Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich, Washington D.C., October 23-25, 1991.

IDENTIFICATION OF DYNAMIC CHARACTERISTICS
OF FLEXIBLE ROTORS AS DYNAMIC INVERSE PROBLEM

N92-13962

W.P.Roisman, L.D.Vajingortin
Khmelnitsky Technological Institute, USSR.

In this work the problem of dynamic and balancing of flexible rotors were considered, wich problems were set and solved as the problem of the identification of flexible rotor systems, or which is the same, the inverse problem of the oscillation theory dealing with the task of the identifying the outside influences and system parameters on the basis of the law of motion known. This approach to the problem allows to disclose the picture of disbalances throughout the rotor-under-test (something that traditional method of flexible rotor balancing, based on natural oscillations, could not provide), and identify dynamic characteristics of the system, which correspond to a selected mathematical model. Eventually, various methods of balancing were developed depending on the special features of the machines as their design, technology and operation specifications. Also, theoretical and practical methods are given for the flexible rotor balancing at far-from-critical rotation frequences, which methods do not necessarily require to know forms of oscillation, dissipation and elasticity-and-inertia characteristics, and to use testing masses.

1. INTRODUCTION

The universal trend of reducing weight and gabarits of flying device engines along with high power requirements has paved way for the wide-spread application of flexible rotors and non-rigid supports. For these types of rotors, dynamics problem dealing, whith the elimination of dangerous resonance states in the area of operating rotation frequences of the machine, becomes vital. For machines under development, which do not have any well-proven analogs it is hardly possible to evaluate in terms of quantity such characteristics as inertia, rigidity and damping capability judging only by the drawing of the machine, for every one of construction elements comes simultaneously as a mass and rigidity, a source of both exiting and extinguishing vibrations, and the assumption of rotor non-deformity is no more valid. This brings us to the point where principally new balancing, technique and dynamic research are required. Now, a good deal of experimental methods are known, which allow to more precisely evaluate the elasticity-and-inertia parameters, deflection curves and rotation frequences while finishing the machine. However, these methods can not always take into account the diversity of influencing factors and dynamic model of the system. In the meanwhile, it is the task of a vital importance to find accurate values of the said parameters, corresponding to the selected mathematical model, thus making this model more effective. Finding values of these parameters for subsequent ascertaining the deflection curve forms and rotation frequences is an extremely important stage of realization of the most balancing methods.

It is well known that now close attention has been given to the problem of development of mathematical or dynamic models of higher accuracy, which models have to reflect real objects, and as many of their real features as possible. However, no calculation scheme can fully reflect the set of properties of the real object (through the vast number of these properties), but it is possible to make them close to reality. Any mathematical models are under risk of being compromized, whatever close they might be to reality, if precisely dynamic characteristics of the machine are unknown. Therefore, identification methods are required, allowing to determine dissipation and elasticity-and-inertia characteristics of the machine on the basis of appropriate experiments, the sought-for parameters being calculated with regard for all more or less

important peculiarities of the machine. By practicing these methods we can
escape the necessity for particularizing and analyzing every one of the
machine's characteristics. Parameters thus identified are all the more valuable
due to the fact that they were defined with regard to the selected idealization
of the real object, that is the said parameters were reduced to the selected
dynamic model describing the real system. Whether the selected dynamic model is
adequate to the real model and whether identified parameters are sufficiently
accurate, one can judge by how much measurred parameters of the real object
differ from those calculated in accord with the identified parameters. Criti-
cal rotation frequences, deflection curve forms, peaks of vibration et cetera
can serve as those criterion parameters. Plus, methods of identification can be
used to set the distribution of unbalances along the rotor axis. As the rotor
eccentricities are included in the equations for disturbed motion of the rotor,
it is possible to create identification algorithms of elasticity-and-inertia and
dissipation characteristics along with the rotor eccentricities at the same
time. The theory of flexible rotor balancing pays much attention to the problem
of computing the values of discrete correcting masses for rotors with pre-set
unbalanse, while the angular and linear values of the unbalance itself were
half-neglected. In the meanwhile, it is obvious that one has to know the
unbalance before getting to the task of finding solution to the set of problems
dealing with dynamic strength.

