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Comparison of Polynomial Approximations and Artificial Neural Nets for Response Surfaces in Engineering Optimization

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ABSTRACT

Engineering optimization problems involve minimizing some function subject to constraints. In areas such as aircraft optimization, the constraint equations may be from numerous disciplines such as structures, aerodynamics, environmental engineering, etc. The transfer of information between these disciplines and the optimization algorithm presents a problem. Response surfaces are a convenient way of transferring information between disciplines to the optimization algorithm. They are also suited to problems which may require numerous re-optimizations such as in multi-objective function optimization or to problems where the design space contains numerous local minima, thus requiring repeated optimizations from different initial designs. Their use has been limited, however, by the fact that development of response surfaces requires a number of initial functional evaluations either at randomly selected or preselected points in the design space. Thus, they have been thought to be inefficient compared to algorithms that sequentially perform functional evaluations closer and closer to the optimum solution. A development has taken place in the last several years which may effect the desirability of using response surfaces. It may be possible that artificial neural nets are more efficient in developing response surfaces than polynomial approximations which have been used in the past. This paper is concerned with this development.

The performance of polynomial approximations and artificial neural nets are compared on a number of test problems. Different number of designs are used to generate polynomial approximations of various orders and to generate different artificial neural nets. The quality of fit of the approximations at the designs and over the region of interest are compared with respect to the number of designs needed to develop the approximations as well as to the number of undetermined parameters associated with the approximations. For polynomial approximations, the number of undetermined parameters involved in the approximation is the number of undetermined coefficients associated with approximation. With artificial neural nets, the number of undetermined parameters is the number of weights in the net.

The problems that are considered are typical to those found in engineering applications. In Example 1, the irregular shape Banana

Function in two variables by Fox [1] is approximated by polynomials of the order 1, 2, 3, and 4 and by artificial neural nets with 1, 2, 4, 6, and 20 nodes on a hidden layer. The shape of the Banana Function, two approximations that were developed, and typical performance comparisons are given at the end of this abstract.

In Example 2, the volume of a fully stress designed 6 bar truss subject to stress, buckling, and size constraints is determined in terms of the coordinates of one node of the truss. This type of response surface could then be used to find the optimum location of that node of the truss. Response surfaces are developed for the truss volume in terms of the coordinate variables using polynomials of order 2, 3, and 4 and artificial neural nets with 3, 5, and 7 nodes on a single hidden layer. Performance of the approximations are compared over a region of interest. The truss in question, the shape of the function being approximated, and a typical performance comparison is given at the end of this abstract.

The first two examples consider approximations of complicated functions in two variables. The third example is concerned with approximations of a less complicated function in 4 variables. A 35 bar truss is considered. The area of the bottom chord is taken to be A_1, the area if the top chord to be A_2, the area of the verticals and the diagonals to be A_3, and the height of the truss to be H. A response surface for the stress in one of the lower chord members is developed in terms of these variables. Polynomials of order 1 and 2 and artificial neural nets with 1, 2, and 3 nodes on the hidden layer are considered. The 35 bar truss and a table comparing the approximations is given at the end of this abstract.

In the fourth example, the same truss is considered but in this case a response surface for the stress in a member of the bottom chord is developed in terms of 15 area variables. A table comparing the approximations is given at the end of this abstract.

This paper yields valuable information as to the number of training sets required for the two types of approximations and to their relative performance. First, with both types of approximations, it was found that it is desirable to use at least 100% more training sets than the number of associated undetermined parameters. Secondly, it was found that performance is controlled by the number of undetermined parameters associated with the approximation.

Currently, selection of artificial neural nets and the number of designs used to train them is done largely by trial and error. Based on the above findings, this paper develops simple rules which can be used to make a reasonable selection of a neural net and the number of training designs required to train it.

