Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III), Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

> A PC-BASED INVERSE DESIGN METHOD FOR RADIAL AND MIXED FLOW TURBOMACHINERY

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1. ABSTRACT

N92-13943

An Inverse Design Method suitable for radial and mixed flow turbomachinery is presented. The codes is based on the 'Streamline Curvature Concept' and is therefore applicable for current PC's from the 286/287- range.

In addition to the imposed aerodynamic constraints, mechanical constraints are imposed during the design process to ensure that the resulting geometry satisfies production considerations and that structural considerations are taken into account .

By the use of Bezier Curves in the geometric modelling, the same subroutine is used to prepare input for both aero & structural files since it is important to ensure that the geometric data is identical to both structural analysis and production.

To illustrate the method a Mixed Flow Turbine Design is shown.

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2. INTRODUCTION

The objective of this paper is to present an inverse design method which can be used on ordinary PC's.

Since the conventional design process for centrifugal and mixed flow turbomachines is an iterative one, with successive changes to the input geometry subjected to the flow analysis, it is evident that the aerodesign process takes considerable time. It can therefore be tempting to apply inverse design principles to ensure that, at the end of a computational task, the resulting geometry satisfies predetermined aero-

The method described is an 'engineering' approach to the inverse design problem where both aerodynamic and mechanical criteria are imposed.

For rotors of radial and mixed flow design, the shroud line aerodynamics is considered the most critical aerodynamically. In the present method the shroud line aerodynamics are 'imposed' and the three-dimensional geometry is evaluated under mechanical constraints dictated by structural and production considerations.

A novel exducer geometry, featuring 'Balanced Work Extraction' is part of the design procedure for Radial/Mixed Flow Turbines.

The complete 3-D Geometry of the rotor is generated in the Inverse Design method. Through extensive use of 'Bezier Curves' in the geometric modelling the same subroutine performs the 'meshing' for the FEM-analysis and prepares the input files for the structural analysis in NASTRAN FEM-system, as well as the geometry definition for production.

3. ANALYSIS

The flow equations, in the form presented here, is a Quasi 3-D ,inviscid approach to the Navier Stokes Equations. The viscous effect, however is included in an approximate way by including the Entropy term in the equations. The spanwise and streamwise effect of losses are simulated by applying a Polytropic Efficiency, which is allowed to vary spanwise.

The basis of the quasi-three-dimensional flow analysis is the division of the flow field into two types of two-dimensional surfaces, as shown in Fig.1, from Ref. 1. The S2 surface, which describes an 'average' meridional flow is governed by the meridional flow equations is described below, while the S1, or blade to blade flow is handled in chapter 3.2.

The terminology is 'commonpractice' in turbomachinery, illustrated on Fig. 2 & 3.

3.1 THE MERIDIONAL FLOW

The meridional Equilibrium Equations has been applied to Hydraulic Francis Turbine Design since early in this century. Applied to Axial Flow Turbomachinery, the equations are termed The Radial Equilibrium Equations. These two forms of the Equations are treated in numerous reports from the last several decades, and for detailed information they are referred to the in Ref. 1,2 & 3.

In the following chapter a short description of the equations is given, explaining how they are integrated into the procedure.

The meridional flow equation takes the following form

$$\frac{dV_{m}^{2}}{dl} + F * V_{m}^{2} + G = 0 \tag{1}$$

where the two terms F and G contain thermodynamic and geometric elements which are dependent on the flow solution itself. Hence an iterative solution is required. If the 1-direction Fig. 2 is normal to the meridional streamline in a vaneless region the terms in Eq. (1) simplifies to:

$$F = \frac{2}{R_c} - \frac{1}{C_p} * \frac{dS}{dl}$$
 (2)

$$G = 2 * \frac{V_u}{R} * \frac{d(R * V_u)}{dl} - 2 * \frac{dH}{dl} + \frac{1}{C_p} * (2 * H - V_u^2) * \frac{dS}{dl}$$
 (3)

Since, in the general case, the meridional streamline location is unknown, it is convenient to fix most of the calculation 'stations' as 'quasiorthogonals' for the iterative flow calculation process. (Ref.3.) while for the rotor trailing edge the code is required to handle curved calculation station.