2. METHODS OF BALANCING AND IDENTIFICATION OF DYNAMIC CHARACTERISTICS OF FLEXIBLE ROTORS

Taking into account rotor flexibility allows to state and solve very impor-
tant (although more complicated) problems which were beyond the rigid rotor
method possibilities, and first of all it allows to find eccentricities of any
masses placed along the rotor axis. But unbalance is not the value to be direct-
ly measured, instead it has to be calculated through some other directly
measured magnitudes connected with the former one by unknown operators. Hence,
it is evident that the only way to find the flexible rotor unbalances lies
through their identification on the results of operating testing of the machine
or any emulating testing. It is noteworthy that along with unbalances, elasti-
city-and-inertia and dissipation characteristics as well as all other characte-
ristics of the identification algorithm can be identified. As practice demons-
trated, complex structure rotors being tested at critical frequences get defor-
med in three-dimensional manner rather than in two-dimensional one, so that the
orthogonality conditions are not valid for them. Therefore, it is necessary to
develop balancing methods on the basis of real deformations at critical
frequences. But, as the critical frequency operation is not safe and it can
affect the strength and life of the construction, it is desirable to develop
balancing methods on natural curve form of the rotor, but at non-critical
friquences and with the restricted number of start-ups. There are sertain types
of machines which require balancing only under operation mode with unchanging
frequency value, while others have to be balanced over the full frequency range.
For each case, individual and economically effective balancing methods and
approaches can be employed. Far from all structures would permit the attaching
of testing masses. For such types of machines it is necessary to employ balan-
cing methods free from testing unbalances. As it is connected with considerable
difficulties to obtain complex object's natural oscillation forms, one should
permanently search for the balancing methods not requiring the said oscillation
forms.

Certain types of designs allow deflection measuring, while other reject the
possibility absolutly. Therefore, balancing methods are needed, resting on the
deflection measuring and support reactions, housing vibrations et cetera.

Finally, in a number of cases a method is necessary which combines all
above-mentioned methods, that is when there is no need to know curve forms and

oscillation frequences or to work at critical frequences of rotor rotation, or
to use testing masses and additional start-ups, or even to know riqidity, mass
or demping parameters of the rotor - one has only to measure general weight and
external geometric dimensions.

The above-stated material proves a necessity for variuos methods of
flexible rotor balancing depended on specific designing, productive, operating,
economic and other features.

Identification algorithms of rotor characteristics of mass and reqidity as
well as their eccentricities were attained on the basis of solutions for the
differential and integral equations of the oscillations, such as Fredholm's
equations of the 2-nd kind [1], giving a description of non-balanced rotor
motion, the rotor having an arbitrary mass-and-rigidity distribution law.

$$y(z) = \omega^2 \cdot \int_0^L m(s) \cdot \alpha(z,s) \cdot (y(s) + e(s)) ds - \omega^2 \cdot \int_0^L I(s) B(z,s) \cdot (y'(s) + \xi(s)) ds, \qquad (1)$$

$$y'(z) = \omega^2 \cdot \int_0^L m(s) \cdot \gamma(z,s) \cdot (y(s) + e(s)) ds - \omega^2 \cdot \int_0^L I(s) \xi(z,s) \cdot (y'(s) + \xi(s)) ds,$$

where $y(z)$, $y'(z)$ - is, respectively, deflection and turning angle of coordinate
z cross-section of the rotor, $y(s)$, $y'(s)$, $m(s)$, $I(s)$, $e(s)$, $\xi(s)$ is, recpecti-
vely, deflection and angle of turning, linear mass, moment of inertia, radial
and angular eccentricities of the coordinate s cross-section of rotor $L, \omega, \alpha, B, \gamma$,
ξ - respectively, the length, angular speed, and influence functions.

Two ways are suggested in respect of the search for solution to these
equations. The first one is to approximize the equations with a system of
linear equations, which are convenient for decription of the motion of rotors
with discrete parameters, while the second way is excellent for description of
rotors with distributed parameters using Gilbert-Shmidt theorem for accompli-
shing expansion in a series witn respect to deflection forms of some parameters.
Both ways of finding the solution would lead you to the balancing methods
resting on natural form of deflection at critical rotation frequences. The
difference between them is that the second way is usable after some restrictions
being imposed upon the distribution function, making the said expansion possible,
while the first way is free from these restrictions, and therefore it covers a
wider range of rotor types.