References

1. Fox, R.L., Optimization Methods for Engineering Design, Addison-Weslely Publishing Company, Reading, Mass (1971).

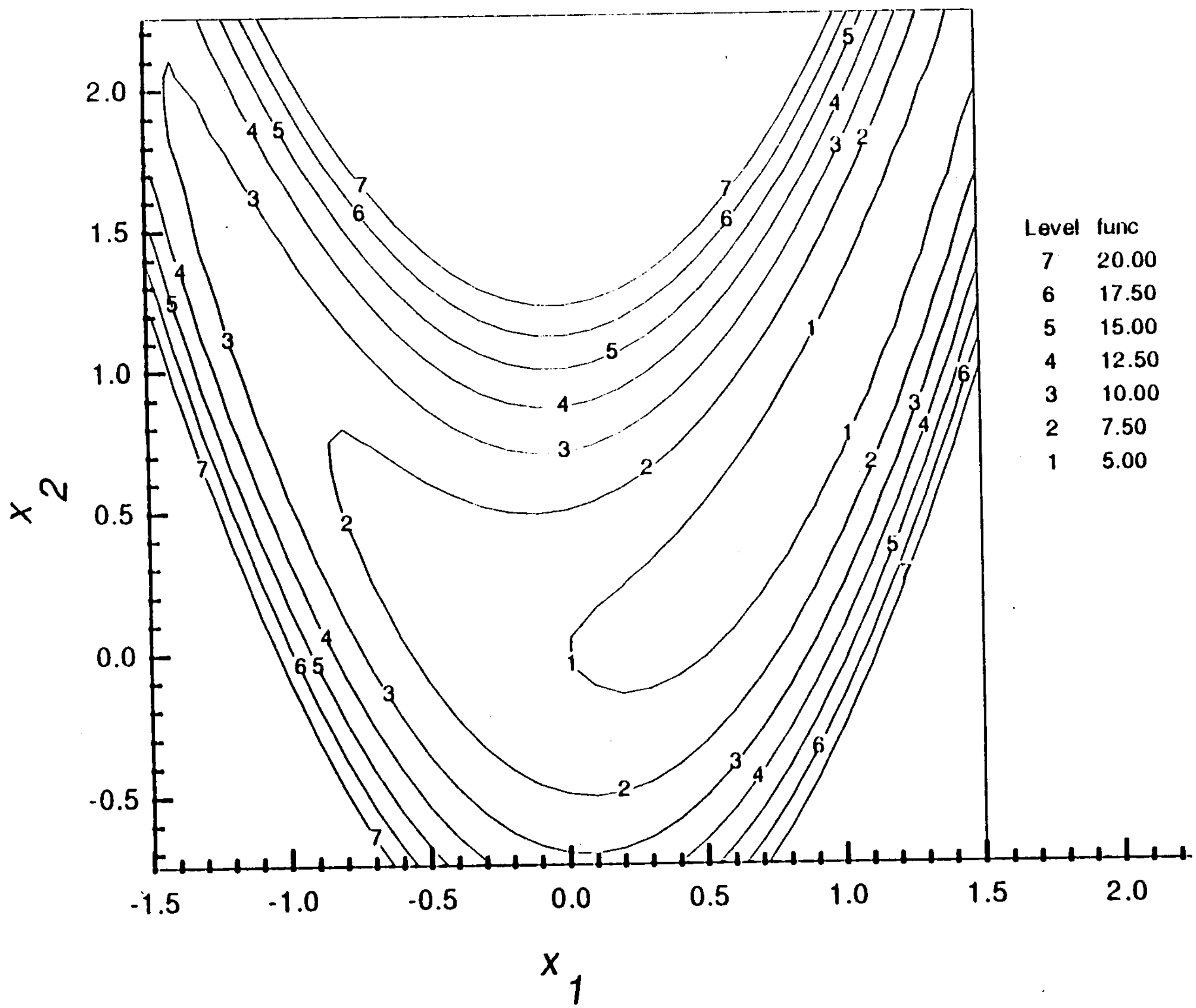

