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Extended Mapping and Characteristics Techniques for Inverse Aerodynamic Design

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Abstract
Some ideas for using hodograph theory, mapping techniques and method of characteristics to formulate typical aerodynamic design boundary value problems are developed. Inverse method of characteristics is shown to be a fast tool for design of transonic flow elements as well as supersonic flows with given shock waves.

Introduction

This paper is intended to illustrate a revitalization of classical tools of theoretical aerodynamics for use on modern graphic workstation computers presently available to the design engineer:

The theoretical methods have their origins in the time before large scale numerical computing became the standard approach for analyzing aerodynamic performance, about two decades ago these tools were already operational for practical aerodynamic tasks. Transonic flows then posed challenging problems and analytical mathematical modelling was used to gain insight into various theoretical and applied questions resulting from nonlinear model equations. The "Hodograph Method" gave answers to many of such type problems. Similarly, the "Method of Characteristics" yielded practical results. Combination of both methods, more recently, has permitted inverse - or at least indirect - formulation and solution of aerodynamic design problems.

Later, numerical methods became more important because of their general applicability but frequently they give only poor insight into a mathematical model underlying a described phenomenon. The previous hodograph approach was somewhat complicated because of mapping procedures, but results still serve as test cases for numerical methods.

Nowadays, while most of the successful analytical methods are used only for educational purposes, we witness another type of tool emerging from developments in computer technology: Graphic Workstations and even PC's provide powerful computation, illustration and documentation of results to the model equations for fast aero analysis and design. Interactive methods are being developed to provide a strong coupling of computer power and speed with the design engineer's experience and strategies to obtain his design goals: For many applications we seem to have the knowledge base and computer hardware to develop a variety of what may be called "Aerodynamic Expert Systems".

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In this situation we may want to recall classical "pre-CFD" methods because, if implemented to those fast graphical desktop computers, they might be modernized and improved to easily give fast first steps for aerodynamic design and optimization, and last not least to serve as educational tools.

In the present paper we illustrate the idea of combining fast classical aero methods with most recent computer and software technology by using hodograph formulations and characteristics to obtain some well known and some new plane and axisymmetric transonic and supersonic flow elements. The fast computation and powerful graphic evaluation of results invite experimenting with conceptual extensions: Here the hodograph method is extended to axisymmetric flows and a method of characteristics for axisymmetric rotational flows will be presented and proposed for use of designing more general three-dimensional flows.

Hodograph-based methods for transonic flows

The following review of an extended hodograph method is focused on transonic applications. For two-dimensional isentropic flow this approach is wellknown in the literature, here the illustration is carried out for plane flow and extended to small perturbation axisymmetric transonic flow - an option widely unknown because the main purpose of the hodograph, linearity, obviously cannot be obtained for axisymmetric flow.

Potential flow models

Isentropic flow assumptions result in a system of PDEs for potential Φ and streamfunction Ψ with D and Q suitably dimensionless density and velocity, and flow angle ϑ and velocity Q independent variables:

$$\Phi_Q = \frac{1}{DQ}(M^2-1)\Psi_\vartheta$$

$$\Phi_\vartheta = \frac{Q}{D}\Psi_Q \qquad (1)$$

Rheograph transformation: Beltrami equations

Here we use a modification of the hodograph variables: System (1) is transformed by using the Prandtl - Meyer angle

$$\nu = \int_{(Q=1)}^{Q} \sqrt{|M^2-1|}\,\frac{dQ}{Q} \qquad (2)$$

as one independent variable instead of the velocity Q. The basic PDEs become

$$\Phi_\nu = jK\Psi_\vartheta$$
$$\Phi_\vartheta = K\Psi_\nu \tag{3}$$

with j = -1 representing subsonic flow (where $\nu < 0$) and j = 1 for supersonic flow (where $\nu > 0$). The coefficient

$$K = \frac{\sqrt{|M^2 - 1|}}{D} \tag{4}$$

is, for isoenergetic flow and with (2), a function of ν only. So far any hodograph problem formulation is just a matter of stretching from a (Q, ϑ) - plane to a (ν, ϑ) - plane.

