$p-78$

# Engineering Calculations for Communications Satellite Systems Planning 

E. Walton, E. Aebker, F. Mata, and C. Reilly

The Ohio State University

# ElectroScience Laboratory 

Department of Electrical Engineering

Columbus, Ohio $\mathbf{4 3 2 1 2}$

Final Report 723160-1
Contract No. NAG-159-20
May 1991

NASA Lewis Research Center
21000 Brookpark Rd.
Cleveland, Ohio 44135


## A. Approved for public release; Distribution is unlimited

## NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

| REPORT DOCUMENTATION PAGE | 1. REPORT NO. | 2. | 3. Reciplent's Accession No. |
| :---: | :---: | :---: | :---: |
| 4. Title and Subtitle |  |  | 5. Heport Date May 1991 |
| Engineering Calculations for Communications Satellite Systems Planning |  |  | - |
| 7. Author(s) <br> E. Walton, E. Aebker, F. Mata, and C. Reilly |  |  | 8. Performing Org. Rept. No. 723160-1 |
| o. Performing Organization Name and Address <br> The Ohio State University <br> ElectroScience Laboratory <br> 1320 Kinnear Road <br> Columbus, OH 43212 |  |  | 10. Project/Task/Work Unit No. |
|  |  |  |  |
|  |  |  | 11. Contract(C) or Grant(G) No. |
|  |  |  | (C) NAG-150-20 |
|  |  |  | (G) |
| 12. Sponsoring Organization Name and Address <br> National Aeronautics and Space Administration <br> Lewis Research Center <br> Cleveland, Ohio 44135 |  |  | 13. Report Type/Period Covered |
|  |  |  | Technical Report |
|  |  |  | 14. |
|  |  |  |  |

15. Supplementary Notes
16. Abstract (Limit: 200 words)

This report describes the final phase of the satellite synthesis project supported by NASA at the Ohio State University. Several methods for generating satellite positionings with improved aggregate carrier to interference characteristics have been studied. Two general methods for modifying required separation values are presented. Also, two methods for improving aggregate $C / I$ performance of given satellite synthesis solutions are presented. A perturbation of the WARC synthesis is presented.
17. Document Analysis a. Descriptors

Optimization
Satellite Communication
Electromagnetic Interference
Orbital Allocation
6. Identifiers/Open-Ended Terms
c. COSATI Fleld/Group
18. Availability Statement
A. Approved for public release; Distribution is unlimited.

| 18. Security Class (This Report) <br> Unclassified | 21. No. of Pages <br> 77 |
| :--- | :--- |
| 20. Security Class (This Page) <br> Unclassified | 22. Price |

## Contents

1 Introduction ..... 1
2 Satellite Synthesis Solutions Using Modified Required Separations ..... 3
2.1 Modifications Based on the Slope of the $\Delta_{i j}$ vs. Single Entry C/I Curve ..... 4
2.2 Modifications Based on $C / I$ Performance of Previous Solutions ..... 8
2.3 Modifications Based Upon Single Entry C/I Performance of the Ex- isting Solution ..... 10
2.4 Modifications Based Upon Aggregate $C / I$ Performance of the Exist- ing Solution ..... 13
3 Improving Aggregate Total-link $C / I$ 's by Posteriori Modification of Scenario Assumptions ..... 16
3.1 Modification of Transmitter Powers ..... 17
3.2 Improvement of Downlink $C / I$ 's by Adaptive Nulling of Interfering Satellites ..... 24
4 Ellipse and Testpoint Considerations ..... 28
4.1 Outlying Testpoints ..... 28
4.2 Isolated Testpoints ..... 30
4.3 Ellipse Plotting Programs ..... 32
5 Solutions With Satellite Positionings Which Are Minimally Per- turbed From The WARC88 Solution ..... 35
A Downlink Interference Power From the EIREB200 Satellite Received at BEN00000's Testpoint \#10 ..... 37
A. 1 Preliminaries ..... 37
A. 2 Geometry ..... 39
A. 3 Power of the EIREB200 Satellite by $C / N$ Requirement ..... 40
A.3.1 Calculation of the Off-axis Angle, $\phi$ ..... 40
A.3.2 The EIREB200 Antenna Plane ..... 42
A.3.3 Finding $\overrightarrow{\mathbf{P}}$; the Satellite-to-Testpoint Vector, Antenna Plane Intersection ..... 43
A.3.4 Antenna Plane Coordinates ..... 43
A.3.5 Calculation of $\phi_{u}$; The Half-power Angle in the Direction of EIREB200's Testpoint \#5 ..... 45
A.3.6 The EIREB200 Required Satellite Power ..... 48
A. 4 Interference Power Received at BENOOOOO's Testpoint \#10 ..... 49
A.4.1 The EIREB200 Satellite Transmit Discrimination in the Direc- tion of BENOOOOO's Testpoint \#10 ..... 49
A.4.2 The Interference Power ..... 52
B Files Recorded on Tape ..... 55
C Description of Files ..... 58
C. 1 Ellipse/rain Attenuation File ..... 58

- Input to DELTA and MISOUP ..... 58
C. 2 Ellipse Key File ..... 59
- Input to DELTA and MISOUP ..... 59
C. 3 Scenario Parameter and Testpoint File ..... 59
- Input to DELTA, MISOUP,PLOTCOI, and PLOTELEV ..... 59
C. 4 Required Separation Matrix (-matrix) ..... 62
- Output of DELTA ..... 62
- Input to SLOT/TOLS/STARS ..... 62
C. 5 Satellite Arc File ..... 62
- Input to SLOT/TOLS/STARS ..... 62
C. 6 Satellite Position/Order File ..... 63
- Output of SLOT/TOLS/STARS ..... 63
- Input to MISOUP ..... 63
- Optional Input to STARS ..... 63
C. 7 C/I Analysis Output File ..... 64
- Output of MISOUP ..... 64
- Input of PLOTCOI and PLOTELEV ..... 64
C. 8 Elevation Angle Output File ..... 64
- Output of PLOTELEV ..... 64
C. $9 C / I$ vs Cumulative Percentage Output File ..... 64
- Output of PLOTCOI ..... 64
C. 10 Existing System Power Data File ..... 65
- Input to SCENARIO PREPROCESSOR ..... 65
D Description of Programs ..... 66
D. 1 DELTA ..... 66
D. 2 SLOT (Satellite Location and Ordering Technique) ..... 67
D. 3 TOLS ..... 68
D. 4 STARS (Synthesis Technique for Allotting Resources to Satellites) ..... 68
D. 5 MISOUP ..... 68
D. 6 PLOTELEV ..... 69
D. 7 PLOTCOI ..... 69
E Program Interaction, Files, and Unit Numbers ..... 71


## List of Figures

2.1 Required separation verses single entry $C / I$ target for several satellite pairs ..... 6
$2.2 \quad \gamma_{i j}$ values verses required separations for all satellite pairs ..... 7
2.3 Aggregate $C / I$ profiles for solutions obtained from SLOT, with $\boldsymbol{\gamma}_{i j}$ modification. ..... 9
2.4 Aggregate $C / I$ profiles for solutions obtained from SLOT, with single entry $\beta_{i j}$ modification ..... 12
2.5 Aggregate $C / I$ changes between the existing and new solutions with undesirable characteristics ..... 14
3.1 Improvement of Aggregate $C / I$ by Modifying Transmitter Powers ..... 20
3.2 Improvement of Aggregate $C / I$ by a Constrained Modification of Transmitter Powers ..... 22
3.3 Convergence Profile When the Power of the Worst Administration is Increased ..... 25
3.4 Convergence Profile When the Power of the Worst Interferer of the Worst Administration is Decreased ..... 26
3.5 Aggregate C/I Profile Showing the Effect of Nulling Worst Interferers on the Downlink ..... 27
4.1 Testpoints and Ellipse of USA13DB1 As Seen From the Satellite at $-56^{\circ}$ ..... 29
4.2 Aggregate $C / I$ profiles comparing the effect of removing the eight worst outlying testpoints ..... 31
4.3 A View of the All the Allotment Satellite Ellipses From $-90^{\circ}$ ..... 32
4.4 Testpoints and Ellipse of CHLOO000 as Seen From the Satellite at $-78^{\circ}$ ..... 33
A. 1 Geometry of the satellite positioning problem ..... 39
A. 2 Vector diagram for the satellite positioning problem. ..... 41
A. 3 The Antenna Plane ..... 42
E. 1 ..... 71

## Chapter 1

## Introduction

This report covers the final phase of the research on the Satellite Synthesis project supported by NASA. Several methods for generating satellite positionings with improved aggregate carrier to interference characteristics have been studied. Chapter 2 describes two general methods for modifying required separation values. In the first method, required separations are modified based on the slope of the required separation verses single entry $C / I$ curve, then the modified separations are used by SLOT to determine a satellite positioning. In this case the modified required separations correspond to a uniform increase in the single entry $C / I$ target for all administration pairs. The second method, modification of required separations based on single entry or aggregate performance of an existing satellite synthesis solution, selectively modifies required separations in order to improve on an existing solution. In Chapter 3 two methods for improving aggregate $C / I$ performance of given satellite synthesis solutions are presented. In the first, transmitter powers of the satellite and/or the earth stations are optimized to result in the maximum worst aggregate $C / I$. In the second method, adaptive nulling of downlink interferers is used to eliminate the downlink interference from a specified number of an administration's worst downlink interferers. In Chapter 4 there is a discussion of some of the peculiarities of the WARC database testpoints and ellipses. The ramifications of the peculiarities are discussed and the OSU approach to dealing with them is presented. Finally,

Chapter 5 briefly discusses the solutions which have been generated and represent a relatively minor perturbation of the WARC synthesis solution.

## Chapter 2

## Satellite Synthesis Solutions Using Modified Required Separations

The keystone in the OSU effort towards satellite synthesis is the pairwise required separation and the concept that if the pairwise required separations, which assure a minimum single entry carrier to interference level, are satisfied then the aggregate $C / I$ targets can, with a high degree of confidence, be met.

If, in the course of trying to determine a feasible satellite positioning, the required separations cannot be satisfied then the approach in the past has been to modify the required separations, decreasing them until a solution satisfying the modified required separations can be determined. Likewise, if a feasible satellite positioning is obtained using the original required separations the prospect remains that an improved solution might be obtained by increasing the required separations.

Increasing and decreasing of the required separations in the two situations described above have been handled by the OSU programs, namely SLOT and SPA, in one of the following two ways.

$$
\begin{gather*}
\Delta_{i j}^{\prime}=w \Delta_{i j} \quad w>0.0  \tag{2.1}\\
\Delta_{i j}^{\prime}=\max \left(\Delta_{i j}+w, 0.0\right) \quad w \in \Re \tag{2.2}
\end{gather*}
$$

In equation (2.1) the required separation for satellites $i$ and $j$ is increased or decreased by the multiplicative factor $w$. In equation (2.2) the required separation is modified by the additive factor $w$. In both cases the factor, $w$, has been constant for all pairs of satellites. It is possible that a constant multiplicative or additive factor may not be the most appropriate for modifying the required separation values. Therefore, other modification schemes have been investigated, schemes which more carefully consider administration pair specific properties to provide for a wiser and more equitable modification of the required separations.

### 2.1 Modifications Based on the Slope of the $\Delta_{i j}$ vs. Single Entry C/I Curve

The construction of a satellite synthesis solution should use separation requirements which are modified equitably. An equitable modification of the separation requirements should result in a uniform increase or decrease in the single entry $C / I$ values corresponding to the modified required separations. Along these lines, a reasonable approach would be to determine a matrix of required separations for a particular single entry target and to run SLOT in an attempt to generate a feasible solution. If a feasible solution cannot be found then one could repeat the process for a smaller single entry target. If, on the other hand, a feasible solution can be determined then the process could be repeated for a larger single entry $C / I$. A conceptually similar, but computationally more efficient method for modification of the required separations is presented in this section.

Define $\gamma_{i j}$, to be incremental change in required separation for a incremental change in the single entry $C / I$ target for satellites $i$ and $j$.

$$
\begin{equation*}
\gamma_{i j}=\frac{\partial \Delta_{i j}}{\partial(C / I)_{s e}} \tag{2.3}
\end{equation*}
$$

Then, given a matrix of required separations, $\Delta_{i j}$, calculated for a single entry $C / I$
target, $(C / I)_{s e}$, the modified required separations, $\Delta_{i j}^{\prime}$, can be defined as

$$
\begin{equation*}
\Delta_{i j}^{\prime}=\max \left[\Delta_{i j}+\gamma_{i j}\left((C / I)_{s e}^{\prime}-(C / I)_{s e}\right), 0.0\right] \tag{2.4}
\end{equation*}
$$

This equation gives the approximate separation required for a single entry carrier to interference of $(C / I)^{\prime}{ }_{s e}$.

