy atom gains very little kinetic energy in the process. Therefore, the  $k$  and  $k'$  integrals in this case are easily performed. The final integral over the electron momentum is just the thermally averaged cross section multiplied by the temperature. So these two terms give:

$$2n_H \int \frac{d^3k}{(2\pi)^3} f_\gamma(k)(k - \epsilon_0)(\sigma_{H\gamma \rightarrow eP}) - n_e n_P T_e (\sigma v)_R (\epsilon_0/T_e)^{1/2} (0.43 + 1/2 \ln \frac{\epsilon_0}{T_e}). \quad (3.26)$$

Finally the Compton term in Eq. (3.20) must be integrated. Note that Compton scattering off the background photons is not important for protons [and obviously not for neutral hydrogen] since the cross section is small than for electrons by a factor of  $(m_e/M)^2$ . Therefore we need only integrate over  $C_{\text{Compton}}$  of electrons. The result is well-known [23]

$$2 \int \frac{d^3p}{(2\pi)^3} (p^2/2m_e) C_{\text{Compton}} = \frac{3n_e(T - T_e)}{2\tau_c} \quad (3.27)$$

where  $T$  is the temperature of the background radiation and  $\tau_c$  is the Compton cooling time

$$\tau_c \equiv \frac{3m_e}{4\rho_\gamma \sigma_{\text{Thompson}}} = \tau_c^0 (T_0/T)^4 \quad (3.28)$$

where the Compton cooling time today (when  $T = T_0$ ) is  $\tau_c^0 = 7.1 \times 10^{19}$  sec. We can collect equations (3.24), (3.26), and (3.27) to write the evolution equation for the electron temperature as

$$\begin{aligned} \frac{3}{2}(2-r) \left\{ \dot{T}_e + 2\frac{\dot{R}}{R}T_e - \frac{\dot{r}}{2-r}T_e \right\} &= 2r \int \frac{d^3k}{(2\pi)^3} f_\gamma(k)(k - \epsilon_0)\sigma_{H\gamma \rightarrow eP} \\ &+ \frac{3}{2}(1-r) \frac{T - T_e}{\tau_c^0} (T/T_0)^4 - (1-r)^2 n_B (\sigma v)_R (\epsilon_0/T_e)^{1/2} (0.43 + 1/2 \ln \frac{\epsilon_0}{T_e}) T_e. \end{aligned} \quad (3.29)$$

#### 4. Photon Spectrum

The set of equations – (3.18), (3.29), and (3.14)– can be numerically integrated to find the photon spectrum and the ionization fraction at all times. We will display results of this numerical work in the next section. Here we focus on the equation for the photon spectrum – Eq. (3.14). For a variety of reasons it is useful to have an analytic expression for the occupation number,  $f_\gamma(y, z)$ . We first derive such an expression and then apply it to three problems that arise when dealing with a non-equilibrium distribution of photons in the early Universe: (i) What is the remnant spectrum of photons today? (ii) How many ionizing photons are present at a given redshift  $z$ ? and (iii) Why and when is recombination to the ground state of hydrogen suppressed? This last point – initially made by Peebles – will be shown to emerge in a straightforward manner from the analytic expression for  $f_\gamma(y, z)$ .

It is useful first to rewrite the photon equation in terms of dimensionless variables. Let

$$y \equiv \frac{k}{k_0} \quad ; \quad 1 + z \equiv \frac{R(t_0)}{R(t)} = \left( \frac{t_0}{t} \right)^{2/3} \quad (4.1)$$

where the last equality follows from our assumption that the Universe is flat and matter dominated. By way of orientation in these new variables, we note that all photons are produced with energy  $k = k_0$  and lose energy due to the redshift, so we are interested only in  $y < 1$ . The main redshifts  $z$  of interest are determined by the neutrino lifetime: roughly, we will be studying the epoch  $z < 300$ . Using the facts that (i) the Universe is matter dominated and therefore  $\frac{\dot{R}}{R} = H_0(1+z)^{3/2}$  ( $H_0$  is the Hubble constant today) and (ii) the baryon and neutrino number densities increase as  $(1+z)^3$ , the photon equation becomes

$$\begin{aligned} & \left\{ -(1+z) \frac{\partial}{\partial z} - y \frac{\partial}{\partial y} + \lambda (\epsilon_0/k_0)^3 \frac{r(z)(1+z)^{3/2} \theta(y - \epsilon_0/k_0)}{y^3} \right\} f_\gamma(y, z) \\ & = \frac{3\pi^2 t_0 n_\nu(t_0)}{2 \tau k_0^3} \delta(y-1) (1+z)^{3/2} \\ & + \lambda \left( \frac{2\pi(1+z)^3}{m_e T_e} \right)^{3/2} n_B(t_0) (1-r)^2 e^{-(k_0 y - \epsilon_0)/T_e} \Theta[y - \epsilon_0/k_0] \end{aligned} \quad (4.2)$$

where

$$\lambda \equiv \frac{n_B(t_0) \sigma_I}{H_0} \quad (4.3)$$

The first term on the right in Eq. (4.2) is the source term due to decays and the second the source term due to recombination. The effect of the ionization sink term is governed

by the dimensionless number  $\lambda$ , which is essentially the ratio of the ionization rate to the Hubble rate. We can get a feel for the magnitude of  $\lambda$  by taking  $n_B(t_0) = 2 \times 10^{-7} \text{ cm}^{-3}$  – the favored value from cosmic nucleosynthesis. Then the ionization rate today [if all baryons were in the form of neutral hydrogen] is  $n_B(t_0)\sigma_I = 3 \times 10^{-14} \text{ sec}^{-1}$ . This is much larger than the expansion rate today, so  $\lambda$  is a very large number.

For the time being we will call the right hand side of Eq. (4.2)  $S(y, z)$  so that the final form of the solution may be used for any sources of interest. We note, though, that  $S$  typically has [at least] two pieces: the source of ionizing photons – in this case the decaying neutrinos – and the photons produced in the process of recombination. To solve this partial differential equation, we first turn it into an ordinary differential equation by introducing a new variable

$$\rho \equiv -\ln y = -\ln \left[ \frac{1+z}{1+z_c} \right]. \quad (4.4)$$

The variable  $\rho$  runs along a curve in the  $(y, z)$  plane; different curves are labelled by different values of  $z_c$ . Along any one curve,

$$\frac{d}{d\rho} = -(1+z) \frac{\partial}{\partial z} - y \frac{\partial}{\partial y} \quad (4.5)$$

so that in terms of  $\rho$ , Eq. (4.2) becomes

$$\begin{aligned} \frac{df_\gamma(\rho)}{d\rho} + \lambda(\epsilon_0/k_0)^3 r \frac{[(1+z_c)e^{-\rho}] [(1+z_c)e^{-\rho}]^{3/2} \Theta(e^{-\rho} - \epsilon_0/k_0)}{(e^{-\rho})^3} f_\gamma(\rho) \\ = S[e^{-\rho}, (1+z_c)e^{-\rho}]. \end{aligned} \quad (4.6)$$

To be explicit, the right hand side here means: take  $S(y, z)$  and evaluate it with  $y$  set equal to  $e^{-\rho}$  and  $1+z$  set equal to  $(1+z_c)e^{-\rho}$ . For a given  $z_c$ , the solution to this ordinary differential equation is elementary:

$$\begin{aligned} f_\gamma(\rho) = \int_0^\rho d\rho' S[e^{-\rho'}, (1+z_c)e^{-\rho'}] \\ \times \exp \left\{ - \int_{\rho'}^\rho d\rho'' \frac{\lambda(\epsilon_0/k_0)^3 r [(1+z_c)e^{-\rho''}] [(1+z_c)e^{-\rho''}]^{3/2} \Theta(e^{-\rho''} - \epsilon_0/k_0)}{e^{-3\rho''}} \right\}. \end{aligned} \quad (4.7)$$

Here we have dropped the boundary term,  $f_\gamma(\rho = 0)$ , since there are no photons with momentum greater than  $k_0$ , i.e. with  $\rho$  negative. This solution in terms of  $\rho$  can be converted back into a solution  $f_\gamma(y, z)$  by eliminating  $\rho$  [via  $\rho = -\ln y$ ] and  $z_c$  [via  $1+z$

$z_c = (1+z)/y]$ . It is also convenient to introduce new dummy variables:  $y' \equiv e^{-\rho'}$  and  $1+z' \equiv (1+z_c)e^{-\rho''} = \frac{1+z}{y}e^{-\rho''}$ . These steps lead to

$$f_\gamma(y, z) = \int_y^1 \frac{dy'}{y'} S[y', (1+z)\frac{y'}{y}] \times \exp \left\{ -\lambda(\epsilon_0/k_0)^3 \left(\frac{1+z}{y}\right)^3 \int_{\max[1+z, (1+z)\frac{\epsilon_0}{k_0 y}]}^{(1+z)y'/y} \frac{dz'}{(1+z')^{5/2}} r(z') \right\}. \quad (4.8)$$

We can go no further without an explicit form for the source terms, but we note that  $\lambda$  is a very large number - of order  $10^3$  - so  $f_\gamma(y, z)$  is negligible unless other factors in the argument of the exponential are very small. That is, ionization - as represented here by the argument of the exponential - plays an important role in suppressing the number of photons coming from any source.