The technique stressed here involves further elliptic or hyperbolic mapping of the variables (ν, ϑ)

$$\nu_s = \vartheta_t$$
$$\nu_t = j\vartheta_s \tag{5}$$

which results in a transformation of (3) to become

$$\Phi_s = jK\Psi_t$$
$$\Phi_t = K\Psi_s \tag{6}$$

Both (3) and (6), and also the Cauchy-Riemann or wave equations (5) are more generally named Beltrami equations. With K now a function of s and t this system of PDEs is linear. The first author has made extensive use of it for transonic airfoil design [1, 2].

For the purpose of illustrating a generalization to axisymmetric flow, we use here the small perturbation version of (5) and (6), with a notation

$$\nu \sim U$$
$$\vartheta \sim VY^{-p_1}$$
$$\Phi \sim X \tag{7}$$
$$\Psi \sim Y^{1+p_1}$$

where for plane 2D flow $p_1 = 0$. Replacing the coefficient K by its leading term near sonic conditions,

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$$jK \sim v^{p_2} \tag{8}$$

with $p_2 = 1/3$ results in the near sonic version of the systems (5) and (6), the compatibility relations

$$\begin{aligned} V_s &= jY^{p_1}U_t \\ V_t &= Y^{p_1}U_s \end{aligned} \tag{9}$$

and characteristic equations

$$\begin{aligned} X_s &= U^{p_2}Y_t \\ X_t &= jU^{p_2}Y_s \end{aligned} \tag{10}$$

Now we see an elegant symmetry of these two coupled pairs of equations, each modeling a generalized axially symmetric potential, [3]. We can distinguish between various types of flow, depending on the parameters j, p_1 and p_2. Linear subsonic (j = -1) or supersonic (j = 1) flow is described by $p_2 = 0$, while transonic flow requires $p_2 = 1/3$, with j both -1 and +1 for mixed type flow. With $p_1 = 0$ or 1 we have plane 2D or axisymmetric flow, respectively. Mapping in various aero or fluid dynamics case studies can so be reduced to one generalized system of basic equations [4, 5]. Any one of p_1 or p_2 being equal to zero yields linear equations, but for near-sonic axisymmetric flow a weak nonlinearity persists, which seems to be the reason why this formulation has not been used for aerodynamic problems, except in the one work by Hassan [6].

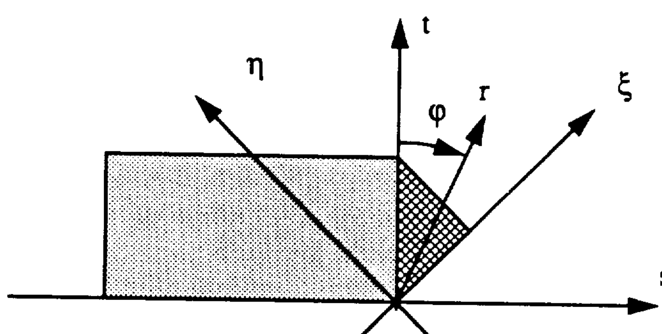

Fig. 1: Rheograph or Characteristics plane: Elliptic (shaded) and hyperbolic (cross-hatched) domain for mixed type model equations

Self-similar solutions

We see the relation of Beltrami equations to conformal and characteristic mapping: singular solutions in classical hydromechanics have helped to understand many aerodynamic phenomena, so we wish to use the system for axisymmetric near-sonic flow also for solving some of its typical features.

Figure 1 illustrates the working plane (s, t): neither physical plane (X, Y) nor hodograph plane (U, V), it is suited for a definition of boundary and initial value problems which require a parametric formulation. In transonic flows, the mixed elliptic/hyperbolic type subdomains require contact along the mapped sonic line, here suitably fixed at s = 0. Earlier applications and illustrations [5] explain the use of the name "Rheograph" and "Characteristics Plane".

Some classical and many new phenomena may be modeled from the general harmonic set of self-similar solutions in polar coordinates

$$
\begin{aligned}
U &= r^{n} \cdot h(\varphi) \\
V &= r^{n+p_1 \cdot b} \cdot k(\varphi) \\
X &= r^{b+p_2 \cdot n} \cdot f(\varphi) \\
Y &= r^{b} \cdot g(\varphi)
\end{aligned}
\qquad (11)
$$

which require only solving a set of four coupled ODEs for the generalized harmonic functions h, k, f and g, with two free parameters n and b.

