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ON DESIGNING FOR QUALITY

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The problem to ensure the required quality level of products and (or) technological processes often becomes more difficult due to that there is no general theory for determining the optimal sets of values of the primary factors, that is, of the output parameters of the parts and units comprising an object and ensuring the correspondance of the object's parameters to the quality requirements. This is the main reason for a number of years being spent for finishing complex articles of vital importance.

To create this theory, one has to overcome a number of difficulties and to solve the following tasks : creating reliable and stable mathematic models showing the influence of the primary factors on the output parameters; finding of accurate solutions when mathematical models are poorly stipulated; creating new technique of assigning tolerances for primary factors with regard to economical, technological and other criteria, the technique being grounded on the solution of the main problem; well-reasoned assignment of nominal values for primary factors which serve as the basic for creating tolerances. Each of the above listed tasks is of independent importance. The present work is an attempt to give solution for this problems. The foregoing problem dealing with quality ensuring in mathematically formalized aspect has been called the multiple inverse problem.

1. INTRODUCTION

When creating any new machine, mechanism, technological, medical and other systems and processes one has to start with presenting the original technical specifications of output parameters.

The above-mentioned technical conditions are normaly represented as rated values and tolerances of output parameters. Then, creators face the problem of designing, manufacturing and finishing up the object so that it could carry out predetermined functions while preserving the output parameters within the range specified by the technical conditions, thus providing the required level of quality.

The values of complex object's output parameters depend on the rated values and tolerances of great number of parts and units, which form the object and provide its functional purposes. These units' and components' parameters are further called as basic parameters or primary factors.

It is well known, that finishing up the object is a long and hard task, especially for a new one that has no analogues. Of course, it would be mostly desirable to get quick solutions for the following problems : does the scheme or construction created meet the specifiend requirements under the suggested technology; if not, which construction and which technology would satisfy the objectives?

We have no possibility to analyse the solutions accomplished in diffirent period. But we have to note, that the steady developtment of sciences and technique makes this problem up continuosly change, making it more complex but all the time up-to-date. Its solution is defined by the level of modern scientific achievements. The formalized description of the problem as well as possible method of its solution are described below.

2. THE STATEMENT OF THE PROBLEM

Let the quality of each object is rated by values of its control output parameters represented by the vector

$$\bar{Y} = \{ Y_1, Y_2, \ldots\ldots, Y_m \}.$$

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To provide the required level of quality the following relations should be valid

$$[y_i] < Y_i < [Y_i], \quad i=1,2,\dots,m, \tag{1}$$

where $[y_i]$ and $[Y_i]$ are lower and upper limits of the parameter Y_i, correspondingly. They are represented by initial technical conditions.

We've going to look for the solution of this problem as a set of values of the primary factors, which can be represented by following inequalities:

$$[x_i] < X_i < [X_i], \quad i=1,2,\dots,n. \tag{2}$$

The belonging of the vector to this set has to provide the fulfiling of the restrictions (1), which are imposed on the output parameters.

We've call the problem, which has just been formulated as the multiple inverse problem. This term emphasizes that the solution of the problem suggests the determination of the set of points (region) in the n-dimensional space of the primary factors. This circumstance differs it from the point inverse problems, which are traditionally solved in many technical bronches. In the point inverse problem only one vector of the primary factors and (or) one collection of model's parameters have to be determined, if the vector $\bar{Y}$ is set beforehand.

3. THE REDUCTION TO THE PROBLEM OF THE OPTIMIZATION

The above mentioned vector $\bar{Y}$ is completely defined by the vector of the primary factors $\bar{X}=\{X_1, X_2, \dots, X_n\}$ and the operator

$$\bar{Y} = \bar{f}(X_1, X_2, \dots, X_n; B_1, B_2, \dots, B_k), \tag{3}$$

which carries out connectioh between the said vectors. The structure of $\bar{f}$ and the vector of parameters of the mathematical model $B_1, B_2, \dots, B_k$ corresponds to the phisical nature of the object and its functional destination.