In our case there are two source terms; we focus first on the dominant one: the photons coming from decaying neutrinos. The contribution to  $f_\gamma(y, z)$  from this term is

$$f_\gamma^{(1)}(y, z) = \int_y^1 \frac{dy'}{y'} \left[ \frac{3\pi^2 t_0 n_\nu(t_0)}{2 \tau k_0^3} \delta(y' - 1) \left( (1+z)y'/y \right)^{3/2} \right] \times \exp \left\{ -\lambda(\epsilon_0/k_0)^3 \left(\frac{1+z}{y}\right)^3 \int_{\max[1+z, (1+z)\frac{\epsilon_0}{k_0 y}]}^{(1+z)y'/y} \frac{dz'}{(1+z')^{5/2}} r(z') \right\} \quad (4.9) \\ = \frac{3\pi^2 t_0 n_\nu(t_0)}{2 \tau k_0^3 y^{3/2}} (1+z)^{3/2} \Theta(1-y) \\ \times \exp \left\{ -\lambda(\epsilon_0/k_0)^3 \left(\frac{1+z}{y}\right)^3 \int_{\max[1+z, (1+z)\frac{\epsilon_0}{k_0 y}]}^{(1+z)/y} \frac{dz'}{(1+z')^{5/2}} r(z') \right\}$$

where the superscript on  $f_\gamma$  refers to the fact that this is the contribution from the first source term. Eq. (4.9) tells us that the photon spectrum at a given energy  $y$  and epoch  $z$  depends on an integration over the amount of neutral hydrogen at redshifts larger than  $z$ . This is reasonable: In the absence of ionization, high energy photons red-shift down and add to the number of low energy photons at later epochs. If these high energy photons are lost through ionization, the low energy number at later epochs is correspondingly suppressed. The integral over  $r(z)$  quantifies this suppression. In short, if we are given  $r(z)$  we can do the integral and determine the photon spectrum. In practice things are not so simple:  $r(z)$  depends on  $f_\gamma$ . Nonetheless, we will see that in several important cases, one can make good approximations which considerably simplify the way in which  $f_\gamma$  depends on  $r$ .

We can use Eq. (4.9) to estimate the photon spectrum today when  $z = 0$ . Since  $z = 0$ , the integral in the argument of the exponential in Eq. (4.9) ranges from  $z' = 0$  to  $1 + z' = 1/y = k_0/k$ . As a first approximation to the effect of ionization, let us assume that the Universe is spontaneously ionized at  $z = z_i$ ; i.e.  $r = 0$  for all  $z < z_i$ . Then the argument of the exponential vanishes as long as  $1 + z_i > k_0/k$ . For energies  $k$  less than  $k_0/(1 + z_i)$ , the argument of the exponential is huge and  $f_\gamma \simeq 0$ . We can write for the photon intensity today

$$\begin{aligned} I &\equiv k \frac{dn_\gamma}{dk d\Omega} = k \frac{2 d^3k f_\gamma(k, z=0)}{(2\pi)^3 dk d\Omega} \\ &= \frac{3}{8\pi} \frac{n_\nu(t_0)t_0}{\tau} \left(\frac{k}{k_0}\right)^{3/2} \Theta(k_0 - k)\Theta(k - k_0/(1 + z_i)). \end{aligned} \quad (4.10)$$

The first part of this expression is the standard formula for the intensity due to a radiatively decaying relic particle [10]. Only the last step function – reflecting the ionization process – is new. Of course the drop in intensity at  $k_0/(1 + z_i)$  is not as dramatic as the step function indicates; in the next section we will show numerical results.

Another question of interest is : At a given redshift  $z$ , how many ionizing photons are present due to the decaying neutrinos? To find this, we must calculate  $\tilde{n}_\gamma$ , as defined in Eq. (3.19),

$$\begin{aligned} \tilde{n}_\gamma^{(1)}(z) &= n_\nu(t_0) \frac{3t_0 \epsilon_0^3 (1+z)^{3/2}}{2\tau k_0^3} \\ &\times \int_{\epsilon_0/k_0}^1 \frac{dy}{y^{5/2}} \exp\left[-\lambda \left(\frac{\epsilon_0(1+z)}{yk_0}\right)^3 \int_{1+z}^{(1+z)/y} \frac{d(1+z')}{(1+z')^{5/2}} r(z')\right]. \end{aligned} \quad (4.11)$$

One simple approximation which is often useful is to neglect completely the effect of ionization on  $f_\gamma$ ; this corresponds to setting the exponential in Eq. (4.11) to 1 so that

$$\tilde{n}_\gamma^{(1)}(z) \rightarrow \tilde{n}_\gamma^{\text{ionized}} = n_\nu(t_0) \frac{t_0}{\tau} (1+z)^{3/2} (\epsilon_0/k_0)^3 \left[ \left(k_0/\epsilon_0\right)^{3/2} - 1 \right]. \quad (4.12)$$

where the superscript reminds us that this approximation should be valid only in an ionized Universe. For if there are very few neutral hydrogen atoms, then the ionization process has a negligible effect on the photon distribution. Note that as  $k_0$  approaches  $\epsilon_0$ , the number of ionizing photons goes to zero: expansion causes the decay-produced photons to lose energy, pushing them below threshold.



The opposite situation occurs when there are many neutral atoms so that a photon produced in a neutrino decay immediately ionizes a neutral atom. This corresponds to a large damping factor in the exponential of Eq. (4.9). In such a situation there is no time for a photon to lose energy via the red-shift before it is absorbed in an ionization process. Therefore, there are almost no photons with energy much less than  $k_0$ . We can take advantage of this fact by expanding the argument of the exponential around  $y = 1$ . This leads to

$$\begin{aligned} \tilde{n}_\gamma^{(1)}(z) &= n_\nu(t_0) \frac{3t_0(1+z)^{3/2}}{2\lambda\tau} \int_0^{\lambda(\epsilon_0/k_0)^3[1-\epsilon_0/k_0]} \frac{d\nu}{\left[1 - \frac{\nu}{\lambda(\epsilon_0/k_0)^3}\right]^{5/2}} \\ &\times \exp\left\{-\nu r(z)(1+z)^{3/2} \left[1 - \frac{11}{4} \frac{\nu}{\lambda(\epsilon_0/k_0)^3} - \frac{1}{2} \left(\frac{d \ln r}{d \ln(1+z)}\right) \frac{\nu}{\lambda(\epsilon_0/k_0)^3} + O(\nu^2/\lambda^2)\right]\right\} \end{aligned} \quad (4.13)$$

where the dummy variable  $\nu \equiv \lambda(\epsilon_0/k_0)^3(1-y)$ . The upper limit of this integral is quite large, effectively  $\infty$ . It is clear that as long as  $r(z)(1+z)^{3/2}$  is of order 1, the dominant contribution comes from  $\nu \approx 1$ . Hence the terms of order  $\nu/\lambda$  do not contribute in this pre-ionization regime. Only when  $r(z)$  becomes very small ( $= O(\lambda^{-1}(1+z)^{-3/2})$ ) do the higher order terms in the argument of the exponential become significant. Hence as long as  $r(z)$  is not too small,

$$\tilde{n}_\gamma^{(1)}(z) \approx \frac{n_\nu(t_0)t_0}{\tau} \frac{3(1+z)^{3/2}}{2\lambda} \int_0^\infty d\nu \exp\left\{-r(z)\nu(1+z)^{3/2}\right\}. \quad (4.14)$$

The remaining integral is trivial so that the un-ionized approximation is

$$\tilde{n}_\gamma^{(1)}(z) \rightarrow \tilde{n}_\gamma^{\text{un-ionized}} = \frac{n_\nu(t_0)t_0}{\tau} \frac{3}{2\lambda r(z)}. \quad (4.15)$$