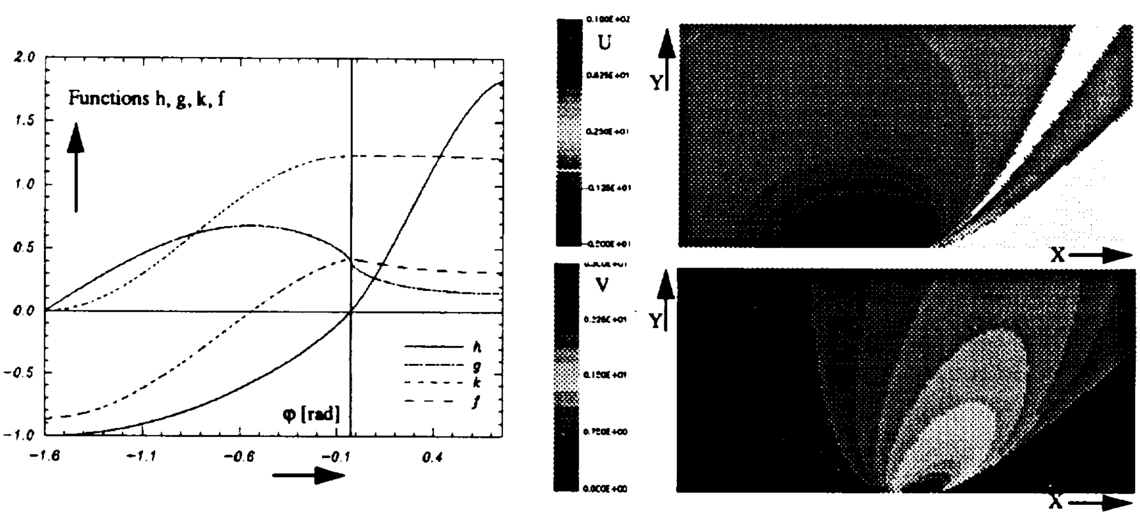

Fig. 2: Quasi-harmonic functions for far-field singularity U,V(X,Y) modeling flow past a body of revolution in sonic free-stream Mach = 1

Example: Guderley's far-field singularity of an axisymmetric body in sonic flow

M. Klein [7] has investigated these coupled potential flow problems calculating some plane and axi-symmetric cases with different exponents n, b. This was done prior to using some gained knowledge for setting up more general boundary/initial value problems for numerical solution of (9) and (10) with a Poisson solver and the method of characteristics on a graphic workstation. One case studied in detail is the solution for simulating the flow past a body of revolution in sonic free-stream. This is a classical transonic problem first solved by Guderley 1954 [8] and elegantly confirmed by Müller & Matschat 1964 [9]. Their work suggests use of (11) with a ratio of the exponents b/n = - 7/9. In fact, it is just this ratio which yields a physically reasonable solution.

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Figure 2 illustrates the result: Graphic CFD postprocessing software is used to show the U and V distribution in the physical meridional plane (X, Y). This example may be used to illustrate the use of working in the Rheograph plane, to understand flow details with nonlinear model equations better and to have more freedom to suitably model boundary values. In transonic flow, meaningful solutions frequently can only be obtained by formulating boundary conditions in an indirect, inverse way, - this is the basic reason why some practical design problems are easier solved in inverse mode.

For the following transonic 2D example ($p_1 = 0$) we return to the systems (5) and (6), the Rheograph equivalent of the 2D full potential equation.

Example: 2D transonic nozzle exit

Equations (5) and (6) for supersonic flow j = 1 transform into compatibility relations

$$\left.\frac{d\vartheta}{d\nu}\right|_{\eta = const} = 1 \qquad \left.\frac{d\vartheta}{d\nu}\right|_{\xi = const} = -1 \tag{12}$$

and characteristic equations

$$\left.\frac{d\Psi}{d\Phi}\right|_{\eta = const} = \frac{1}{K} \qquad \left.\frac{d\Psi}{d\Phi}\right|_{\xi = const} = -\frac{1}{K} \tag{13}$$

which are the basis for a rapid linear method of characteristics. Implemented on a graphic workstation, solutions may be obtained and visualized extremely fast, we use the method to set up a knowledge base for interactive transonic design expert systems with advanced graphic pre- and postprocessing. Flexible geometry input for boundary conditions was used by Gentner [10] to define a 2D sonic throat and the downstream accelerated exit flow. With initial data for Mach number, flow angle and physical coordinates along the t-axis (Fig. 1) a first calculation determines the solution of (12) and with $K(\nu(\xi,\eta))$ available, the second step is the solution of (13).

