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VARIATIONAL FORMULATION OF HYBRID PROBLEMS FOR FULLY 3-D TRANSONIC FLOW WITH SHOCKS IN ROTOR

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ABSTRACT:

Based on Refs [3,4], the unified variable-domain variational theory of hybrid problems for rotor-flow [1,2] is extended to fully 3-D transonic rotor-flow with shocks, unifying and generalizing the direct and inverse problems. Three variational principle (VP) families have been established. All unknown boundaries and flow discontinuities (such as shocks, free trailing vortex sheets) are successfully handled via functional variations with variable domain, converting almost all boundary and interface conditions, including the Rankine-Hugoniot shock relations, into natural ones. This theory provides a series of novel ways for blade design or modification and a rigorous theoretical basis for finite element applications and also constitutes an important part of the optimal design theory of rotor-bladings [6]. Numerical solutions to subsonic flow by finite elements with self-adapting nodes given in Refs[16,19,22] show good agreement with experimental results.

NOMENCLATURE:

A	total area of boundary surfaces.
A_1	inlet & outlet surfaces (Fig.1).
A_2	periodic boundary surfaces (Fig.1): $A_2 = A_{2u} \cup A_{2d}$, $A_{2u} = A'_{2u} \cup A''_{2u}$, $A_{2d} = A'_{2d} \cup A''_{2d}$.
A_{2d}	free trailing vortex sheets.
A_3	all solid boundary walls: $A_3 = A_a \cup A_b = A^*_3 \cup A^{**}_3$.
A_a	hub- & casing annular walls: $A_a = A^*_a \cup A^{**}_a$.
A_b	blade surfaces: $A_b = A^*_b \cup A^{**}_b$.
$dA_{r,\varphi,z}$	components of elementary area $d\vec{A}$ in r-, φ-, z- directions respectively, $d\vec{A} = dA \cdot \vec{n}$.
A_s	shock surfaces.
a	sound speed.
$\vec{C}, \vec{W}$	absolute, relative flow velocity respectively.
κ, m	specific heat ratio C_p / C_v and $m = (\kappa - 1)^{-1}$
M	relative Mach number.
$\vec{n}$	outward normal unit vector.
p	dimensionless pressure
q	mass flux $\rho \Lambda$.

r,φ,z and $\vec{i}_r,\vec{i}_\varphi,\vec{i}_z$,

cylindric coordinates fixed on rotor and the corresponding unit vectors.

$\mathscr{R},S$ rothalpy & entropy.

$\vec{s},\delta\vec{s}$ boundary position vector & its variation.

$\vec{U}$ $\vec{U}=\vec{\omega}\times\vec{r}$.

dv elementary volume $dv=rd\varphi drdz$.

ϕ potential function

$\vec{\omega}$ angular speed of the rotor.

$\vec{A}_c,\vec{A},\vec{A}_u$ demensionless forms of $\vec{C},\vec{W},\vec{U}$ respectively.

ρ dimensionless density.

Subscripts:

a,b annular and blade walls respectively.

d,u downstream and upstream respectively.

m circumferentially averaged value.

n,τ normal & tangential components.

pr prescribed.

r,φ,z radial, azimuthal and axial components respectively.

p pressure blade surface.

s suction blade surface.

$'$, $''$ parameters on $(A'_{zu}$, $A'_{zd})$ and $(A''_{zu}$, $A''_{zd})$ respectively.

--,+ parameters just before & behind the interface or shock respectively.

Superscripts:

o restricted variation [10].

*,** known and unknown portions of the boundary respectively.

1. INTRODUCTION

Nowadays the design of advanced turbomachinery would be impossible without using advanced aerodynamic theory and thereupeon based computational methods. During the last decade much progress has been made in this field and a detailed state-of-the-art review is given in Ref.[5], which reveals that with few exceptions, e.g. Refs [1,2,11-17], most of work done to date, however, are concerned with the direct (analysis) problem, quasi-3-D flow model and mainly finite difference, finite volume and streamline curvature methods. Owing to the lack of exact (classical) variational principles (VPs) for rotor-flow finite element methods (FEM) used so far are exclusively based either on Galerkin approach or on approximate VPs for the linearized problem. It is the great progress and the widespread and fruitful applications of the FEM in solid mechanics that motivated the present author in the mid 1970's to start a systematic search for VPs in fluid mechanics in general [17,18] and in 3-D turbomachine flow theory in particular [1-4,8,11,12] with special emphasis on inverse and hybrid problems in order to provide both a new rigorous, sound theoretical foundation for FEM in computational aerodynamics of turbomachinery and a number of novel rational versatile ways for new blade design or old blade modification. The hybrid problem is the one which, being a unification as well as generalization of the traditional direct and inverse problems, is capable of combining the merits of the two, while eliminating their shortcomings [1,2,11]. As a result, a lot of VP families have been established first for the direct problem [8] and the hybrid problem [11-13,16] of quasi-3-D cascade flow. In extending them to