The solution of Eg. 1 is performed by Direct Integration

$$V_{m}^{2} = \exp -\{ \int F * dl \} * [V_{m,hub}^{2} - \int G * \exp -\{ \int F * dl \} * dl]$$
 (4.a)

$$=V_{m,hub}^{2}*e^{-\int F \cdot dl} - e^{-\int F \cdot dl}*\int G * e^{\int F \cdot dl}*dl$$
 (4.h)

where the integration is performed from hub to the streamlin in question. The constant of integration is set by the continuity equation:

shroud
$$\dot{W} = \int_{hub} 2 * \pi * R * V_m * \cos(\delta) * \rho * \tau * C_d * dl$$
(5)

where the angle (Fig.2)

the blade blockage (when inside bladerows)

$$\tau = 2 * \pi * R - Z_h * t_h \tag{7}$$

and the Discharge Coefficient C_d is basically sized to take care of boundary layer displacements effects

3.2. BLADE TO BLADE FLOW

The aerodynamic blade loading can be derived by relating the change of moment of momentum for a flow-filament to the torque exerted by the pressure difference blade-to-blade (Fig.2)

$$dp*Z_b*dn*dm*R = d\dot{W}*\frac{d(R*V_u)}{dm}*dm$$
 (8)

The filament massflow can be expressed as

$$d\dot{W} = 2 * \pi * R * V_m * \rho * dn \tag{9}$$

If the assumption (to be revised below) is made, that the flow is incompressible and linear blade-toblade, the following expression relates the velocity difference to the pressure difference $dp_{stat} = \frac{1}{2} * \rho * (W_{ss}^2 - W_{ps}^2) = 2 * \rho * \overline{W} * \Delta W$

$$dp_{\text{stat}} = \frac{1}{2} * \rho * (W_{\text{ss}}^2 - W_{\text{ps}}^2) = 2 * \rho * \overline{W} * \Delta W$$
 (10)

By introducing this expression together with equation (9) into equation (8), an approximate expression for the suction side velocity can be formulated

$$W_{ss} = \overline{W} + \Delta W = \overline{W} + \frac{\pi}{Z_b} * \frac{V_m}{\overline{W}} * \frac{d(R * V_u)}{dm}$$
 (11)

By means of the relative flow angle definition

$$\beta = \arccos(V_{\perp}/W) \tag{12}$$

equation (11) can be rearranged to give an approximate expression for the Suction Side Velocity

$$W_{ss} = \frac{V_m}{\cos(\beta)} (-/+) \frac{\pi}{Z_b} * \cos(\beta) * \frac{d(R*V_u)}{dm}$$
 (13)

In our iterative design procedure we use the above mentioned linear approximation only in the first iteration. For the subsequent iterations

$$W = W_{rr} + AI * \theta_{Pirch}^2 + BI * \theta_{Pirch}$$
 (14)

and the two constants, A1 & B1, are evaluated so that, with compressibility Eq.(8) is satisfied. By integrating the massflow density blade-to-blade input for Eq. (5) is evaluated and the difference between the S1 flow surface, and the blade surface is determined.

The above formulation is similar to the SFC-concept (Stream-Function- Coordinate) method described by Professor G.S.Dulikavich in Ref.5, however, less ambiguous due to the intended use of a PC. For the blade to blade solution

there are three areas of major concern, namely the blade inlet, the blade exit, and splitter if present. Blade Inlet

Since for radial and mixed flow turbomachinery we are normally dealing with high solidity blades in the rotors a 'channel flow' approach gives reasonable results (Ref. 1 & 4). For our Mixed flow turbine we selected an 'optimum' inlet blade angle by setting the 'slipfactor'=.85 in the following relation

$$\beta_{blade} = \arctan(1 - \frac{1 - \phi * \tan(\beta)}{\mu})$$
where $\phi = \frac{v_m}{U}$ is Flow Coefficient, $\mu = \text{'slippfactor'}$ (15)

Blade Exit

In our mixed flow turbine rotor, where the flow is close to axial, the 'Cutta Condition' implies (when transonic effects are excluded) that the aerodynamic bladeloading is zero at the trailing edge, which is reflected by introducing zero gradient in the imposed (R^*V_u) at the trailing edge. The difference between the S1 and blade average pitch must be corrected according to some deviation 'rules' based on experimental evidence.