Figure3 once more stresses the difference between hodograph and Rheograph or Characteristics plane: The flow structure may map into a multivalued hodograph, while the Rheograph may be controlled to show a single-valued characteristics grid.

The result with a non-symmetrical exit contour designed by prescribing velocity distribution along the nozzle axis is depicted in Figure 4. The idea here was the combination of (known and well-developed) potential flow modeling with mapping transformations based on hodograph theory (also known but considered complicated), and the use of powerful workstations (helping with rapid computation and graphic visualization). The above isentropic model equations are either linear or weakly nonlinear. In the following, design problems involving non-isentropic flow will also be solved by the method of characteristics.

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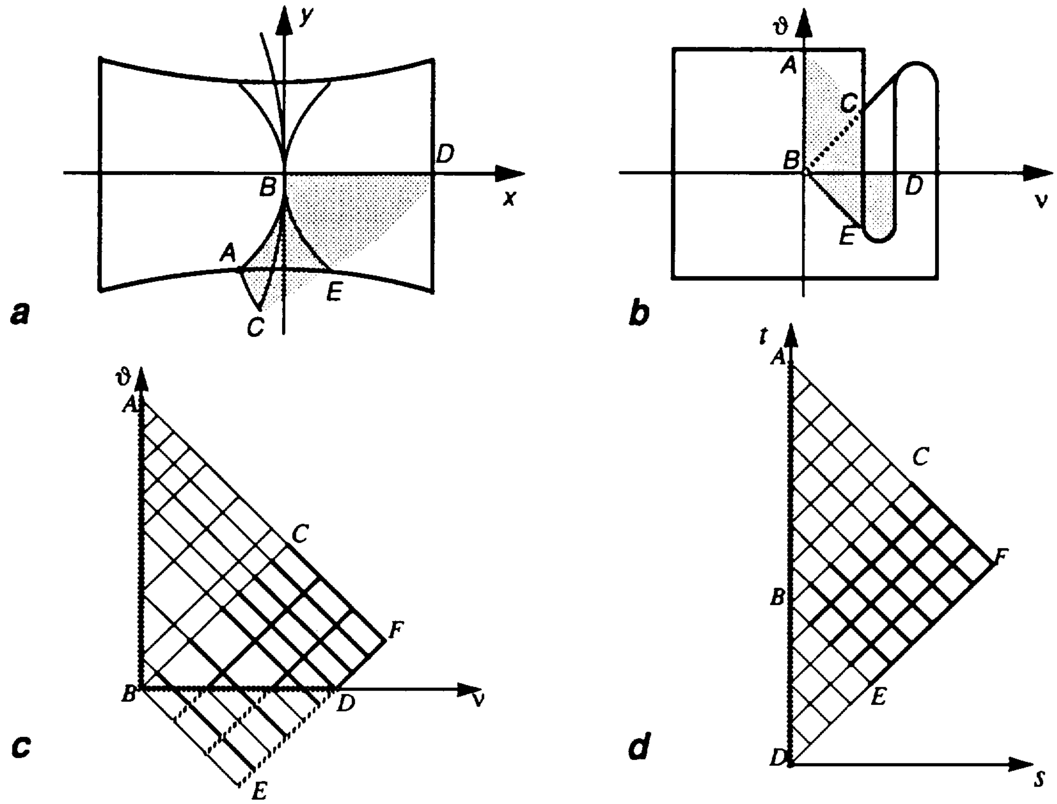

Fig. 3: Unfolding the multi-valued mapping of a Laval-nozzle (a) supersonic hodograph (b, c) to single-valued triangular domains (c) in the Rheograph plane

