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Expected Antenna Utilization and Overload

E. C. Posner
Office of Telecommunications and Data Acquisition

This article develops the trade-offs between the number of antennas at a DSN Deep-Space Communications Complex and the fraction of continuous coverage provided to a set of hypothetical spacecraft, assuming random placement of the spacecraft passes during the day. The trade-offs are fairly robust with respect to the randomness assumption. A sample result is that a three-antenna complex provides an average of 82.6 percent utilization of facilities and coverage of nine spacecraft that each have 8-hour passes, whereas perfect phasing of the passes would yield 100 percent utilization and coverage. One key point is that sometimes fewer than three spacecraft are visible, so an antenna is idle, while at other times, there aren't enough antennas, and some spacecraft do without service. This point of view may be useful in helping to size the network or to develop a normalization for a figure of merit of DSN coverage.

I. Introduction

In deciding how much antenna (and signal processing) capability is appropriate for the DSN, user requirements need to be known. One also needs to know how the spacecraft to be supported are distributed in the sky, or rather in the duration of their pass over a single DSN Deep-Space Communications Complex. This article assumes a random independent pass distribution. This might hold over many years, but in a particular year or even decade there will be bunching—for example, the outer planets at southern declination during the 1980s. Or, there might be several orbiters around the same planet, e.g., Mars, but not in the same antenna beam. Thus, actual near-term facilities' decisions need actual view period knowledge. In the long term, though, randomness may be a good assumption.

The differences between the model presented here and the actual DSN are as follows. First, this article considers one Complex in isolation, instead of all three Complexes together. This would give a correct picture if the view periods at each Complex were the same, and all spacecraft required continuous coverage. The view periods are not the same, of course, so to find total coverage, one would have to sum the coverage at each Complex. Second, no distinction is made here between types of facilities at a Complex, e.g., between 34-m high efficiency (HEF) and 70 m. This leads to a harder problem than the one considered here. However, the methods of this article are relevant for the 70 m alone for spacecraft that can only be supported by the 70 m. A third, and the most significant, difference between the model and reality is that the model assumes that all spacecraft require continuous coverage. Some may and some may not require coverage, and passes can be moved around to meet requirements or critical spacecraft events can be scheduled to occur when a facility is in view. To find the fraction of requirements that can be supported in the realistic situation is a harder problem to state and solve than the one considered here. Nevertheless, this article is useful because it still gives an estimate for the expected time when there is nothing to track at all, an important parameter to know. Finally, a fourth difference is that actual view periods are not random-independent, but occur according to mechanical laws. As argued above, for the long term, randomness may be as good an assumption as can be made. The assumption that all pass lengths are equal is not essential in what follows, but it does simplify the formulas; it is not counted as a difference between the simple model and reality.

Thus, it is useful to consider the problem for random passes. At the very least, this can be used as a calibrator of the actual utilization, sort of a zero point for a figure of merit on the adequacy of DSN facilities. This article develops the random-pass model and applies it to cases involving realistic numbers of spacecraft and antennas. The methods used provide expected values quite easily. To determine probabilities such as "what is the probability that three antennas can service 90 percent of five spacecraft with random 12-hour passes?" is harder and will not be attempted here. Note also that continuous coverage requirements are assumed since this is a model that can be solved and often occurs.

Section II develops the model, Section III solves it for expected values, and Section IV works out some cases of relevance to the size of typical DSN deep-space mission sets. Section V, the concluding section, compares the theoretical results with some actual spacecraft view periods. The results on percent coverage for the actual view periods are not too far off the theoretical expected values.

II. The Model

Think of the 24-hour day as the circumference of a circle, and the passes as connected intervals of arc. The circumference length is assumed equal to 1, and the length of a pass is β (where $0 < \beta < 1$), which is the same for all spacecraft in this model, although this assumption is not essential to solve the model. The circumference of a circle is used as if each previous and succeeding day had the same spacecraft visibilities. Although the visibility periods or pass times precess somewhat from one day to the next, this is a minor effect.

As in Fig. 1, there are n spacecraft corresponding to n intervals of arc $X_i(\omega)$, where $0 \leq \omega < 1$ is the phase around the circle and the random variable X_i is 0 if the spacecraft is not visible and 1 if it is visible. It is noted here that the expected value method does not require these intervals of arc to be connected, even though they are connected for deep-space spacecraft.

The n spacecraft are also viewed as n independent random variables $\{X_i\}$ defined on the circle. The pass lengths are all assumed to be equal to β, so that the expected value of the $0-1$ random variable X_i is β. The reason is that β is the probability that the pass contains a particular point on the circle, since the β of the circle is covered by spacecraft i. The method used in this article would work only if the *expected* pass length were β; it is not actually required that the pass lengths be β with certainty, nor even that a "pass" be connected. What *is* important in this model is that the spacecraft is to be supported every time it is visible—for example, no data dumps. With a minor modification of this method, the lengths of the passes can be spacecraft dependent, as was said above. Another assumption is that all antennas are assumed equivalent here—for example, no distinction is made between the 34-m and 70-m antennas.