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fuly 3-D flow the major difficulties encountered were how to capture all possible unknown flow discontinuities, such as shock waves, free trailing vortex sheets and the unknown portions of the blade- and/or annular walls in inverse and hybrid problems. Subsequently, a series of VPs in terms of potential or stream functions for the direct problem of fully 3-D transonic potential and rotational flows with shocks in rotors have been developed in Refs [3,4], and furthermore, a unified variational theory of various hybrid problems for fully 3-D incompressible rotor flow has been presented in Ref.[1] and extended to compressible flow in Ref.[2], thereby a very powerful mathematical apparatus "the functional variation with variable domain" being used to full advantage for handling abovementioned flow discontinuities. Successful numerical validations of such a theory have been carried out in Refs[16,19,22] by using a new finite element with self-adapting nodes.

In the present paper, based on Refs [3,4], the unified variable-domain variational theory of hybrid problems for rotor-flow of Refs [1,2,16] is extended to fully 3-D transonic flow with shock waves in rotors of axial, radial- and mixed-flow types.

2. BASIC AEROTHERMODYNAMIC EQUATIONS

Consider the fully 3-D subsonic and transonic potential, steady relative flow of an inviscid fluid past a rotating blading with constant angular speed $\vec{\omega}$ (Fig.1).

For such potential flows the nondimensional governing aerodynamic equations have the following form [3,4,7]:

Continuity equation:

$$\nabla \cdot (\rho \vec{A}) = \frac{1}{r}\left\{\frac{\partial(\rho r A_r)}{\partial r}+\frac{\partial(\rho A_\varphi)}{\partial \varphi}+\frac{\partial(\rho r A_z)}{\partial z}\right\}=0 \tag{1}$$

Irrotationality of the absolute flow:

$$\left\{\frac{\partial \Phi}{\partial r}, \quad \frac{1}{r}\frac{\partial \Phi}{\partial \varphi}, \quad \frac{\partial \Phi}{\partial z}\right\}=\left\{A_r, \quad (A_\varphi+A_u), \quad A_z\right\} \tag{2}$$

First law of thermodynamics:

$$\frac{p}{\rho}+\frac{1}{2m}(A^2 - A_u^2)=1 \tag{3}$$

Homentropic equation:

$$p=\rho^\kappa \tag{4}$$

Eliminating p in Eq.(3) via Eq.(4) yields:

$$\rho=\left\{1 -\frac{1}{2m}(A^2 - A_u^2)\right\}^m \tag{3'}$$

Using Eqs. (2) & (3'), a full potential equation can be obtained from Eq.(1) [2,3,4,7]:

$$(1 - M_r^2)\frac{\partial^2 \Phi}{\partial r^2}+\frac{1-M_\varphi^2}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2}+(1 - M_z^2)\frac{\partial^2 \Phi}{\partial z^2} - \frac{2M_r M_\varphi}{r}\frac{\partial^2 \Phi}{\partial r\partial \varphi}$$

$$\frac{2M_z M_\varphi}{r}\frac{\partial^2 \Phi}{\partial z\partial \varphi} - 2M_r M_z\frac{\partial^2 \Phi}{\partial r\partial z}+\frac{1+(M_\varphi+M_u)^2}{r}\frac{\partial \Phi}{\partial r}=0 \tag{5}$$

where

$$\rho^{1/m}=1 -\frac{1}{2m}[(\frac{\partial \Phi}{\partial r})^2+(\frac{1}{r}\frac{\partial \Phi}{\partial \varphi} - A_u)^2+(\frac{\partial \Phi}{\partial z})^2 - A_u^2], \tag{3"}$$