2.1. TESTING MASS BALANCING

There exists more general solution. Taking into account that a defection at
any rotation frequences can be represented by the sum of deflections (which
deflections are multiplyed by some constant factors), it is possible to employ
the method of balancing on natural deflection forms at any other rotation
frequency at which the rotor deflection can be detected; and doing this you can
use a single testing mass system whith a singe start-up of the machine.

Really, carring out the rotor deflection measurments at far-from-critical
frequences (first measurment is made on the rotor with initial unbalance, the
second one-on the rotor with testing unbalances system whose eccentricities are
similar to the measured elasticity line of the rotor) and accomplishing the
expansion of these deflections and eccentricities in series, you can find the
components of eccentricities of the counterbalances, and the whole system $e(z)$.
As balancing frequences are subjected to no restrictions except for as indica-
tors of the rotor deformity, this common method when particularized by critical
frequences, turns into well-known balancing method at critical frequences.

In those cases when natural deflection forms of a rotor are not known, the
received information can be processed by means of expansion in series related to

any orthonormal system of fuctions such as sine series as shown below:

$$y(z) = \frac{Ao}{2} + \sum_{k=1}^{n} A_k \cdot \sin \frac{\Pi kz}{L},$$ (2)

$$h(z) = \frac{Bo}{2} + \sum_{k=1}^{n} B_k \cdot \sin \frac{\Pi kz}{L},$$ (3)

$$\dot{y}(z) = \frac{Do}{2} + \sum_{k=1}^{n} D_k \cdot \sin \frac{\Pi kz}{L},$$ (4)

$$e(z) = \frac{Fo}{2} + \sum_{k=1}^{n} F_k \cdot \sin \frac{\Pi kz}{L},$$ (5)

where $y(z)$ and $\dot{y}(z)$ - is the deflection caused by the initial unbalance, and the deflection appeared after attaching the testing unbalance system on the rotor, respectively; $h(z)$ and $e(z)$ - are the systems of testing and initial eccentricities, respectively; Ai, Bi, Di, Fi - coefficients of the expansions.

Assuming the unbalance-to-deflection ratio for similar expansion members to be linear, we can obtain:

$$Fi = AiBi/(Di-Ai).$$

After the curves of eccentricity projection on the two inter-perpendicular planes have been determined their vector sum can be found, which allows to obtain the form of the curve depicting the distribution of the initial eccentricities.

It is noteworthy that you can use not only deflections functions to implement these methods, but also their derivatives such as cross-section turning angles, mechanical tensions, and relative deformation; note that the highest form of unbalance reveals itself in a more apparent manner with the deflection derivatives $y^{(i)}$ that with the deflection itself, as can be seen from the expression

$$y^{(i)} = \frac{\Pi^i}{L^i} \cdot \sum_{k=1}^{\infty} A_k \cdot k^i \cdot \sin \frac{\Pi kz}{L}, \quad i=1,2,3,4.$$

2.2. BALANCING WITHOUT TESTING MASSES

This group of methods ensues from the first method of finding solution for Fredholm equation and suggests the eccentricity identification on basis of static coefficients of influence. The coefficient of influence is the value of deflection (or turning angle) of the i-cross-section caused by unity force (or bending moment) applied to the k-cross-section. The main idea of the method is like the following: deflections and turning angles or one of these parameters are measured at noncritical rotation frequencies and their projections on two inter-perpendicular planes are substituted into equations (6); equations (6) are solved for unknown projections of eccentricities.

$$y_i = \sum_{k=1}^{n} m_k \cdot \alpha_{ik} \cdot \omega^2 \cdot (y_k + e_{yk}) - \sum_{k=1}^{n} I_k \cdot B_{ik} \cdot \omega^2 \cdot (y'_k + \xi_{yk}),$$ (6)

$$y'_i = \sum_{k=1}^{n} m_k \cdot \chi_{ik} \cdot \omega^2 \cdot (y + e_{yk}) - \sum_{k=1}^{n} I_k \cdot \mathcal{E}_{ik} \cdot \omega^2 \cdot (y' + \mathcal{E}_{yk}),$$

Similar equations may be written and solved for the second projection of deflections.

This group of methods gave birth to some methods, that can be distinguished by the parameters to be measured such as deflections, turning angles, support reactions, and vibrations.