Until now we have focused on the source term for photons coming from decaying neutrinos. Now let us consider the source term due to the process  $ep \rightarrow H\gamma$ . The contribution of this process to the photon occupation number can be obtained by inserting the last term on the right hand side of Eq. (4.2) into our general expression for  $f_\gamma$  (Eq. (4.8)):

$$\begin{aligned} f_\gamma^{(2)}(y, z) &= \int_y^1 \frac{dy'}{y'} \exp\left\{-\lambda(\epsilon_0/k_0)^3 \left(\frac{1+z}{y}\right)^3 \int_{\max[1+z, (1+z)\frac{\epsilon_0}{k_0 y}]}^{(1+z)y'/y} \frac{dz'}{(1+z')^{5/2}} r(z')\right\} \\ &\times \lambda n_B(t_0) \left(\frac{2\pi}{m_e}\right)^{3/2} \left(\frac{[(1+z)y'/y]^3}{T_e[(1+z)y'/y]}\right)^{3/2} \\ &\times (1 - r[(1+z)y'/y])^2 \exp\left\{-\frac{k_0 y' - \epsilon_0}{T_e[(1+z)y'/y]}\right\} \Theta(y' - \epsilon_0/k_0). \end{aligned} \quad (4.16)$$

This term can be shown to be much smaller than  $f_\gamma^{(1)}$ . However, it does have one important implication for the recombination process. Consider the contribution of this term to  $\tilde{n}_\gamma$ , the number density of ionizing photons. We show in Appendix B that, if the electron temperature is small,

$$\tilde{n}_\gamma^{(2)} = n_B(z) \frac{(1-r)^2}{r} \frac{0.8(\sigma v)_R}{\sigma_I} \frac{1}{1 + \frac{\epsilon_0}{\lambda(1+z)^{3/2} r T_e}}. \quad (4.17)$$

This contribution to  $\tilde{n}_\gamma$  must be included in the evolution equation for  $r$  (Eq. (3.18)), the right hand side of which is

$$\begin{aligned} & n_B(\sigma v)_R (\epsilon_0/T_e)^{1/2} (0.43 + 1/2 \ln \frac{\epsilon_0}{T_e}) (1-r)^2 - \tilde{n}_\gamma \sigma_I r \\ &= n_B(\sigma v)_R (\epsilon_0/T_e)^{1/2} (0.43 + 1/2 \ln \frac{\epsilon_0}{T_e} - \frac{0.8}{1 + \frac{\epsilon_0}{\lambda(1+z)^{3/2} r T_e}}) (1-r)^2 - \tilde{n}_\gamma^{(1)} \sigma_I r \end{aligned} \quad (4.18)$$

The new term – coming from  $\tilde{n}_\gamma^{(2)}$  – reduces the recombination rate. In fact, if  $\lambda(1+z)^{3/2} r T_e / \epsilon_0$  is very large, the factor of 0.8 exactly subtracts out recombination to the ground state. The fact that recombination to the ground state is strongly suppressed was first noted by Peebles [24] in the context of the standard cosmological scenario and recently emphasized by Asselin et al. [9] in the context of decaying particle scenarios. The reason for the suppression is straightforward: Any recombination to the ground state produces an ionizing photon. If the ionization time is very short compared to the Hubble time – i.e. if  $\lambda(1+z)^{3/2} r T_e / \epsilon_0$  is very large – then this photon will quickly ionize another neutral atom. Thus the total ionization fraction remains unchanged. Only when  $r$  becomes very small does the ionization time become larger than the expansion time; at that point recombination to the ground state also becomes relevant.

## 5. Numerical Results

In this section, we present numerical solutions to the coupled equations –(3.18), (3.29), and (4.8). For definiteness, we will first present results and discuss the details of a 30 eV neutrino with a lifetime of  $10^{24}$  seconds. The qualitative behavior is independent of the exact neutrino parameters in the parameter space of interest ( $27.2 \text{ eV} < m_\nu < 37.5 \text{ eV}$  and  $10^{22} \text{ sec} < \tau < 10^{26} \text{ sec}$ ), though, obviously, the exact quantitative results will differ.

First, we shall consider the evolution of the ionization state of the Universe. In Figure 1 we have plotted the neutral hydrogen fraction,  $r$ , versus  $1+z$ . The most striking feature

of this plot is the precipitous drop in  $r$  – four orders of magnitude – while  $z$  changes by one or two. This suggests that reionization was *spontaneous* at a  $z_i$  of about 17. If we plot the complementary quantity,  $1 - r$ , a different picture emerges. In Figure 2 we have plotted the free electron ratio,  $1 - r$ , versus redshift. The salient characteristic of this plot is the *gradual* change in  $1 - r$  until total ionization. Is the ionization spontaneous or not? That depends upon what question is being asked. If one is interested in how often CBR photons scatter off free electrons, then the important scale is the scattering rate of the photons which is proportional to  $n_e \sigma_{\text{Thompson}}$ , where  $n_e = (1 - r)n_B$ . The opacity – roughly the scattering rate divided by the Hubble rate – changes gradually, as suggested by Fig. 2. This is the issue analyzed and the conclusion reached by Scott, Rees, and Sciama [25]. On the other hand, we are interested in how reionization affects the spectrum of decay photons. The relevant scale is  $n_H \sigma_I$ , where  $n_H = r n_B$ . The relevant plot is Fig. 1 in which it is seen that the opacity changes very rapidly for these photons.

The scale which characterizes the ionization rate is  $\tilde{n}_\gamma \sigma_I$ . In Figure 3 we have plotted the ratio of ionizing photons to baryons as a function of redshift. The two solid lines correspond to the two sources of ionizing photons: decaying neutrinos (Eq. (4.11)) and recombinations (Eq. (4.17)). This plot corroborates the claim made in the last section that the recombination source is always less important than the decaying particle source, and thus its contribution to the photon spectrum is negligible. The importance of this term lies in the correction it provides to the recombination rate. In the pre-ionization regime, when the ratio of these photons to neutral atoms is much smaller than unity, every recombination photon immediately reionizes a neutral atom. Thus direct recombination to the ground state is suppressed. However, in the post-ionization regime direct recombination to the ground state is unsuppressed. It is reassuring that in our formalism this comes out naturally.

The shape of the recombination photon density curve provides insight about the various mechanisms driving recombination. Early on, when the Universe is nearly neutral, there is very little recombination, and hence  $n_\gamma^{(2)}$  is very small. As the Universe becomes reionized recombinations become more common, so there is a rise in  $n_\gamma^{(2)}$ . Eventually, due to the Hubble expansion, the free electron-proton density turns around again, and the number of recombination photons correspondingly drops. The rise in  $n_\gamma^{(2)}$  at  $z \sim z_i$  corresponds to the physical fact that recombination [to all levels] is an important process at this epoch. As an example of this, we can estimate what  $z_i$  would have been in the absence of recombination. Then, once the number of photons produced by neutrinos was equal to the

number of baryons, the Universe would be ionized. That is, a simple estimate would give  $n_\nu t_i / \tau = n_B$ , or  $z_i = ((3/11)t_0 / \eta\tau)^{2/3}$ . With the parameters under consideration, this estimates  $z_i \simeq 45$ . The fact that  $z_i$  is actually closer to 20 illustrates that recombination delays the onset of full ionization[26].

We have also included in this figure the two approximations to  $\tilde{n}_\gamma^{(1)}$  discussed in the last section. The dashed curve corresponds to Eq.(4.15), and the dotted curve to Eq. (4.12). It is evident from these plots that each approximation is extremely good in a particular epoch. The first approximation is virtually exact up until  $z \approx z_i$ . The reason this approximation is so good in the un-ionized regime is straightforward. Let's rewrite Eq. (4.15) in the more transparent notation,

$$\tilde{n}_\gamma^{(1)} \approx \frac{n_\nu(t)/\tau}{n_H(t)\sigma_I} \quad (5.1)$$

This says that the number of photons is approximately equal to the rate of neutrino decay divided by the rate of ionization. When the ratio of photons to neutral atoms is so small that every photon will immediately ionize a neutral atom then the Hubble expansion is unimportant, and the approximate equality is virtually exact. With our sample parameters it turns out that even at  $z = z_i$ , there are about 160 neutral atoms for each decay photon, so it is not surprising that the approximation is so good up until this point. It is only when the number of neutral atoms per photon is of order unity and less, i.e. for  $z < z_i$ , that approximation (5.1) breaks down. We see here that this is an extremely good approximation until  $r$  falls below  $10^{-4}$ . If one is interested in calculating the density of free electrons then this approximation is sufficient. However, if one is interested in a calculation of the neutral atom fraction for  $z < z_i$ , then this approximation breaks down too soon.