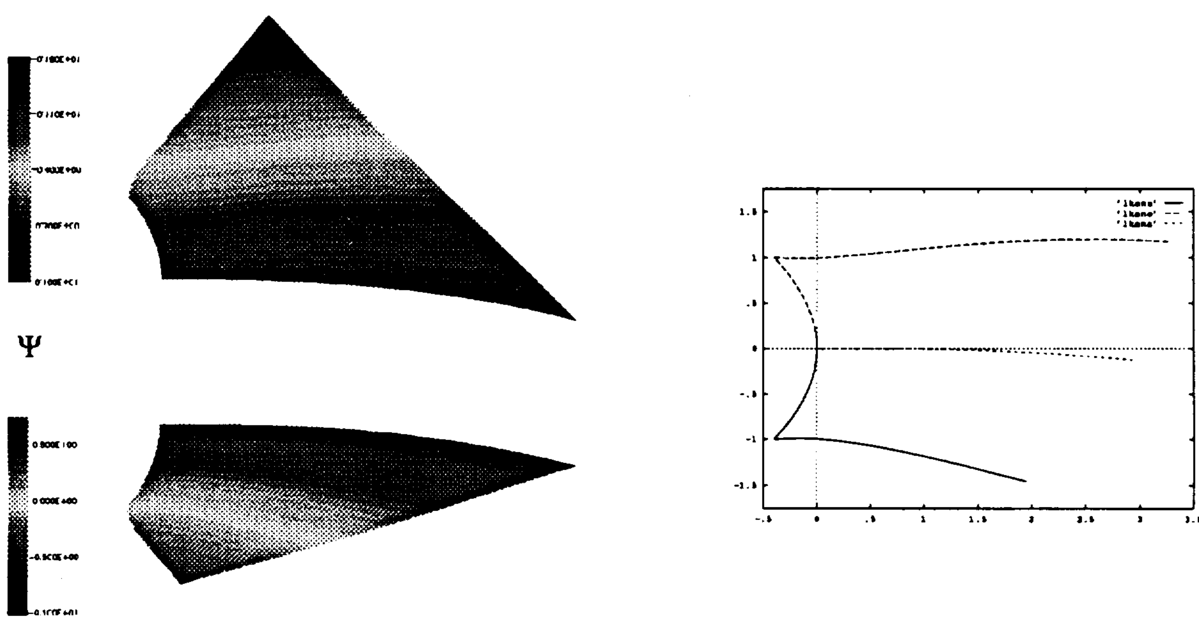

Fig. 4: Laval nozzle exit designed from sonic line Cauchy data and velocity distribution (Mach, flow angle) along a curved axis. Color graphics illustrate stream function / contour .

Supersonic flows with controlled shock waves

Before we apply the method of characteristics to a problem involving oblique shocks, it should be illustrated that given initial data in the Rheograph working plane directly relate to Cauchy data in the physical plane, the marching direction starting from AB and progressing towards C runs approximately normal to the resulting local flow direction, Fig. 5. We call this and related numerical approaches to compute the flow field "Cross - (stream) Marching". This will be useful for supersonic design applications where we seek to control the shape and strength of occurring shock waves.