2.3. PARAMETRIC IDENTIFICATION OF FLEXIBLE ROTORS

In order to escape the procedures of obtaining static coefficient of influence as well, a group of identification algorithms was suggested allowing to find the unknown elasticity-and-inertia characteristics related to the mathematical model depicting the real rotor.

Let's consider the universally known dependences of the deflection theory:

$$M''_{zz}(z,\omega) = q(z,\omega) = m(z) \cdot \omega^2 \cdot (y(z,\omega) + e_y(z)), \tag{7}$$

where bending moment $M(z,\omega)$ at the rotation frequance ω is equal to

$$M(z,\omega) = K(z,\omega) \cdot EI(z), \tag{8}$$

The rotor axis curvature is determined from the expression

$$K = y'' / (1+(y')^2)^{3/2}, \tag{9}$$

and $q(z,\omega)$ - is inertial load, E - is Young's modulus.

Taking into account the resistance forces we can obtain (denoting the total moment of these forces through $f(z,\omega)$):

$$f(z,\omega) + M(z,\omega) = K(z,\omega) \cdot EI(z).$$

Using relations (7-9) we can obtain after some manipulations:

$$\frac{1}{m(z)} \cdot f''_{zz}(z,\omega) + \alpha_0(z) \cdot K''_{zz}(z,\omega) + 2\alpha_1(z) \cdot K'_z(z,\omega) +$$

$$+ \alpha_2(z) \cdot K(z,\omega) - \omega^2 \cdot e(z) = \omega^2 \cdot y(z,\omega),$$

where

$$\alpha_i(z) = \frac{1}{m(z)} \cdot \frac{d^{(i)}[EI(z)]}{dz^i}, \quad i=0,1,2.$$

Let's reprezent the function $- f''_{zz}(z,\omega)/m(z)$ as expanded in the series on z. Setting a finite number for members of the series we assume that the sum of the abolished members would not violate the pre-set deflection measuring accuracy. Hence,

$$\sum_{k=0}^{n} \delta_k(z) \cdot \omega^k + \alpha_0(z) \cdot K''_{zz}(z,\omega) + 2\alpha_1(z) \cdot K'_z(z,\omega) + \alpha_2(z) \cdot K(z,\omega) -$$

$$- \omega^2 \cdot e_y(z) = \omega^2 \cdot y(z,\omega).$$

The objective of consequent manipulations is to find the value of fuctions $\delta_k(z)$, $k=0,1,\ldots,n$; $e_y(z)$; $\alpha_i(z)$, $i=0,1,2$ for some fixed point $z=a$. Thus we have n+5 unknown values requiring for their determination the same number of equations.

Then, we obtain the sought-for system of n+5 equations with the same number of unknown values, putting down the previous equation for each value of ω_j at the point z=a.

$$\sum_{k=0}^{n} \delta_k(a) \cdot \omega_j^k + \alpha_0(a) \cdot K''_{zz}(z,\omega_j) \Big|_{z=a} + 2\alpha_1(a) \cdot K'_z(z,\omega_j) \Big|_{z=a} +$$

$$+ \alpha_2(a) \cdot K(a,\omega_j) - e_y(a)\omega_j^2 = \omega_j^2 \cdot y(a,\omega_j), \qquad j=1,2,\ldots n+5. \tag{10}$$

When we find the solution for this system we'll be able to determine the unknown values at the pre-set point. Keeping in mind that this point z=a was selected arbitrary we create the similar system for any other point z, obtaining thus the sought-for values at this point.

In this manner we obtain functions $e_y(z)$ and $\alpha_i(z)$ $(i=0,1,2)$.

Carring out the similar manipulations for $e_x(z)$ we obtain the value of the unbalance vector:

$$D(z) = M(z) \cdot \sqrt{e_x^2(z) + e_y^2(z)},$$

as well as the angle formed by this vector and OY-axis.

$$tg\,\gamma = e_x(z)/e_y(z).$$

Finding the solution for the system of equations gives us not only the eccentricity value but also $\alpha_i(z)$, $i=0,1,2$ and $\delta_k(z)$, $k=0,1,\ldots,n$

The known values of $\delta_k(z)$ allow to determine a total moment of resistance, while α_i give reduced masses and regidities of the rotor

$$m(z) = M \cdot \exp\left(\int_0^z (\alpha_1/\alpha_0)dz\right) \Big/ \left(\alpha_0(z) \cdot \int_0^L \exp\left(\int_0^z (\alpha_1/\alpha_0)du\right)dz/\alpha_0(z)\right),$$

$$EI(z) = \alpha_0(z) \cdot m(z),$$

where M - is the mass of the rotor.