The post-ionization approximation, Eq. (4.12), becomes accurate a few redshifts after the un-ionized approximation becomes invalid. Recall from the discussion in the last section that this approximation neglects ionization totally. In Fig. 1, we see, however, that  $r$  continues to drop even in the post-ionization regime. The residual fraction of neutral atoms is not strictly frozen, but rather continues to get smaller, so there still is some ionization happening. Since the ratio of the ionization rate to the Hubble rate decreases at lower redshifts, this approximation becomes virtually exact. Nonetheless, there is a regime, albeit a fairly small one, in which neither approximation obtains, so numerical integration of the photon spectrum, eq (3.19), is critical.

The DDM scenario predicts a diffuse spectrum today. Armed with the ionization history of the Universe in the reionization epoch, we are able to calculate it, by numerically integrating Eq. (4.9) with  $z = 0$ . Others who have considered the DDM scenario [10] have ignored this effect.

In Fig. 4 we have plotted the correct DDM spectrum (solid line). For comparison's sake we have also plotted the uncorrected spectrum, i.e. the one which ignores ionization effects (dashed line). The two spectra are identical for photon energies greater than a few eV. At a low photon energy (for our sample parameters at about  $k = .9$  eV), the DDM flux drops quite abruptly to zero. This is readily understood in light of previous discussions. At  $z \approx z_i$  there is a precipitous drop in opacity. Photons today with energies  $k < \frac{k_0}{1+z_i}$  would have been emitted while the Universe was still opaque, i.e. when  $z > z_i$ . Essentially every photon produced in this pre-ionization regime is used up in the process of reionization. Photons today with energy  $k > \frac{k_0}{1+z_i}$  were produced in a transparent Universe, so accordingly travel freely. In the last section this was approximated as a step function at  $k = k_0/(1+z_i)$ . The numerical result verifies that this is a reasonable approximation.

Fig. 4 also shows several sets of data points. The two in the low energy regime [27] are representative of upper limits in the cosmic flux: no detection has been made. Any predicted cosmic flux must be lower than this. It is seen that the upper limit is far above the predicted levels even in the absence of ionization. Therefore, while data in this regime may ultimately be used to detect the radiation emitted by unstable neutrinos and in particular the characteristic drop in intensity due to ionization by early photons, the present upper limit is several orders of magnitude away from this goal.

The second set of data points in Fig. 4 lie between 6 – 10 eV [28]. They purport to be actual measurements of a cosmic background flux with all local contaminants subtracted off. These measurements are much closer to the spectrum produced in the DDM scenario for two reasons: first, the observed magnitude is an order of magnitude or so lower than the upper limit at 3 eV and, second, the predicted flux is larger here, since it grows as  $k^{3/2}$ . Therefore, this regime is most likely to be of use in detecting photons from unstable neutrinos. The only unfortunate aspect of this is that the sharp drop in intensity due to ionization occurs at lower energies than this most favorable regime.

Since the predicted spectrum is a function of  $(m_\nu, \tau)$ , the observed points can be turned into constraints on the neutrino mass and lifetime. We will see shortly that this set

of constraints complements the constraints coming from the Gunn-Peterson test to allow only a small window in parameter space.

For completeness we include a plot of electron temperature versus redshift (Figure 5). It is seen that the temperature does not change very drastically. In fact the temperature remains low enough so that collisional ionization can be justifiably neglected. We also note that the thermal evolution is insensitive to the initial temperature. Compton scattering insures that the electron temperature tracks the photon temperature until the decays set in.

The second prediction of the DDM scenario is that there is a relic abundance of neutral hydrogen. When Sciama and others [5] first considered the DDM hypothesis in light of Gunn-Peterson, they required simply that total reionization occur by  $z = 4.7$  since this is the epoch of the earliest test [29]. As discussed earlier, the approximations they made do not allow a precise calculation of the neutral atom abundance for  $z < z_i$ . By computing  $r$  all the way to  $z = 0$ , we can simply calculate

$$n_H(z) = r(z)n_B(z) = r(z)fn_B(t_0)(1+z)^3 \quad (5.2)$$

where  $f$  is the clumping factor defined in Eq. (2.6). This is plotted in Fig. 6 for  $f = 1$ .

The importance of this prediction is that the neutral hydrogen density can be probed observationally via the Gunn-Peterson test. In figure 6 we have also plotted the upper limits on the neutral hydrogen density coming from three different Gunn-Peterson tests at  $z = 4.7, 4.1$  and  $2.64$  [29][30] [2].

The major uncertainty in our calculation of the neutral hydrogen abundance stems from the uncertainty in clumping. Throughout we have assumed a homogeneous and isotropic Universe, but clearly once quasars, whose observation allows one to deduce the neutral hydrogen density in the IGM, are formed (and possibly earlier) the Universe is no longer homogeneous. While the data point at  $1+z = 5.7$  appears to impose the least stringent requirement, we have included it since at higher redshifts it is safer to assume that  $f$  is of order unity.

Finally, we are in a position to summarize the constraints on the mass and lifetime of the dark matter particle in the DDM scenario. In Figure 7, we have collected the constraints coming from the Gunn-Peterson tests (with  $f = 1$ ) and from the UV flux data. We have presented the constraints coming from two G-P tests [29][30]. The dashed line corresponds to the constraint coming from the  $z = 4.1$  quasar. This appears to impose the

strictest constraint, leaving only region A in parameter space. In particular the smallest allowed mass is 28.5 eV with a  $2 \times 10^{23}$  sec. lifetime. The dotted line, corresponding to the  $z = 4.7$  Gunn-Peterson constraint, allows region B in parameter space, as well. This does not rule out, for example, a 27.7 eV dark matter particle (Sciama's preferred value [7]), provided the lifetime is larger than  $2.5 \times 10^{23}$  seconds. We have included this less stringent constraint because it is the one in which our approximations – homogeneity and no other sources heating the electrons – are most likely to be valid.

One feature of this plot should be explicitly noted. That is the fact that as  $m_\nu$  approaches 27.2 eV the upper limit on the lifetime drops steeply. It turns out that in order to satisfy the Gunn-Peterson tests, it is necessary that some ionization take place in the ionized regime. As discussed in section 4, the number of ionizing photons in the ionized regime goes to 0 as  $k_0$  approaches  $\epsilon_0$ . The only way to compensate for this reduction in ionizing photon number is to lower the lifetime. Consequently, in order to satisfy the Gunn-Peterson tests, for masses close to 27.2 eV, the lifetime of the decaying particle must be short.

More realistic accounting of clumping and electron heating may well modify some of these quantitative conclusions. We note, though, that the three sets of observations – of the Hubble parameter; diffuse photon spectrum; and neutral hydrogen density – complement each other. Measurement of the Hubble parameter (or the age of the Universe) seems to require the neutrino mass to be *less* than 37.5eV. Measurement of the diffuse photon spectrum constrains the neutrino lifetime to be *greater* than  $10^{23}$  seconds. The Gunn-Peterson tests complement these two by providing an *upper* limit on the neutrino lifetime and a *lower* limit on the neutrino mass.

## 6. Conclusion

We have quantitatively analyzed a scenario of reionizing the Universe. The numerical results allow accurate comparison between theory and observation. In particular, we are able to derive strict constraints on the parameters of the decaying neutrino scenario – i.e. on the neutrino mass and lifetime – by requiring the Universe to be ionized enough to satisfy the Gunn-Peterson tests and by limiting the ultraviolet radiation produced to be under the observational limits.

Several aspects of this work may be of use in other reionization scenarios and in more general work on the early Universe.

- (i) We have derived an expression, Eq. (4.8), for the photon spectrum due to a general source in the presence of a background of neutral hydrogen atoms.
- (ii) A qualitative feature of the photon spectrum in such a situation is a sharp drop in the spectrum today. Photons with energies lower than this cut-off would have been produced before the Universe was ionized. Hence, they were immediately absorbed in the ionization process and are no longer present today.
- (iii) The number of neutral hydrogen atoms drops dramatically at the time of ionization. This is completely consistent with the point made by several groups that the number of free ions rises gradually. In our language  $r$  changes rapidly while  $1 - r$  changes very slowly.