As a rule, the industrial, physical, economical and other considerations allow to indicate the widest bounderies of the set of the possible values of the primary factors. Then, the relations (3) enriched with these new bounderies can be presented as a system

$$\begin{cases} Y_i = f_i(X_1, X_2, \dots, X_n; B_1, B_2, \dots, B_k), \quad i=1,2,\dots,m, \\ C_i < X_i < D_i, \quad i=1,2,\dots,n, \end{cases} \tag{4}$$

keeping in mind the co-ordinate form of the operator (3).

It should be noted, that the structure of the functions and the sets of the primary factors' values may be various. For example, one of the primary factors X_i can have the descrete or even finite set of values. Then, this fact must be reflected in (4) by such relations as $X_i = 1,2,\dots,N$.

The system (4) determines some curved region in the n-dimensional space of the primary factors. In geometrical sense, if you'd find some sets in the form of (2), you would inscribe n-dimensional parallelepiped in the said curved region.

This problem has more than one solution due to the fact that countless set

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of this kind of parallelepipeds may be inscribed into the above-mentioned region. Every one of them can be entirely determined by two conditions. The first one comprises the point $\bar{X}_0 = \{X_{10}, X_{20}, \ldots, X_{no}\}$ which is known to belong to the region and corresponds to one of the object's basic version with these nominal inner characteristics. The second condition is represented by set of lower δ_i and upper Ω_i deviations of the primary factors from their basic values corresponding to the boundaries of the tolerance zones of the primary factors, that is to the technology chosen.

The following relations are evident here:

$$X_{io} - \delta_i < X_i < X_{io} + \Omega_i, \quad i=1,2,\ldots,n. \tag{5}$$

But not every solution (5) of the problem formulated above can be realized in practice because of various constructive, technological, economical and other considerations. The high cost of production or the absense of the necessary equipment, components, materials, performers of required qualification, peculiarities of the object can serve the sourses of the troubles.

These considerations can be analytically formulated by the criteria expressed through the deviations of the primary factors from their basic values

$$F_i = F_i(\delta_1, \delta_2, \ldots, \delta_n, \Omega_1, \Omega_2, \ldots, \Omega_n), \quad i=1,2,\ldots,L. \tag{6}$$

It is evident that of the above mentioned parallelepipeds the most acceptable for the practical implementation are those, in which the criteria (6) or some of them are optimized and another of the criteria are added to the restrictions in (4).

Various criteria of the tolerances optimization are possible. The cost function is the most important. Sinse this functional dependence on the current tolerance values is usually unknown, it might be possible to replace it by in some sense equivalent criteria. For example, it might be possible to demand the maximizing of every or some tolerances.

Then the criteria (6) will look like following

$$\max_i(\delta_1, \delta_2, \ldots, \delta_n, \Omega_1, \Omega_2, \ldots, \Omega_n) \rightarrow \max.$$

Thus, the problem of providing the pre-determined level of the object's quality is drawn to the multicriteria optimization problem with the certain restrictions. The deviations δ_i and Ω_i are to be determined in this problem so that the restrictions (1) are valid for the region (5).

4. NECESSARY POINTS OF THE SOLUTION

The way to the solution of the formulated problem is connected with some difficulties. The overcoming of these difficulties has to be the necessary points of the solution.

4.1. MATHEMATICAL MODELLING

In practice there are cases, when the model required for writing down the functional part of the restrictions (1) is known. But as a rule, complicated objects can have either unknown (inaccurate) parameters or unknown structure or both. Therefore it is necessary to develop an easy-operated approach to the task of setting functional dependences of primary factors and output parameters having reliable coefficients reduced to the object model.

The algorithm-creating technique for urgent mathematical modelling can be ground on the active-controlled influence on the object.

Let the calculated model of the experimental sample be represented as

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$$\begin{cases} Y_i = f_i(X_1, X_2, \ldots, X_n; B_1, B_2, \ldots, B_k), \\ i = 1, 2, \ldots, L. \end{cases} \tag{7}$$

At the beginning we suppose that the structures of the functions fi as known, but parameters B_i are not.