In the most general case, a number of equations required for determining all unknown values and the same number of rotation frequences at which deflection values are measured, is equal to n+5. In practice, things are more simple.

If you know the law under which the resistance forces are changing, you need 4 equations; if this law is accompanied by a law of rigidity changing you need only 3 equations; with uniform shaft - 2 equations are enough; and if you know elasticity-and-inertia characteristics you'll need only one equation.

When you find it convenient to use a certain method such as, for example, the electric strain-gauging for relative deformation measuring and (or) parameter stability tracking during the operating period, there are parametric identification algorithms for eccentricities and flexural rigidity (or eccentricities only) based on the relative deformation being measured, and then the transition is made to the values of curvature, tension, bending moments, turning angles and deflections.

2.4. PARAMETRIC IDENTIFICATION ON RELATIVE DEFORMATIONS

Let us use Hocke's law $\sigma = \varepsilon \cdot E$ and linear differential equation for curve axis of the rotor

$$M_z = EI \cdot y''_{zz} = EI \cdot \varepsilon/t, \tag{11}$$

where M_z - is a bending moment, ε - is a relative deformation, t-is the distance from the neutral axis to the fibres for which the ε - measurment is taken.

For multidisk rotor which can to any degree of accuracy approximate (by means of adjusting a number of disks) a rotor with an arbitary mass distribution, nonbalanced forces are equal to

$$P_i = m_i \cdot (e_i + y_i) \cdot \omega^2, \tag{12}$$

where y_i can be determined by double integration of y''.

Bending moment for an arbitrary cross-section of the rotor is equal to the sum of all moments of external forces (including the support reactions) applied to a single side (left or right) from the section examined

$$M_{zi} = \sum_{k=1}^{n} b_{ik} \cdot p_k, \tag{13}$$

where b_{ik} - coefficients depending on the distances from the unbalanced forces to the supports and cross-section being examined; b_{ik} are calculated in advance.

Accomplishing the ε - measurments at some non-critical frequences for n sections we can determine the values of bending moments for these sections using (11); then unbalanced forces can be found from (12). If these forces would now be compensated with the appropriate counterbalances, the balancing would be correct only for the angular speed ω.

To carry out the balancing over the full speed range, you have to determine eccentricities e_i using (12). But you can choose another way of searching for the solution.

$$\sigma_i = \varepsilon_i \cdot E = M_{zi}/W_i = \sum_{k=1}^{n} b_{ik} \cdot P_k/W_i, \tag{14}$$

where $W_i = \dfrac{I_i}{R_i}$ - is an axial moment of resistance for section i;

R_i - radius of the rotor's cross-section.

The same can be written down differently

$$\mathcal{E}_i \cdot EI_i /t_i = \sum_{k=1}^{n} b_{ik} \cdot (y_k + e_k)\omega^2 , \quad i=1,2,\dots,n \qquad (15)$$

Now, if the e_k and EI_i values are considered unknown, you have to get two times equations then was represented by system (15); this can be obtained by measuring the values of $\mathcal{E}$ for all sections at some other non-critical angular speed ω_2. It can be shown that equations of the (15) type are independent.

2.5. STABILITY OF IDENTIFICATION ALGORITHMS

Thus, all balancing methods requiring no testing masses, are based on the identification of unbalances within the framework of inverse problems of dynamics. In this connection, the stability of identification algorithms was researched on the static influence factors known. The whole research is published in works [4] and [5]. These works also hold all major results. Fredhom's equation in the matrix form looks like the following (gyroscopic moments neglected):

$$\bar{y} = A(\bar{y} + \bar{e})\cdot \omega^2 \qquad (16)$$

where $\bar{y}$ and $\bar{e}$ are vectors of n-dimension; A-square matrix of nxn dimension, whose elements are the products of the static unfluence coefficients by the masses of corresponding disks.

This model can be practically applied in cases when small measuring errors of the values in equetion (16) (the values are measured experimentally) cause similarly small eccentricity calculation errors.