### Acknowledgements

We would like to thank J. Bernstein, A. Dekel, G. Feinberg, K. Freese, J. Frieman, and M. Turner for fruitful discussions and E. Nieuwood and I. Soibelman for computational advice. SD would like to thank A. Guth and the Center for Theoretical Physics at MIT for their hospitality while some of this work was done. The work of SD was supported in part by DOE and by NASA under Grant No. NAGW-1340 at Fermilab and by the U.S. Department of Energy under Grant No. DE-FG02-84ER40158 with Harvard University. JJ acknowledges the Sloan Foundation under Grant No. 26623 and the NSF for partial support.

### Appendix A. Ionization From Excited States

Here we justify our claim that direct ionization of excited hydrogen atoms can be neglected. It is useful to first review the standard recombination scenario as described by Peebles [24]. Recombination occurs when there are essentially no thermal photons available to ionize ground state neutral atoms. In fact the only photons around with energy greater than a Rydberg come from recombination to the ground state. These photons immediately reionize the atoms, effectively cancelling every recombination to the ground state, such that neither recombination or ionization involving ground state atoms needs to be included in the ionization equation. The important physical processes are direct recombination to excited states, ionization of excited atoms, radiative transitions between atomic levels and the Hubble expansion. Peebles derived the following equation to describe the ionization ratio,

$$\frac{dr}{dt} = \left[ \alpha_{exc} n_B (1 - r)^2 - \beta_{exc} r \frac{n_{exc}}{n_H} e^{-(10.2 \text{ eV}/T)} \right] C \quad (\text{A.1})$$



where  $C$  is a time-dependent factor (defined explicitly in [24]) which incorporates the effects of the physical processes mentioned above;  $\alpha_{exc}$  is the excited state recombination coefficient, of order  $(\sigma v)_R(\epsilon_0/T_e)$ ;  $\beta_{exc}$  is the rate of ionization of excited atoms; and  $n_{exc}$  is the number density of hydrogen atoms in excited states. The ionization rate is well-approximated by  $n_{3.4}\sigma_I$  where  $n_{3.4}$  is the number of photons with energy greater than 3.4 eV. The upshot is that even though the the fractional population of excited states is small, since there is an exact cancellation between ionization and recombination of ground state atoms, the relevant physics concerns the excited states.

In the reionization epoch,  $z < 300$ . all of these physical processes are still taking place, so the two terms in (A.1) in principle should be included in the ionization ratio equation. (We note that through out this epoch, the multiplicative factor,  $C$ , is equal to unity.) However, in this epoch there is a source of photons energetic enough to ionize ground state atoms – decays. Thus to Eq. (A.1) we need to add the ionization term,  $-\tilde{n}_\gamma^{(1)}\sigma_I n_{gs}/n_B$ , where  $n_{gs}$  is the number density of hydrogen atoms in the ground state.

Now let us focus on the ionization terms. Because of the exponential suppression, the ionization term of (A.1) – i.e. the second term on the right hand side – is clearly negligible. We need to examine the new term,  $\tilde{n}_\gamma^{(1)}\sigma_I n_{gs}/n_B$  and determine whether ionization of excited atoms by decay photons needs to be included. We will be justified in neglecting these ionizations if the per volume rate of ionization of excited atoms is much smaller than the per volume rate of ionization of ground state atoms,

$$\frac{\beta_{exc}n_{exc}}{\tilde{n}_\gamma^{(1)}\sigma_I n_{gs}} \approx \frac{n_{3.4}n_{exc}}{\tilde{n}_\gamma^{(1)}n_{gs}} \ll 1. \quad (\text{A.2})$$

In the opaque era,  $z > z_i$ , it is easy to estimate Eq. (A.2). First of all,  $n_{3.4} \sim \tilde{n}_\gamma^{(1)}$  since the photons immediately ionize and do not get redshifted. Secondly, because there are virtually no resonant decay photons, the fraction of excited atoms is given by the expression derived by Peebles,

$$\frac{n_{exc}}{n_{gs}} \approx \left( \frac{K\alpha_{exc}n_e^2}{1 + K\Lambda n_H} \right) \quad (\text{A.3})$$

where  $K \equiv \frac{\pi^2}{(\epsilon_0 - \epsilon_1)^3 H}$  and  $\Lambda = 8.227 \text{sec}^{-1}$  is the decay rate of the  $2s$  state. The ratio in Eq. (A.3) is negligibly small, always less than  $10^{-15}$ , so the condition (A.2) is easily satisfied. For  $z < z_i$ , when photons travel freely through the universe and get redshifted, then  $n_{3.4}$  is larger than  $\tilde{n}_\gamma^{(1)}$  by up to two orders of magnitude. However, even though there are some resonant photons around to pump atoms from the ground state to the excited states, adding another term to Eq. (A.3), the fractional population of excited states is still negligibly small, so criterion (A.2) is again satisfied.

## Appendix B. Recombination Photons

In this appendix we calculate the number density of ionizing photons coming from recombination. This is

$$\begin{aligned}
\tilde{n}_\gamma^{(2)}(z) &= \frac{\epsilon_0^3}{\pi^2} \int_{\epsilon_0/k_0}^{\infty} \frac{dy}{y} f_\gamma^{(2)}(y, z) \\
&= \left( \frac{2\pi(1+z)^3}{m_e} \right)^{3/2} \frac{\lambda n_B(t_0) \epsilon_0^3}{\pi^2} \int_{\epsilon_0/k_0}^{\infty} \frac{dy}{y} \int_y^1 \frac{dy'}{y'} \\
&\quad \exp \left\{ -\lambda (\epsilon_0/k_0)^3 \left( \frac{1+z}{y} \right)^3 \int_{1+z}^{(1+z)y'/y} \frac{dz'}{(1+z')^{5/2}} r(z') \right\} \\
&\quad \left( \frac{y'}{y} \right)^{3/2} \frac{\left( 1 - r[(1+z)y'/y] \right)^2}{\left( T_e [(1+z)y'/y] \right)} \exp \left\{ -\frac{k_0 y' - \epsilon_0}{T_e [(1+z)y'/y]} \right\},
\end{aligned} \tag{B.1}$$

where we have inserted  $f_\gamma^{(2)}$  from Eq. (4.16). The exponential  $e^{-(k_0 y' - \epsilon_0)/T_e}$  is very small except when  $y'$  is very close to  $\epsilon_0/k_0$ . But  $y' \geq y \geq \epsilon_0/k_0$ , so  $y$  must also be close to  $\epsilon_0/k_0$  and, of course, close to  $y'$  as well. We can therefore expand  $y'$  about  $y$  and expect the leading terms to give the largest contributions. Consider, for example, the integral in the argument of the first exponential:

$$\int_{1+z}^{(1+z)y'/y} \frac{dz'}{(1+z')^{5/2}} r(z') = \frac{r[1+z]}{(1+z)^{3/2} y} (y' - y) \left[ 1 + O(y' - y) \right]. \tag{B.2}$$

Since  $y' - y \sim T_e/\epsilon_0$ , the higher order terms may be dropped. Keeping only the leading terms amounts to setting  $y' = y$  everywhere in the integrand of Eq. (B.1) except in the exponentials, where only the linear terms in  $y' - y$  need be retained. Therefore,

$$\begin{aligned}
\tilde{n}_\gamma^{(2)}(z) &\simeq \left( \frac{2\pi(1+z)^3}{m_e} \right)^{3/2} \frac{\lambda n_B(t_0) \epsilon_0^3}{\pi^2} \frac{(1-r)^2}{T_e^{3/2}} \int_{\epsilon_0/k_0}^{\infty} \frac{dy}{y^2} \int_y^1 dy' \\
&\quad \times \exp \left\{ -\frac{\lambda (\epsilon_0/k_0)^3 (1+z)^{3/2} r}{y^4} (y' - y) - \frac{k_0 y' - \epsilon_0}{T_e} \right\} \\
&= \left( \frac{2\pi(1+z)^3}{m_e T_e} \right)^{3/2} \frac{\lambda n_B(t_0) \epsilon_0^3 (1-r)^2}{\pi^2} \int_{\epsilon_0/k_0}^{\infty} dy \frac{e^{-(k_0 y - \epsilon_0)/T_e}}{\frac{k_0 y^2}{T_e} + \frac{\lambda r (\epsilon_0/k_0)^3 (1+z)^{3/2}}{y^2}}.
\end{aligned} \tag{B.3}$$

Neglecting terms of order  $T_e/\epsilon_0$ , the  $y$ -integral can be performed by setting  $y = \epsilon_0/k_0$  (except in the exponential), leading to

$$\tilde{n}_\gamma^{(2)}(z) \simeq [n_B(t_0)(1+z)^3] \frac{(1-r)^2}{r} \left[ \frac{\epsilon_0^2 (2\pi)^{3/2}}{\pi^2 m_e^{3/2} T_e^{1/2}} \right] \left[ 1 + \frac{\epsilon_0}{\lambda (1+z)^{3/2} r T_e} \right]^{-1}. \tag{B.4}$$

The first factor in square brackets is just  $n_B(z)$  since the baryon number density scales as  $(1+z)^3$ . The second factor in square brackets is the ratio of the ground state recombination cross section to the ionization cross section:  $(\sigma v)_0/\sigma_I$ . The ground state recombination cross section, though, is  $g(\epsilon_0)(\sigma v)_R$  where  $g$  is the gaunt factor, roughly 0.8 at threshold. We therefore recapture Eq. (4.17).