Figure 5. Fox's banana function

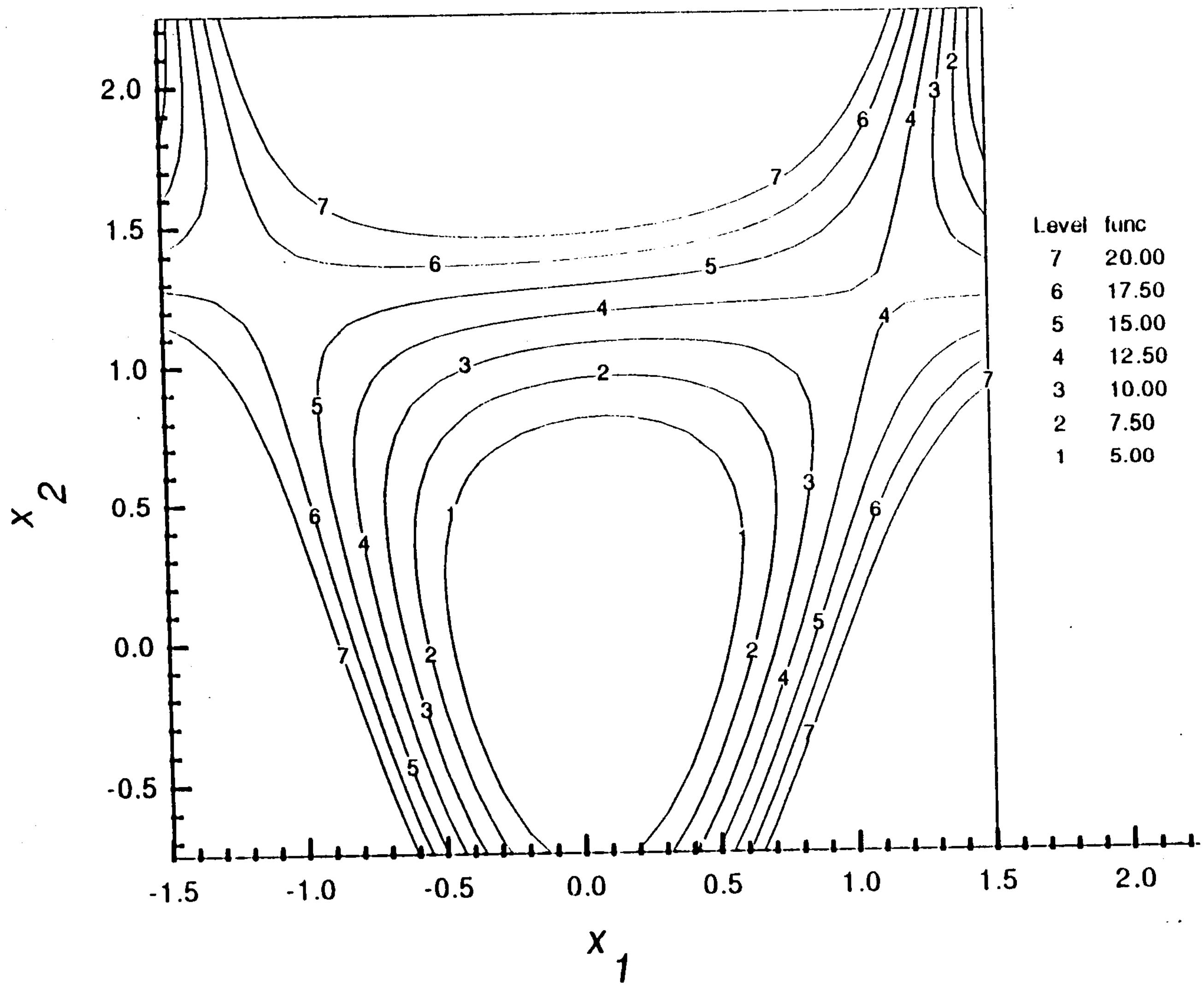

Figure 7. Third order polynomial approximation

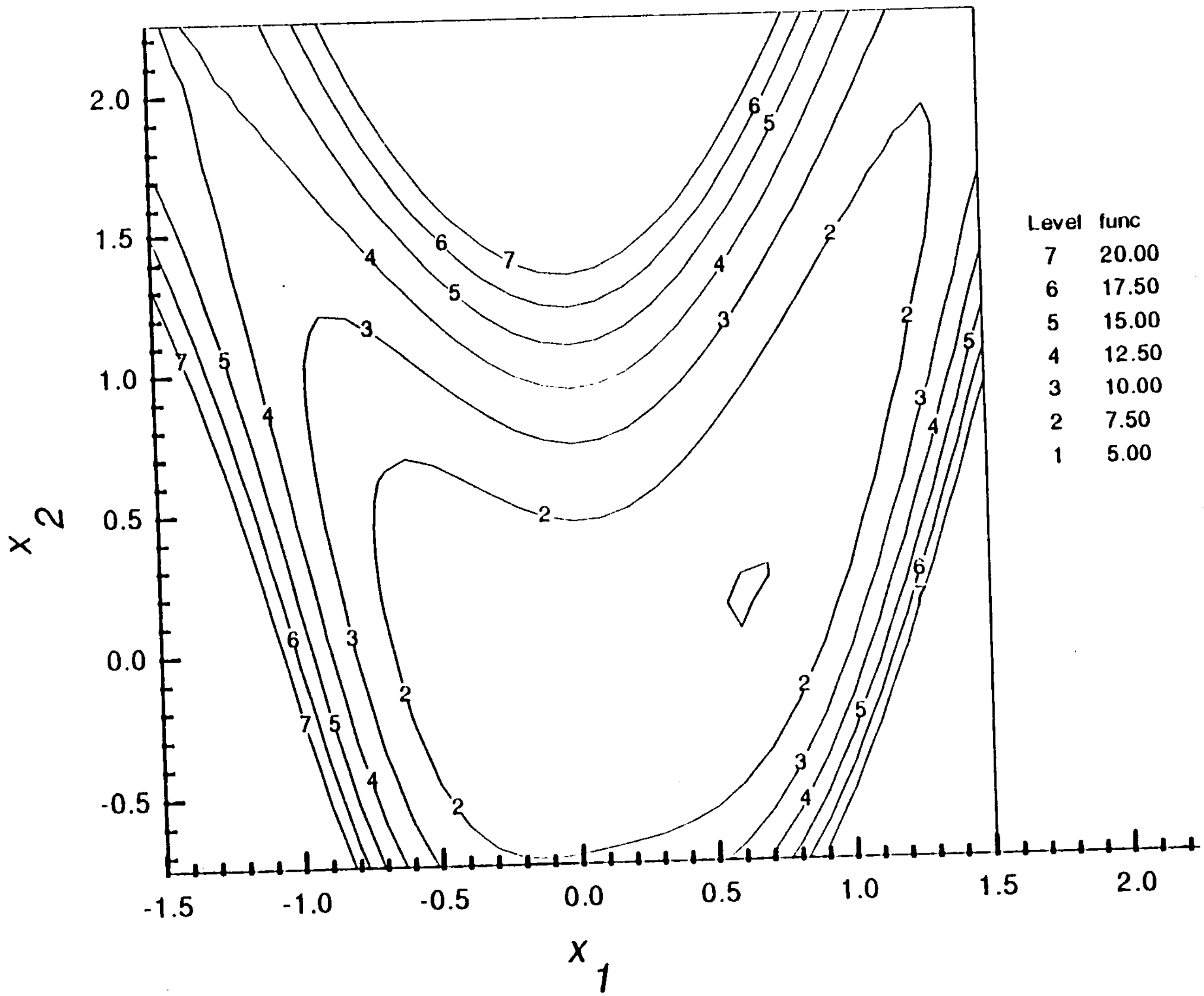