The program DELTA is used for generating the $\gamma_{i j}$ values. DELTA determines a required separation for a pair of satellites at each potential orbital longitude. Within DELTA, once the required separation has been determined 0.2 degrees is added to the satellite separation and the new single entry $C / I$ values are calculated. The change in satellite separation divided by the change in single entry $C / I$ determines the $\gamma_{i j}$ value.

Figure 2.1 is a plot of $\Delta_{i j}$ verses the single entry $C / I$ target for a several pairs of administrations. Since $\gamma_{i j}$, the slope of this curve, is not constant as the single entry $C / I$ target varies it is important that the $\gamma_{i j}$ values be determined for a single entry target close to that which will give a feasible solution. In other words, the $\gamma_{i j}$ values accurately describe the slope of the $\Delta_{i j}$ curve only about the particular target, $(C / I)_{s e}$, for which they were calculated. The $\gamma$ values may still be useful for other ranges of single entry $C / I$ target, but they less accurately represent the actual values. It is possible that with further study a simple function describing the variation can be discerned, therefore extending the range over which a particular $\gamma_{i j}$ can be used accurately.

Figure 2.2 is a plot of $\gamma_{i j}$ verses $\Delta_{i j}$ for all the pairs of administrations. It demonstrates that the relationship is not random, but rather that it is distinctly nonrandom. In fact, it is likely that the $\gamma$ and $\Delta$ values can be related through the antenna patterns, particularly as $\Delta$ gets large.

The $\gamma_{i j}$ values were incorporated into the version of SLOT which implements a binary search to determine the maximal value of $\delta$, the binary search variable, generating a solution which is feasible with respect to the modified separation requirement, $\Delta^{\prime}{ }_{i j}=\max \left(\Delta_{i j}+\delta, 0.0\right)$. With the incorporation of $\gamma$ values the modified


Figure 2.1: Required separation verses single entry $C / I$ target for several satellite pairs


Figure 2.2: $\gamma_{i j}$ values verses required separations for all satellite pairs
required separations can be calculated as

$$
\begin{equation*}
\Delta_{i j}^{\prime}=\max \left(\Delta_{i j}+\delta \gamma_{i j}, 0.0\right) \tag{2.5}
\end{equation*}
$$

Relating this equation to equation (2.4), in this formulation $\delta$ can be thought of as the increase or decrease in the single entry $C / I$ target. The modified required separations are determined to yield an approximately uniform change in the single entry target.

This implementation was tested using an initial required separation matrix calculated for a 26 dB single entry target $C / I$. It was found to result in a minimum aggregate total link $C / I$ of 6 dB with the maximum $\delta$ equal to -8.04 . Upon closer examination it became clear that the relatively few satellite pairs with very large $\Delta_{i j}$ values made finding a feasible solution quite difficult, hence the relatively large negative value for the maximum $\delta$. By restricting the size of the initial required separations much better satellite synthesis solutions were obtained. In particular, if the initial required separations are limited to $8^{\circ}$ then a solution with a minimum aggregate total link $C / I$ of 18 dB and a maximum $\delta$ of -0.37 was obtained. The $C / I$ profiles of the two solutions are shown in Figure 2.3.

The improvement obtained by implementing a limit on the initial required separations is dramatic. For comparison, the $C / I$ profile for the WARC88 solution is also plotted.

### 2.2 Modifications Based on C/I Performance of Previous Solutions

In the runs presented above no use is made of information provided by existing satellite synthesis solutions. It is possible that advantage could be made of the information contained in such solutions. For example, it may be possible to modify existing solutions based on the $C / I$ performance of such solutions. The pairwise separation of two satellites might be increased if the single entry $C / I$ values for the


Figure 2.3: Aggregate $C / I$ profiles for solutions obtained from SLOT, with $\gamma_{i j}$ modification.
pair are low relative to the values for the other pairs in the plan. Likewise, the pairwise separation for two satellites with relatively large single entry $C / I$ values might be reduced.

Implicit in this method is the assumption that there is a quality in the previous solution which if possible should be preserved, with better $C / I$ performance, in the new solution. The WARC88 planned allotment was used as the existing solution for most of this work. This synthesis solution has been accepted by the nations participating in the WARC conference. It is desirable to maintain the character of the WARC88 solution in development of alternative solutions. An alternative plan which represents a modest perturbation of the WARC plan, and not a wholesale reworking of the plan, is likely to be most acceptable.

The modification techniques begin by examining the existing solution and determining all pairwise satellite separations. Then the initial required separations are defined as the minimum of the separations identified from the existing solution and the required separation values identified using DELTA.

$$
\begin{equation*}
\Delta_{i j}=\min \left(\Delta_{i j} \text { DELTA }, \Delta_{i j} \text { existing }\right) \tag{2.6}
\end{equation*}
$$

The desired locations used by SLOT in the generation of the perturbed satellite positioning are the satellite positions from the existing solution. Therefore, because of the definition of the required separations SLOT will identify the existing solution as a feasible solution. In order to improve upon the previous solution the required separations are modified according to the $C / I$ performance of the existing solution.

### 2.3 Modifications Based Upon Single Entry $C / I$ Performance of the Existing Solution

The first modification method calls for the required separations to be modified according to the single entry $C / I$ values for the existing solution. The single entry $C / I$ values, $(C / I)_{i j}$, are obtained using the analysis program, MISOUP. In simple
terms, the form of the required separation modification is

$$
\begin{equation*}
\Delta_{i j}^{\prime}=\max \left(\Delta_{i j}+\gamma_{i j} \delta \beta_{i j}, 0.0\right) \tag{2.7}
\end{equation*}
$$

where $\gamma_{i j}$ is the factor described in Section $2.1, \delta$ is the binary search variable, and $\beta_{i j}$ is defined as

$$
\begin{align*}
& \beta_{i j}=\frac{\left(32-(C / I)_{i j}\right)^{k}}{\sigma} \quad \text { if }(C / I)_{i j}>37 \text { or }(C / I)_{i j}<27 \\
& \beta_{i j}=0 \quad \text { otherwise } \tag{2.8}
\end{align*}
$$

The value 32 in the expression for $\beta_{i j}$ represents the nominal single entry $C / I$ target corresponding to the aggregate total link allotment target of 26 dB . If the single entry $C / I$ 's of the existing solution are significantly smaller (larger) than the 32 dB target then the $\beta_{i j}$ values are positive (negative) and in equation (2.7) serve to increase (decrease) the required separations.

The exponent, $k$, in equation (2.8) is chosen to be either 1 or 3 for all synthesis runs. These choices preserve the sign on the quantity $32-(C / I)_{i j}$. The factor, $\sigma$, in the denominator is used to scale the $\beta_{i j}$ values.

Figure 2.4 shows the results of two SLOT runs, one using $k=1$ and $\sigma=32$, and the other using $k=3$ and $\sigma=20000$. In both cases the modified required separations were required to be greater than two-thirds of the $\Delta_{i j}$. The SLOT runs give very similar results, and both runs give substantially better results than the WARC88 solution.

The modification of required separations using the $\beta_{i j}$ values is selective in the sense that only those administration pairs with excessively large or small single entry $C / I$ 's have modifications to their required separations. This is unlike the $\gamma_{i j}$ modification method presented in Section 2.1 since the modification is not uniform. Ideally, an administration which initially had a large aggregate $C / I$ would not, after running SLOT with modified required separations, become the administration with the worst aggregate $C / I$. Likewise, while it is desirable to increase the $C / I$ 's of


Figure 2.4: Aggregate $C / I$ profiles for solutions obtained from SLOT, with single entry $\beta_{i j}$ modífication
the administrations with the worst $C / I$ values, it is not necessarily desirable that those administrations, after running SLOT with modified required separations, have excessive aggregate $C / I$ 's. These concerns are dependent upon what might be called the stability of the modification process. If one were to consider multiple iterations of the process described above, where each new solution becomes an existing solution to serve as a basis for another new solution, it is desirable for the process to converge to a final solution. If the combination of $k$ and $\sigma$ is improperly chosen then the solutions have been shown to diverge. In particular, Figure 2.5 shows the profile of changes in aggregate $C / I$ 's which might be considered undesirable. The aggregate $C / I$ values of the existing solution are on the left and attached by a straight line to the corresponding $C / I$ values in the new solution. Only the $C / I$ values showing the most change are plotted. The changes in Figure 2.5 suggest an instability due to either a $k$ which is too large or a $\sigma$ which is too small.

### 2.4 Modifications Based Upon Aggregate $C / I$ Performance of the Existing Solution

The $\beta_{i j}$ values used to modify the required separation in the previous section are based on the single entry $C / I$ performance of the existing solution. Several runs were also made in which the $\beta_{i j}$ values were also dependent upon the aggregate $C / I$ performance of the existing solution. In particular, the $\beta_{i j}$ values between the 7 administrations with the worst aggregate $C / I$ 's and the other administrations were doubled relative to the values obtained from equation (2.8). The results, however, showed no significant improvement over the method based on single entry $C / I$ 's alone.

Both of the methods presented in this section were capable of generating solutions with much better performance than the WARC88 solution based on the worst aggregate $C / I$ 's. We have made many runs using the above methods in varying ways, and many times developed solutions with a minimum aggregate $C / I$ of 18 dB


Figure 2.5: Aggregate $C / I$ changes between the existing and new solutions with undesirable characteristics
or better, yet none of the generated solutions has had a minimum aggregate $C / I$ that exceeded 19 dB . It appears that a solution with a minimum aggregate $C / I$ just under 19 dB is the best that can be obtained from the positioning algorithms with the given scenario assumptions. In the next section two methods for further improvement in the aggregate $C / I$ values, based upon a modification of the scenario assumptions, are presented.

## Chapter 3

## Improving Aggregate Total-link $C / I$ 's by Posteriori Modification of Scenario Assumptions

OSU's efforts in the satellite synthesis project have centered on the selection of a suitable geosynchronous orbital positioning of the satellites assuming a set of fixed scenario parameters. These scenario parameters include satellite and earth station antenna patterns, antenna gains, and transmitter powers. Given an allotment it is, of course, possible to improve a satellite's carrier to interference ratio by modifying any or all of the design parameters. In the actual implementation of the allotment it is reasonable to assume that administrations will make these modifications to improve the performance of their communication systems as long as the changes do not significantly degrade the performance of the other satellite systems.

In this section, two methods for improving the carrier to interference results are presented. The first method allows for a modification of the satellite and earth station transmitter powers. The second method allows for adaptive nulling of interfering satellites on the downlink. Both methods are used after a satellite positioning has been established.

### 3.1 Modification of Transmitter Powers

Changes in scenario parameters were mentioned as potentially improving $C / I$ values. A change in the transmitter powers is the one change which provides a linear change in carrier and interference signal strength. This fact makes optimization of carrier to interference ratios based on changes in the transmitter powers particularly easy to implement and to analyze.

The OSU analysis program, MISOUP, determines normalized uplink and downlink carrier and interference powers for each testpoint/satellite pair. This information is output by MISOUP in the uplink and downlink, carrier and interference matrices.

Let $n=$ number of administrations. $K=\{1, \ldots, n\}$, with $k \in K$, is the set of administrations for which $C / I$ values are to be computed. $I=\{1, \ldots, n\}$, with $i \in I$, is the set of interfering administrations. Also, let $n t p s(m)=$ number of testpoints in $m$. So that $J(k)=\{1, \ldots, n t p s(k)\}$ with $j \in J$, is the set of testpoints of administration k and $L(i)=\{1, \ldots, n t p s(i)\}$ with $l \in L$ is the set of testpoints for administration i.