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## Figure Captions

1. The neutral hydrogen ratio as a function of redshift in a universe with a decaying neutrino of mass 30 eV and lifetime  $10^{24}$  sec.

2. The free electron ratio as a function of redshift in a universe with a decaying neutrino of mass 30 eV and lifetime  $10^{24}$  sec.

3. The density of ionizing photons as a function of redshift. The solid lines are exact solutions:  $\tilde{n}_\gamma^{(1)}$  (decay photons) is labelled (1) and  $\tilde{n}_\gamma^{(2)}$  (recombination photons) is labelled (2). The dashed line corresponds to the un-ionized approximation, Eq. (4.15), and the dotted line to the ionized approximation, Eq. (4.12).

4. The predicted photon spectrum due to a decaying neutrino with mass 30 eV and lifetime  $10^{24}$  sec. The solid line is the correct predicted spectrum. The dashed line ignores corrections due to ionization effects. The arrows mark upper limits and the crosses mark observations, discussed in the text.

5. The electron temperature as a function of redshift in a universe with a decaying neutrino of mass 30 eV and lifetime  $10^{24}$  sec.

6. The predicted amount of neutral hydrogen as a function of redshift in the DDM scenario with a neutrino mass 30 eV and lifetime  $10^{24}$  sec. The clumping factor,  $f$ , has been set equal to 1. The data points are upper limits: any prediction must be *below* these levels.

7. The allowed values of  $m_\nu$  and  $\tau$  when the clumping factor  $f$  is set to 1. The solid line represents the constraint coming from observations of the diffuse UV background. The dashed line represents the constraint coming from the  $z = 4.1$  Gunn-Peterson test, and the dotted line from the  $z = 4.7$  Gunn-Peterson test. Region A satisfies all three constraints. Region B appears to be ruled out by the  $z = 4.1$  test, though it satisfies the  $z = 4.7$  and the UV constraints.