Figure 8. Artificial neural net--6 node hidden layer

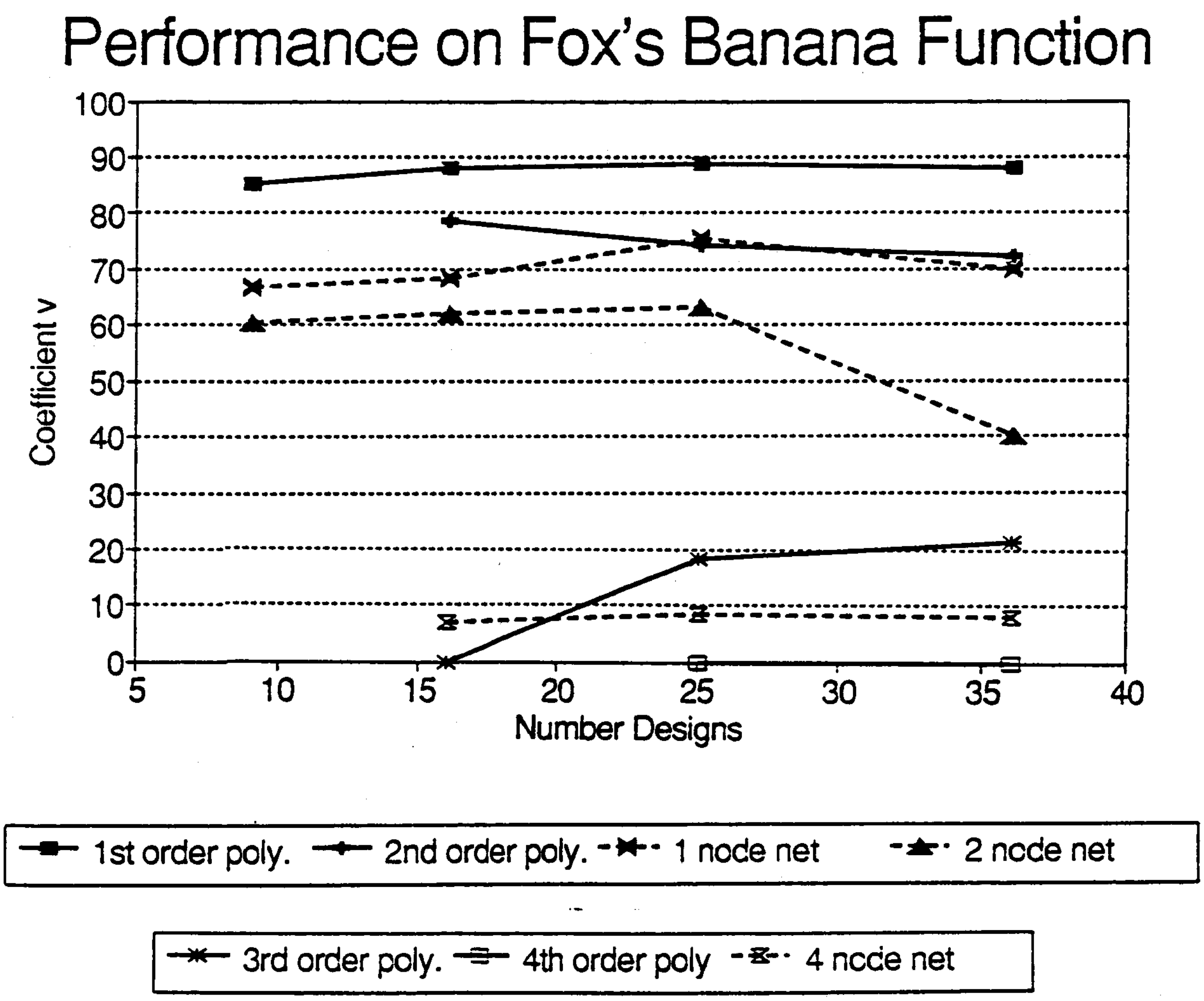

Figure 10. Non-dimensional RMS error

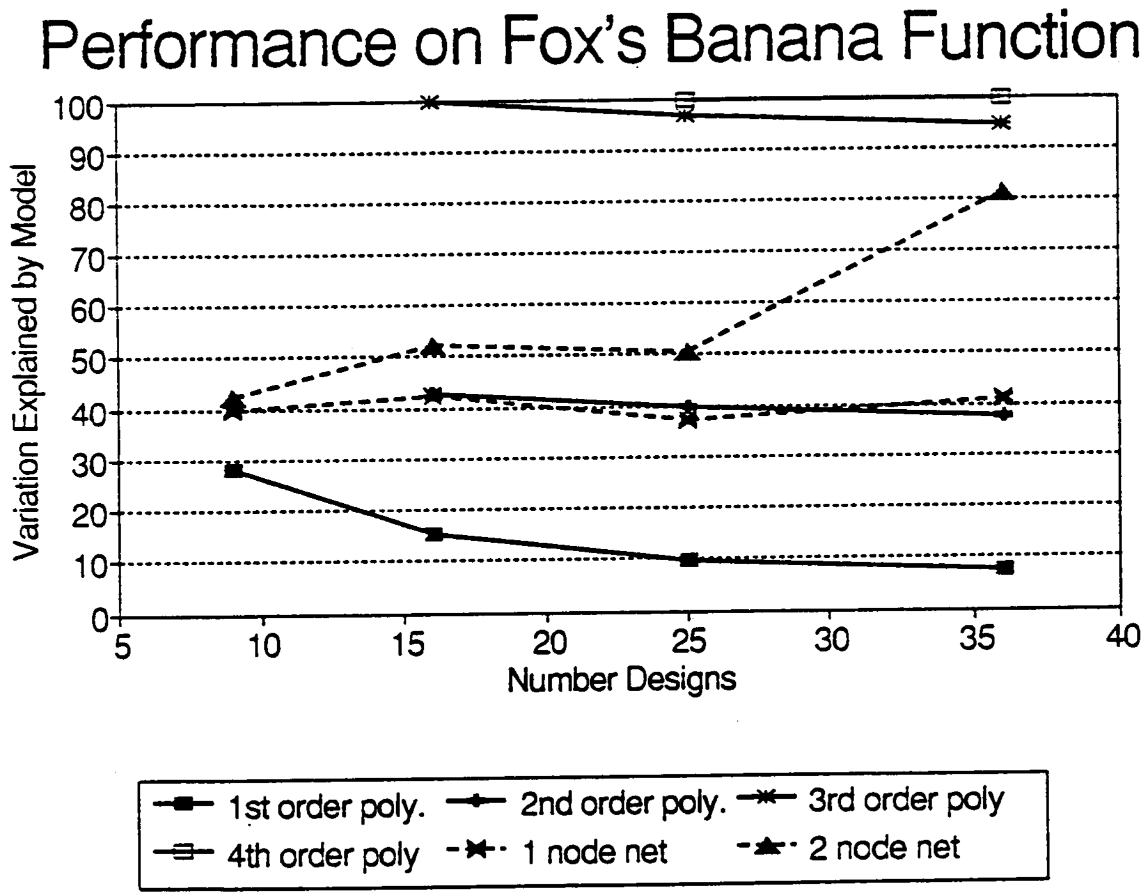

