Formulation of the Linear Model from the Nonlinear Simulation for the F18 HARV

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The F18 HARV is a modified F18 aircraft which is capable of flying in the post-stall regime in order to achieve superagility. The onset of aerodynamic stall, and continued into the post-stall region, is characterized by nonlinearities in the aerodynamic coefficients. These aerodynamic coefficients are not expressed as analytic functions, but rather in the form of tabular data. The nonlinearities in the aerodynamic coefficients yield a nonlinear model of the aircraft's dynamics. Nonlinear system theory has made many advances, but this area is not sufficiently developed to allow its application to this problem, as many of the theorems are existance theorems and that the systems are composed of analytic functions. Thus, the feedback matrices and the state estimators are obtained from linear system theory techniques. It is important, in order to obtain the correct feedback matrices and state estimators, that the linear description of the nonlinear flight dynamics be as accurate as possible.

The nonlinear simulation is run under the Advanced Continuous Simulation Language (ACSL). The ACSL simulation uses FORTRAN subroutines to interface to the look-up tables for the aerodynamic data. ACSL has commands to form the linear representation for the system. This is a two step process. The first step is to trim the system, which is identical to trimming the aircraft. This involves calculating the trim input, u^*, for a given trim state, x^*, such that the derivatives of the state vector, $\dot{x}$, are zero. The state space for the simulation is $x = (u, w, q, \theta)^T$ The second step is to calculate the Jacobians of the state transisiton map, $f(x)$ with respect to x at x^*, and the input map, $g(x, u)$ with respect to u at x^* and u^*. For analytic functions this is accomplished by calculating the partial derivatives of the functions, but since analytic functions do not exist, and ACSL can not perform symbolic calculations, the Jacobians are formed by numeric differentiation. ACSL uses the central difference method to perform the numeric differentiation.

$$\frac{\partial f_i(\mathbf{x})}{\partial x_j}\Big|_{\mathbf{x}=\mathbf{x}^*} = \frac{f_i(\mathbf{x}^* + \delta_j \hat{\mathbf{e}}_j) - f_i(\mathbf{x}^* - \delta_j \hat{\mathbf{e}}_j)}{2\delta_j} \tag{1}$$

Where the perturbation vector is defined as $\delta = (U\cos(\alpha)/100, U\sin(\alpha)/100, 5°/s, 2°)^T$. This is used to calculate the A matrix for the linear representation, $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$ with $\tilde{x} = x - x^*$ and $\tilde{u} = u - u^*$. Similarly the B matrix is calculated by the numerical differentiation of $g(\cdot)$. The result of this is that the A and B matrices are not only functions of x^* and u^*, but also of δ.

The state transition map, $f(x)$, and the input map, $g(x, u)$, are derived from the equations of motion for the system. A predominant part of these equations are based on the aerodynamic forces and moments. These forces and moments are calculated via the standard methods by using a nondimensional coefficient. These coefficients are the results of wind tunnel testing and they are stored in data files. The data points are generally functions of two variables, α and Mach number. Occasionally, a third variable is added such as control surface deflection. For this work, emphasis was placed on the α-Mach space variables. There are different sets of α vaules and Mach number values at which the coefficients are known. For example, C_{m_0} are known for $\alpha \in \{-12., -4., -2., 0., \ldots\}$ and Mach number $\in \{0.2, 0.6, 0.8, 0.85, \ldots\}$ but C_{m_q} are known for $\alpha \in \{-4., 0., 4., \ldots\}$ and Mach number $\in \{0.2, 0.6, 0.8, 0.9, \ldots\}$. To calculate the particular coefficient, with α and Mach number either equal or not equal to values in the respective sets, a second order Lagrange interpolation is performed. If the desired α and M are both equal to elements in the respective sets, then the Lagrange interpolation yields the coefficient from the table. This is accomplished first by bracketting

the α and Mach number, M, between availible α_i and M_j, such that $\alpha_i < \alpha \leq \alpha_{i+1}$ and $M_j < M \leq M_{j+1}$. Letting $\Delta\alpha = \alpha_{i+1} - \alpha_i$ and $\Delta M = M_{j+1} - M_j$, and $C_{i,j}$ be the coefficient C at α_i and M_j, the following expression yeilds $C(\alpha, M)$.

$$C(\alpha, M) = \frac{C_{i,j}(\alpha_{i+1} - \alpha)(M_{j+1} - M) + C_{i,j+1}(\alpha_{i+1} - \alpha)(M - M_j)}{\Delta\alpha\Delta M} +$$

$$\frac{C_{i+1,j}(\alpha - \alpha_i)(M_{j+1} - M) + C_{i+1,j+1}(\alpha - \alpha_i)(M - M_j)}{\Delta\alpha\Delta M} \tag{2}$$

It can be seen from the form of Eq. 2 that for a constant α or M, the plot for the resulting one variable function $C(\cdot)$ is a series of connected straight line segments. Calculation of the various aerodynamic coefficients, not including coefficients in $\dot\alpha$, and summation into the total force and moment calculation are perform by the FORTRAN subroutine SFAERRF. SFAERRF takes into account the various sets of α and M that are availible.

When the system is linearized about some trim point and the eigenvalues are compared to those of the DMS simulation, some differences arise. The eigenvalues also exhibited a dependence on δ. With the need for accurate linear representations for the system, the following work addressed the problem of how to minimize the differences of the two systems.

Due to the dependence on the perturbation size, δ, in the numerical calculation of the Jacobian, elimination of the numerical differentiation would address this problem and possibly assist in minimizing the differences between the two models. The concept was to express the tabular data in a known functional form, that could have its derivative calculated analytically by a FORTRAN subroutine. Various possible functional forms were examined, such as the fourth order Lagrange, full order Lagrange, and quadratic, cubic, quartic splines. These functions were C^1 or greater over the entire α-Mach space, being at least C^1 on the boundaries of different α-Mach cells. The full order Lagrange interpolation and the various splines produced oscillations that were not desirable. The fourth order Lagrange interpolation required a data window, which developed discontinuities in the derivatives at the window boundaries. Applying these techniques to the many data tables during each iteration would be computer intensive, and discontinuities would still exist in the derivatives on the cell boundaries.

An intermediate approach has been followed. Numerical differentiation has been retained with a larger perturbation step size, and the data space for "sparcely" distributed data has been augmented. The larger perturbation step size is opposite to the expected limit in calculating a derivative. The use of the increased perturbation size has the effect of decreasing the magnitude of the second derivative of the particular coefficient. The perturbation step size has been set equal to one half of the minimum parameter step size, in this study $\delta_\alpha = 1°$. The α perturbation which is useful for operations with the aerodynamic data space, but α is not a state space variable but it is related to the state variable w. A function $\delta_w = f(\delta_\alpha)$ was calculated and included in the formulation of the linear model. The augmenting of the data space also provides the for the minimization of the second derivative. The data space is augmented by calculating intermediate values for the particular coefficient by using a fourth order Lagrange interpolation. For example for a function of one variable defined at $\alpha = 0, 4, 8, 12, \ldots$, a value of the function can be estimated at $\alpha = 2, 6, 10, \ldots$ by using the nearest four data points.

These techniques have been applied in a staged way to the ACSL simulation. The modified perturbation step size was applied to the linearization of the system, as were the unmodified perturbation step size. Then the data for C_{m_q} was augmented by using the fourth order Lagrange interpolation for intermediate data points in α. Nine different α values were set, and the longitudinal eigenvalues were calculated by each of the three methods at each α. The differences between the various longitudinal eigenvalues were small, except for two cases. The locus of the eigenvalues, as a function of α was generated as a result of this work.