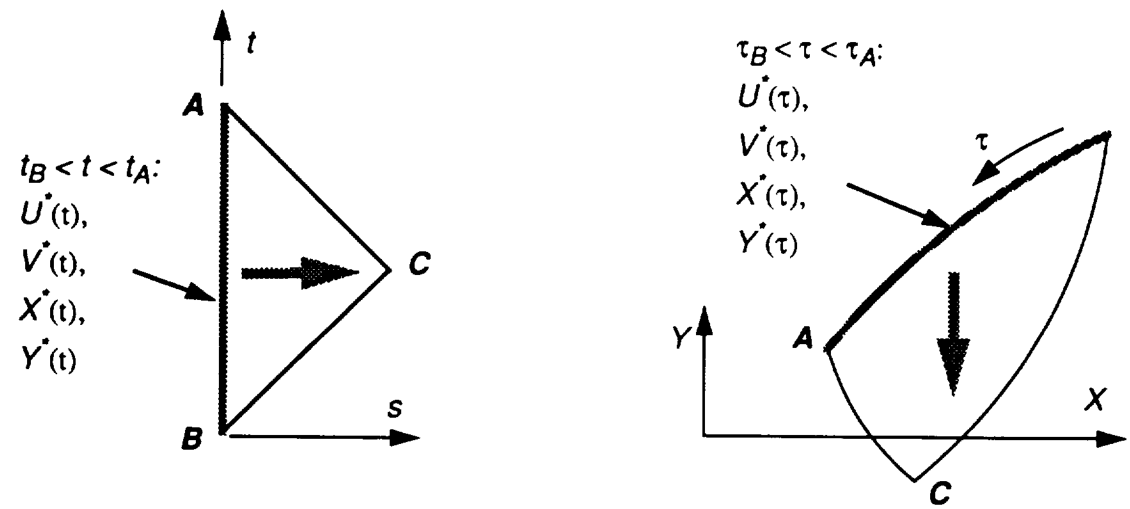

Fig. 5: Cauchy Initial data in characteristics plane (s, t) and in physical plane (X, Y)

Cross-Marching from given shock waves

As can easily be seen from a flow field with an oblique shock wave and its supersonic post-shock characteristics, there is only the possibility of Cross-Marching since the initial data do not allow for marching downstream, Fig. 6. A portion AB of oblique shock wave determines a flow field ABC and a limited portion AD of the contour compatible with the given shock wave. A larger region of dependence ABEF and contour ADG are obtained if also the flow at a segment BE at the axial exit station is prescribed, see [11] for some remarks about numerical consequences of such given input.

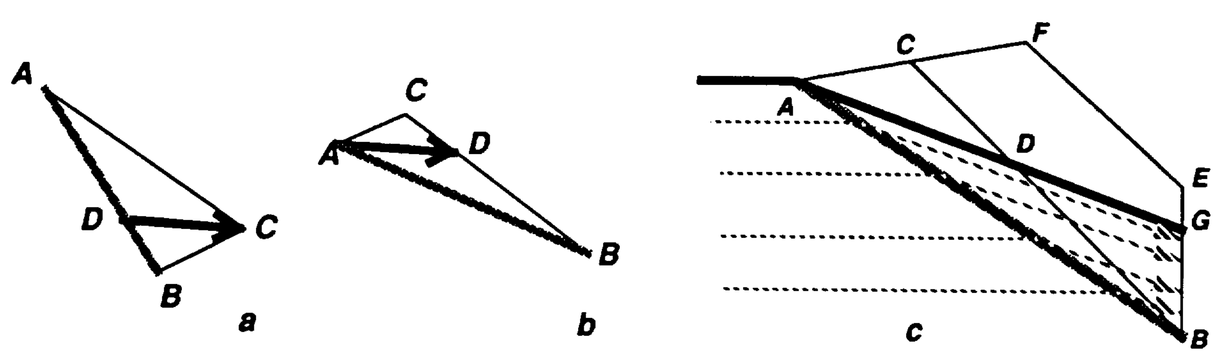

Fig. 6: Basic steps of downstream marching (a) and Cross-Marching (b), depending on initial data curve AB. Computing the flow behind oblique shocks (c) requires Cross-Marching.

Numerical methods of characteristics have been developed for plane and axisymmetric , for isentropic and rotational flows. Prescribing arbitrary shock waves results in rotational flow because of shock curvature. Cauchy initial data for flow field computation therefore require coordinates, velocity components and entropy distribution along the prescribed shock geometry. For Cross-Marching (Fig. 6b), the iterative calculation of entropy convection along the streamlines requires an extrapolation of data BD→C, while for the usual downstream marching (Fig. 6a) an interpolation of data A→D←B is needed.

The following two examples were obtained with a new numerical Cross-Marching method of characteristics for axisymmetric isentropic or rotational flow by the second author [12]. A flexible input geometry generator and workstation implementation lay ground for further extensions and use for aerodynamic design tasks.

Example: Segment of a conical flow field

As a first example for the new method of characteristics a part of the flow field past a circular cone is computed. Input data are the upstream Mach number, a set of coordinates of and post-shock conditions behind the given conical shock wave with given angle. Fig. 7 illustrates the characteristic grid, a selected integrated streamline and reveals a limit line singularity along a ray through the cone vertex, well within the solid cone which is compatible with the shock cone and Mach number. The case is well-suited for checking the accuracy in comparison with the solution of the Taylor-Maccoll ODE; graphic visualization of the Mach number and flow angle distribution must show constant values along rays through the cone vertex, though its location is not part of the input data.