Figure 11. Variation of function explained by model

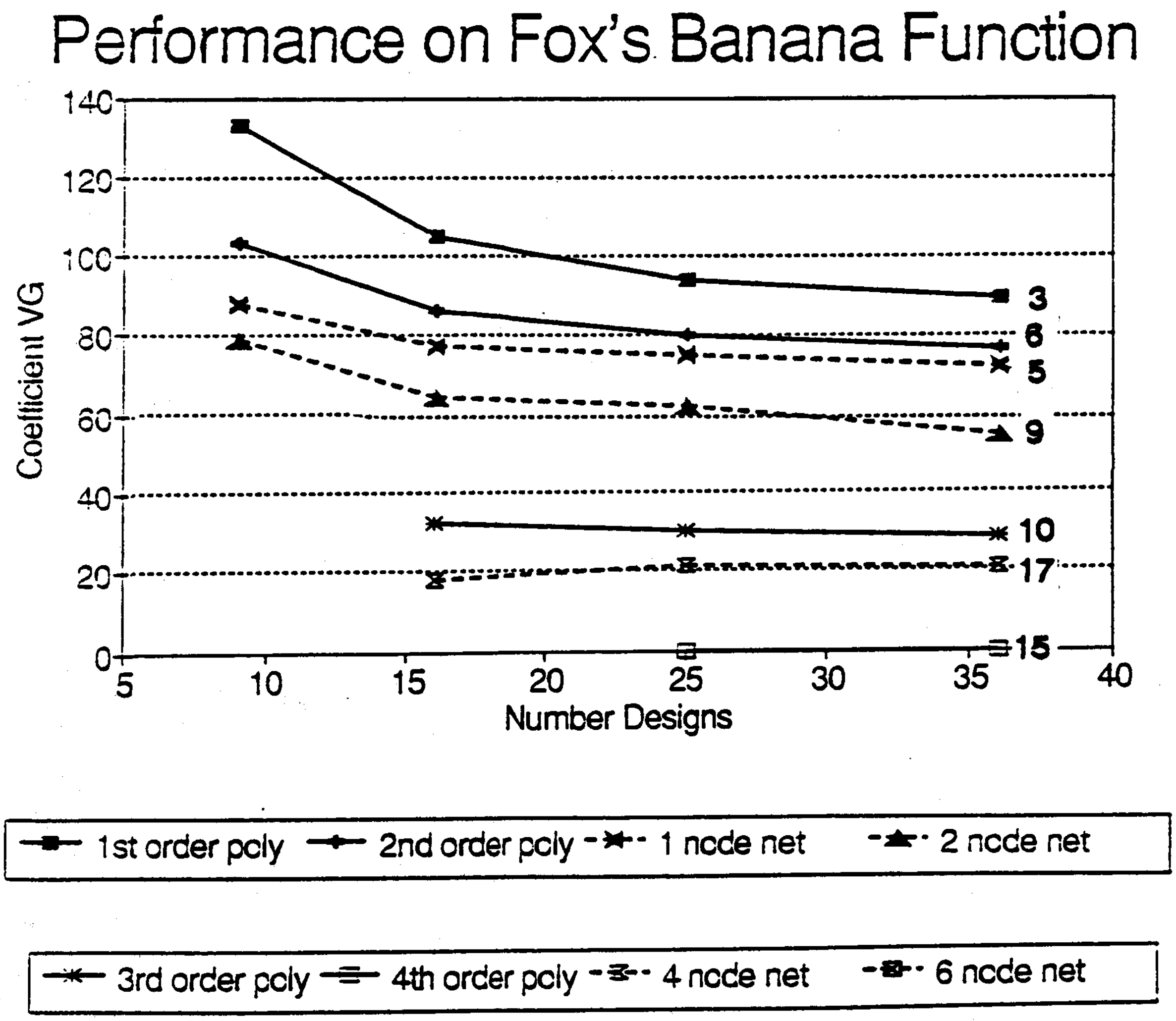

Figure 12. Non-dimensional RMS error at grid

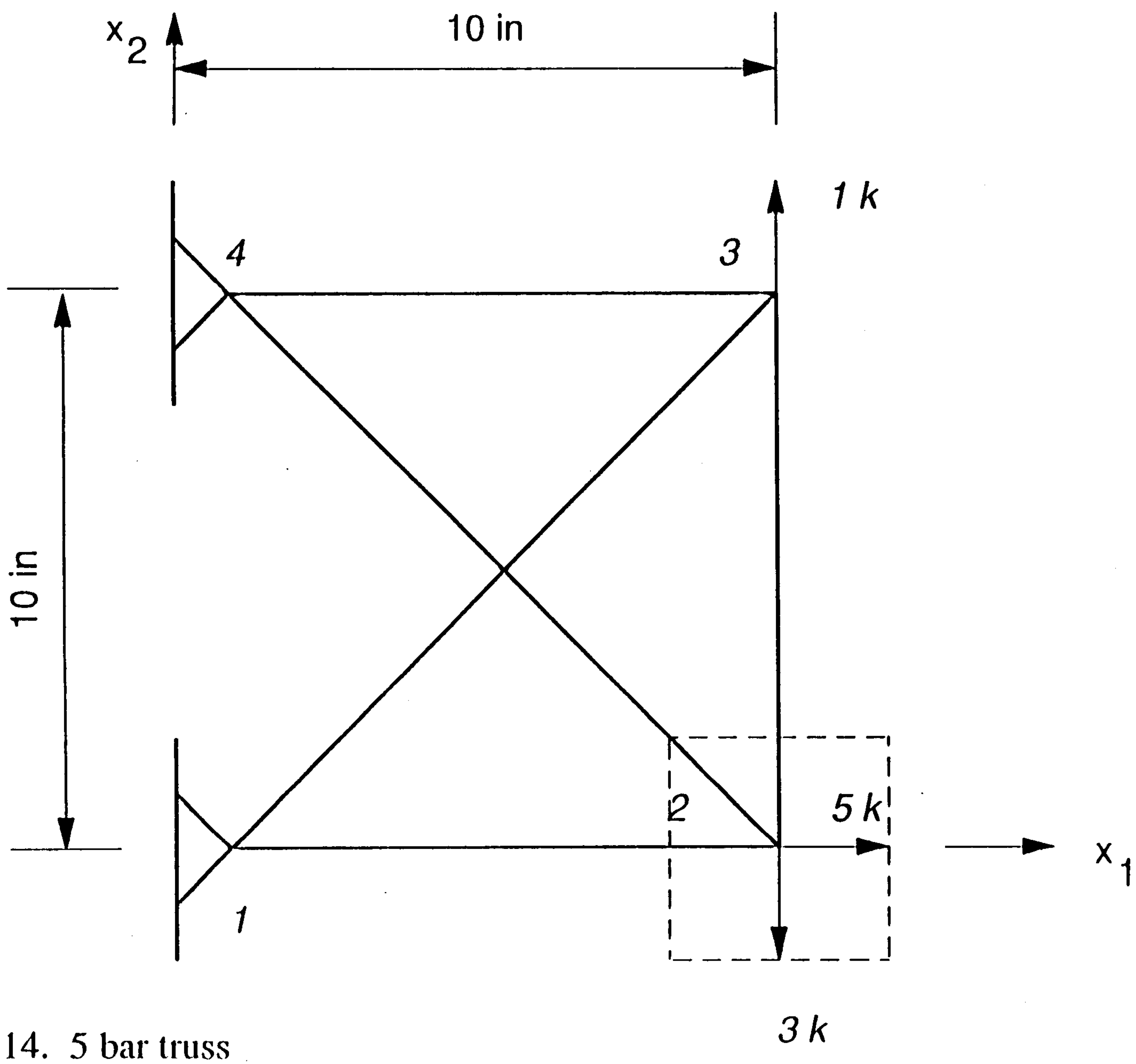