If the output parameters' values and some primary factors are substituted in (7) then k of unknown values of primary factors and j of coefficients of the model should satisfy (7) together with the substituted values. As a rule, the system (7) is not completely defermined (that is k+j>L) and admits of countless set of solutions. However, keeping in mind that the object operates and really exists, it would be natural to find the solution fully responsive to the given object. Hence, it is necessary to complete the problem by means of some additional experiments. The method of test parameters suggested here is bound to provide the afore mentioned completion.

To put it into practice, k+j-L additional components should be employed or varied in the object under examination, or the object should be exposed to the same number of the test modes of functioning, the modes belonging to the set of modes specified by initial technical conditions. Thus, the operation of the object is regulated in active manner. The influence of the said test components (or modes) in accord with components whose parameters are being identified, allow the measuring of missing values of the output parameters make the system (7) complete, and identify the missing factors and coefficients of the model, that is to find the solution of the inverse point problem.

In case the structure of functions f_i is unknown we recommend to disintegrate them by series according to any complete system of functions, for example, the series

$$\begin{cases} Y_i = B_{io} + \sum_j B_{ij} \cdot X_j + \sum_{j,k} B_{ijk} \cdot X_j \cdot X_k + \ldots, \\ i = 1, 2, \ldots, L \end{cases} \tag{8}$$

and to identify coefficients by several sequential stages. At the beginning, we suggest to determine the coefficients of linear approximation using required quantity of test parameters. Then, after employing additional test parameters, the functions Yi are selected which are adequate to the object. For the rest of Yi functions the square and higher appoximation are considered. It is easy to show that the process is converging and the number of stages usually does not exceed two or three.

Putting the method into practice one has to start with considering the output parameters of large units as primary factors and to sort out those vitally influencing the functioning of the object. Then, the functional dependence of sorted out characteristics on the smaller units is ascertained in similar manner and so on. This approach based on the principle of hierarchy allows to operatively adjust the model to the object under examination with regard to the degree of its idealization and functioning conditions, and eliminates the necessity of the registration and analysis of the inessential primary factors.

However, the hierarchical principle of modelling can be employed only if the output parameters of separate units can be measured at every cascade. If this possibility is not provided by the design the method of multy-cascade modelling can be used. Let us assume that the interconnection exists between separate units (cascade) and an output parameter, that is we know the function $Y = f(\varphi_1, \varphi_2, \ldots, \varphi_s)$, where $\varphi_i = \varphi_i(X_{i1}, X_{i2}, \ldots, X_{it})$ - output parameter of

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i unit. Then, fixing the values of the primary factors of all cascades but one, and measuring the output parameter Y, we can create the model of every cascade and combine them into the common model of the object capable to varying parameters of all units [I].

We would like to emphasize that this manner of mathematical modelling is in itself a particular case of the inverse problem solution.

4.2. PROVIDING THE MODEL STABILITY

However models are practically important only if the faults of the experimental input information are not likely to cause intolerably large faults of the values being determined, that is the models should be stable. In [2] it is shown how the stability of the model should be determined with regard to all or some factors, as well as the necessary proof is placed to estimate the relative fault of parameters identified with the help of linear model:

$$\|\Delta\bar{X}\| / \|\bar{X}\| \leqslant C(A)\cdot\|\Delta\bar{Y}\| / \|\bar{Y}\| + [C(A)]^2\cdot\|\Delta A\| / \|A\|. \tag{9}$$