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$$M_r = W'_r / a = \frac{\partial \Phi}{\partial r} / \rho^{1/2m},$$

$$M_\varphi = W'_\varphi / a = (\frac{1}{r} \cdot \frac{\partial \Phi}{\partial \varphi} - \Lambda_\nu) / \rho^{1/2m},$$

$$M_z = W_z / a = \frac{\partial \Phi}{\partial z} / \rho^{1/2r},$$

$$M_u = \omega r / a = \Lambda_u / \rho^{1/2m}.$$

In the present paper, just as in Refs [1,2], the following three types of 3-D hybrid problems $(H_A \times H_A)$, $(H_B \times H_A)$ & $(H_C \times H_A)$ are studied in detail, where, as defined in Refs [1,2], e.g. $(H_C \times H_A)$ denotes such a hybrid problem in which a hybrid problem of type C (H_C) is posed on the blade surface, while a hybrid problem of type A (H_A) is posed on the annular walls (Fig.2a). In other words, the first symbol characterizes the problem type on the blade surface, while the second symbol ----- that on the annular wall. As for the hybrid problems H_A, H_B,...., they are defined in Table I for 2-D cascades [11,12,16], for the annular walls they are defined similarly [1,2].

Table I. Problem Classification (for the Blade Surface)

Types	Given conditions	
	Geometric	Aerodynamic
H_A	Part of airfoil form	Pressure distribution along remaining part
H_B		Blade loading distribution $(p_p - p_s)$
H_C	Airfoil thickness	Pressure distribution along suction surface
H_L	distribution	Velocity difference distribution $(A_s - A_p)$
D	All cascade geometry	none
I	none	Pressure distribution along airfoil contour

Of course, the abovementioned three 3-D hybrid-problem types are only some typical ones taken as examples for consideration herein. Generally speaking, 3-D hybrid problems encompass a much wider variaty of types. They provides the designer with a series of novel design tools, which enable him to choose the most suited problem types or their combinations for meeting various practical design conditions at hand (e.g. aerodynamics, cooling, strength-vibrational and technological requiements etc). As pointed out in Ref.[1], the three hybrid problems studied herein per se embrace also very comprehensive special cases, which, to a large extent, are capable of fulfilling various practical requirements of blade design and which can be made even much broader by posing different problem types on different portions of the blade (or annular) wall (Fig.2b).

3. VP FAMILY FOR THE $(H_A \times H_A)$-PROBLEM

In the development to follow, starting from the VPs for the direct problem given in Refs [3,4] and employing the functional variation with variable domain

[1,4], we present a unified variational theory of hybrid problems for fully 3-D subsonic and transonic flows with shocks.

Proceeding similarly to Ref.[1] leads to the following results without going into details.

VP_I: The solution to the $(H_A \cdot H_A)$-problem of 3-D sub- and transonic rotor-flow makes the functional J_I stationary: $\delta J_I = 0$, where Φ, A_3^{**}, A_{2d}, A_s should be varied independently.

$$J_I(\Phi, A_2^{**}, A_{2d}, A_s) = I_I + L + L^{AA},$$ (6)

where

$$\kappa I_I = \iiint_{(V)} p\,dv = \iiint_{(V)} \{1 - \frac{1}{2m}[(\frac{\partial\Phi}{\partial r})^2 + (\frac{1}{r}\frac{\partial\Phi}{\partial\varphi})^2 + (\frac{\partial\Phi}{\partial z})^2 - \frac{2A_2}{r}\cdot\frac{\partial\Phi}{\partial\varphi}]\}^{\kappa m}\,dv$$

$$L = \iint_{(A_1)} (q_n)_{pr}\,\Phi\,dA - \iint_{(A_{2u}')} (\Phi^{"} - \Phi^{'} - \Delta\Phi_u)(\rho A_n)^{'}\,dA$$

$$- L^{AA} = \iint_{(A_u^{**})} (\frac{p_m}{\kappa})_{pr}\vec{s}\cdot d\vec{A} + \iint_{(A_b^{**})} (\frac{p}{\kappa})_{pr}\vec{s}\cdot d\vec{A}.$$

With all unknown boundaries or interface A_3^{**}, A_s and A_{2u} treated by the method of functionnal variations with variable domain [1,4] the following set of stationarity conditions for J_I can be derived from $\delta J_I = 0$:

Euler's eq.: Eq.(5)