The following data is available from MISOUP.

$$
\begin{aligned}
& C_{\text {down }}(k, j)=\text { Downlink carrier power for testpoint }(\operatorname{tp}) j \text { of administra- } \\
& \text { tion(admin) } k \\
& C_{u p}(k, j)=\text { uplink carrier power for tp } j \text { of admin } k \\
& I_{\text {down }}(k, j, i)=\text { Downlink interference power from admin } i \text { 's satellite, at } \\
& \text { tp } j \text { of admin } k \\
& I_{u p}(k, i, l)=\text { Uplink interference power from } \operatorname{tp} l \text { of admin } i, \text { at the sat } \\
& \text { of admin } k
\end{aligned}
$$

Given these single entry carrier and interference values, the aggregate uplink, downlink, and total-link $C / I ' s \quad \forall k \in K, \forall j \in J(k)$ are computed as

$$
\begin{align*}
& C / I_{u p}(k, j)=\frac{C_{u p}(k, j)}{\sum_{I} \max _{I}\left(I_{u p}(k, i, l)\right)}  \tag{3.1}\\
& C / I_{\text {down }}(k, j)=\frac{C_{\text {down }}(k, j)}{\sum_{I}\left(I_{\text {down }}(k, j, i)\right)} \tag{3.2}
\end{align*}
$$

$$
\begin{equation*}
C / I_{\text {total }}(k, j)=\frac{C / I_{\text {down }}(k, j) C / I_{u p}(k, j)}{C / I_{\text {down }}(k, j)+C / I_{u p}(k, j)} \tag{3.3}
\end{equation*}
$$

The method applies a minimax method in order to achieve improved carrier to interference performance for a given satellite ordering. The method performs

$$
\begin{equation*}
\max _{\substack{C_{\text {up }}(K, J) \\ C_{\text {down }}(K)}}\left(\min _{K, j} C / I_{\text {total }}(k, j)\right) \tag{3.4}
\end{equation*}
$$

It does so by increasing the transmitter power, and therefore the carrier power for the up or down link of the testpoint/satellite pair with the worst $C / I$ value. If the aggregate uplink $C / I$ is worse than the aggregate downlink $C / I$ for the worst testpoint/satellite pair then the power of the testpoint's earth station is increased by a factor of $1.25(1 \mathrm{~dB})$. Likewise, if it is the aggregate downlink $C / I$ which is worst then the satellite transmitter power is increased by 1 dB . The method for performing the maximization of equation (3.4) follows:

1. Set $\operatorname{scale}_{u p}(k, j)=1.0$ and $\operatorname{scale}_{\text {down }}(k)=1.0 \forall k, j$
2. Determine $k_{w o r s t}$ and $j_{\text {worst }}$, the administration and testpoint for which $C / I_{\text {total }}(k, j)$ is a minimum.
3. Modify carrier powers according to (a) or (b)
(a) If $C / I_{u p}\left(k_{\text {worst }}, j_{w o r s t}\right)<C / I_{\text {down }}\left(k_{\text {worst }}, j_{\text {worst }}\right)$ then
i. $s c a l e_{u p}\left(k_{w o r s t}, j_{w o r s t}\right)=\operatorname{scale}_{u p}\left(k_{w o r s t}, j_{w o r s t}\right) * 1.25$
ii. $C_{u p}\left(k_{\text {worst }}, j_{\text {worat }}\right)=C_{u p}\left(k_{\text {worst }}, j_{\text {wor st }}\right) * 1.25$
iii. $I_{u p}\left(i, k_{\text {worst }}, j_{\text {worst }}\right)=I_{u p}\left(i, k_{\text {worst }}, j_{\text {worst }}\right) * 1.25 \quad \forall i \in I$
iv. Recalculate $C / I_{u p}(k, j)$ and $C / I_{\text {total }}(k, j) \quad \forall k \in K, \forall j \in J(k)$
(b) If $C / I_{\text {down }}\left(k_{w o r s t}, j_{w o r s t}\right)<C / I_{u p}\left(k_{\text {worst }}, j_{\text {worst }}\right)$ then
i. $\operatorname{scale}_{\text {down }}\left(k_{\text {worst }}\right)=\operatorname{scale}_{\text {down }}\left(k_{\text {worst }}\right) * 1.25$
ii. $C_{\text {down }}\left(k_{\text {wor st }}, j\right)=C_{\text {down }}\left(k_{\text {worst }}, j\right) * 1.25 \quad \forall j \in J\left(k_{\text {worst }}\right)$

$$
\text { iii. } I_{\text {down }}\left(i, l, k_{\text {worst }}\right)=I_{\text {down }}\left(i, l, k_{\text {wor st }}\right) * 1.25 \quad \forall i \in I, \forall l \in L(i)
$$

iv. Recalculate $C / I_{\text {down }}(k, j)$ and $C / I_{\text {total }}(k, j) \quad \forall k \in K, \forall j \in J(k)$
4. If not finished, goto (2).
5. scale $e_{u p}(k, j)=$ scale $_{u p}(k, j) /\left(\min _{K, J(k)} \operatorname{scale}_{u p}(k, j)\right) \quad \forall k \in K, \forall j \in J(k)$ $\operatorname{scale}_{\text {down }}(k)=\operatorname{scale}_{\text {down }}(k) /\left(\min _{K}\right.$ scale $\left._{\text {down }}(k)\right) \quad \forall k \in K$
6. Stop

The resulting values of $\operatorname{scale} e_{u p}(k, j)$ and $s c a l e_{\text {down }}(k)$ are the factors by which the earth station and satellite powers should be increased. For the optimal solution the algorithm should be run until

$$
\begin{equation*}
\operatorname{scale}_{u p}(k, j)>1.0 \quad \text { and } \quad \text { scale }_{\text {down }}(k)>1.0 \quad \forall k \in K, \forall j \in J(k) \tag{3.5}
\end{equation*}
$$

For this work an iteration count was used as an alternate stopping condition to limit the amount of computer time used.

Figure 3.1 shows the results of an experiment in which the algorithm outlined above was used to improve the $C / I$ values of one of OSU's satellite synthesis runs for the Ku band with existing system satellites. 3000 iterations of the power optimization algorithm were performed. The plot shows the improvement in the minimum aggregate $C / I$ which is achieved by the power optimization algorithm. It also demonstrates the leveling effect which is characteristic of the minimax optimization method.

The implementation of this method described above does not constrain the satellite or earth station powers in any way. In fact, the solution shown in Figure 3.1 requires that these powers be increased by up to 50 dB . This is not a reasonable increase in power. Therefore, constraints were added to the optimization. The uplink and downlink power increases are limited to a user specified value. With this constrained method, if the aggregate uplink $C / I$ is smaller than the aggregate


Figure 3.1: Improvement of Aggregate $C / I$ by Modifying Transmitter Powers

Table 3.1:

| Max. Uplink <br> Increase | Max. Downlink <br> Increase | \# of <br> Iterations | Minimum <br> Agg. $C / I(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: |
| 1X | 1X | - | 18.48 |
| 1X | 2X | 6 | 20.43 |
| 1X | 5 X | 35 | 21.67 |
| 1X | 10 X | 44 | 21.73 |
| 2X | 1X | 10 | 19.14 |
| 5X | 1 X | 22 | 19.59 |
| 10X | 1X | 30 | 19.75 |
| 2X | 2X | 49 | 21.49 |
| 5X | 2X | 154 | 22.28 |
| 10X | 2 X | 325 | 22.48 |
| 10X | 10 X | 394 | 23.01 |

downlink $C / I$ then the power of the worst uplink testpoint is increased up to some maximum value. If the uplink power is already at the maximum permitted level then the downlink power is increased. The downlink power is also constrained by a prescribed limit. If the aggregate downlink $C / I$ is smaller than the aggregate uplink $C / I$ then the power of the satellite with the worst aggregate $C / I$ is increased up to some maximum value. If the downlink power is already at the maximum permitted level then the uplink power of the worst testpoint is increased, up to a prescribed limit. If both the uplink and the downlink increase of the testpoint/satellite pair with the minimum aggregate total link $C / I$ are at their limits then the algorithm is complete.

Several runs were made using the constrained power optimization method. These are summarized in Table 3.1. This table indicates the maximum acceptable uplink and downlink power increases, the resulting minimum aggregate $C / I$, and the number of iterations to achieve the optimum solution. A sample $C / I$ profile is plotted in Figure 3.2.

A characteristic which is evident in these runs, and in runs for all three of the


Figure 3.2: Improvement of Aggregate C/I by a Constrained Modification of Transmitter Powers
other satellite positionings which have been similarly studied, is that an increase in downlink power is generally more effective than an increase in uplink power at increasing the minimum aggregate $C / I$ value. For example, the initial doubling of the satellite power increases the minimum aggregate $C / I$ from 18.48 dB to 20.43 dB , while a similar increase in the earth station power only increases the minimum aggregate $C / I$ to 19.14 dB . This result agrees with the observation that it is generally the downlink $C / I$ which is most critical in the calculation of the total link $C / I$.

It is reasonable to assume that power increases would be more easily introduced for earth station transmitters than for satellite transmitters. Unfortunately, the data indicates that, in general, a greater benefit is obtained from an increase in satellite power than from an increase in earth station power. It may be possible to steer the satellite position solutions in such a way as to cause the uplink to dominate in the calculation of the aggregate total link $C / I$, thus potentially increasing the utility of the power optimization method for improving the minimum $C / I$. For example, a method discussed in the next section, adaptive nulling of interfering satellites, is effective at reducing the downlink interference. It thereby increases the relative contribution of the uplink $C / I$ in the calculation of the total link $C / I$ and increases the effectiveness of the power optimization method.

Decreasing the powers of the worst interferers of the administration with the worst aggregate total link $C / I$ is very similar in concept to increasing the powers of the administrations with the worst $C / I$. This approach was tested, but for several reasons was determined to be inferior. First, the power of the worst interferer can not actually be decreased. All powers must be maintained at levels which satisfy carrier to noise criteria. For example, if we wish to effectively reduce an earth station power by 10 dB this is possible only by increasing the power of all other earth stations by 10 dB . The optimization performed by increasing the powers of the administrations with the worst $C / I$ results in a relatively few administrations with power increases, and of the power increases only one satellite and one earth
station have the maximum increase. When the power of the worst interferer in decreased the result is that nearly all of the satellites and earth stations have the maximum increase in power. The former situation is the most desirable.

A second reason which argues for increasing the power of the administration with the minimum $C / I$ rather than decreasing the power of the worst interferers of that administration is evident by considering the case where two administrations are each others worst interferers. If we increase their powers they will remain each others worst interferers, however both will have increased $C / I$ 's since the relative contribution from all the other administrations is reduced. If, on the other hand, the powers of the worst interferers had been decreased then the two administrations would have their powers decreased relative to all the other administrations, and their $C / I$ 's would begin to decrease. Figures 3.3 and 3.4 illustrate this effect. In Figure 3.3 the power of the administration with the worst $C / I$ is increased. The algorithm converges rather rapidly to a pseudo steady-state with a variability of about 0.5 dB . This variability is due to the fact that powers are increased in increments of 1 dB . The plot in Figure 3.4 shows the reduction in minimum C/I which occurs after an initial increase when the powers of the worst interferers are reduced.

### 3.2 Improvement of Downlink $C / I$ 's by Adaptive Nulling of Interfering Satellites

Interference on the downlink is from isolated locations in space, namely the geosynchronous orbital locations of the interfering satellites. Adaptive array technology should enable the receiving antenna of an earth station to direct nulls in the directions of its worst interféring satellites. Equipping earth station receivers with adaptive arrays improves the downlink, and therefore total link carrier to interference ratios with no adverse effect on the other administrations.

The effect of nulling a prescribed number of worst downlink interferers for each administration was investigated for four satellite positionings. The results presented


Figure 3.3: Convergence Profile When the Power of the Worst Administration is Increased


Figure 3.4: Convergence Profile When the Power of the Worst Interferer of the Worst Administration is Decreased


Figure 3.5: Aggregate C/I Profile Showing the Effect of Nulling Worst Interferers on the Downlink
in Figure 3.5 are typical. There is an expected general increase in the aggregate $C / I$ 's.

The nulling of interfering satellites is performed on a given satellite ordering. It is likely that a positioning scheme which leads to a dominant downlink $C / I$, with the downlink interference concentrated in only a few satellites, would also result in a most beneficial application of the adaptive nulling. This avenue was not investigated. However, the power optimization technique described in the previous section, used in coordination with the adaptive nulling scheme, does increase nulling scheme. Power optimization is perhaps most suitable for increasing uplink $C / I$ values while the adaptive nulling is suitable only for downlink nulling. The two methods are therefore somewhat complementary, each method enhances the effectiveness of the other.

## Chapter 4

## Ellipse and Testpoint Considerations

### 4.1 Outlying Testpoints

In several instances some testpoints of an administration fall outside of the ellipses specified in the ellipse database file. In the $\mathrm{Ku}+$ Ex scenario there are 71 testpoints in 22 administrations which are not enclosed by the ellipse. By far, the two worst cases are the U.S. existing satellites, USA13EB1 and USA13DB1. The testpoints and ellipse as seen from the USA13DB1 satellite are shown in Figure 4.1. These figures were prepared using the program, ELLPLOT, described in Section 4.3.