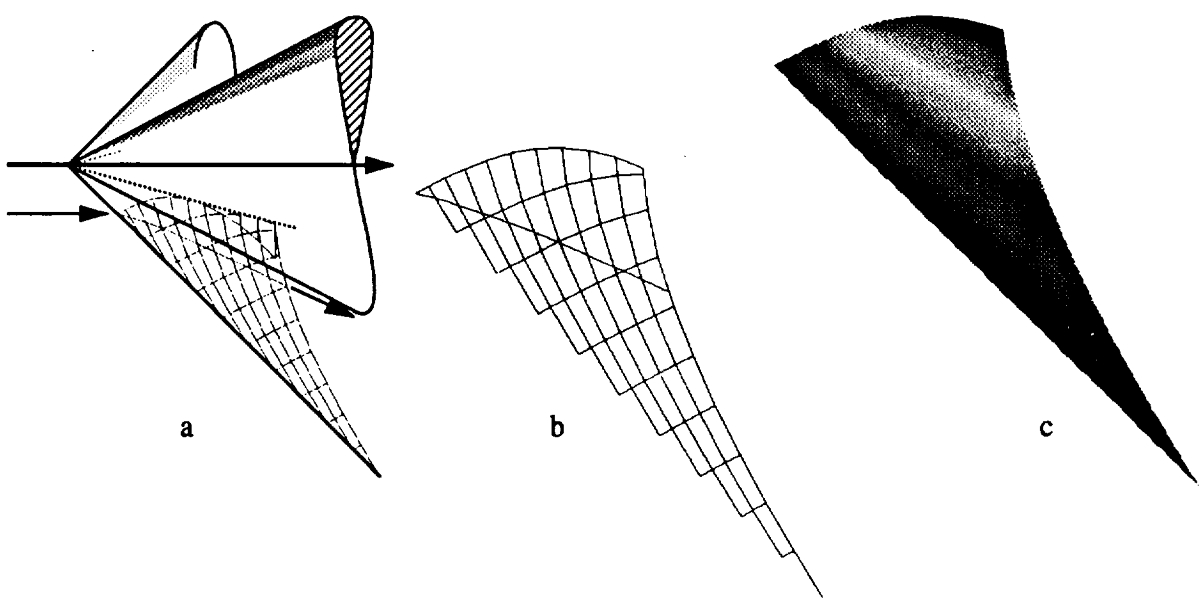

Fig. 7: Ideal gas ($\gamma = 1.4$) flow past a circular cone. Given Mach = 2, shock angle = 45^o (compatible with a solid cone of 27.32^o). Choice of shock segment size relative to axial distance: Characteristics grid (a) with or (b) without limit cone of ~16.3^o. Every third grid line shown. Color graphics (iso - Mach) for flow field conicity check (c).

Computation time on a Sun Sparc Station: 6 seconds.

Example: Segment of a flow field downstream of a curved shock

A slight variation of the input shock segment geometry brings rotation to the flow field downstream of the shock. Color graphic visualization of the velocity and pressure distribution shows a strong deviation from conical structure, Fig. 8.