In Fig. 1, intervals of time during the day (intervals of arc on the circle) are shown where no spacecraft is trackable because none is visible. There are regions where a certain number of spacecraft are visible: only one spacecraft, exactly two, exactly three, and exactly four (the maximum as assumed in the figure). All antennas (say there are three) would be idle during times when there is no spacecraft to track, two of the three would be idle during times with only one spacecraft visible, one would be idle during times with only two spacecraft visible, and all three would be busy and all visible spacecraft being tracked during times when exactly three spacecraft are visible. When four spacecraft are visible, all three antennas would be busy, but one spacecraft would not be tracked. In this case, some requirements are not being met. The rest of the article finds the expected fractions of time for all these conditions of visibility and antenna utilization.

III. Analytic Expressions

Recall that $X_i(\omega) = 0$ or 1 depending upon whether spacecraft i (one of n spacecraft) is not visible or is visible at "time" ω on the circle of circumference 1. The expected value

$$E(X_i) = \beta \tag{1}$$

for all i, where β is the relative pass length. Thus, $\beta = 1/2$ corresponds to 12-hour passes. The expectation is β, because all rotations of the pass around the circle are equally likely to be selected by the random pass generator.

The random fraction $U_0(\omega)$ of the circle, or day, where none of the n spacecraft is visible is

$$U_0(\omega) = \prod_{i=1}^{n} [1 - X_i(\omega)] \tag{2}$$

This is because $U_0(\omega) = 1$ precisely for these ω points where all $X_i(\omega) = 0$, i.e., no spacecraft is visible. The expected fraction α_0 of a day where no spacecraft is visible, by the independence assumed for the random variables $X_i(\omega)$, is

$$\alpha_0 = E[U_0(\omega)] = E\prod_{i=1}^{n}[1 - X_i(\omega)]$$

$$= \prod_{i=1}^{n} E[1 - X_i(\omega)] = (1-\beta)^n$$

so

$$\alpha_0 = (1-\beta)^n \tag{3}$$

More generally, let $U_k(\omega)$ be the random fraction of a day during which exactly k spacecraft are visible, where $0 \le k \le n$. It is given by the following, not very useful, expression:

$$U_k(\omega) = \sum_{i_1}\sum_{i_2}\cdots\sum_{i_k}\prod_{l=1}^{k} X_{i_l}(\omega) \times \prod_{j \ne \text{any } i_l} [1 - X_j(\omega)] \tag{4}$$

This merely makes $U_k(\omega) = 1$ precisely when exactly k spacecraft (such as $i_1, \cdots, i_k$) are visible to the tracking station. Let α_k be the expected time during which exactly k spacecraft are visible. On taking the expectation, Eq. (4) becomes the more useful result

$$\alpha_k = E[U_k(\omega)] = \sum_{i_1}\cdots\sum_{i_k}\beta^k(1-\beta)^{n-k}$$

which becomes

$$\alpha_k = \binom{n}{k}\beta^k(1-\beta)^{n-k}, \qquad \text{for } 0 \le k \le n \tag{5}$$

Now suppose there are r interchangeable antennas at the Complex. Here $1 \le r \le n$ is of interest, since all spacecraft are supported all the time when $r = n$. What are the expected total requirements E_r that are not met? Good units to use for E_r are spacecraft days. However, the metric E_r does not consider exactly *which* of the spacecraft are not being supported. The expression for E_r is

$$E_r = \sum_{k=r+1}^{n} (k-r)\alpha_k, \qquad \text{for } 1 \le r < n \tag{6}$$

Here $k - r$ is the number of visible spacecraft not being tracked. Using Eq. (5), this becomes

$$E_r = \sum_{k=r+1}^{n} (k-r)\binom{n}{k}\beta^k(1-\beta)^{n-k}, \text{ for } 1 \le r \le n \tag{7}$$

This sum can also start at $k = r$.

As a check, when $r = 0$ (no antennas), $E_r = n\beta$ (the total number of spacecraft days visible). Equation (7) gives

$$E_0 = \sum_{k=1}^{n} k\frac{n!}{k!(n-k)!}\beta^k(1-\beta)^{n-k}$$

$$= \sum_{k=1}^{n}\frac{n(n-1)!}{(k-1)![n-1-(k-1)]!}\beta^k(1-\beta)^{[n-1-(k-1)]}$$

$$= n\beta\sum_{l=0}^{n-1}\binom{n-1}{l}\beta^l(1-\beta)^{n-1-l}$$

$$= n\beta[\beta + (1-\beta)]^{n-1}$$

$$= n\beta$$

where the sum above is evaluated by the binomial theorem. Therefore, this checks.