As currently implemented all of the satellite EIRPs are determined so as to just meet the downlink $C / N$ requirement for the worst case testpoint of an administration. The worst testpoints of USA13EB1 and USA13DB1 are 21.68 dB and 24.16 dB down from the satellite on axis gain respectively. In the case where all testpoints are enclosed by the ellipse the worst-case discrimination should be -3 dB . The offending satellites have their powers increased between 18 and 21 dB over what is normally necessary in order to meet the $C / N$ requirements, with the result that the other satellites suffer increased downlink interference.

As a means of dealing with this potential problem the maximum additional satellite power has been set to 3 dB . In other words, the increase in satellite power to


Figure 4.1: Testpoints and Ellipse of USA13DB1 As Seen From the Satellite at $-56^{\circ}$
account for discrimination between a satellite and its own ground station is a maximum of 3 dB . The desired signal at the earth station is also considered to be no worse than 3 dB down from the on-axis signal for the purpose of the interference calculation for the outlying testpoints. These changes have been made in both DELTA and in MISOUP.

The effect of outlying testpoints on the $C / I$ was tested by removing the eight testpoints which are more than 4.7 dB down from the satellite on-axis gain. The analysis program, MISOUP was then rerun and the resulting $C / I$ 's compared to the $C / I$ 's when all testpoints were used. The largest improvement in aggregate total link $C / I$ was 1.6 dB , the next largest change was 0.2 dB , and all the other $C / I$ 's changed by less than 0.1 dB . The results seem to indicate that the outlying testpoints are not particularly important either as a source of interference to other administrations or as the point of minimum $C / I$ for its administration.

The effect of outlying testpoints was also tested by repeating the entire synthesis process without the eight worst outlying testpoints. DELTA was used to generate a new required separation matrix, SLOT was used to generate a synthesis solution, and MISOUP was used to analyze the results. Figure 4.2 shows that no substantial improvement in aggregate $C / I$ 's was achieved when the outlying testpoints were removed.

### 4.2 Isolated Testpoints

In a few instances, an administration is composed of a large main area along with an isolated testpoint far from the main area. Such is the case for administrations with island territories. The ellipse required to cover both the main area and the isolated testpoint may be quite large. It is likely that in such instances it would be advantageous to cover the isolated testpoint by a spot beam. This would allow the main area to be covered by a smaller ellipse, and would reduce the interference presented by the satellite to other administrations.


Figure 4.2: Aggregate $C / I$ profiles comparing the effect of removing the eight worst outlying testpoints


Figure 4.3: A View of the All the Allotment Satellite Ellipses From -90 ${ }^{\circ}$

Figure 4.3, generated using WORLDPLOT which is described in the next section, is a plot of the ellipses for allotment system satellites as seen from -90 degrees longitude. The very large ellipse towards the bottom is for Chile. The ellipse covers not only Chile, but also an island territory in the Pacific. As shown in Figure 4.4, it is clear that the large beam transmitted by the Chilean satellite can cause difficulties in developing a satisfactory allotment plan.

### 4.3 Ellipse Plotting Programs

There are two programs for plotting ellipses as they are seen from the satellite. In both programs the ellipses are projected onto the antenna plane, which is perpendicular to the beam axis between the satellite and the aimpoint. The antenna plane intersects the beam axis at the aimpoint.

In the first program, ELLPLOT, the testpoint and ellipse data for all administra-

### 0.8 DEGREES



Figure 4.4: Testpoints and Ellipse of CHL00000 as Seen From the Satellite at $-78^{\circ}$
tions is read from the testpoint and ellipse databases. The user is then prompted for an administration number. The administration numbers correspond to the position of the administration in the key file. The administrations are currently numbered from 1 to 283. After supplying the administration number the user is prompted for the satellite longitude. The satellite longitude must lie between the westmost and eastmost longitudes, which are supplied to the user by the program. After supplying the longitudes, the ellipse and testpoints are plotted. The testpoints are projected onto the antenna plane using a perspective projection. The view of the testpoints and ellipse is scaled so as to fill the screen. Figures 4.1 and 4.4 are examples of this type of plot.

The second program, WORLDPLOT, plots all the ellipses which can be viewed on the earth's surface from a particular orbital longitude. All satellites which can potentially reside at the specified longitude are placed there and their ellipses are plotted.

## Chapter 5

## Solutions With Satellite Positionings Which Are Minimally Perturbed From The WARC88 Solution

In Section 2 solutions based on the existing WARC88 synthesis solution were presented. These solutions were generated using SLOT with $\beta_{i j}$ modification, and with the WARC88 satellite positions as the desired locations. These satellite positionings are perturbed relatively little from the WARC88 positionings. In particular, for the two runs in Figure 2.4 the maximum deviations between a satellite position and the position of the satellite in the WARC88 solution are $6.6^{\circ}$ and $6.0^{\circ}$. The corresponding totals of the deviations for all the satellites are $164.8^{\circ}$ and $173.5^{\circ}$ respectively. These two solutions have minimum aggregate $C / I$ values which are about 14 dB better than the WARC88 results.

We have generated other positionings which represent only a slight perturbation of the WARC88 positioning. In particular, one solution generated using $\gamma_{i j}$ modification of the required separations and feasible arcs restricted to $10^{\circ}$ about the WARC88 positions had a maximum deviation of $1.6_{0}$, a total deviation of $9.5^{\circ}$, and had a minimum aggregate $C / I$ which was improved by 6.28 dB over the minimum $C / I$ for the WARC88 solution.

It is clear that the OSU programs, with modifications discussed in this report, can produce solutions which are similar to the WARC88 solution, and therefore perhaps more universally acceptable, but which have improved aggregate $C / I$ characteristics.

## Appendix A

# Downlink Interference Power From the EIREB200 Satellite Received at BEN00000's Testpoint \#10 

## A. 1 Preliminaries

Carrier power from EIREB200's satellite must be enough to produce a carrier-to-noise ratio of at least 15 dB at each of the five service area testpoints. It appears that testpoint \#5 is farthest from the aimpoint, so the satellite's power should depend on meeting the $C / N$ requirement there. The Friis transmission equation describes the carrier power received at a given testpoint as

$$
\begin{equation*}
C=\frac{P_{S A T} G_{S T} D_{S T}\left(\phi, \phi_{0}\right) G_{E R}(\varphi) \lambda^{2}}{(4 \pi L)^{2}} \tag{A.1}
\end{equation*}
$$

and the noise power is given by

$$
\begin{equation*}
N=k T B \tag{A.2}
\end{equation*}
$$

where

- $P_{S A T}$ is the satellite power (W)
- $G_{S T}$ is the on-axis gain of the satellite's transmitting antenna
- $D_{S 7}\left(\phi, \phi_{0}\right)$ is the relative gain below maximum of the satellite's transmitting antenna in the direction of the testpoint
- $\phi$ is the off-axis angle to the testpoint (degrees)
- $\phi_{U}$ is the off-axis half-power angle in the direction of the testpoint (degrees)
- $G_{E R}(\varphi)$ is the gain of the testpoint's receiving antenna in the direction of its off-axis angle $\varphi$
- $\lambda$ is the carrier wavelength ( m )
- $L$ is the distance from the satellite to the testpoint (m)
- $k$ is Boltzmann's constant ( $\mathrm{J} / \mathrm{K}$ )
- $T$ is the testpoint's receiver noise temperature ( K ) and
- $B$ is the testpoint's receiver noise bandwidth ( Hz ).

In decibels, the $C / N$ ratio is
$C / N_{d B}=P_{S A T_{d B}}+G_{S T_{d B}}+D_{S T_{d B}}\left(\phi, \phi_{0}\right)+G_{E R_{d B}}(\varphi)+10 \log \lambda^{2}-10 \log \left[(4 \pi L)^{2} k T B\right]$.

Solving for $P_{S A T_{d B}}$ gives
$P_{S A T_{d B}}=C / N_{d B}-G_{S T_{d B}}-D_{S T_{d B}}\left(\phi, \phi_{0}\right)-G_{E R_{d B}}(\varphi)-10 \log \lambda^{2}+10 \log \left[(4 \pi L)^{2} k T B\right]$.

If attenuation due to rain is considered, the satellite's power must be boosted to maintain the $15 \mathrm{~dB} C / N$ requirement. Letting $A_{R_{d B}}$ be the rain attenuation factor in decibels, $P_{S A T_{d B}}$ with rain attenuation is
$P_{S A T_{d B}}=C / N_{d B}-G_{S T_{d B}}-D_{S T_{d B}}\left(\phi, \phi_{0}\right)-G_{E R_{d B}}(\varphi)-10 \log \lambda^{2}+10 \log \left[(4 \pi L)^{2} k T B\right]+A_{R_{d B}}$.


Figure A.1: Geometry of the satellite positioning problem

## A. 2 Geometry

Now consider the earth as a unit sphere centered in a cartesian coordinate system. The unit vectors in this system are $\hat{x}$, corresponding to ( $0^{\circ}$ longitude, $0^{\circ}$ latitude), $\hat{y}$, corresponding to ( $90^{\circ} \mathrm{E}, 0^{\circ}$ latitude) and $\hat{z}$, corresponding to the North Pole. Let $\overrightarrow{\mathbf{A}}$ be the aimpoint position vector of a particular beam's service area, let $\overrightarrow{\mathbf{T}}$ be a testpoint position vector for a particular testpoint of the beam's service area and let $\overrightarrow{\mathbf{O}}$ be the orbitral position vector of the beam's satellite. This geometry is illustrated in Fig. A.1.

The transformation to this coordinate system from earth coordinates (longitude, latitude) is:

$$
\begin{align*}
x & =\cos (\text { longitude }) \cos (\text { latitude }) \\
y & =\sin (\text { longitude }) \cos (\text { latitude })  \tag{A.6}\\
z & =\sin (\text { latitude })
\end{align*}
$$

Using this transformation, the vector $\overrightarrow{\mathbf{A}}$ to the aimpoint of EIREB200's service area $\left(0.3^{\circ}, 46.8^{\circ}\right)$ is

$$
\begin{align*}
\overrightarrow{\mathbf{A}} & =\hat{\mathbf{x}} \cos \left(0.3^{\circ}\right) \cos \left(46.8^{\circ}\right)+\hat{\mathbf{y}} \sin \left(0.3^{\circ}\right) \cos \left(46.8^{\circ}\right)+\hat{\mathbf{z}} \sin \left(46.8^{\circ}\right) \\
& =\hat{\mathbf{x}} 0.6845+\hat{\mathbf{y}} 0.0036+\hat{\mathbf{z}} 0.7290, \tag{A.7}
\end{align*}
$$

and the vector $\overrightarrow{\mathbf{T}}$ to EIREB200'S testpoint \#5 $\left(-7.0^{\circ}, 58.0^{\circ}\right)$ is

$$
\begin{align*}
\overrightarrow{\mathbf{T}} & =\hat{\mathbf{x}} \cos \left(-7.0^{\circ}\right) \cos \left(58.0^{\circ}\right)+\hat{\mathbf{y}} \sin \left(-7.0^{\circ}\right) \cos \left(58.0^{\circ}\right)+\hat{\mathbf{z}} \sin \left(58.0^{\circ}\right) \\
& =\hat{\mathbf{x}} .5260-\hat{\mathbf{y}} .0646+\hat{\mathbf{z}} .8480 \tag{A.8}
\end{align*}
$$

The vector $\overrightarrow{\mathbf{O}}$ to the EIREB200 satellite orbital position at $-31.0^{\circ}$ longitude is given by

$$
\begin{align*}
\overrightarrow{\mathbf{O}} & =6.6105\left[\hat{\mathbf{x}} \cos \left(-31.0^{\circ}\right)+\hat{\mathbf{y}} \sin \left(-31.0^{\circ}\right)\right] \\
& =\hat{\mathbf{x}} 5.666-\hat{\mathbf{y}} 3.405 . \tag{A.9}
\end{align*}
$$

Note that $\overrightarrow{\mathbf{O}}$ in Equation (A.9) has no $z$-component and has its magnitude increased by 6.6105. This is because the geosynchronous orbit lies in the equatorial plane, 6.6105 earth radii from the earth's center. (See Fig. A.1)