Taking into account the measuring errors we can represent system (16) in following form:

$$\bar{y} + \Delta \bar{y} = (\omega^2 + \Delta\omega^2)\cdot (A+\Delta A) \; (\bar{y} + \Delta\bar{y} + \bar{e} + \Delta \bar{e}),$$

where Δ - are the measuring errors.

In process of evaluating the relative error of the eccentricity identification we can see from the following expression

$$\frac{\| \Delta \bar{e} \|}{\| \bar{e} \|} \leq C(A)\cdot C \left(\frac{E}{\omega^2} - A\right)\cdot \frac{\| \Delta \bar{y} \|}{\| \bar{y} \|} + \frac{1}{\omega^2}[(C(A)] \cdot \frac{2 C((E/\omega^2)-A)}{\|(E/\omega^2)-A\|} \cdot \frac{\| \Delta A \|}{\| A \|} +$$

$$+ \frac{1}{\omega^2}\cdot C(A)\frac{C((E/\omega^2)-A)}{\|(E/\omega^2)-A\|}\cdot \frac{|\Delta\omega^2 |}{\omega^2}$$

(where C-is the stipulation number of any square matrix B, which number is equal to the product of the straight matrix norm by the norm of invers matrix,that is, $C(B)= \| B \|\cdot \| B^{-1} \| \geq 1$, E - is the unit matrix), that the selected model is

theoretocally stable, but as in reality we can not assume that measurment errors are likely to be less than any pre-set values, this problem is reduced to the task of evaluating the solution accuracy, which is defined by the stipulation numbers of the matreces involved.

We succeded in trying to disclose the physical sense of the matrix A stipulation number. It can be evaluated initially by the ratio of squared maximum and minimum natural rotation frequences for given discrete model. At this point, a descrepancy has came over: on the one side, trying to approximate the real rotor by increasing a number of masses, we bring the dynamic model still closer to the real structure; on the other side we increase the calculation error due to the growth of the stipulation number. Gyroscopic moments (when included in the scheme) also contribute to the growth of the calculation errors.

This is the sourse for obtaining quantitative relations between balancing accuracy on the one hand and measuring devices and a number of masses approximating the real rotor on the other hand. These relations allow to determine the third factor on the two others. For example, you can select measuring devices of required accuracy knowing the balancing accuracy and the planes of correction.

To get the required accuracy under a high stipulation number you can use the possibility to pass from the one identification algorithm to another one, for example, from system of equations of the fourth special case of the method described in section 2.3, with only one equation suggested.

3. EXAMPLE

We are going to analyse the results of the research and balancing of aero-engine compressor rotor on static influence coefficients. The disk-and-drum type compressor rotor (Fig.1) consists of ten separate disks bearing operating fan blades on their rims. Factory balancing was carried out in usual way in the "rigid rotor" mode for the two correction planes on a balancing machine at 800 rpm with operating frequences within 10000...12500 rpm.

In the process of exploitation some defects emerged such as deformation of the rear shaft, pin joint breakage, unpermissibly high resonance vibration level of the whole aero-engine.

Varios calculating techniques for natural oscillation frequences did not bring any reliable results due to the absence of precise data on the local rigidities of rotor as well as on the support pliabilities.

To increase the calculating scheme effectiveness, static tests of a number of rotors of this type were carried out, and precise values of static influence coefficients were determined over all ten stages. The first critical rotation frequency for this rotor fixed supports (the frequency was calculated on static influence coefficients) turned out to be 11000 rpm. Practically this value coincides with the third peak of vibration of the amplitude-and-frequency characteristic of the rotor (Fig.2). Peaks of vibration in the region of 4200 rpm and 8300 rpm are connected with resonance oscillation of "rigid" rotor on pliable supports.

To check whether the precise values of the elasticity-and-inertia characteristics (reduced to the selected model) were used effectively,natural oscillation frequency of this very rotor was calculated, but the calculation scheme assumed only one general mass-that of the whole rotor (M=115,4 kg) with the static influence coefficient in the centre-of-mass cross-section. The schematization error of the calculation of the first natural oscillation frequency turned out to be not more than 1,5%.