This estimation, thus, is represented by the number of stipulation C(A) and the faults of characteristics and elements A being measured. Estimation (9) allows to explain the decreasing stability of the model while the degree of A is growing. In other words, it states the necessity to search the compromize between the desire to give thorough description of the object using large number of factors and ensuring the stability of the model. The estimation (9) showes that the model can be regularized not only by way of influencing the A operator, which in real production environments can not always be available for the various reasons. Not less efficient regularization can be achieved by way of influencing the Y vector of parameters being measured, which method is based on the statistical nature of the vector. To achieve this, you have to carry out a great number of Y measurments,insert the value of the vector into the calculated model and count the realization of every one of identified parameters. Mathematical expectations of parameters values calculated on the base of these realizations are assumed as true values of these parameters. The estimation of the number of realizations sufficient to ensure the accuracy of the method

$$n \leqslant t^2\cdot(C(A)\cdot\sigma_1 / \|\bar{Y}\| + C^2(A)\cdot\sigma_2 / \|A\|)\,/\delta^2 \tag{10}$$

(where σ_1 , σ_2 - mean root squere deviations of the vector components and matrex A correspondingly, t - Student's coefficient) shows that the described method of statistical solution is efficient when coupled with methods of influencing the A operator [2].

Estimation (9) places interest in pure practical aspect, since it states the functional interdependence of economical factors (accuracy of the method and accuracy of measuring facilities), thus making it possible to choose one of these requirements to provide the two others set apriory.

5. PROBLEMS, CONNECTED WITH OPTIMIZATION

The concrete optimizing method can be chosen from the sufficiently wide collection of the detailly developed optimizing algorithms. It is evident that the results of the criteria optimization depends on the basic version chosen, i.e. on the point $\bar{X}o$. Here we offer some recommendations connected with it.

5.1. CONSTRUCTING THE REGIONS

We suggest that the algorithm is implemented through making proper regions spreading from basic point with step-by-step checking the validity of the restrictions (1). This basic point often can be determined out of physical or practical considerations. But there are cases when this point is uknown, and the problem of the seeking becomes very difficult one.

5.2. ON CHOISE OF THE BASIC POINTS

Because of great number of random and unpredictable situations that may occur during the manufacturing and exploiting of the object, and due to non-

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stability of properties of construction materials, the characteristics of the object may be treated as random values. Then we can estimate the true values using the method of confidential intervals [3], provided that the law of distribution is known.

For a long time, the normal distribution law or its modification was considered the best approximation for which the majority of statistical criteria and estimations can be applied. However, a lot of practical problems have turned up lately which give strong evidences that the normal distribution law is not so universal as it was thought. The situations emerging during the study of real process bear evidences that a good deal of the object's parameters' distributions deviate from the normal distribution, moreover they often have more than one summit of the probabilities' function densities. Therefore the physical essence and new technical schemes of the processes of this sort are disclosed in [4]. The schemes are based on the method of representation of each random value selection in the form of the set of subselections, combined by some dominant causes for diversity of values of the quantity under examination. Here some examples of such kind of situations are illustraited and it is shown that more often than not the situations of this type can be depicted by Gauss functions' linear combinations with some weight coefficients Pi assigned to estimate the contribution of each subselection to total selection of the realized random values.

$$f(X; a_1, a_2, \ldots a_n; S_1, S_2, \ldots, S_n; P_1, P_2, \ldots P_n) =$$

$$= \sum_i P_i \cdot S_i^{-1} \cdot (2\Pi)^{-0,5} \cdot \exp\left(-(X-a_i)^2 / 2S_i^2\right). \tag{11}$$

In [4], various methods for finding the unknown parameters of the function (11) are described, depended on the required accuracy of calculation and the selected criterion of approximation histograms. The values thus determited define the integral function which in turn makes it possible to write down the equations for locating the permissible [X] value:

$$W = P\{X < [X]\} = \sum_i P_i \cdot S_i^{-1} \cdot (2\Pi)^{-0,5} \cdot \int_{-\infty}^{[x]} \exp(-(X - a_i)^2 / 2S_i^2) dX. \tag{12}$$

When processing the experimental data one has employ well-founded technique for compiling histograms to prevent, on the one hand, the probability of missing considerable part of the distribution by too large spacing intervals, or having to deal with unimportant subselections which may turn up under too small spacing intervals, on the other hand. It is good idea to start making a histogram with the smallest possible spacing interval which can be compared with the measuring accuracy, and to approximize the histogram using function (11) having the number of additieves equal to the number of spacing intervals. The already known weight parts which turned out to be less than pre-set probability $\alpha = 1 - W$ give you an indication of unimportant subselections joined with the contiguous subselections. Then, the spacing interval tends to gradually increase, and the whole procedure is carried out over again untill each weight part is made comparable with α.