Natural boundary conditions (B.C.):

on A_1: $\rho A_n = (q_n)_{pr}$,

on A_{2u}: $(\rho A_n)^{'} = (\rho A_n)^{"}$, $\Phi^{"} = \Phi^{'} - \Delta\Phi_u$,

leading to the circumferential periodicity of all flow parameters.

on A_{2d}: $(\rho A_n)^{'} = (\rho A_n)^{"} = 0$, $p^{'} = p^{"}$,

They are just the interface conditions on the free trailing vortex sheets.

on A_s: Using $\Phi_- = \Phi_+$ as an essential (enforced) interface condition, we have $(\partial\Phi/\partial\tau)_- = (\partial\Phi/\partial\tau)_+$, that is, the tangential velocity components at both sides of the shock are equal:

$$(\vec{A_\tau})_- = (\vec{A_\tau})_+$$ (7a)

So we obtain the following natural interface conditions:

$$(\rho A_n)_- = (\rho A_n)_+$$ (7b)

$$(p/\kappa + \rho A_n^2)_- = (p/\kappa + \rho A_n^2)_+$$ (7c)

In addition, from Eq.(3) we can write

$$\mathfrak{R}_- = \mathfrak{R}_+$$ (7d)

Obviously, Eq-s (7a)-(7d) are none other than the well-known Rankine-Hugoniot shock relations [3,4].

on A_3^{**}: $\rho A_n = 0$.

on A_a^{**}: $\rho A_n = 0$, and $p_m = (p_m)_{pr}$.

on A_b^{**}: $\rho A_n = 0$, and $p = p_{pr}$.

Thus, it has been shown that from this VP I actually the full potential

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equation (5) together with almost all boundary conditions for the 3-D ($H_A \cdot H_A$)-problem can be derived naturally, and all unknown surface (e.g. shocks, free trailing vortex sheets and unknown walls) can be determined using e.g. FEM.

Applying a constraint-removing transformation [17], the above VP I can be extended to the following generalized VP(GVP).

GVP II: The solution to the above 3-D ($H_A \cdot H_A$)-problem makes the following functional J_{II} stationary: $\delta J_{II}=0$, with independent variations of Φ, $\vec{A}$, p, ρ, A^{**}, A and A_{2d}.

$$J_{II}(\Phi,\vec{A},p,\rho,A^{**}_s,A_{2d})=I_{II}+L+L^{AA}, \tag{8}$$

where

$$-I_{II}=\iiint_{(V)}\left\{\rho\vec{A}\cdot\nabla\Phi - \frac{1}{2}(A^2+A_d^2+2A_dA_v)+\frac{mp}{K}[1 - \ln(p \cdot \rho^\kappa)] - m\rho\right\}dv$$

In a way similar to the above one it can be shown that from $\delta J_{II}=0$ the following set of natural conditions results:
Eular's equations: Eqs (1)-(4).
Natural B.C.: All the same as those of VP I.

Subgeneralized VPs (SGVPs):

Via a constraint-recovering transformation [17], from GVP II a family of subgeneralized VP's can be derived, one of which is the foregoing VP I.

4. VP FAMILY FOR THE ($H_B \times H_A$)-PROBLEM

In this case the B.C. on the annular walls remain the same as those of the ($H_A \times H_A$)-problem, while the B.C. on the blade surface become:
(i) Blade thickness distribution given by

$$\varphi_p - \varphi_s = g_r(r,z); \tag{9A}$$

(ii) blade-loading distribution given by

$$\frac{p_p - p_s}{K}=g_p(r,z) \tag{9B}$$

where $g_r(r,z)$ and $g_p(r,z)$ are prescribed functions.

Proceeding in just the same way as in the foregoing section, we can establish the following VP family for the ($H_B \times H_A$)-problem, which differs from that for the ($H_A \times H_A$)-problem only in that the boundary integral term L^{AA} now should be replaced by the following L^{BA}:

$$L^{BA} = \int\int_{(A_a^{**})} (\frac{p_m}{K})_{p}\vec{s}\cdot d\vec{A}+\int\int_{(A_b)_s}g_p \vec{s}\cdot d\vec{A}, \tag{10}$$

imposing Eq.(9A) as an essential B.C.. In Eq.(10) the symbol $(A_b)_s$ stands for the suction blade surface. In this way we obtain the following VP family.