Therefore, we decided to use the said single-mass model for balancing in the region of the first critical rotation frequency, due to difficulties connected with attaching correction masses to all stages of the rotor. Maximum deflection (y=0,15 mm) value of the eighth stage was assumed for the eccentri-

city identification at 10500 rpm.   -7

On these data the value of $e=0,17 \cdot 10$ m was found from the expression

$y = \omega \cdot m(y+e) \cdot \alpha$. Correcting mass was brought into on the score of the eighth stage fan blade, which was replaced. Balancing results are shown on Fig.2.

The carried out research allowed to improve the design and balancing technology of the rotor and to eleminate the indicated defects.

Eccentricity identification based on measured deflections of the rotor using static influence coefficients was also carried out by Bradjko A.I. [6] employing a computerized imitation model, laboratory physical model and a natural rotor of compressor on an accelarating vacuum stand.

Table 1 holds data on mass of stages, static influence coefficients and deflections of the 5-mass rotor, which he balanced.

For $\omega^2 = 0,274 \cdot 10^6$ 1/c^2 the following values of eccentricities were obtained:

$$e_1 = 77,4 \cdot 10^{-6} \text{ m}; \quad e_2 = 89,9 \cdot 10^{-6} \text{ m}; \quad e_3 = 105 \cdot 10^{-6} \text{ m};$$

$$e_4 = 79 \cdot 10^{-6} \text{ m}; \quad e_5 = 59,5 \cdot 10^{-6} \text{ m}.$$

Correcting masses were attached to all of the five stages.

As a result of the balancing that was carried out, the maximum rotor deflections were diminished almost by 4 times, and housing vibration were diminished by 2,5 times.

CONCLUSIONS

The problem studying dynamics and high-frequency balancing of flexible rotor systems can be set and solved as the task of identification of elasticity-and-inertia characteristics and eccentricities corresponding to a selected calculating model within the framework of the inverse problem of the oscilation theory.

On the basis parametric identification of the flexible rotor systems on the measured vibration parameters of products was developed, providing simultaneous determining of the mass, rigidity, and demping characteristics of the rotor and its eccentricities as well.

The identification algorithms obtained on measured parameters of products allowed to develop three groups of flexible rotor balancing, which don't require knowing rotor oscillation forms or operating at critical angular speeds:

- with emploing only system of testing masses and a single testing start-up;

- without employing testing masses and start-ups, grounding known and unknown elasticity-and-inertia characteristics.

The accomlished research on stability and accuracy of the suggested identification algorithms allows to have optimal relations between the required balancing accuracy, measuring instruments and dynamic model of the system.