## A. 3 Power of the EIREB200 Satellite by $C / N$ Requirement

## A.3.1 Calculation of the Off-axis Angle, $\phi$

Summarizing the important position vectors for the EIREB200 calculation:

$$
\begin{array}{ll}
\overrightarrow{\mathbf{O}}=\hat{\mathbf{x}} 5.666-\hat{\mathbf{y}} 3.405 & \text { Orbital location } \\
\overrightarrow{\mathbf{A}}=\hat{\mathbf{x}} .6845+\hat{\mathbf{y}} .0036+\hat{\mathbf{z}} .7290 & \text { Aimpoint } \\
\overrightarrow{\mathbf{T}}=\hat{\mathbf{x}} .5260-\hat{\mathbf{y}} .0646+\hat{\mathbf{z}} .8480 & \text { Testpoint \#5 }
\end{array}
$$

Since $\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}$ is the beam axis vector from the satellite to the aimpoint and $\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}$ is the vector from the satellite to testpoint \#5, the angle $\phi$ between the beam axis


Figure A.2: Vector diagram for the satellite positioning problem.
and the vector to testpoint \#5, as seen by the EIREB200 satellite, is

$$
\begin{equation*}
\phi=\cos ^{-1} \frac{\langle\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\rangle}{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|\|\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\|} \tag{A.10}
\end{equation*}
$$

where $\langle$,$\rangle is the standard 3$-space inner product and $\|\cdot\|$ is the 3 -space norm operator. Now,

$$
\begin{aligned}
\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\| & =\sqrt{(.6845-5.666)^{2}+(.0036+3.405)^{2}+(.729)^{2}} \\
& =6.080(\text { earth radii }) \\
\|\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\| & =\sqrt{(.526-5.666)^{2}+(-.0646+3.405)^{2}+(.848)^{2}} \\
& =6.189 \text { (earth radii) }
\end{aligned}
$$

and

$$
\begin{aligned}
\langle\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\rangle= & (.6845-5.666)(.526-5.666)+(.0036+3.405)(-.0646+3.405) \\
& +(.729)(.848) \\
= & 37.610 \text { (earth radii) }{ }^{2}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\phi & =\cos ^{-1} \frac{37.610}{(6.080)(6.189)} \\
& =1.678^{\circ} .
\end{aligned}
$$



Figure A.3: The Antenna Plane

## A.3.2 The EIREB200 Antenna Plane

Consider a plane containing the service area aimpoint $\overrightarrow{\mathbf{A}}$ and normal to the beam axis, as seen in Fig. A.3. This plane will be referred to as the [?]. The aimpoint is designated as the origin antenna plane of a new antenna plane coordinate system, whose position vectors are described with primed coordinates. The ellipse agreed upon for EIREB200 at WARC88 will be projected onto this plane from the EIREB200 satellite. The half-power angle $\phi_{0}$, in the direction of testpoint \#5, can then be calculated. First, the intersection of the satellite-to-testpoint vector, $\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}$, with the antenna plane will be found. Call the vector from the earth's center to this intersection $\overrightarrow{\mathbf{P}}$.

## A.3.3 Finding $\overrightarrow{\mathbf{P}}$; the Satellite-to-Testpoint Vector, Antenna Plane Intersection

As seen in Fig. A.3,

$$
\begin{align*}
\|\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}\| & =\frac{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|}{\cos \phi}  \tag{A.11}\\
& =\frac{6.080}{\cos \left(1.678^{\circ}\right)} \\
& =6.082(\text { earth radii })
\end{align*}
$$

The unit vector in the direction of $\overrightarrow{\mathbf{P}}$ as seen from $\overrightarrow{\mathrm{O}}$ is the same as one in the direction from $\overrightarrow{\mathbf{O}}$ to $\overrightarrow{\mathbf{T}}$. Let this unit vector be called $\hat{\mathbf{u}}$ so that $\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}$ may be expressed as $\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}=\hat{\mathbf{u}}\|\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}\|$. First,

$$
\begin{align*}
\hat{\mathbf{u}} & =\frac{\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}}{\|\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\|}  \tag{A.12}\\
& =\frac{\hat{\mathbf{x}}(.526-5.666)+\hat{\mathbf{y}}(-.0646+3.405)+\hat{\mathbf{z}}(.848)}{6.188} \\
& =-\hat{\mathbf{x}} .8306+\hat{\mathbf{y}} .5398+\hat{\mathbf{z}} .1370 .
\end{align*}
$$

Then

$$
\begin{align*}
\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}} & =\hat{\mathbf{u}}\|\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}\|  \tag{A.13}\\
& =-\hat{\mathbf{x}} 5.0517+\hat{\mathbf{y}} 3.283+\hat{\mathbf{z}} .8332
\end{align*}
$$

Finally, in geocentric coordinates,

$$
\begin{align*}
\overrightarrow{\mathbf{P}} & =\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{O}}+\overrightarrow{\mathbf{O}}  \tag{A.14}\\
& =\hat{\mathbf{x}} .6143-\hat{\mathbf{y}} .1220+\hat{\mathbf{z}} .8332
\end{align*}
$$

The next step is to find $\overrightarrow{\mathbf{P}}$ in antenna plane coordinates.

## A.3.4 Antenna Plane Coordinates

The development of Subsection A.3.3 gives the intersection of $\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}$ with the antenna plane in terms of the original geocentric coordinate system. The ellipse data used in
the satellite positioning problem are given in terms of antenna plane coordinates, so it will be necessary to make a change of basis for $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{P}}$ before $\phi_{0}$ can be found. Let the antenna plane basis vectors be $\hat{\mathbf{x}}^{\prime}, \hat{\mathbf{y}}^{\prime}$ and $\hat{\mathbf{z}}^{\prime}$ where the primed unit vectors are defined as follows:

- $\hat{\mathbf{z}}^{\prime}$ is the unit vector normal to the antenna plane in the direction of $\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}$
- $\hat{\mathbf{y}}^{\prime}$ is $\perp$ to $\hat{\mathbf{z}}^{\prime}$ and $\|$ to the equatorial plane
- $\hat{\mathbf{x}}^{\prime}$ is $\perp$ to $\hat{\mathbf{y}}^{\prime}$ and $\hat{\mathbf{z}}^{\prime}$ forming a right-handed cartesian coordinate system.

Let $\hat{\mathbf{x}}^{\prime}, \hat{\mathbf{y}}^{\prime}$ and $\hat{\mathbf{z}}^{\prime}$ be given in geocentric coordinates as

$$
\begin{align*}
\hat{\mathbf{x}}^{\prime} & =\hat{\mathbf{x}} x_{\dot{x}^{\prime}}+\hat{\mathbf{y}} y_{\dot{x}^{\prime}}+\hat{\mathbf{z}} z_{\dot{x}^{\prime}} \\
\hat{\mathbf{y}}^{\prime} & =\hat{\mathbf{x}} x_{\hat{\mathbf{y}}^{\prime}}+\hat{\mathbf{y}} y_{\hat{y}^{\prime}}+\hat{\mathbf{z}} z_{\hat{y}^{\prime}}  \tag{A.15}\\
\hat{\mathbf{z}}^{\prime} & =\hat{\mathbf{x}} \\
x_{\dot{z}^{\prime}} & +\hat{\mathbf{y}} y_{\tilde{z}^{\prime}}+\hat{\mathbf{z}} z_{\dot{z}^{\prime}} .
\end{align*}
$$

From the definition above, $\hat{\mathbf{z}}^{\prime}$ will be given by

$$
\begin{align*}
\hat{\mathbf{z}}^{\prime} & =\frac{\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}}{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|} \\
& =\frac{\hat{\mathbf{x}}(.6845-5.666)+\hat{\mathbf{y}}(.0036+3.405)+\hat{\mathbf{z}}(.729)}{6.080} \\
& =-\hat{\mathbf{x}} .8193+\hat{\mathbf{y}} .5606+\hat{\mathbf{z}} .1199 \tag{A.16}
\end{align*}
$$

For $\hat{\mathbf{y}}^{\prime}$ to be $\|$ to the equatorial plane its $\hat{\mathbf{z}}$ coefficient must be zero. Thus, $z_{\dot{\nu}^{\prime}}=0$ and

$$
\hat{\mathbf{y}}^{\prime}=\hat{\mathbf{x}} x_{\hat{y}^{\prime}}+\hat{\mathbf{y}} y_{\hat{y}^{\prime}} .
$$

If $\hat{\mathbf{y}}^{\prime}$ is $\perp$ to $\hat{\mathbf{z}}^{\prime}$, their inner product is zero. Thus,

$$
\left\langle\hat{\mathbf{y}}^{\prime}, \hat{\mathbf{z}}^{\prime}\right\rangle=x_{\hat{y}^{\prime}}(-.8193)+y_{\hat{y}^{\prime}}(.5606) \stackrel{\text { set }}{=} 0
$$

gives

$$
y_{\hat{y}^{\prime}}=1.46147 x_{\hat{y}^{\prime}}
$$

and

$$
\begin{equation*}
\hat{\mathbf{y}}^{\prime}=\hat{\mathbf{x}} x_{\hat{y}^{\prime}}+\hat{\mathbf{y}} 1.46147 x_{\dot{y}^{\prime}} \tag{A.17}
\end{equation*}
$$

The value of $x_{\hat{y}^{\prime}}$ may be found by forcing $\hat{\mathbf{y}}^{\prime}$ to be of unit length. To this end, let

$$
\begin{aligned}
\sqrt{\left(x_{\hat{y}^{\prime}}\right)^{2}+\left(1.46147 x_{\hat{y}^{\prime}}\right)^{2}} & =1 \\
x_{\hat{y}^{\prime}}^{2}(1+2.1359) & =1 \\
x_{\hat{y}^{\prime}} & =.5647
\end{aligned}
$$

where the positive square-root is assumed. Plugging this result back into Eq. (A.17) gives

$$
\begin{equation*}
\hat{\mathbf{y}}^{\prime}=\hat{\mathbf{x}} .5647+\hat{\mathbf{y}} .8253 \tag{A.18}
\end{equation*}
$$

Finally, in order to form a right-handed coordinate system, $\hat{\mathbf{x}}^{\prime}$ can be found as the cross-product $\hat{\mathbf{y}}^{\prime} \times \hat{\mathbf{z}}^{\prime}$. Thus,

$$
\begin{align*}
\hat{\mathbf{x}}^{\prime} & =\left|\begin{array}{ccc}
\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\
.5647 & .8253 & 0 \\
-.8193 & .5606 & .1199
\end{array}\right| \\
& =\hat{\mathbf{x}} .0989-\hat{\mathbf{y}} .0677+\hat{\mathbf{z}} .9927 \tag{A.19}
\end{align*}
$$

Equations (A.16), (A.18) and (A.19) can be combined using matrix notation to get

$$
\left[\begin{array}{crc}
.0989 & -.0677 & .9927 \\
.5647 & .8253 & 0 \\
-.8193 & .5606 & .1199
\end{array}\right]\left[\begin{array}{c}
\hat{\mathbf{x}} \\
\hat{\mathbf{y}} \\
\hat{\mathbf{z}}
\end{array}\right]=\left[\begin{array}{c}
\hat{\mathbf{x}}^{\prime} \\
\hat{\mathbf{y}}^{\prime} \\
\hat{\mathbf{z}}^{\prime}
\end{array}\right]
$$

or

$$
\begin{equation*}
\mathbf{U} \overrightarrow{\mathbf{B}}=\overrightarrow{\mathbf{B}}^{\prime} \tag{A.20}
\end{equation*}
$$

where the obvious correspondences are made. Note that $\mathbf{U}$ is the unitary transition matrix from unprimed to primed coordinates.