And now the recommendation on the selection of the basic point rest on the following ideas.As basic point we can select the point belonging to the space of primary factors and having one of the mathematical expectation as the first co-ordinate, the mathematical expectation being that of random value depicting the distribution of the first primary factor. Analogically, the second co-ordinate will be connected in the similar way with the second primary factor, and so on.

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5.3. CHECKING THE VALIDITY OF THE RESTRICTIONS

When making regions spreading from basic point checking the validity of the restrictions (1) during each step of the optimization can be accomplished on the set of uniformly distributed points belonging to the created region of points. But in some practical situations this checking technique can be simplified. For example, when partial derivatives of the functions (3) have invarriable signs then the checking may be accomplished only for the tops of the region.

5.4. CHOOSING THE OPTIMAL BASIC VERSION

As the number of the basic points can be more than one it is natural to realise the optimizing algorithm for each of the possible basic versions separately, and to choose the most optimal of them as regards to criteria (6).

6. POSSIBLE APPLICATIONS

This approach which generally formalizes the problem of optimal ensuring of technical conditions requirements for output parameters of the article or technological process allows, in the first place, to ascertain the interlinkage of problems connected with selection of the object's basic version determined by rated values of its primary factors, with the problem of setting disigning and technological tolerances for them depended on the restrictions of the technical conditions for the object output parameters. This practicularly provides for the study of various selection possibilities concerning the already known and finished units, processes and technological decisions which might be utilized in the article or technological process being created.

Thus we are granted the possibility of formulating and solving the problem of synthesizing some of the design versions of articles, having optimal sensetivity towards manufacturing and operational deviations of their primary factors, that is we can directly link the selection of the object basic version to specific features of its practical implementation.

Secondly, this approach allows of formalizing a great quantity of important promiscuous special problems of design, manufacturing and testing procedures regardless of the technical branch of application.

The same conditions are capable of procuring recommendations for setting tolerances for both primary factors of the article as a whole and its separate units during design, manufacturing and finishing procedures. It allows also of carrying out the selective machine assembly by way of sorting out the object's components and materials by real values of their parameters which are sure to create the most favourable combinations.

When it comes to serial production it is possible by means of multiple inverse problem with regard to statistical origin of parameters, to effect diagnostics dealing with the yield (or the percentage of waste articles) and allowing of setting conditions providing the technical conditions requirements.

The same approach is effective when solving other types of problems. For instance, we can check the possibility of attaining the desired values of all or some output parameters under given design or technological conditions, which stands for finding a solution of a relative multiple inverse problem. If the solution doesn't exist or it's out of reasonable limits in designer's or productive engineer's point of view, that means that the given object fails to meet the requirements if technical conditions therefore it is necessary to take to searching for completely new designing or technological decisions based on different principles.

This approach is also good to cover not only the article as a whole but its components as well.

Thus the approach under examination is a natural reflection of the set of real situations emerging at the stage of design, manufactoring and finishing articles.

7. EXAMPLE

To check up the versatality of the above described theory multiple inverse

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problem was formuleted and solved, the problem being applied to various branches of technical engineering including:

- providing the strength and air-water proof quality for radioelectronic elements [5];
- enhancing the stability of the output parameters of the articles of the secondary radiodetection (airplane answering devices) [1];
- lowering to preset level of vibrating activity of gas-turbine engines and turbopump assembly unites [2,6-8,11];
- assigning of well-reasoned tolerances for the residual unbalance during balansing and assembling of rotors [4,9];
- developing of balancing technique for flexible rotors [10].