Splitter blades

In our design method we must be capable of determining the position of rotor splitter blades for an imposed optimised suction side velocity. It should be evident from the above equation (13) that

the splitter blade must be 'unloaded' to leave a smooth suction side velocity on the neighbouring 'mainblade'.

3.3. FORMULATION OF THE INVERSE DESIGN PROCEDURE.

A practical turbomachine design system must meet both

- -aerodynamic
- -structural (stress, vibration, cyclic load e.t.c)
- -and geometric constraints imposed by the method of producing.

Since none of these requirements are secondary, they are handled in three different chapters.

3.3.1. THE AERO-PART OF THE INVERSE DESIGN.

In the aero-design of turbomachines the shroudline suction side relative velocity is considered the most critical part of the flowpath, regarding the boundary layer behaviour. This suction side velocity can be controlled by the distribution of the following parameters.

Since the suction side velocity cannot be 'dictated' for the whole 3-D geometry for a practical rotor

$$V_m, \beta, Z_b, \frac{d(R*V_u)}{dm}$$

design (Ref.6) the rest of the flowpath is defined from mechanical constraints. Also the Rotor Exit Flow Quality is imposed to enhance the succeeding diffusor performance.

Based on the anticipated gradient in efficiency from hub to shroud, the required temperature gradient at rotor exit is determined by Eq. (16)

With the assumption of Axisymetric Stream Surfaces (Chapter 3)

$$\frac{P_{Rotor Extt}}{P_{Rotor Inlet}} = \left[\frac{T_{Rotor Extt}}{T_{Rotor Inlet}}\right]^{\left(\frac{\kappa * \eta_p}{(\kappa - 1)}\right)}$$
(16)

the Euler turbomachinery equation is applied along the meridional streamline

$$T_{Rotor Extt} - T_{Rotor Inlet} = \omega * [R * V_{u,Rotor Extt} - R * V_{u,Rotor Inlet}]$$
(17)

The novel exducer configuration with slanted trailing edge shown on Fig.3 & 8, allows controlled rotor exit bladeangles without violating structural considerations.

To ensure the performance (total-to-static) above conventional turbine designs a conical diffusor with 'centerbody' is required (Fig.5) since strong 'counterswirl' near hub results from the design.

The Inverse Design Procedure to be implemented consists of the following steps:

- a) Define a 'first guess' meridional flowpath 'Grid'.
- b) From estimated efficiency the streamwise distribution of (R^*V_u) is determined including rotor exit (Equation (16)&(17).

- c) Impose a Smooth Meridional Velocity field in the grid. The value of the imposed meridional velocity along the shroud, together with the the $(R*V_u)$ distribution, determine the 'Critical' shroudline suction side velocity according to Eq. (13)
 - d) Evaluate the F- and G- terms of equation 2 & 3.
- e) Integrate equation (1) (Eq.(4)), with the constant of integration is set by the imposed shroud meridional velocity.
- f) Integrate RHS of Equation (5) with the velocity resulting from e) above. Correct ,with a relaxation factor, the meridional streamline position according to the integrated massflow fraction. The lack of continuity dictates how the meridional flowpath is altered during the iterations. In other words, the rotor hub, (or shroud) is allowed to 'migrate' during the convergence to satisfy the continuity equation (ultimately).

g)Impose the 'Mechanical Constraints' (To be defined in

section 3.3.2 & 3.3.3 below) .The characteristics of the blade depends on the S1 surface which is determined by integrating the relative flow angle:

$$\theta_{SI} = \int \frac{\tan \beta}{R} dm = \int \frac{\omega * R - V_u}{R * V_m} dm = \int \left(\frac{\omega}{V_m} - \frac{V_u}{R * V_m}\right) dm \tag{18}$$

The S1 surface /blade surface relations were mentioned under 3.2 above. The polar angle derivative along the integration path for Eq 1 is required to evaluate the Bladeforce Term.

- h) Special attention is required for the rotor exit, where the air angle is dictated from the requirement of a prescribed $(R*V_u)$, according to Eq 16 & 17, due to the imposed total pressure. Depending on the mechanical restriction the trailing edge may have to 'migrate' during the design process.
- i) With revised flowfield information the F- and G- terms in eq.1 is updated, and the computational procedure returns to e) above.