## A.3.5 Calculation of $\phi_{0}$; The Half-power Angle in the Direction of EIREB200's Testpoint \#5

Since $U$ of Eq. (A.20) is the transition matrix from geocentric to EIREB200 antenna plane coordinates, the coordinate vectors $\mathbf{O}_{\vec{B}^{\prime}}, \mathbf{A}_{\vec{B}^{\prime}}$ and $\mathbf{P}_{\vec{B}^{\prime}}$ may be found by applying $U$ as a linear operator to $\mathbf{O}_{\vec{B}}, \mathbf{A}_{\vec{B}}$ and $\mathbf{P}_{\vec{B}}$, respectively. A translation is is
performed first however, to make $\overrightarrow{\mathbf{A}}_{\boldsymbol{B}^{\prime}}$ the origin of the new system. Thus,

$$
\left.\begin{array}{c}
\mathbf{A}_{\vec{B}^{\prime}}=\mathrm{U}\left[\begin{array}{l}
(.6845-.6845) \\
(.0036-.0036) \\
(.7290-.7290)
\end{array}\right] \\
=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right], \\
\mathbf{O}_{\vec{B}^{\prime}}=\mathrm{U}\left[\begin{array}{cc}
(5.666-.6845) \\
(-3.405-.0036) \\
(0 & -.7290)
\end{array}\right] \\
= \\
=\left[\begin{array}{cc}
.0989 & -.0677 \\
.5647 & .8253 \\
-.8193 & 0 \\
.5606 & .1199
\end{array}\right]\left[\begin{array}{c}
4.9815 \\
-3.4086 \\
-.7290
\end{array}\right] \\
-.0002 \\
-.0001 \\
-6.0796
\end{array}\right] \quad \$ \mathrm{c}
$$

and

$$
\begin{aligned}
\mathbf{P}_{\vec{B}^{\prime}} & =\mathrm{U}\left[\begin{array}{c}
(.6143-.6845) \\
(-.1220-.0036) \\
(.8332-.7290)
\end{array}\right] \\
& =\left[\begin{array}{ccc}
.0989 & -.0677 & .9927 \\
.5647 & .8253 & 0 \\
-.8193 & .5606 & .1199
\end{array}\right]\left[\begin{array}{c}
-.0702 \\
-.1256 \\
.1042
\end{array}\right] \\
& =\left[\begin{array}{c}
.1050 \\
-.1433 \\
-.0004
\end{array}\right] .
\end{aligned}
$$

Rounding off and reattaching the antenna plane unit vectors yields

$$
\begin{align*}
& \overrightarrow{\mathbf{A}}_{\vec{B}^{\prime}}=0 \\
& \overrightarrow{\mathbf{O}}_{\vec{B}^{\prime}}=-\hat{\mathbf{z}}^{\prime} 6.0796  \tag{A.21}\\
& \overrightarrow{\mathbf{P}}_{\vec{B}^{\prime}}=\hat{\mathbf{x}}^{\prime} \cdot 1050-\hat{\mathbf{y}}^{\prime} \cdot 1433 .
\end{align*}
$$

Now, from WARC88, the ellipse for EIREB200 is described by

$$
\text { Major axis }: 3.61^{\circ}
$$

```
minor axis : 1.75
Orientation : 145.*
```

where the Orientation angle is measured counterclockwise from $\hat{\mathbf{y}}^{\prime}$ as seen from $\overrightarrow{\mathbf{O}}_{\vec{B}^{\prime}}$. The equation for this ellipse in the antenna plane is

$$
\begin{align*}
\frac{\left(x^{\prime \prime}\right)^{2}}{\left[\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\| \tan \left(1.75^{\circ} / 2\right)\right]^{2}}+\frac{\left(y^{\prime \prime}\right)^{2}}{\left[\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\| \tan \left(3.61^{\circ} / 2\right)\right]^{2}} & =1 \\
\frac{\left(x^{\prime \prime}\right)^{2}}{[6.08(.01527)]^{2}}+\frac{\left(y^{\prime \prime}\right)^{2}}{[6.08(.03151)]^{2}} & =1 \\
\frac{\left(x^{\prime \prime}\right)^{2}}{.00862}+\frac{\left(y^{\prime \prime}\right)^{2}}{.0367} & =1 \tag{A.22}
\end{align*}
$$

where the double-primed coordinates are related to the primed coordinates by

$$
\left[\begin{array}{l}
x^{\prime \prime}  \tag{A.23}\\
y^{\prime \prime}
\end{array}\right]=\left[\begin{array}{cc}
\cos (\text { Orientation }) & -\sin (\text { Orientation }) \\
\sin (\text { Orientation }) & \cos (\text { Orientation })
\end{array}\right]\left[\begin{array}{l}
x^{\prime} \\
y^{\prime}
\end{array}\right]
$$

Using this rotation, the double-primed coordinates for $\overrightarrow{\mathbf{P}}$ are

$$
\begin{aligned}
x^{\prime \prime} & =.1050 \cos 145^{\circ}-(-.1433) \sin \left(145^{\circ}\right) \\
& =-.0038
\end{aligned}
$$

and

$$
\begin{aligned}
y^{\prime \prime} & =.1050 \sin 145^{\circ}+(-.1433) \cos 145^{\circ} \\
& =.1776
\end{aligned}
$$

The angle between $\overrightarrow{\mathbf{P}}-\overrightarrow{\mathbf{A}}$ and the $y^{\prime \prime}$ axis is

$$
\begin{align*}
\tan ^{-1} \frac{x^{\prime \prime}}{y^{\prime \prime}} & =\tan ^{-1} \frac{-.0038}{.1776} \\
& =-1.226^{\circ} \tag{A.24}
\end{align*}
$$

To find the ellipse intersection of the ray extending from $\overrightarrow{\mathbf{A}}$ through $\overrightarrow{\mathbf{P}}, x^{\prime \prime}$ and $y^{\prime \prime}$ must satisfy

$$
\frac{x^{\prime \prime}}{y^{\prime \prime}}=-.0214
$$

and the ellipse equation. Assuming $y^{\prime \prime}$ is positive, as before,

$$
\begin{equation*}
x^{\prime \prime}=-.0214 y^{\prime \prime} \tag{A.25}
\end{equation*}
$$

and Eq. (A.22) becomes

$$
\frac{\left(-.0214 y^{\prime \prime}\right)^{2}}{.00862}+\frac{\left(y^{\prime \prime}\right)^{2}}{.0367}=1
$$

which reduces to

$$
y^{\prime \prime}=.1914
$$

Also, from Eq. (A.25),

$$
\begin{aligned}
x^{\prime \prime} & =(-.0214)(.1914) \\
& =-.0041
\end{aligned}
$$

The length from $\overrightarrow{\mathbf{A}}$ to the ellipse intersection is

$$
\sqrt{(.1914)^{2}+(-.0041)^{2}}=.19144(\text { earth radii })
$$

Finally, the angle $\phi_{0}$ is twice the angle between $\overrightarrow{\mathbf{A}}$ and the point on the ellipse in the direction of $\overrightarrow{\mathbf{P}}$.

$$
\begin{aligned}
\phi_{0} & =2 \tan ^{-1} \frac{.19144}{6.080} \\
& =3.607^{\circ}
\end{aligned}
$$

## A.3.6 The EIREB200 Required Satellite Power

Referring to data from WARC88 for EIREB200,

$$
\begin{aligned}
C / N_{d B} & =15(\mathrm{~dB}) \\
G_{S T_{d B}} & =36.44(\mathrm{~dB}) \\
G_{E R_{d B}}\left(\varphi=0^{\circ}\right) & =49.4(\mathrm{~dB}) .
\end{aligned}
$$

With

$$
\begin{aligned}
\frac{\phi}{\phi_{0}} & =\frac{1.678^{\circ}}{3.607^{\circ}} \\
& =.465
\end{aligned}
$$

and with EIREB200 using satellite antenna pattern \#2,

$$
\begin{aligned}
D_{S T_{d B}}\left(\phi, \phi_{0}\right) & =-12(0.465)^{2} \\
& =-2.59 \mathrm{~dB}
\end{aligned}
$$

The downlink Ku -band rain attenuation for EIREB200 with its satellite at $-31^{\circ}$ longitude is 24.34 dB , as read from the WARC88 ellipse file. The rain attenuation is adjusted according to

$$
\begin{aligned}
A_{R_{d B}} & =\min \left(24.34(0.1 / 0.01)^{-0.41}, 8\right) \\
& =8 \mathrm{~dB}
\end{aligned}
$$

Plugging this and all other required values back into Eq. (A.5) gives

$$
\begin{align*}
P_{S . A T_{d B}=}= & 15-36.44-(-2.59)-49.4-10 \log \lambda^{2}+10 \log \left[(4 \pi L)^{2} k T B\right]+8 \\
= & -60.23-10 \log \lambda^{2}+10 \log \left[(4 \pi\|\overrightarrow{\mathbf{T}}-\overrightarrow{\mathbf{O}}\|)^{2} k T B\right] \\
= & -60.23-10 \log \left[\left(3 \times 10^{8} / 11.2 \times 10^{9}\right)^{2}\right] \\
& +10 \log \left[\left(4 \pi\left[6.188 \times 6.3787 \times 10^{6}\right]\right)^{2}\left(1.38 \times 10^{-23}\right)(346)\left(1 \times 10^{6}\right)\right]  \tag{A.26}\\
= & 1.91 \mathrm{~dB}
\end{align*}
$$

## A. 4 Interference Power Received at BEN00000's Testpoint \#10

A.4.1 The EIREB200 Satellite Transmit Discrimination in the Direction of BENOOOOO's Testpoint \#10

Now consider the power produced at Benin's testpoint \#10. This testpoint, at ( $2.85^{\circ}$ longitude, $12.35^{\circ}$ latitude), is chosen because it appears to be closest to the

EIREB200 footprint. Letting the vector from the earth's center to this testpoint be $\overrightarrow{\mathbf{T}}_{2}$, the transformation of Eq. (A.7) gives the geocentric coordinates expression

$$
\begin{aligned}
\overrightarrow{\mathbf{T}}_{2} & =\hat{\mathbf{x}} \cos \left(2.85^{\circ}\right) \cos \left(12.35^{\circ}\right)+\hat{\mathbf{y}} \sin \left(2.85^{\circ}\right) \cos \left(12.35^{\circ}\right)+\hat{\mathbf{z}} \sin \left(12.35^{\circ}\right) \\
& =\hat{\mathbf{x}} .9757+\hat{\mathbf{y}} .0486+\hat{\mathbf{z}} .2139 .
\end{aligned}
$$

The vector from the EIREB200 satellite to BENOOOOO's testpoint \#10 is

$$
\begin{aligned}
\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}} & =(.9757-5.666) \hat{\mathbf{x}}+(.0486+3.405) \hat{\mathbf{y}}+(.2139) \hat{\mathbf{z}} \\
& =-\hat{\mathbf{x}} 4.6903+\hat{\mathbf{y}} 3.4536+\hat{\mathbf{z}} .2139
\end{aligned}
$$

Now,

$$
\begin{aligned}
\left\|\overrightarrow{\mathrm{T}}_{2}-\overrightarrow{\mathrm{O}}\right\| & =\sqrt{(-4.6903)^{2}+3.4536^{2}+.2139^{2}} \\
& =5.8286(\mathrm{earth} \text { radii) }
\end{aligned}
$$

and

$$
\begin{aligned}
\left\langle\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathrm{T}}_{2}-\overrightarrow{\mathbf{O}}\right\rangle & =(-4.9815)(-4.6903)+(3.4086)(3.4536)+(.7290)(.2139) \\
& =35.2926 \text { (earth radii) }
\end{aligned}
$$

Thus the angle, $\phi_{2}$, between the EIREB200 beam axis and $\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathrm{O}}$ is

$$
\begin{aligned}
\phi_{2} & =\cos ^{-1} \frac{\left\langle\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}\right\rangle}{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}\right\|} \\
& =\cos ^{-1} \frac{35.2926}{(6.080)(5.8286)} \\
& =5.190^{\circ}
\end{aligned}
$$

Calculating the intersection of $\overrightarrow{\mathbf{T}}_{\mathbf{2}}-\overrightarrow{\mathrm{O}}$ with the EIREB200 antenna plane is next. Call this intersection $\overrightarrow{\mathbf{P}}_{2}$. Now,

$$
\begin{aligned}
\overrightarrow{\mathbf{P}}_{2}-\overrightarrow{\mathbf{O}} & =\hat{\mathbf{u}}\left\|\overrightarrow{\mathbf{P}}_{2}-\overrightarrow{\mathbf{O}}\right\| \\
& =\frac{\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}}{\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}\right\|} \frac{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|}{\cos \phi_{2}} \\
& =-\hat{\mathbf{x}} 4.9127+\hat{\mathbf{y}} 3.6174+\hat{\mathbf{z}} .2240 .
\end{aligned}
$$

So, with geocentric coordinates,

$$
\begin{align*}
\overrightarrow{\mathbf{P}}_{2} & =\overrightarrow{\mathbf{P}}_{2}-\overrightarrow{\mathbf{O}}+\overrightarrow{\mathbf{O}} \\
& =\hat{\mathbf{x}} .7533+\hat{\mathbf{y}} .2124+\hat{\mathbf{z}} .2240 \tag{A.27}
\end{align*}
$$