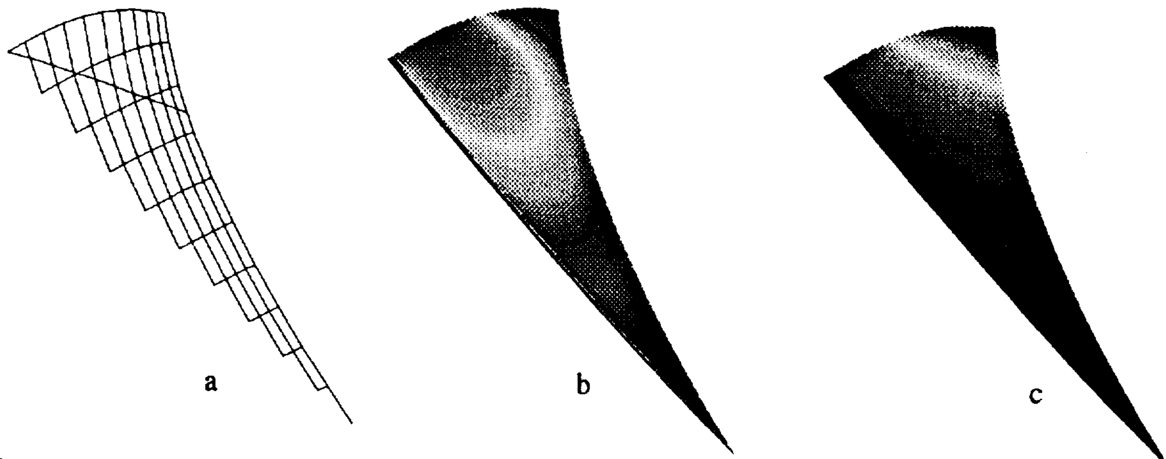

Fig. 8: Flow with a curved shock wave. Given Mach = 2, shock angle varies from 50^{o} to 45^{o}. Characteristics grid, surface streamline (a). Color graphics for iso-Mach (b) and iso-flow angle (c)

Design of three-dimensional flow fields

The exploitation of plane and axisymmetric inviscid flow fields for the definition of flow patterns generated by three-dimensional bodies in supersonic flow has been used since about three decades when Nonweiler [13] created the first "waveriders". In recent years renewed interest originated in such configurations for generic lifting aerospace transport vehicles and supersonic inlet shapes [14]. The first author recently contributed an idea to this research which is aimed in generalizations of waverider shape definition by applying conical flow solutions with constant shock strength but axial distance of the shock segments varying along span [11]. The idea is based on the assumption of a local axisymmetry in every 3D flow, which is well defined in an"osculating plane" if the shock wave is known, Fig. 9. The method to generate three-dimensional configurations requires little more effort than evaluating one Taylor-Maccoll conical flow solution. Only the Mach number, shock angle, leading edge and shock profile in the exit plane need to be prescribed. Based on this method a very rapid interactive design code was developed by Center et al [15]. Numerical analysis with an Euler code shows a striking agreement of the numerically captured shock location with the design solution, which makes it worthwhile to further develop such techniques.

The method of characteristics with Cross-Marching developed here may be used for such further development, with the possibility of exploiting also rotational flow fields to find 3D body contours. This method is then equivalent to solve the 3D Euler equations in an inverse design mode, rapidly carried out in an interactive fashion on the workstation.

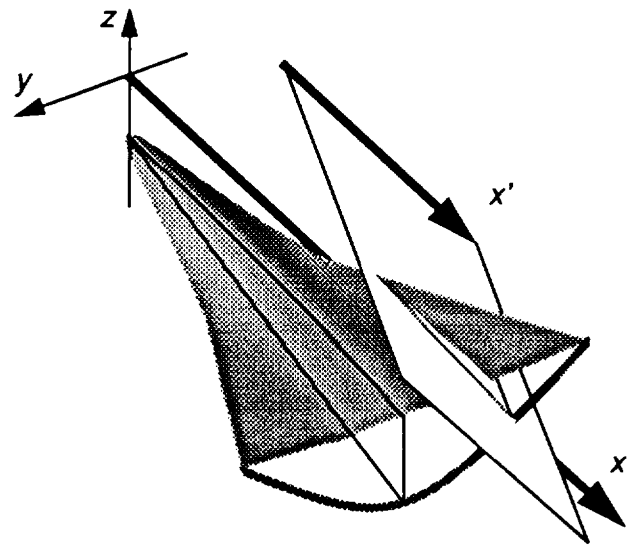

Fig. 9: Local conicity in osculating plane of a flow behind given rule surface shock wave: Design of generalized super/hypersonic waveriders.

Conclusion

We have tried to illustrate some ideas to extend classical theoretical methods for the aerodynamics of inviscid, compressible flows. The purpose is an implementation of these tools to develop software on fast modern workstation computers which enables the design aerodynamicist to perform rapid early stage design studies with aerodynamic expert systems, but also to develop these techniques toward convincing educational programs for students. Transonic and supersonic aerodynamics require inverse problem formulations if flow properties should be used optimally for design goals. This was shown using the method of characteristics for inverse applications.

Acknowledgement
Numerical codes were written or adapted to interactive workstation versions and combined with CFD color graphics postprocessing at DLR Göttingen by Birgit Seitz, Thomas Gentner and Michael Klein.

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