The obtained results were used for research of dynamic and high-frequency balancing of a turbopump assembly unit, a turbogenerator, rotors of gas turbine engine compressors, and they allowed to considerably lower the vibration level, deflections and tensions in the parts of flexible rotor systems, thus increasing life and reliability of products.
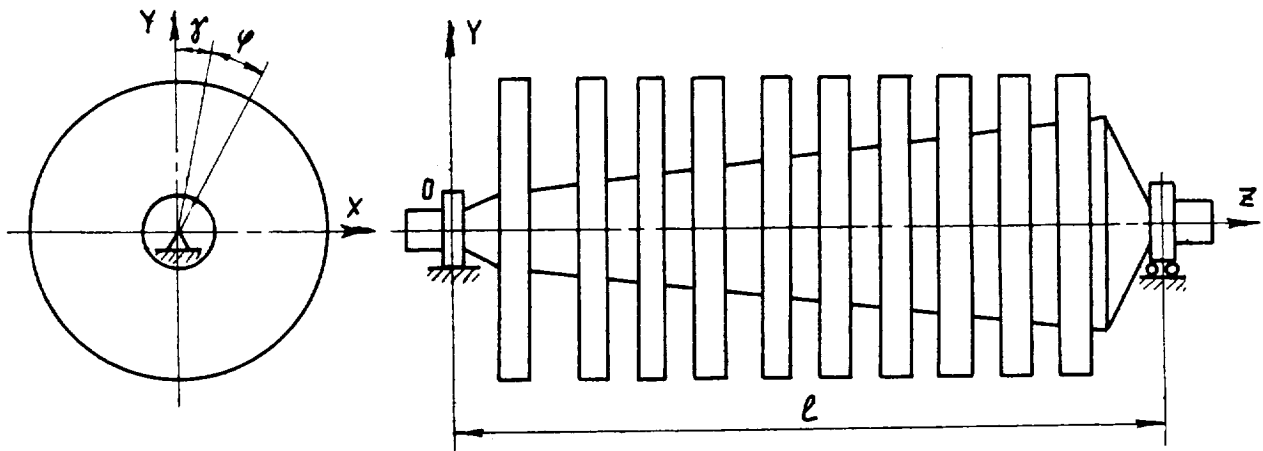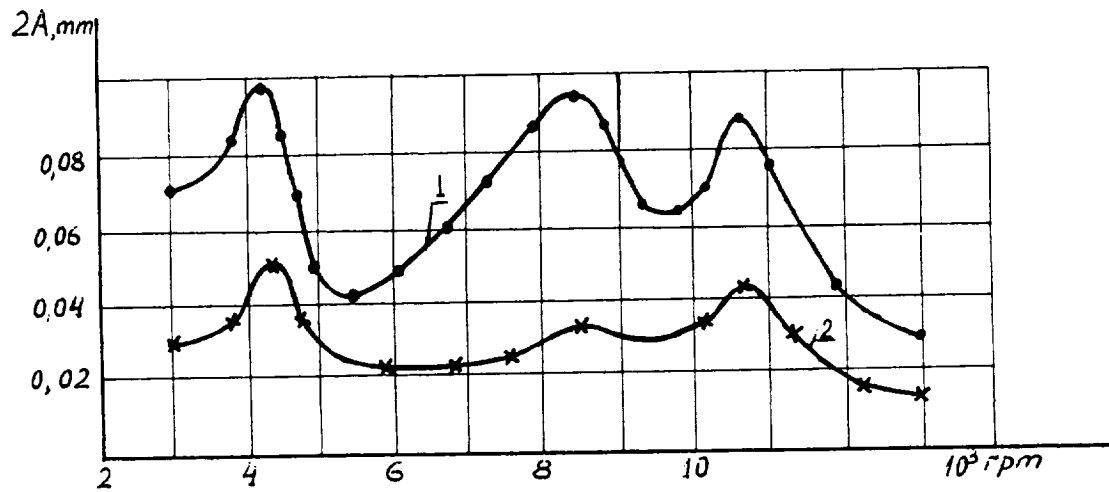
Fig.1. The rotor scheme.

Fig.2. Amplitude-and-frequency characteristics of the rotor.
1 - before the balancing; 2 - after the balancing

468

Third International Conference on Inverse Design Concepts and Optimization in Engineering Sciences
(ICIDES-III). Editor: G.S. Dulikravich. Washington D.C., October 23-25, 1991.

Table 1

Data for identification of eccentricities of a five-masses rotor

N	masses m_i, kg	deflections $y_i \cdot 10^{-5}$ m	static coefficients of influence $\alpha_{ik} \cdot 10^{-9}$, m/N				
			1	2	3	4	5
1.	9,03	7,87	9,2	7,4	6,23	4,8	2,77
2.	9,96	11,38	7,45	9,0	8,95	7,3	5,25
3.	12,32	11,16	5,3	7,85	9,88	8,5	6,8
4.	12,53	11,37	4,2	7,0	8,62	9,7	8,98
5.	17,6	10,35	2,62	4,67	7,6	9,43	10,8

REFERENCE LIST

1. Lovitt U.V. Linear integral equations. -NTI-publishers, Moscow, 1933.
2. Levit M.E., Roizman W.P. Vibration and balancing of the aero-engines rotors. -Mashinostroyeniye Publishers, Moscow, 1970.
3. Roizman W.P., Vajingortin L.D. Invention certificates # 1921852 and # 1961887. The discription of the inventions.
4. Roizman W.P., Vajingortin L.D. Selected problems of the theory of flexible rotor balancing. Part of the book: Elastic and hydroelastic oscillations of assembly parts and whole structures, Nauka Publishers, Moscow, 1979, p. 55-63.
5. Roizman W.P., Vajigortin L.D., Guditch R.V. Statictic identification algorythms of objects and processes. Mashinovedenie, N5, Nauka, Moscow, 1984, p.43-45.
6. Bradjko A.I., Roizman W.P. Computer-assisted balancing of the rotors of hydro-turbine engines. Selected works collection titled: Theory and practical of balancing technique, Mashinostroyeniye Publishers, Moscow, 1973, p.137-145.