To get $\overrightarrow{\mathbf{P}}_{2}$ 's coordinates in terms of the EIREB200 antenna plane, the coordinates of Eq. (A.27) are translated to the EIREB200 aimpoint and then operated on by $U$ of Eq. (A.20). This gives

$$
\begin{aligned}
\mathbf{P}_{2_{B^{\prime}}} & =\mathrm{U}\left[\begin{array}{c}
(.7533-.6845) \\
(.2124-.0036) \\
(.2240-.7290)
\end{array}\right] \\
& =\left[\begin{array}{ccc}
.0989 & -.0677 & .9927 \\
.5647 & .8253 & 0 \\
-.8193 & .5606 & .1199
\end{array}\right]\left[\begin{array}{c}
.0688 \\
.2088 \\
-.5050
\end{array}\right] \\
& =\left[\begin{array}{c}
-.5086 \\
.2112 \\
.0001
\end{array}\right]
\end{aligned}
$$

Rounding off gives, in antenna plane coordinates,

$$
\overrightarrow{\mathbf{P}}_{2}=-\hat{\mathbf{x}}^{\prime} .5086+\hat{\mathbf{y}}^{\prime} .2112
$$

Rotating to ellipse coordinates via Eq. (A.23),

$$
\begin{aligned}
x^{\prime \prime} & =-.5086 \cos 145^{\circ}-(-.2112) \sin 145^{\circ} \\
& =.2955 \\
y^{\prime \prime} & =-.5086 \sin 145^{\circ}+(.2112) \cos 145^{\circ} \\
& =-.4647
\end{aligned}
$$

Finding the $x^{\prime \prime}$ and $y^{\prime \prime}$ coordinates which satisfy the ratio

$$
\frac{x^{\prime \prime}}{y^{\prime \prime}}=\frac{.2955}{-.4647}=-.6359
$$

and the ellipse equation (Eq. (A.22)) will give the antenna plane intersection of the EIREB200 in direction of BEN00000's testpoint \#10. Here, assume $x^{\prime \prime}$ is positive so
that

$$
\begin{equation*}
x^{\prime \prime}=-.6359 y^{\prime \prime}, \tag{A.28}
\end{equation*}
$$

where $y^{\prime \prime}$ is negative. Plugging into the ellipse equation gives

$$
\frac{\left(-.6359 y^{\prime \prime}\right)^{2}}{.00862}+\frac{\left(y^{\prime \prime}\right)^{2}}{.0367}=1
$$

which reduces to

$$
y^{\prime \prime}=-.11612
$$

assuming the negative square-root. From Eq. (A.28),

$$
x^{\prime \prime}=(-.6359)(-.11612)=.07384
$$

Finally, the half-power angle in the direction of BENOOOOO's testpoint \#10 is

$$
\begin{aligned}
\phi_{0_{2}} & =2 \tan ^{-1} \frac{\sqrt{(.07384)^{2}+(-.11612)^{2}}}{\|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{O}}\|} \\
& =2.594^{\circ} .
\end{aligned}
$$

Referring again to satellite transmit antenna pattern \#2 with

$$
\frac{\phi_{2}}{\phi_{\mathrm{O}_{2}}}=\frac{5.190^{\circ}}{2.594^{\circ}}=2.001
$$

gives

$$
\begin{align*}
D_{S T_{d B}}\left(\phi_{2}, \phi_{\mathrm{U}_{2}}\right) & =-[22+20 \log (2.001)] \\
& =-28.025(\mathrm{~dB}) \tag{A.29}
\end{align*}
$$

## A.4.2 The Interference Power

In order to use the Friis transmission equation to find the interference power from EIREB200 at BENO0000, the directivity $G_{E R}(\varphi)$ must be found. $\varphi$ may not be exactly zero in this case since the dish at BENOOOOO's testpoint \#10 is not pointed directly
at the EIREB200 satellite. Let $\overrightarrow{\mathrm{O}}_{2}$ be the vector to the orbital position of BENOOOOO's satellite at $-30.6^{\circ}$ longitude.

$$
\begin{aligned}
\overrightarrow{\mathbf{O}}_{2} & =6.6105\left[\hat{\mathbf{x}} \cos \left(-30.6^{\circ}\right)+\hat{\mathbf{y}} \sin \left(-30.6^{\circ}\right)\right] \\
& =\hat{\mathbf{x}} 5.690-\hat{\mathbf{y}} 3.365 .
\end{aligned}
$$

The off-axis angle $\varphi$ to EIREB200's satellite, as seen from the BENOOOOO testpoint is

$$
\varphi=\cos ^{-1} \frac{\left\langle\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}_{2}\right\rangle}{\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}\right\|\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}_{2}\right\|}
$$

Since

$$
\begin{aligned}
\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}_{2} & =\hat{\mathbf{x}}(.9757-5.690)+\hat{\mathbf{y}}(.0486+3.365)+\hat{\mathbf{z}}(.2139) \\
& =-\hat{\mathbf{x}} 4.7143+\hat{\mathbf{y}} 3.4136+\hat{\mathbf{z}} .2139 \\
\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}_{2}\right\| & =\sqrt{(-4.7143)^{2}+(3.4136)^{2}+(.2139)^{2}} \\
& =5.824 \text { (earth radii) }
\end{aligned}
$$

and

$$
\begin{aligned}
\left\langle\overrightarrow{\mathrm{T}}_{2}-\overrightarrow{\mathbf{O}}, \overrightarrow{\mathrm{T}}_{2}-\overrightarrow{\mathrm{O}}_{2}\right\rangle & =(-4.6903)(-4.7143)+(3.4536)(3.4136)+(.2139)(.2139) \\
& =33.946 \text { (earth radii) }
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\varphi & =\cos ^{-1} \frac{33.946}{(5.8286)(5.824)} \\
& =0.452^{\circ} .
\end{aligned}
$$

From WARC88 then,

$$
\begin{align*}
G_{E R_{d B}} & =10 \log \left[\eta(\pi D / \lambda)^{2}\right]-0.0025(D \varphi / \lambda)^{2} \\
& =10 \log \left[.7\left(\frac{3 \pi}{\left(3 \times 10^{8}\right) /\left(11.2 \times 10^{9}\right)}\right)^{2}\right]-0.0025\left[\frac{(3)(0.452)}{\left(3 \times 10^{8}\right) /\left(11.2 \times 10^{9}\right)}\right]^{2} \\
& =42.97 \mathrm{~dB} \tag{A.30}
\end{align*}
$$

Using the Friis transmission equation, an expression for the interference power received from the EIREB200 satellite at BENO0000's testpoint \#10 may now be found. In decibels, this expression is

$$
\begin{equation*}
I=P_{S A T_{d B}}+G_{S T_{d B}}+D_{S T_{d B}}\left(\phi_{2}, \phi_{0_{2}}\right)+G_{E R_{d B}}(\varphi)+10 \log \left(\lambda^{2}\right)-10 \log \left[(4 \pi L)^{2}\right] \tag{A.31}
\end{equation*}
$$

where $P_{S . A T_{d B}}$ is given by Eq. (A.26), $G_{S T_{d B}}$ is known to be $36.7(\mathrm{~dB}), D_{S T_{d B}}\left(\phi_{2}, \phi_{\mathrm{O}_{2}}\right)$ comes from Eq. (A.29) and $G_{E R_{d B}}(\varphi)$ is given by Eq. (A.30) above. Plugging these results into Eq. (A.31) gives finally,

$$
\begin{align*}
I= & 1.91+36.44-28.025+42.97+10 \log \lambda^{2}-10 \log \left[(4 \pi L)^{2}\right] \\
= & 53.295+10 \log \left[\left(3 \times 10^{8} / 11.2 \times 10^{9}\right)^{2}\right] \\
& -10 \log \left[\left(4 \pi\left\|\overrightarrow{\mathbf{T}}_{2}-\overrightarrow{\mathbf{O}}\right\|\right)^{2}\right] \\
= & 21.853-10 \log \left[\left(4 \pi\left[5.8286 \times 6.3787 \times 10^{6}\right]\right)^{2}\right] \\
= & -151.54 \mathrm{~dB} \tag{A.32}
\end{align*}
$$

## Appendix B

## Files Recorded on Tape

## Programs

1. Ellipse preprocessor used to generate ellipse file in OSU format from NASA ellipse data.
2. Scenario preprocessor used to generate scenario data file from ORBIT data files.
3. DELTA program to calculate required satellite separations.
4. SLOT program to find a satellite positioning/ordering that is likely to place satellites near a specified or default desired location.
5. TOLS program to find a satellite positioning/ordering that is likely to place satellites as far as possible from a specified or default location.
6. STARS program to find feasible allotment plans.
7. MISOUP program that performs carrier to interference analysis of allotment plans.
8. PLOTCOI program which generates a sorted list of aggregate total link $C / I$ values.
9. PLOTELEV program which generates the worst case satellite elevation angles and maximum attainable satellite elevation angles.

## Input Files

10. Existing system power data file satellite powers for existing system satellites.
11. Ellipse key file (See Appendix C.)
12. Ellipse/rain attenuation data file (See Appendix C.)
13. Scenario data file for Ku band, allotment systems only (See Appendix C.)
14. Scenario data file for Ku band, allotment and existing systems (See Appendix C.)
15. Scenario data file for C band, allotment systems only (See Appendix C.)
16. Scenario data file for $C$ band, allotment and existing systems (See Appendix C.)
17. Required separation matrix (See Appendix C.)
18. Satellite arc file (See Appendix C.)
19. Satellite position/order file (See Appendix C.)
20. Permutation file input file for STARS program.
21. $C / I$ analysis output file (See Appendix C.)
22. $C / I$ vs. cumulative percentage output file (See Appendix C.)
23. Elevation angle output file (See Appendix C.)

## Programs

24. SLOT with binary search (See Appendix D.)
25. TOLS with binary search (See Appendix D.)

Tape Parameters:<br>Recording Density - 1600 bpi<br>Format - EBCDIC<br>Record length - 80 characters<br>Blocksize - 800 characters<br>Labels - standard

## Appendix C

## Description of Files

## C. 1 Ellipse/rain Attenuation File

## - Input to DELTA and MISOUP

This file contains the ellipse and rain attenuation data for all administrations. In a scenario including only a few administrations DELTA and MISOUP will select the appropriate data from this file. The Ellipse/rain attenuation file should, therefore, not require editing.

This file is generated by the ellipse preprocessor program from the NASA ELLIPSE.DATA and CONFKEY2.DATA files. Data for each satellite longitude for an administration is contained in one record. The format for this record is listed below.

| $\frac{\text { Column }}{2-6}$ |  |
| :---: | :--- |
| Content |  |
| $7-14$ | Record number (I5) |
| $15-21$ | Satellite area name (A8) |
| $22-28$ | Boresight longitude (F7.2) |
| $29-34$ | Boresight latitude (F6.2) |
| $35-39$ | Ellipse major axis (F5.2) |
| $40-44$ | Ellipse minor axis (F5.2) |
| $45-51$ | Orientation angle (F7.2) |
| $52-57$ | Rain attenuation downlink C band (F6.2) |
| $58-63$ | Rain attenuation uplink C band (F6.2) |
| $64-69$ | Rain attenuation downlink Ku band (F6.2) |
| $70-75$ | Rain attenuation uplink Ku band (F6.2) |

The rain attenuation values in this file are obtained as the maximum of the rain attenuation values over all testpoints of the administration.

## C. 2 Ellipse Key File

## - Input to DELTA and MISOUP

This file is used by MISOUP and DELTA as an aid in inputing the ellipse data from the ellipse file. The format for this file is the same as that for ORBIT's key file.

## Column Content

2-9 Service area name (A8)
11-16 Record number of first ellipse for service area (I6)
18-23 Integer west longitude of ellipses (I6)
25-30 Integer east longitude of ellipses (I6)
32-37 Number of ellipses for service area (16)
This file must contain exactly the service areas in the ellipse file, in the order found in the ellipse file.

## C. 3 Scenario Parameter and Testpoint File

## - Input to DELTA, MISOUP,PLOTCOI, and PLOTELEV

This file contains information only for the administrations in a particular run. The first three lines contain information which is common to all administrations. Administration-specific parameters and testpoints are then listed in eight records per administration. The format for this file is listed as the following.

The order of administrations in this file must agree with the order of administrations in the ellipse/rain attenuation file. This file might contain, however, only a subset of the administrations in the ellipse/rain attenuation file. This file is generated using the scenario file generator program.