Each of the above-listed applications is complex enough in itself, and it would take more time and space than we have,to give their full description here. Therefore, the present work is an attempt to throw light upon general ways of finding solutions to multiple inverse problem, and the new approach to the problem is illustrated by brief example showing the way to lower vibrating activity of a turbopump assembly unit. We also supplyed the example with necessary references to the sources containing more detailed description of the statements placed here.

Turbopump assembly units with high-speed rotors are widely used in varius branches of industry including rocket production, aircraft building, chemical industry and so on. As it has been found out that the device-under-test had enhanced vibration caused by rotor unbalance, the task was to lower the vibration and the rotor deformation; to put the rotor bearings load within the threshold of 300 N, in particular, by way of assigning the appropriate residual values of eccentricity for the most massive parts attached to the rotor shaft.

7.1. GETTING AN EFFECTIVE MATHEMATICAL MODEL

The turbopump assembly unit shaft, rotating in two supporting bearings, carries two compressor impellers and axial turbine disk. These are very points of heavy masses fraught with possible unbalance; which consideration served a basic reason for choosing the "three-masses" calculation scheme shown on Fig.1.

To make the mathematical model of the rotor oscillations, corresponding to this scheme, more effective, we have accomplished the identification of the rotor parameters includinq stiffness, and mass and inertia characteristics, using the method of testing parameters which in our case, are four different values of the speed of rotation ω_j , where j = 1,2,3,4.

The rotation of the rotor is described with the help of the integral-and-differential equations of the bending theory [10]. The resulting equations for the three cross-sections of the rotor link the unknown values of stiffness EI, mass m and eccentricity e with the rotor deflection y (the equations being created for the two inter-perpendicular planes).

$$\alpha_0 \cdot K''_{zz}(z,\omega_j) + 2\alpha_1 \cdot K'_z(z,\omega_j) + \alpha_2 \cdot K(z,\omega_j) - e_y \cdot \omega_j^2 = \omega_j^2 \cdot y, \quad j = 1,2,3,4, \qquad (13)$$

where

$$\alpha_i = \alpha_i(z) = \frac{1}{m} \cdot \frac{d^{(i)}(EI)}{dz^i}, \quad i = 0,1,2, \qquad (14)$$

$K(z,\omega) = y''/(1+(y')^2)^{3/2}$ – is the curvature of the rotor elastic curve,

z – is the axial co-ordinate of a cross-section.

Then, we accomplished the measurments of the diflection values at the above

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mentioned points for the whole range of the rotation frequency (in our case 0-18000 rpm), and selected four specific values - 14100, 15000, 15600, and 16000 rpm.

After that, two components of the defections at each of the points (each component being a projection to one of the said inter-perpendicular planes) were substituted to the two systems of equations (13).

7.2. CHECKING UP THE STIPULATION AND PROVIDING THE ACCURACY OF SOLUTIONS

Before dealing with the system of equations (13) we calculated their stipulation numbers, which numbers turned out to be within the range of 3,2...6,7.

Consequently, [13] the expected error of the solution might be as high as 134% provided that the 15% measuring devices accuracy is achieved. To rise the accuracy of calculation we employed our statistical method for ensuring stability of mathematical models [2]. In our example, the measuring operation was carried out over 50 times, the result being that we found the mathematical expectations of the values and phases of the deflections. The averaged values were substituted to the equations (13). The solution brought us the following results:

For the first cross- section:

$$e_x = -5\cdot 10^{-6}\ m;\ e_y = 5{,}84\cdot 10^{-6}\ m;\ \alpha_0 = 185{,}65\ m^4/s^2;$$

$$\alpha_1 = 270{,}37\ m^3/s^2.$$

For the second cross-section:

$$e_x = -9{,}0\cdot 10^{-6}\ m;\ e_y = 1{,}7\cdot 10^{-6}\ m;\ \alpha_0 = 710{,}67\ m^4/s^2;$$

$$\alpha_1 = -247{,}18\ m^3/s^2.$$

For the third cross-section:

$$e_x = -6{,}2\cdot 10^{-6}\ m;\ e_y = 30\cdot 10^{-6}\ m;\ \alpha_0 = 280{,}83\ m^4/s^2;$$

$$\alpha_1 = 680\ m^3/s^2.$$

The values of α_0 and α_1, thus found for each of the cross-sections, made it possible to determine the values of the rotor reduced mass and stiffness according to the formulae

$$m = M\cdot \exp\left(\int_0^z \alpha_1/\alpha_0\, dz\right) / \left(\alpha_0 \cdot \int_0^L \exp\left(\int_0^z (\alpha_1/\alpha_0)du\right)dz/\alpha_0\right),\quad EI = \alpha_0\cdot m,$$

where M is the rotor mass.

For the first cross-section:

$$EI_1 = 4147\ N/m^2;\ m_1 = 0{,}22\cdot 10^{-2}\ kg/m;\ M_1 = 4{,}05\ kg.$$

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For the second cross-section:

$EI_2 = 15954\ N/m$; $m_2 = 0{,}2 \cdot 10^{-2}\ kg/m$; $M_2 = 2{,}74\ kg$.

For the third cross-section:

$EI_3 = 23988\ N/m$; $m_3 = 0{,}83 \cdot 10^{-2}\ kg/m$; $M_3 = 11\ kg$.

These values of EI and m in their turn, allowed us to calculate the critical frequences of the rotor oscillations for the first and second forms correspondingly:

$$n_1 = 16600\ rpm, \quad n_2 = 25080\ rpm.$$

Experimentally measured value was 16100 rpm, which means that the calculation error did not exceed 3%.

7.3. FINDING PERMISSABLE VALUES FOR ACCENTRICITIES OF THE IMPELLERS AND THE TURBINE DISK

Let us denote $\bar{\Delta} = \{\Delta_1, \Delta_2, \ldots, \Delta_n\}$ - vector, whose coordinates are represented with the product of disk masses by their residual eccentricities.

The vector of residual deflection values should satisfy the equation

$$\bar{y} = \omega^2 \cdot A\bar{y} + \omega^2 \cdot A'\bar{\Delta} \quad ,$$

where A' - matrix of pliabilities α_{ik}, created by means of the Mohr metnod [12] on the basis of already known values of stiffness EI, A - matrix composed of the products $\alpha_{ik} \cdot M_k$, where M_k - mass of the disks.

Hence,

$$\bar{y} = \omega^2 \cdot (E - \omega^2 \cdot A)^{-1} \cdot A'\bar{\Delta} \quad ,$$

where E is unit matrix.

Now, the equation for the support reactions looks like

$$R_i = \sum_k b_{ik} \cdot (M \cdot y_i + \Delta_i)\omega^2, i = 1,2.$$

where b_{ik} - are known values represented through distances from the disks fitting planes to the corresponding support plane. The following inequalities serve here as conditions of the (1) type:

$$|R_i| < 300\ N, \quad i = 1,2. \qquad (15)$$

The acceptable value of defections is limited to 0,1 mm value.

The criteria of optimization are

$$\Delta_i \rightarrow \max, \quad i = 1,2,3.$$

It is easy to understand that the basic point $\bar{\Delta}_0$ represents the ideal situation, that is when rated values of the eccentricity equals to 0 ($\bar{\Delta}_0$ = { 0; 0; 0}).

When trying to create the three-measured expansions around this point it is

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quite satisfactory to check the conditions (15) for rotation frequences in the region of critical value 16000 rpm.

We accomplished all calculations resting on the algorithm set forth in [13], and found out that the residual eccentricity values should not exceed 0,0008 mm.

Considering that it would be extremely difficult to put this condition into practice, and costly too (plus loss of balancing during operation possible), it has been decided that with given construction of the assembly unit the above-formulated problem has no practical solution and some other method should be used for lowering vibrating activity of the unit.