Figure 1

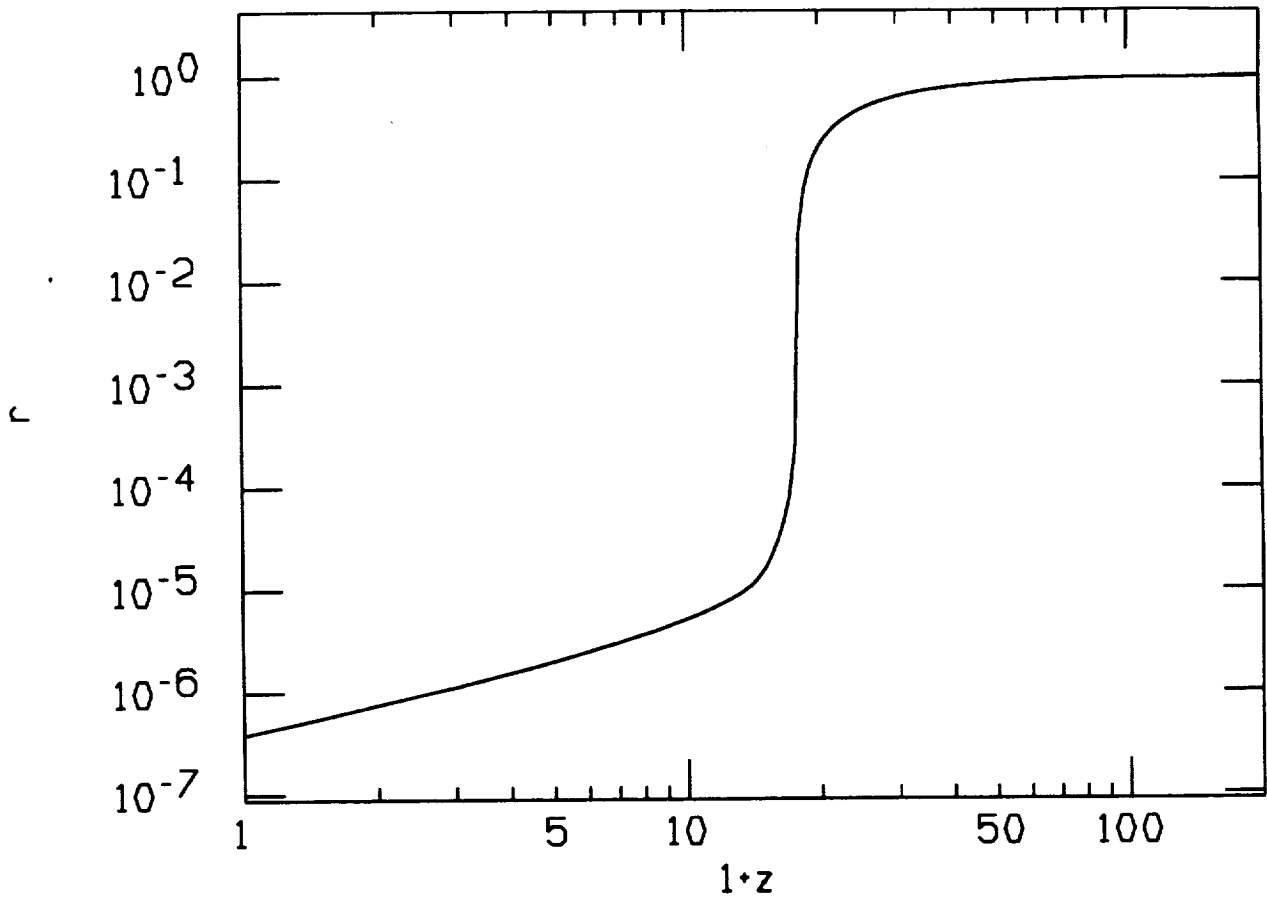




Figure 2

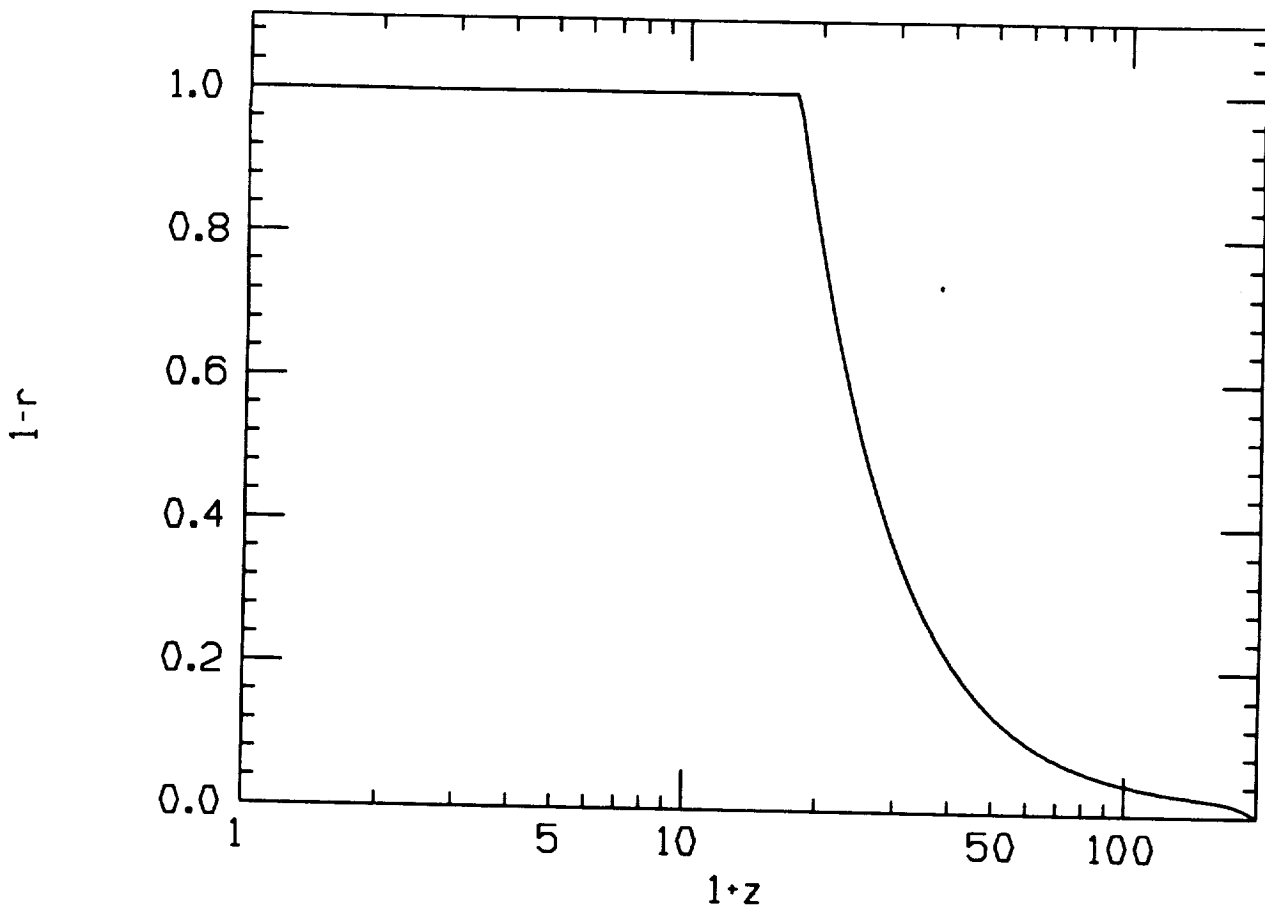


Figure 3

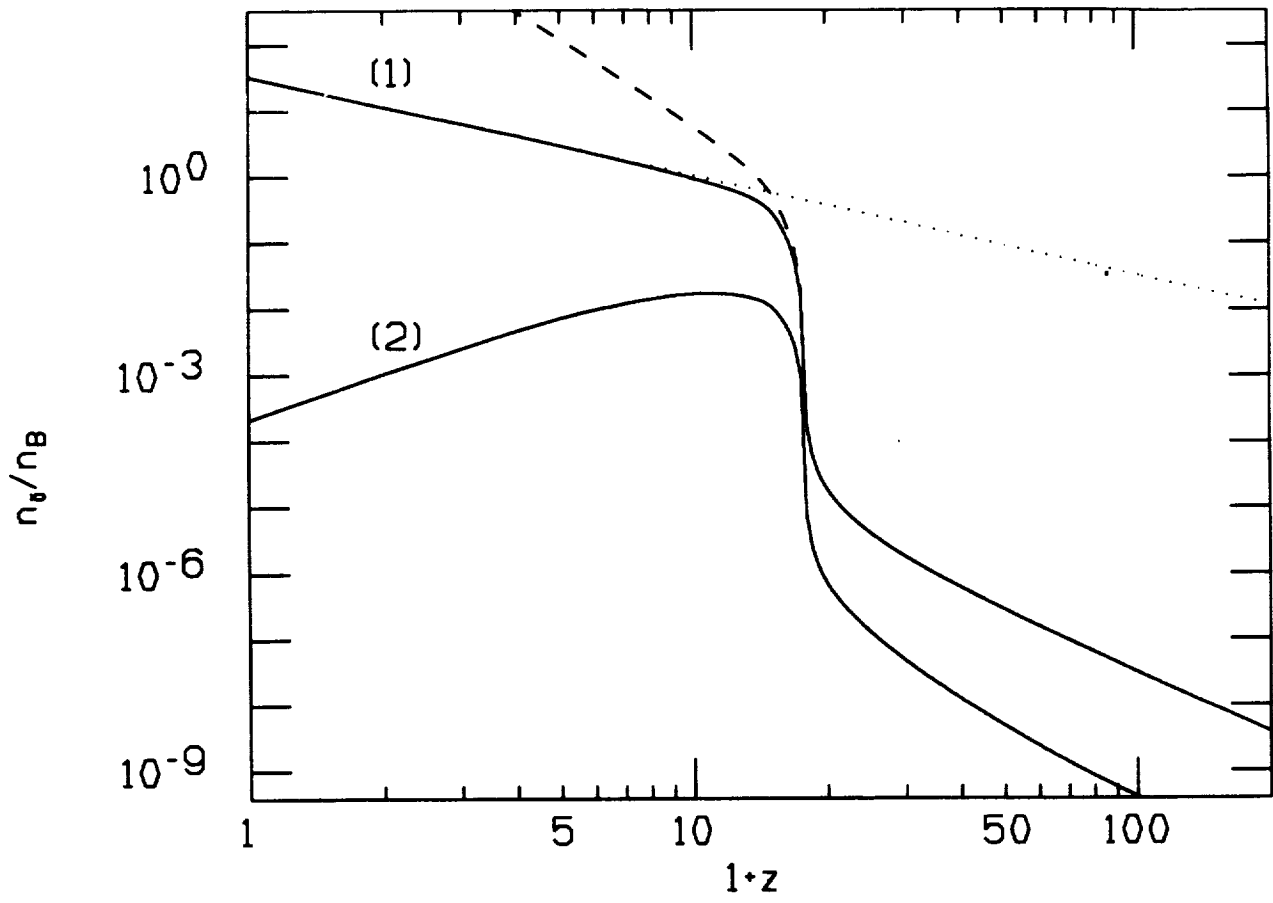