Figure 14. 5 bar truss

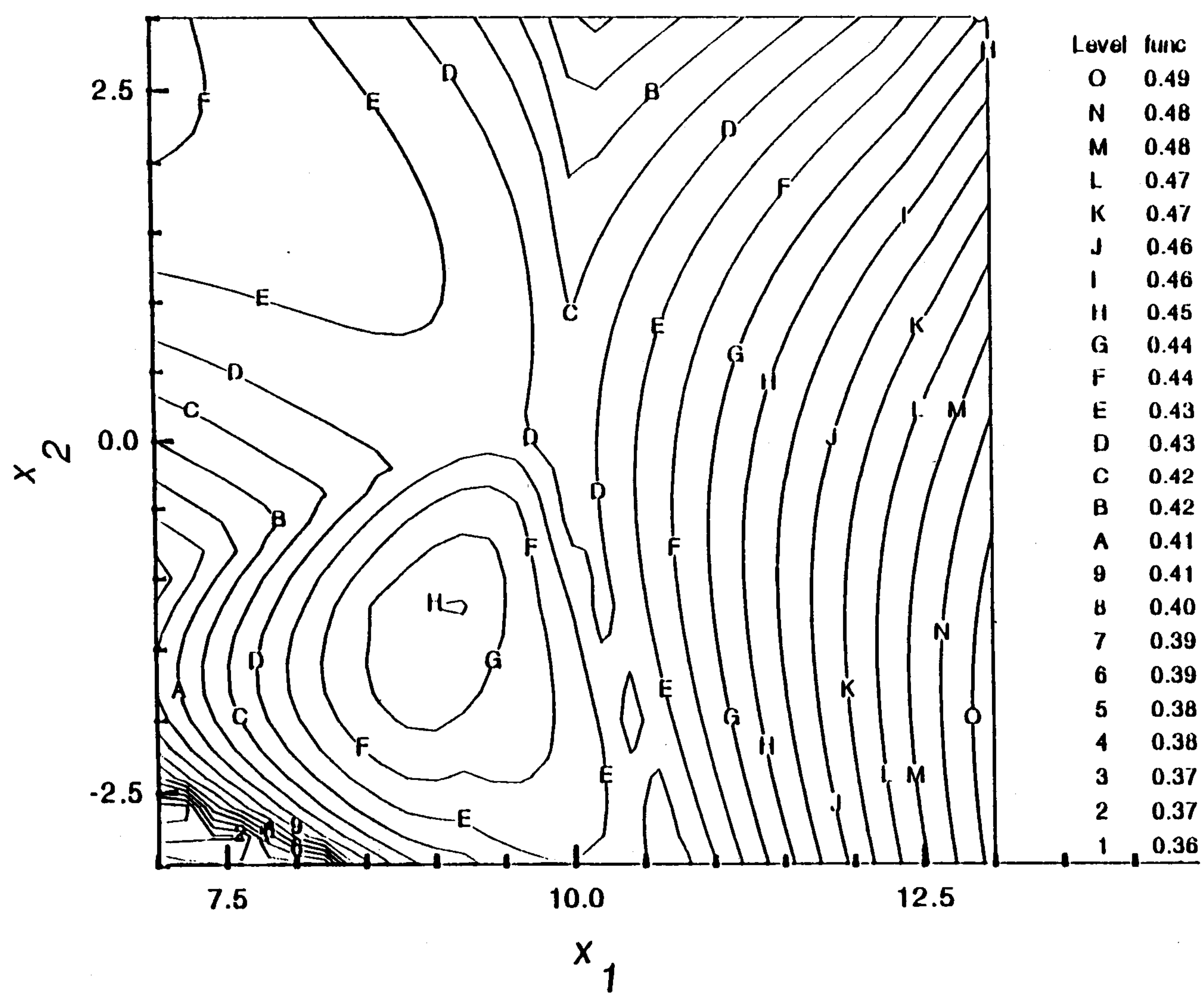

Figure 15. Optimum volumes

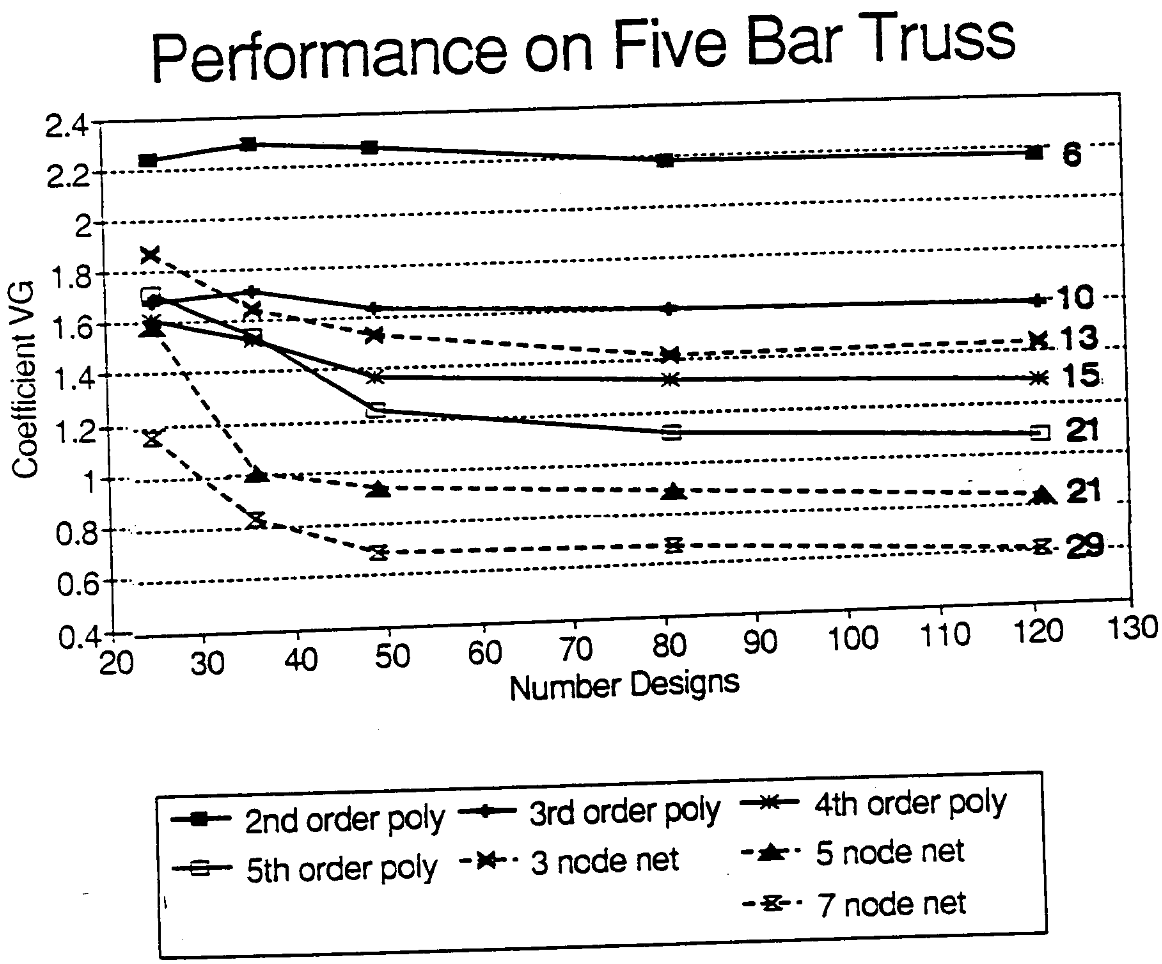

Figure 18. Performance of approximations