VP III: The solution to the ($H_B \times H_A$)-problem of 3-D sub- and transonic rotor-flows makes the functions J_{III} stationary: $\delta J_{III}=0$, thereby Φ, A_a^{**}, A_s, A_b and A_{2d} should be varied independently.

$$J_{III}(\Phi,A_a^{**},A_b,A_s,A_{2d})=I_I+L+L^{BA}, \tag{11}$$

It is easy to verify that from $\delta J_{III}=0$ the same Euler's equation and natural B.C. set as those of VP I can be derived with the only exception that the natural

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B.C. on the blade surface A_b has now become Eq.(9B).

GVP IV: The solution to the above 3-D $(H_B \times H_A)$-problem makes the following functional J_{IV} stationary: $\delta J_{IV}=0$ with independent variations of Φ, $\vec{A}$, p, ρ, A_a^{**}, A_b, A_s and A_{2d}.

$$J_{IV}(\Phi,\vec{A},p,\rho,A_a^{**},A_b,A_s,A_{2d})=I_{II}+L+L^{bA}, \tag{12}$$

From $\delta J_{IV}=0$ the same Euler's equations and natural B.C. set as those in GVP II follow except only that the natural B.C. on the blade surface has become Eq. (9B).
SGVP Family: By means of a constraint-recovering transformation [17] a family of SGVPs can be derived from the GVP IV, including also the VP III.

5. VP FAMILY FOR THE $(H_C \times H_A)$-PROBLEM

Here the B.C. on the annular walls still remain unchanged as before, while the B.C. on the blade surface are now given as follows:
(i) blade thickness distribution given by Eq.(9A):
(ii) pressure distribution along the suction blade surface:

$$p_s = [p_o(r,z)]_{p_r} \tag{13}$$

To establish the VP family for the $(H_C \times H_A)$-problem, we proceed similarly as above. It turns out that this VP family differs from that for the $(H_A \times H_A)$-problem only in that the boundary integral term L^{AA} should be replaced by the following L^{bA}:

$$L^{bA} = \int\int_{(A_a^{**})} (\frac{p_\pi}{K})_{p_r}\vec{s}\cdot d\vec{A} + \int\int_{(A_b)_s} (\frac{p_r}{K}-\frac{p_s}{K})\,\vec{s}\cdot d\vec{A}, \tag{14}$$

while the Eq.(9A) should be treated as an enforced B.C.. In Eq.(14) the superscript 'o' denotes that the 'restricted variation' [10] should be taken. Thus, we have:
VP V: For the $(H_C \times H_A)$-problem $\delta J_V=0$ with independent variations of Φ, A_a^{**}, A_b, A_s and A_{2d} holds, and

$$J_V(\Phi,A_a^{**},A_b,A_s,A_{2d})=I_I+L+L^{CA}, \tag{15}$$

VP VI: For the $(H_C \times H_A)$-problem $\delta J_{VI}=0$ with independent variations of Φ, $\vec{A}$, p, ρ, A_a^{**}, A_b, A_s and A_{2d} holds, and

$$J_{VI}(\Phi,\vec{A},p,\rho,A_a^{**},A_b,A_s,A_{2d})=I_{II}+L+L^{CA}, \tag{16}$$

SGVP Family: Applying the constraint-recovering transformation [17], we can derive a SGVP family for the $(H_C \times H_A)$-problem from GVP VI, including also the VP V.

It can be shown similarly as in previous sections that the Euler's equations and the naturall B.C. sets of the VP V, GVP VI and its derived SGVP family are the same as those of the VP I, GVP II and its derived SGVP family respectively, except that the natural B.C. on the blade surface now has become Eq.(13).

6. SOME GENERAL REMARKS

1) It is easy to see that the traditional direct problem $(D \times D)$ [3,4] and inverse

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problem (1×1) are simply two special cases of the $(H_A \times H_A)$-problem, corresponding to $\Lambda_3^* = 0$ and $\Lambda_3^* = 0$ respectively. Accordingly, by setting $\Lambda_3^* = 0$ all VPs developed herein reduce to those presented previously in Refs [3,4].

2) If, alternatively, the B.C. on the upstream periodic boundary A_{2u} (namely $\Phi'' = \Phi' + \Delta\Phi_u$) is imposed as essential B.C., the boundary integral terms on A_{2u} involved in L of all VPs should be dropped accordingly.