This process continues until some criterion of convergence is satisfied.

3.3.2. STRUCTURAL CONSIDERATIONS

The total structural life criteria cannot be analyzed during a PC- based inverse design procedure, since rather complex FEM- analysis is required. To ensure that the 'first guess' of the aero-defined blade respect some basic stress criteria (Creep e.t.c) a simple 2-D stress model can be performed during the inverse design. When second order terms are neglected the following equations applies for the maximum radial Stress near hub

$$\sigma_{Max} = \frac{1}{t_{\theta,Hub}} \int_{Hub}^{Shroud} \delta_{Cf} dr + -\frac{M_{Hub}}{I_{x-x}} * \frac{1}{2} * t_{\theta,Hub}$$
 (19)

where the centrifugal force and bending to be integrated (Eq. 20 & 21) are

$$\delta_{Cf} = \rho_{Material} * t_{\theta} * \omega^{2} * R \tag{20}$$

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences (ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

$$M_{Hub} = \int_{Shroud}^{Hub} \delta_{Cf} *R * (\theta - \theta_{Hub}) * dR$$
 (21)

and the second moment of inertia of the hub section (Eq. 22) is

$$I_{x,Hub} = \frac{1}{12} * t_{\theta,Hub}^3 \tag{22}$$

This analysis require little additional code and can be performed during the inverse design procedure. For our Mixed Flow turbine rotor geometry, this integration was performed in 'Section B-B' and in 'Section A-A' to determine a tangential blade thickness ratio compatible with the materials creep life data. Equation (19) can be solved 'inversely' and the resulting 'constant creeplife blade' results in a 'Eifel Tower' blade shapes of the type seen in Fig.8. For critical designs like this radial turbine and centrifugal compressor of Ref.11, is vital that the structural analysis is performed with the same geometry as the geometry defined for production. The geometry definition of radial and mixed flow rotors is a typical case for 'special purpose' software, and it is logical that the aerodesigner prepare the complete geometry definition files for the FEM-program input, as illustrated on Fig.8. These geometries are defined in the same subroutine with the constants determined in the designprosess. In subsequent structural analysis temperature, heat transfer cefficient e.t.c must be added.

3.3.3. GEOMETRIC CONSTRAINTS DUE TO PRODUCTION.

It is important that the 3-D blade geometry which is output of the inverse design is compatible with an available/economic production method. The two manufacturing methods which is common for radial and mixed flow turbomachines is Flank Milling and Casting.

For the Production of Castings, there is a close connection between the requested thickness distribution, material quality requirement and scrapprate. Due to this the relative thickness ratio for the tip vary with size .As a consequence the optimum bladenumber reduces, and the meridional flowpath length increase with reduced size to conserve the aerodynamic blade loading, Eq. 8, 13, Fig. 9. Ref.14

The 'Cold Rig' version of the mixed Flow Turbine in question has been 'Flank milled' in a 5-Axis Controlled Milling Machine. Further 'Flank milling' is a candidate for the production of the forms for 'Lost Vax' casting process and it is a good method for high performance Centrifugal compressors with transonic inducers.

The 'Flank Milling' production process is illustrated on Fig. 3, where it can be shown that the production process will impose mechanical constraints on the blade geometry in the direction of the 'Cutter Centerline'.

The blade surface definition, and the machining process is illustrated on Fig.3, Section C-C, which is seen normal to the cutter centerline for one particular position along the 'Cutter Path'.

Since in the general case, a rotor blade is 'twisted' from hub to shroud, it is evident that the cutter direction (In woorkpice Coordinates) are different at shroud and 'near hub'. As a consequence, the contact line of the cutter spans an angle from hub to shroud and the blade surface are 'undercut' compared to the stright line a) to b). The deviation from this generatix half way from hub to shroud is close to

$$\delta = R_{Cutter} * [1 - \cos(.5 * (\overline{\beta}_{Hub} - \overline{\beta}_{Shroud}))]$$
 (23)

where the bladeangles is taken in a plane normal to the Cutter Centerline.

For the Mixed flow Turbine, and for compressors as shown on Fig.9 & 10(Ref.11), undercutt can be comprensated for when defining the blade for FEM-analysis by using a slightly different 'Cutter 'Path' for the geometry definition as compared to the 'Cutter Path' defined for machining.