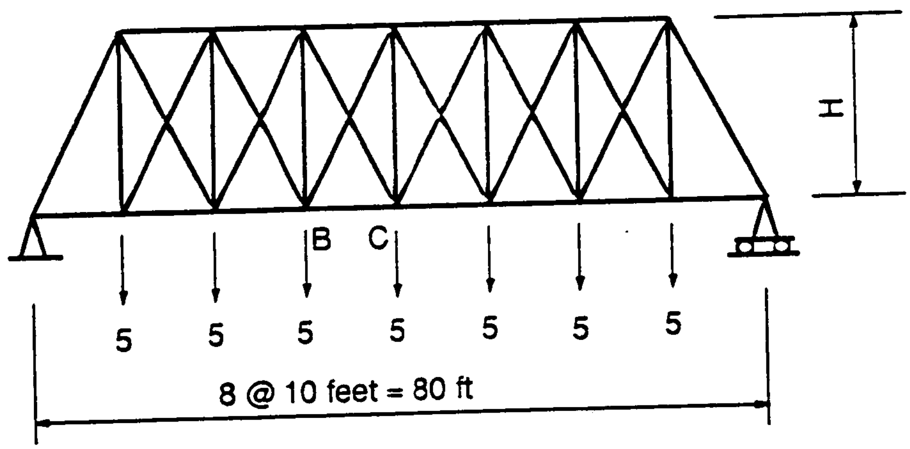

Figure 19. 35 bar truss

Table III. Performance of Approximations on the 35 Bar Truss Response Surface.

Description	Number of undetermined parameters	coefficient v_G
1st Order Polynomial	5	8.37
Neural Net, H=1	7	7.56
Neural Net, H=2	13	3.75
2nd Order Polynomial	15	2.41
Neural Net, H=3	19	2.19