3) An alternatve approach to handling free trailing vortex sheets A_{2d} is also possible by taking formally no variation of A_{2d}, though A_{2d} is unknown, but the interface conditions on A_{2d} (namely $A_n' = A_n'' = 0$, $p' = p''$) are enforced as essential ones [21].

4) As stressed in Ref.[1], sufficient attention should be paid to a rational choice of the position-variation $\delta \vec{s}$ of the unknown boundaries A_3^* and A_{2d} for facilitating the practical computation of $(\delta\vec{s} \cdot d\vec{A})$ & δJ, $(i = I \sim VI)$. Some recommendations on this point are available in Ref.[1] and, of course, also valid for the present case.

For better shock-capturing a special finite element with self-adaptive build-in discontinuities is very promising and is now being under development.

The numerical solutions to the problems $[(H_B + D) \times D]$ & $[(H_C + D) \times D]$ obtained for subsonic flow by finite elements in Refs.[19,22] show good agreement with experimental results.

8. CONCLUSIONS

The unified theory of 3-D hybrid problems of Refs [1,2] has been extended to transonic flow with shocks. This theory is primarly aimed at providing, firstly, a new rigorous theoretical basis of blade design for use in FEM and other direct variational methods (e.g. Ritz's method, Kantorovich's method) and, secondly, a wide variety of new rational versatile ways for new blade design and old blade modification. It also constitutes an important ingredient of the optimal design theory of 3-D rotor-bladings [6]. Based on the VPs for the direct problem of 3-D rotational flow [3,4], the present theory can be extended also to 3-D rotational flow. This will be presented in a companion paper.

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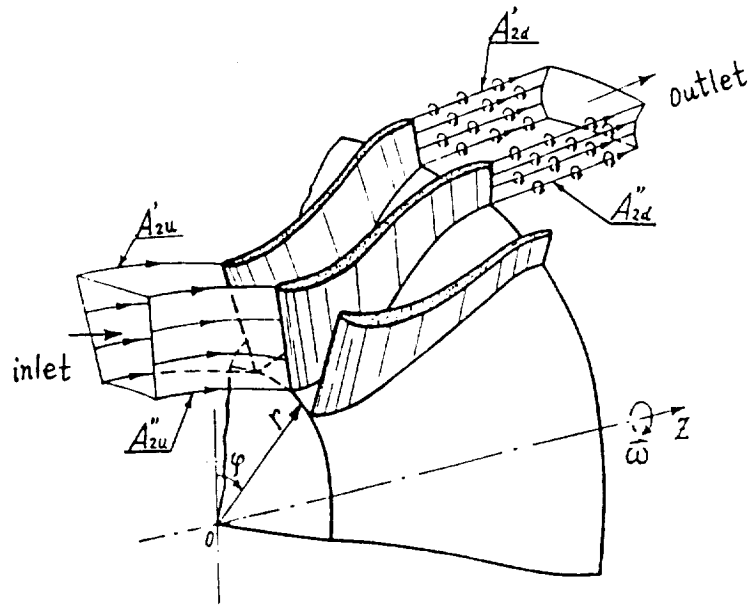
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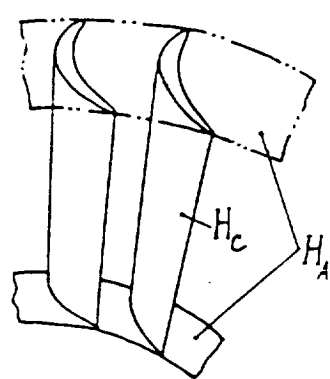Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
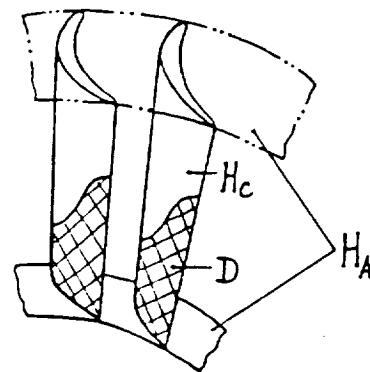(ICIDES-III). Editor: G.S. Dulikravich. Washington D.C., October 23-25, 1991.

Fig.1 Fully 3-D rotor flow

Fig.2a (H_c - H_A)-problem

Fig.2b [(H_c +D) · H_A]-problem