The rather obvious requirement that a practical cutter has to pass between the blades to be machined does put restraints on the selection of number of blades and possition of splitters.

4. RESULT OF THE INVERSE DESIGN

4.1. A Mixed Flow Turbine

The presented design method has been utilized, during the development period, for several turbomachines from the Centrifugal Compressor for an 'Ultra Small Jet Engine' in 1988, Ref. 14, Fig 9, to the Radial Inflow Turbine currently in the design phase Fig. 8.

The mixed flow turbine used to illustrate the inverse design method, Figs. 3 to 7, was designed for a Total to Static Pressure ratio of 2.05.

For the particular spool a high rpm was required due to the Compressor Efficiency, size, and cost. Applying typical 'turbocharger turbine geometry' would result in low total-to-static efficiency (Ref. 8 & 12), consequently a mixed flow turbine was designed for this application. Fig.5 shows the turbine rig which has been designed by ARTI in Praha, and Fig. 4 shows a photo of the turbine rig rotor, 'Flank Milled' at ARTI. The rig is currently in the manufacturing process and 'Cold Rig' tests are scheduled later this year.

Due to the combination of conical flowpath and 'almost' radial element blades, some freedom exist in the selection of rotor inlet tipspeed and 'Design Charts' for hydraulic Fransis Turbines could to a certain degree be utilized.

A design procedure as described in 3.3.1 with the restriction of 'Flank Milling' was performed with different combination of bladenumber and splitterlocation. The final design geometry shows the meridional velocity profiles in Fig 5.3 and the relative machnumbers in Fig 5.4. By imposing a 'kink' in the R*V_u distribution in the splitter blade trailing edge region, a quite uniform suction side machnumber is obtained on the whole mainblade, and the deceleration near the trailing edge suction side should give low boundary layer growth (Ref.13).

Since both meridional curvature and aerodynamic blade loading are drastically lower than for High Pressure Ratio Radial Inflow Turbines the resulting 3-D effects, which is not taken care of in the quasi-3-D formulation, should be moderate. It is, however evident that a reliable design procedure for this type of turbines needs feedback from the 'real flow effect' regarding the deviation and loss characteristics. Since the basic Quasi 3-D procedure when properly 'calibrated' for efficiency and deviation, predict the static pressure along a compressor shroud as shown on Fig.11, the same procedure should apply for the lightly loaded turbine.

4.2. Computational Times:

The Computer code described has been used on computers ranging Homecomputer (Fig 1988) through 286/287 (for the Mixed Flow Turbine 90) to 486 type (Radial Inflow Turbine Fig .22,1991). It is difficult to give 'honest' figures for the performance of the code for several reasons:

- -The code is seldom started from 'scratch'. Based on previous experience a tentative streamline pattern and a tentative meridional velocity level can be estimated as a 'first guess'. This reduces the time for obtaining satisfactory convergence drastically.
- -The grid required varies with the type of task.
- -The performance depends on how the computer is configured.
- -An engineer seldom runs a program to the convergence level which a mathemathician would.

Comparison of several codes for turbomachinery flow analysis are given in Ref.1.b) Since both grid and computers vary a direct comparison is difficult. Since the basic characteristic of a streamline is that both the first and second derivative (Slope & Curvature) is included in the information it is logical that the grid can be quite coarsh for SC-procedure.

It would be a task for ICIDE to define a list of 2-D and 3-D turbomachinery geometries which could be used to evaluate different methods, since several factors in addition to relaxation, grid size and number of iterations affects the accuracy and time used.

For the 286/287 Mixed Flow Turbine Fig. 4 & 5,a total of 33 Meridional 'stations' vere used. The first 10 'stations' were used in the nozzles, which vere also inversly designed. In the rest of the flowpath, 23 additional 'stations' vere used , and 9 meridional streamlines vere used including hub & shroud. This task took typically 50 minutes on the 286/287 Laptop.

For the design of a axial/radial diffuser similar to that on Fig. 5 a smaller grid had to be used to include a simple boundary layer code (Ref. 13). For that design (not shown in this report) a optimum boundary layer shape factor were the basis for the geometry definition.