Particularly, we suggested and realized the high-frequency balancing technique comprising the rotor eccentricities identification on the basis of deflections measured at the three sections, and compensation of the deflections by counterbalances.

While so doing we determined the values of unbalances Di for rotor under-test using already known values of the eccentricity projections

$$D_i = M_i \cdot \sqrt{e_{xi}^2 + e_{yi}^2} .$$

Also, orientation of the vectors in relation to a projection plane is represented by angles

$$\varphi_i = \operatorname{arctg} (e_{yi} / e_{xi})$$

It's turn out that $D_1 = 273$ g.cm, $\varphi_1 = 95^o$;

$D_2 = 2,48$ g.cm, $\varphi_2 = 170^o$; $D_3 = 30,6$ g.cm, $\varphi_3 = 102,3^o$.

Fig.2. represents values of the rotor deflection for the section III experimentally measured in initial state and after applying three correcting counterbalances used as compensators, whose values have just been calculated.

In general, the balancing procedure gave the following results: the rotor deflections, withing the range of frequences 2000-18000 rpm. lowerd by 6 times, vibration amplitude of supports lowered by 4 times, support reactions lowered by 4,5 times, the rotor shaft static strains fell by 3,5 times, while dynamic strains-by 3,5 times.

8. SUMMARY

The results of the accomplished are characterized with a concrete tendency for industrial application and can be used for: chosing optimal basic versions of objects, components and parts; assigning optimal, and economically and technologically reasonable tolerances for functional parameters of parts and units being produced; assigning optimal operating conditions for assembling and adjusting technological processes; accomplishing diagnostic of technical condition of objects and their components; identifying real values of the parameters of objects, and the distribution laws for the errors of creating of parameters. Finding solutions to this problem allows to cut investments and save time for finishing objects and controlling their quality in the process of manufacturing, and on the basis of pre-set criteria, to rate the output parameters (quality characteristics) of the object as a whole and its components and parts as well.

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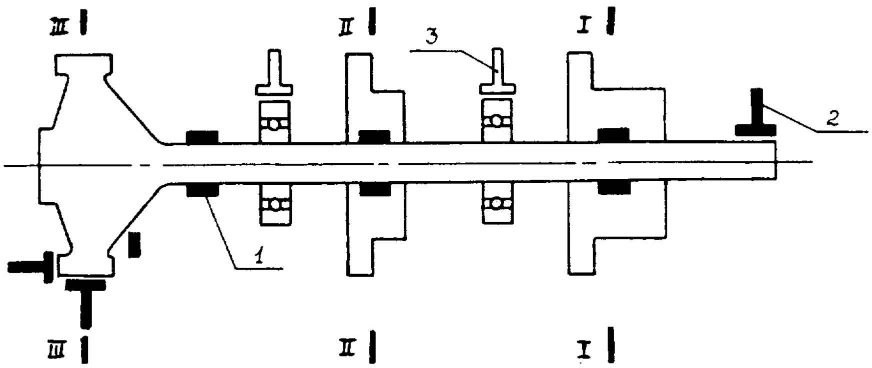

Fig.1. The scheme of the rotor preparation
1. strain resistors, 2. sensors of movements, 3. vibration sensors

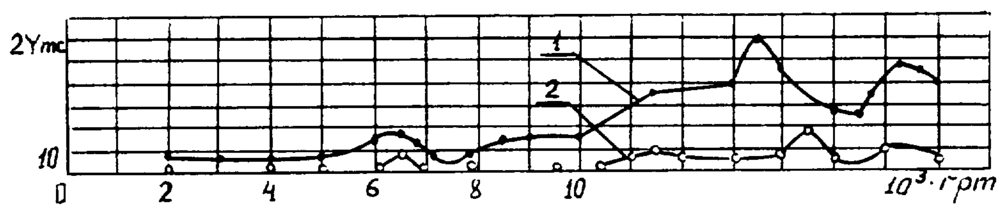

Fig.2. The dependence of the rotor deflections on motion frequency (III cross-section). 1 - before balancing, 2 - after balancing

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