Figure 4

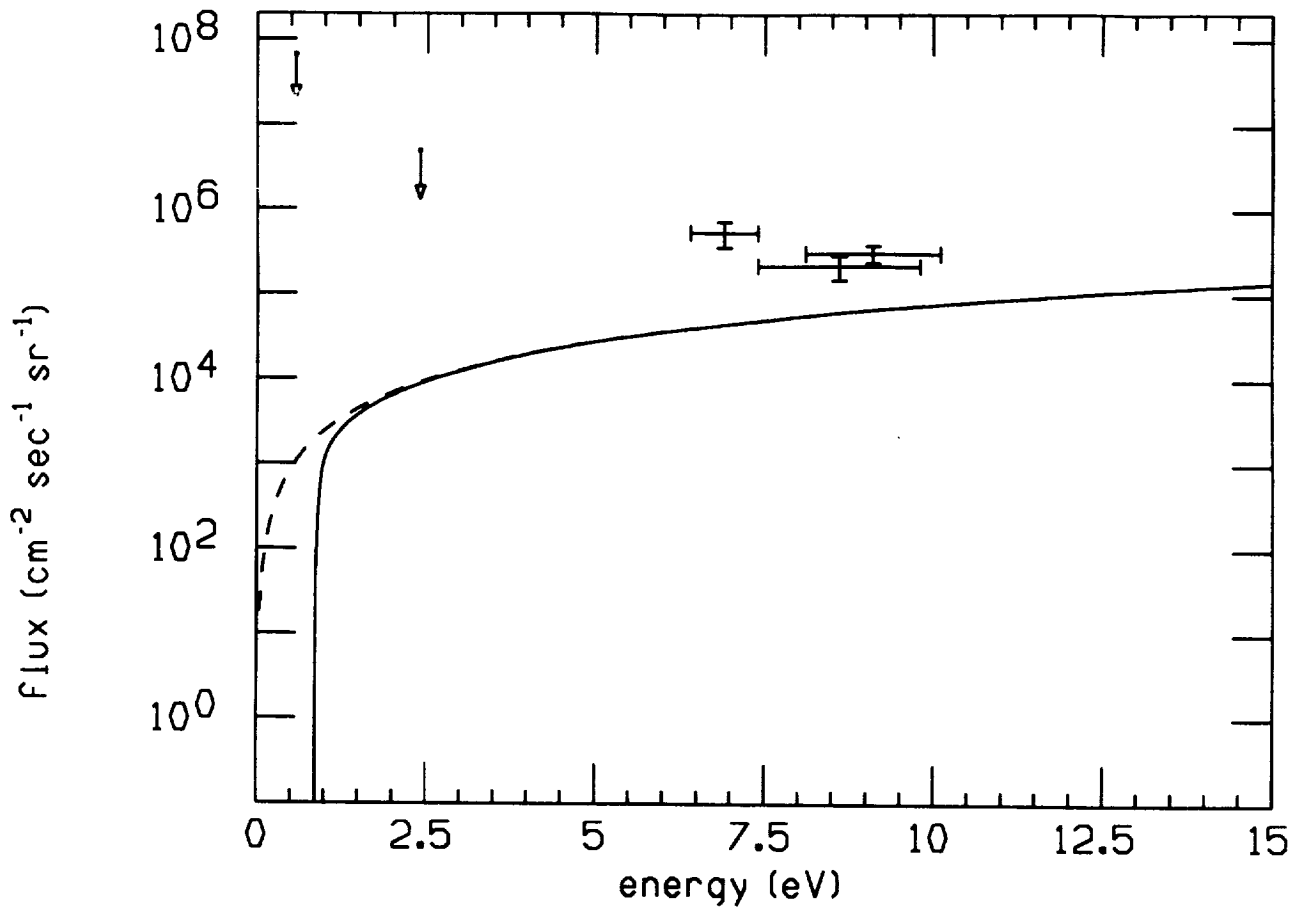


Figure 5

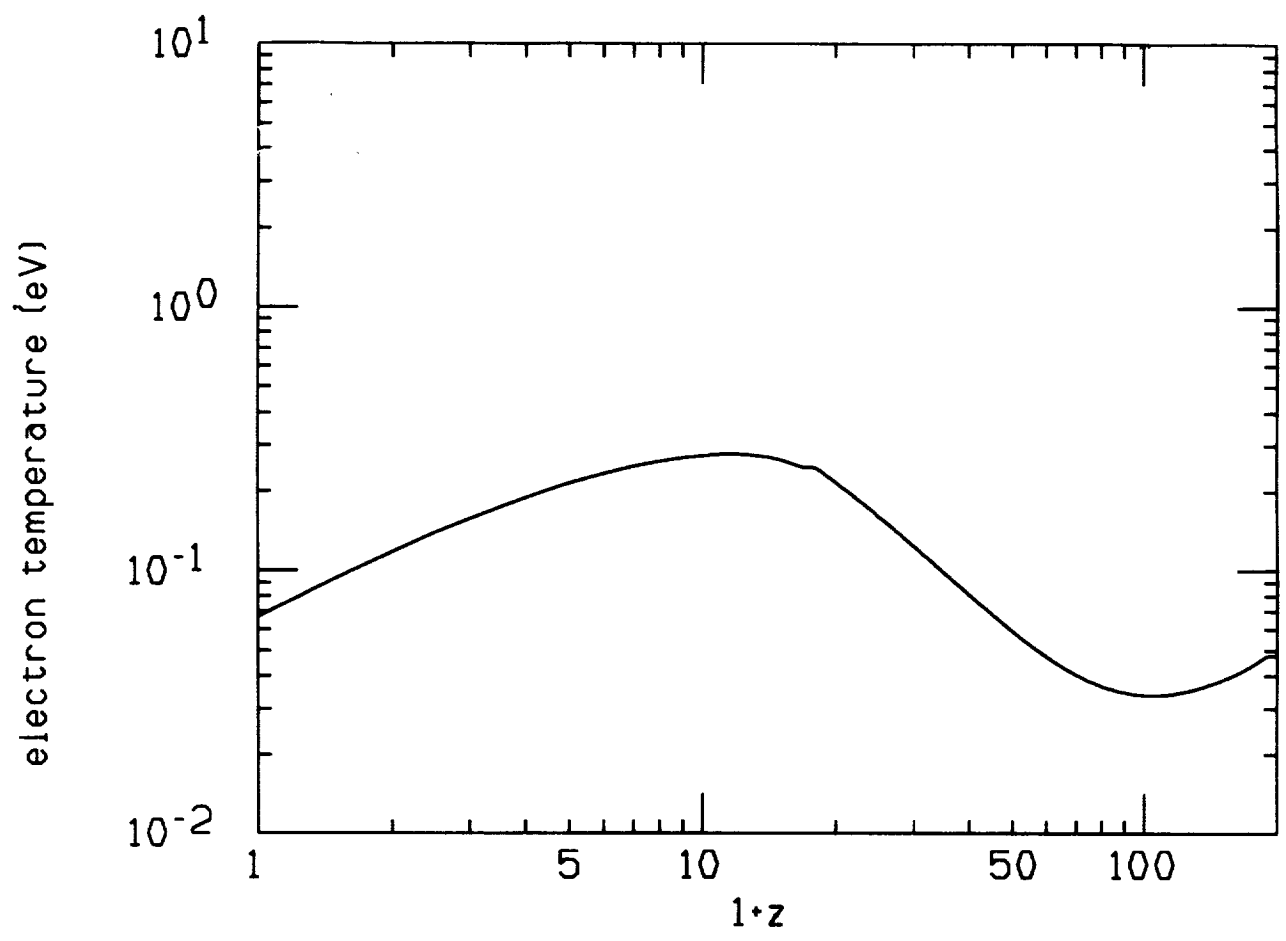


Figure 6

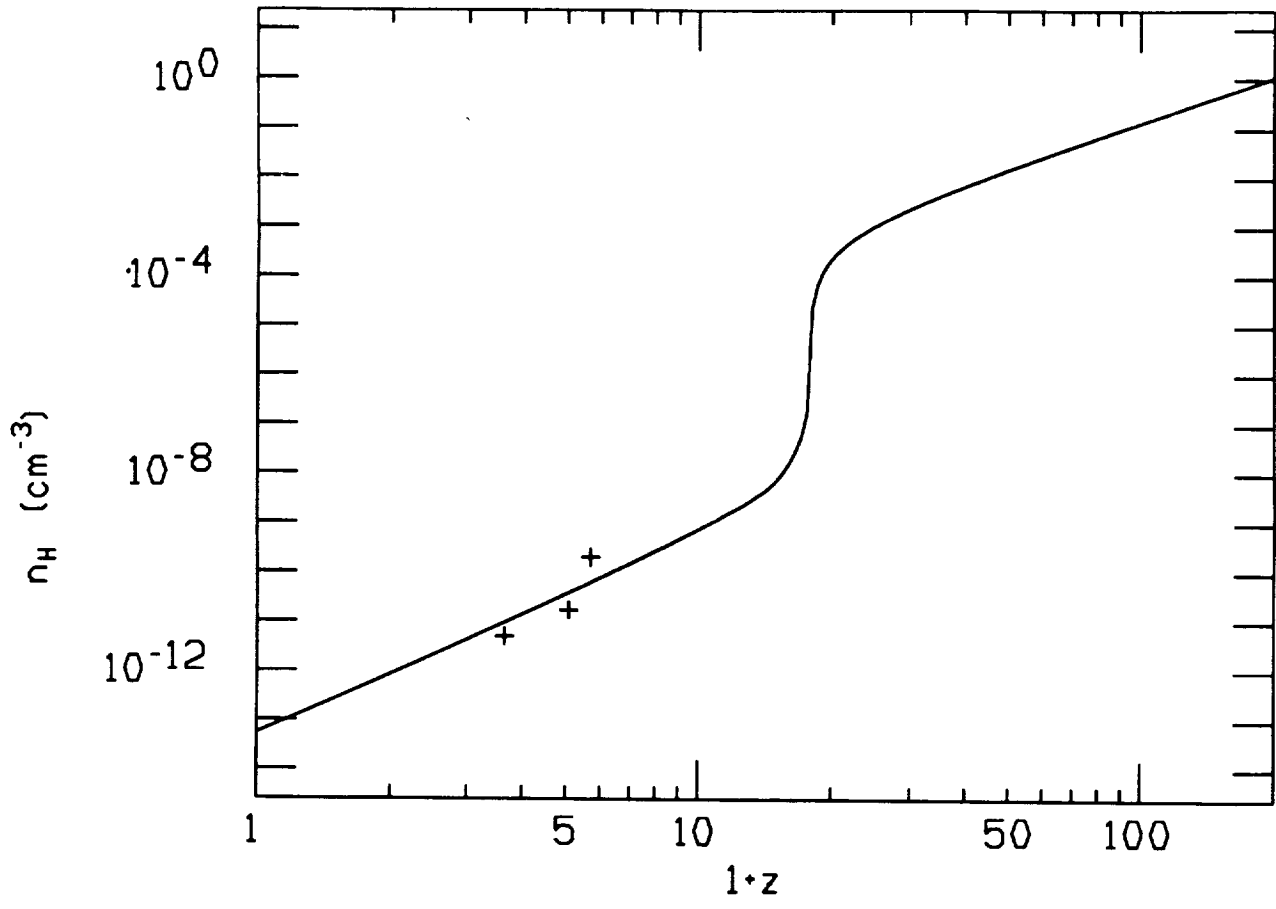


Figure 7