Currently the code is running under the Microsoft Professional Development System 7.1.(QuickBASIC Extended, which is a very convenient development environment). This allows the DOS-barriere of 640 K to be broken by using EXTENDED or EXPANDED memory. In this case a simple 2-D boundary layer integration procedure (Ref.13) can be included in the present code together with a (35*9) meridional grid, at the 'cost' of a speed reduction of some 50% compared to the 486 '640K-DOS speed', and typical CPU is 12 minutes for a Turbine shown on Fig.8.

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6. ACKNOVLEDGEMENT

The permission by R.J.Mowill/MOWILL TURBINMOTOR A/S to use the Mixed Flow Turbine to illustrate the use of the method is appreciated.

The enthusiastic Turbomachinery team at ARTI in Praha are acknowledged for the hardware efforts. (Ref. Figs. 4 & 5)

- and especially my vife Gerd, for her patience with my "inverse" sparetime activity --may the result be more sparetime!

7. FIGURES

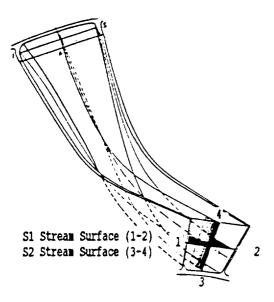


Figure 1. Stream Surfaces

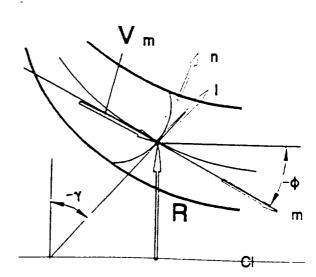


Figure 2. Coordinate System Definition

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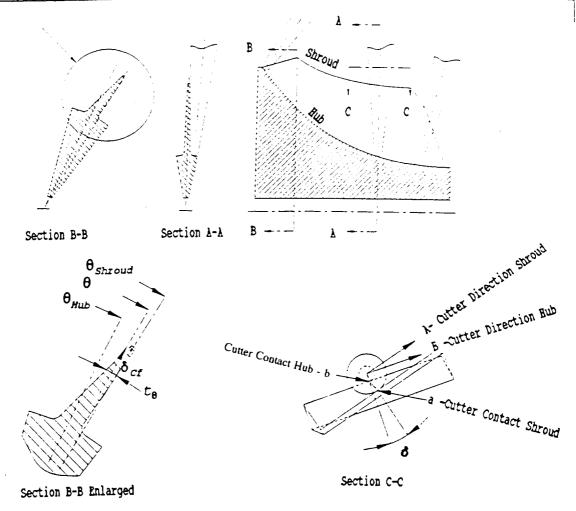


Figure 3. Geometric Constraints for 'Flank Milling'



Figure 4. Mixed Flow Turbine Rig Rotor ('Flank Milled')

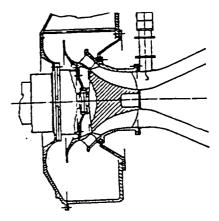


Figure 5. Turbine Rig Meridional View

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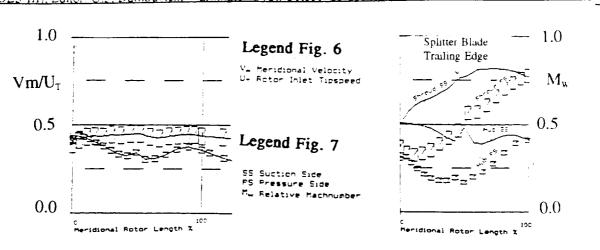


Figure 6. Meridional Velocity

Figure 8. Radial Inflow Turbine (with Rotor FEM-Meshing)



Figure 10. High Pressure Ratio Compressor (Reproduced from Ref. 11)

Figure 7. Relative Machnumbers



Figure 9. Centrifugal Compressor (Ref.14)

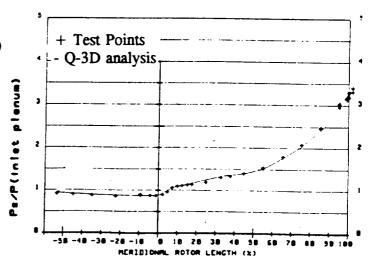


Figure 11. Comparison between Q-3D & Test (Compressor Static Pressure)