Data which either not read or not used by the current version of the OSU programs is notes with the following keys:

* data is neither read nor used by OSU programs
*     * data is read but not used by OSU programs

| Row | Column |  | Content |
| :---: | :---: | :--- | :--- | :--- |
| 1 | $1-10$ |  | Number of service areas (I10) |
| 1 | $16-25$ |  | Receiver noise bandwidth (Hz) (F10.2) |
| 1 | $26-35$ |  | Maximum rain attenuation (dB) (F10.2) |
| 1 | $36-45$ |  | Rain option flag (1=ON,0=OFF) (I10) |
| 1 | $46-55$ |  | Rain outage percentage (F10.2) |
| 1 | $56-65$ |  | Spacing between calculated ellipses (I10) |
| 1 | $66-75$ |  | Bathtub skipper flag (1=SKIP,0=NO SKIP) (I10) |
| 2 | $1-10$ | $*$ | Standardized single entry C/I criteria (dB) (F10.4) |
| 2 | $11-20$ | $*$ | Standardized earth antenna side-lobe slope (dB) (F10.4.) |
| 2 | $21-30$ | $*$ | Total available arc west limit (deg) (F10.4) |
| 2 | $31-40$ | $*$ | Total available arc east limit (deg) (F10.4) |
| 2 | $41-50$ |  | Minimum half-power beamwidth (deg) (F10.4) |
| 2 | $51-60$ | $*$ | Searching step size (F10.4) |
| 3 | $1-10$ | $*$ | Maximum satellite transmit antenna gain (F10.4) |
| 3 | $11-20$ |  | Satellite antenna efficiency (F10.4) |
| 3 | $21-30$ |  | Earth station antenna efficiency (F10.4) |
| 3 | $31-40$ | $*$ | Difference between single entry and agg C/I (F10.4) |
| 3 | $41-50$ | $*$ | Minimum elevation angle (deg) (F10.4) |
| 3 | $51-60$ | $*$ | Minimum power level for earth transmit (F10.1) |
| 3 | $61-70$ | $*$ | Minimum power level for sat transmit (F10.1) |



## C. 4 Required Separation Matrix (-matrix)

## - Output of DELTA

## - Input to SLOT/TOLS/STARS

This file contains a matrix of pairwise separations required to meet single-entry $C / I$ targets.

Example : The delta matrix for 11 satellites would have the format below.

| 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 | 1,8 | 1,9 | 1,10 | 1,11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,3 | 2,4 | 2,5 | 2,6 | 2,7 | 2,8 | 2,9 | 2,10 | 2,11 |  |
| 3,4 | 3,5 | 3,6 | 3,7 | 3,8 | 3,9 | 3,10 | 3,11 |  |  |
| 4,5 | 4,6 | 4,7 | 4,8 | 4,9 | 4,10 | 4,11 |  |  |  |
| 5,6 | 5,7 | 5,8 | 5,9 | 5,10 | 5,11 |  |  |  |  |
| 6,7 | 6,8 | 6,9 | 6,10 | 6,11 |  |  |  |  |  |
| 7,8 | 7,9 | 7,10 | 7,11 |  |  |  |  |  |  |
| 8,9 | 8,10 | 8,11 |  |  |  |  |  |  |  |
| 9,10 | 9,11 |  |  |  |  |  |  |  |  |
| 10,11 |  |  |  |  |  |  |  |  |  |

The required separation matrix contains only one listing per affiliated set (one listing per satellite) and is consistent with the satellite arc file in both number and order of satellites.

## C. 5 Satellite Arc File

## - Input to SLOT/TOLS/STARS

DELTA identifies the affiliated sets and creates an arc file which contains only one listing per affiliated set, with the arc of an affiliated set being the intersected arc of its members. The number of satellites will be less than the number of administrations when affiliated sets are used. The SLOT/TOLS desired arc flag indicates to SLOT whether desired locations are to be calculated or input. The default is for calculated desired locations. If desired locations are to be input they must be inserted after the east arc limit for each satellite.

The number and order of satellites listed in this file is consistent with the required separation matrix also generated by DELTA. The format for this file is indicated below.


## C. 6 Satellite Position/Order File

- Output of SLOT/TOLS/STARS
- Input to MISOUP


## - Optional Input to STARS

This file lists the satellite positions as determined using SLOT, TOLS, or STARS. The listing by SLOT and TOLS is in an east to west order so that this file may also be used as an input initial ordering for STARS.

The first line in this file is the number of feasible solutions. This indicates to MISOUP how many complete solutions are to be read from the file. SLOT and TOLS will always place a 1 on this line to serve as a default value. STARS is capable of outputting multiple feasible solutions. If a STARS output contains more than one feasible solution (the actual number is listed at the end of the file) the first line can be edited to enable analysis of more than one solution. The satellite number is determined from the order of satellites in the satellite arc file.


## C. 7 C/I Analysis Output File

## - Output of MISOUP

- Input of PLOTCOI and PLOTELEV

This file lists the results of the carrier to interference analysis for each administration, including worst single entry, aggregate, and total link aggregate values.

MISOUP has the capability of analyzing multiple feasible solutions (STARS output). The results from each analysis are listed in this file.

## C. 8 Elevation Angle Output File

## - Output of PLOTELEV

This file lists the worst-case elevation and maximum possible worst-case elevation angle for each service area in the current scenario. The format of this file is given below. Row 1 contains the number of service areas.

Each remaining row contains the information for a particular service area.

## Column Content

2-9 Service area name (A8)
14-20 Maximum possible worst-case elevation (deg) (F7.4)
25-31 Satellite worst-case elevation angle (deg) (F7.4)

## C. $9 \quad C / I$ vs Cumulative Percentage Output File

## - Output of PLOTCOI

This file contains a sorted list of the aggregate $C / I$ values. Row 1 contains the number of service areas. Each remaining row contains the information for a particular service area.

## Column Content

2-8 Cumulative percentage (values between 0 and 100) (F7.2)
12-18 Aggregate $C / I$ value (F7.2)

## C. 10 Existing System Power Data File

 - Input to SCENARIO PREPROCESSORThe existing system satellite powers listed in this file are obtained from the WARC88 documentation. The OSU programs do not currently use the actual existing satellite powers. Rather, the powers of the existing system satellites are calculated to provide the minimum downlink $C / N$ at the worst case testpoint.

## Appendix D

## Description of Programs

## D. 1 DELTA

This program calculates the minimum required separation between two satellites that will guarantee that a specified single-entry interference criterion is met. In performing the calculations, issues such as rain attenuation, affiliated sets, and inhomogenities are addressed. In the present version of this program there is a option which directs DELTA to calculate required separations for longitudes only at the extremes of a satellites feasible arc. Required separations for satellite positions at these extremes are typically larger than for satellite positions closer to the center of the arc. The required separation for a pair of satellites is the maximum, over all tested longitudes, of the required separations at each test longitude. Testing only the extremes of the satellite feasible arcs results in a large computational savings.

Since existing satellites are fixed, the required separation output by DELTA for a pair of existing systems is zero. For existing systems having only a downlink or only an uplink beam the total link single entry $C / I$ is simply the uplink or downlink $C / I$ respectively. The satellite and earth station powers are determined based on uplink and downlink $C / N$ criteria.

Some of the testpoints in the WARC data set fall outside of the specified ellipses. Since satellite powers are calculated to provide the minimum $C / N$ criteria for the worst case testpoint of its service area, the satellite transmitter powers for the ad-
ministrations with outlying testpoints are higher than would normally be expected. To cope with the peculiarities in the ellipse/testpoint data the magnitude of the maximum transmit discrimination from a satellite to its own testpoint is taken to be 3 dB .

## D. 2 SLOT (Satellite Location and Ordering Technique)

This program is designed to find an ordering of satellites that can be used to initialize our synthesis program STARS (see description below). The solution found using SLOT may also be analyzed directly by MISOUP. The program attempts to enforce the angular separation requirements determined by DELTA and successively positions the satellite with the fewest remaining feasible longitudes at the longitude nearest to its desired location. (For each satellite, the feasible longitudes are one decidegree apart.) If the program is unsuccessful at satisfying all separation requirements, the required separations are multiplied by 0.9 , and the process is repeated. In such a case, the ordering may not be feasible, but it is determined using separations that are proportional to the actual required separations.

The tape also includes a modified SLOT program which incorporates a binary search in a scheme to modify required separations. With each iteration over the binary search variable, $\delta$, the required separations used by SLOT, $\Delta_{i j}^{\prime}$, are given by, $\Delta_{i j}^{\prime}=\Delta_{i j}^{\prime}+\delta$, where the, $\Delta_{i j}^{\prime}$, are the required separations calculated by DELTA. The routine searches between $\delta=-10.23$ and $\delta=10.23$. Initially $\delta=0.0$. This method allows satellite separations to be increased when a feasible solution exists, and required separations to be reduced when SLOT cannot successfully determine a feasible solution.

## D. 3 TOLS

TOLS is a modification of SLOT in which the satellite with the fewest remaining feasible locations is positioned at the longitude farthest from the specified or default "undesired" location rather than nearest the desired location. When a feasible solution is not found the required separations are multiplied by 0.9 and the process is repeated.

A binary search version of TOLS has also been provide on the tape.

## D. 4 STARS (Synthesis Technique for Allotting Resources to Satellites)

This program is designed to find an allotment plan that allots longitudes to satellites that are as near as possible to their desired locations, in total. The program begins by ordering the satellites. An ordering can be provided from SLOT or TOLS, for example, or a default ordering by desired locations can be used. The allotment plan for the initial ordering is solved by solving a linear program. Then the order of the satellites is modified by permuting the members of small groups of adjacent satellites. Each new ordering is evaluated and is compared to the best ordering found so far in terms of the total deviations between the satellites' allotted and desired locations. Improved solutions are recorded. The process continues until it is determined that no permutation of a small group of adjacent satellites will provide an improved solution. The feasible solutions found are recorded so that they maybe analyzed by MISOUP.

## D. 5 MISOUP

This program, which is modeled after the analysis program SOUP, is designed to analyze the feasible synthesis solutions found using SLOT, TOLS, or STARS. DELTA addresses rain attenuation, affiliated sets, and inhomogeneous systems. For each al-
lotment plan, it reports the worst single-entry and aggregate carrier-to-interference ratio for each satellite on the downlink, the uplink, and the total link. For existing systems having only a downlink or only an uplink beam the total link $C / I$ is simply the aggregate downlink or aggregate uplink $C / I$ respectively. The satellite and earth station powers are determined based on uplink and downlink $C / N$ criteria.

Some of the testpoints in the WARC data set fall outside of the specified ellipses. Since satellite powers are calculated to provide the minimum $C / N$ criteria for the worst case testpoint of its service area, the satellite transmitter powers for the administrations with outlying testpoints are higher than would normally be expected. To cope with the peculiarities in the ellipse/testpoint data the magnitude of the maximum transmit discrimination from a satellite to its own testpoint is taken to be 3 dB .

## D. 6 PLOTELEV

This program determines the worst-case elevation angle of a satellite. The elevation angle is worst case with respect to the testpoints of the service area. Testpoints for the service area are read from the scenario data file. The satellite longitudes are obtained from the MISOUP output file. This program also determines the maximum possible worst-case elevation angle for the satellite within the arc specified in the scenario data file. The worst-case elevation angle and maximum possible worst-case elevation angle are output to the file PLOTELEV.DAT. These results can be plotted in a $X$ vs. $Y$ scatter plot to give an indication of how the worst-case elevation angle for the satellites compare with the maximum possible worst-case elevations.

## D. 7 PLOTCOI

This program reads the aggregate $C / I$ values from the MISOUP output file, sorts them from smallest to largest, and outputs a file PLOTCOI.DAT with the $C / I$ 's vs
cumulative percentage. The data in this file may be plotted as aggregate $C / I$ vs cumulative percentage to give an indication of the aggregate $C / I$ performance of an allotment.

## Appendix E

## Program Interaction, Files, and Unit Numbers



```
9- Scenorio file
10- Required separation matrix
10-Required separation
12- Satellite position/order file
13-Analysis output file
13- Analysis output f
15-Elevation angle output file
16-C/I us cumulative % output rile
```

NOTE: The numbers inside each program block are the unit numbers for the input and output files. The numbers outside of the program blocks correspond to the files listed above.


Figure E.1:
$\qquad$

```
~
```

```
\[
-
\]
```

$$
\cdots
$$

$$
-
$$