The expected fraction of requirements not supported, F_r, is merely

$$F_r = \frac{E_r}{n\beta} \tag{8}$$

The expected number of antennas that are idle when r antennas are installed at a Complex, I_r, is easily found, too

$$I_r = \sum_{i=0}^{r-1}(r-i)\alpha_i \tag{9}$$

Here $r-i$ is the number of antennas that are idle when i spacecraft are visible, where $0 \le i \le r$. Using Eq. (5), this becomes

$$I_r = \sum_{i=0}^{r-1}(r-i)\binom{n}{i}\beta^i(1-\beta)^{n-i} \tag{10}$$

The above sum can, of course, be run up to r.

As a check, the average number of antennas in use, $r - I_r$, plus the average number E_r of spacecraft visible but not being tracked, must equal $n\beta$, the number of spacecraft days

$$\sum_{i=0}^{r} i\alpha_i + r\sum_{i=r+1}^{n}\alpha_i + \sum_{i=r+1}^{n}(i-r)\alpha_i = \sum_{i=0}^{n} i\binom{n}{i}\beta^i(1-\beta)^{n-i}$$

The latter sum is the expected number of heads i in n independent coin flips when the probability of a head is β. This expectation is of course $n\beta$, which checks. This identity is useful in computation, so it is stated below:

$$r - I_r + E_r = n\beta \tag{11}$$

IV. Performance Metrics

Let, for example, $r = n-1$, i.e., there is one less antenna than spacecraft. Equation (7) becomes

$$E_{n-1} = \beta^n$$

If $n = 4$ spacecraft and $r = 3$ antennas, while $\beta = 1/2$ (12-hour passes), it follows that $E_3 = (1/2)^4 = 1/16$. The average fraction of requirements not supported is $E_r/n\beta = E_3/(4\times 1/2) = E_3/2 = 1/32$, or 3-percent unsupported spacecraft on average. To state this another way, to achieve this 97-percent support, an I_3 must be tolerated, from Eq. (10), of

$$I_3 = \sum_{i=0}^{2}(3-i)\binom{4}{i}\cdot\frac{1}{16} = \frac{3}{16} + \frac{2\cdot 4}{16} + \frac{1\cdot 6}{16} = \frac{17}{16}$$

On average, 1-1/16 antennas (out of three) must stand idle to provide 97-percent coverage of four spacecraft, each of which has 12-hour passes. This is true even noting that on average only two of the four spacecraft are visible and yet there are three antennas. The facilities' utilization is only $[3-(17/16)]/3 = 31/48 = 64.6$ percent. On average, one has to tolerate a 64.6-percent facilities' utilization in tracking time to provide 97-percent support to four spacecraft with 12-hour passes per day, each of which must be fully supported.

Table 1 presents and Fig. 2 graphs for $\beta = 1/2$ (12-hour passes) the fraction of tracking requirements not met $E_r/n\beta$ and the antenna-idle fraction I_r/r as a function of r for $n = 3$ to 7 spacecraft, r going from 1 to 7. This shows what price has to be paid in apparently idle facilities (ignoring maintenance, upgrades, and radio/radar astronomy, etc.) in order to meet a given fraction of continuous coverage requirements. The idle capacity curves are almost linear. It is clear that there does need to be some apparent surplus capacity in the Network to achieve good coverage.

V. Comparison With History

An experiment was performed using view period data for 1990 and $\beta = 1/3$ (8-hour passes). Specifically, a random day of year was picked (Greenwich Mean Time, GMT, day 191, July 10, 1990), and the visibility of $n = 9$ particular deep-space spacecraft (actually, eight spacecraft and one planet) over Goldstone was found. The eight deep-space spacecraft were Pioneers 10 and 11, Voyagers 1 and 2, Magellan, Galileo, the International Comet Explorer (ICE), and Giotto; the planet was Saturn. All passes would have been longer than 8 hours, but the 8-hour interval (perhaps involving the days before or after) centered at maximum elevation was used because the case being considered was $\beta = 1/3$. Pioneer 12 (Venus Orbiter) was not included because it orbits Venus, while Magellan was nearly in Venus orbit (insertion on August 10, 1990). This situation was not "random." In fact, the view periods for Giotto and Saturn were accidentally virtually identical.

Also, there was a much larger time with no spacecraft visible than had been expected. All indications are that these targets bunched.

The expected fraction of time α_k of the 24 hours during which k spacecraft would be visible according to Eq. (5) is presented for $0 \leq k \leq 9$ in Table 2, together with the actual fraction $\hat{\alpha}_k$ as determined from the DSN view-period database evaluated for July 10, 1990, by the TDA Mission Support and DSN Operations Office. The raw-data view-period midpoints are presented in Table 3. The results are not far off from the expected values. This is especially noteworthy considering two facts. First, if there were a time when all nine spacecraft were visible, then there would be at least 8 hours when no spacecraft at all was visible: $\hat{\alpha}_9 > 0 \Rightarrow \hat{\alpha}_0 > 1/3$. Second, the view periods as shown above were quite bunched, as observed above. The three antennas actually covered 76.9 percent of the requirements on that day, compared with the expected 82.6 percent. Thus, the fraction obtained is quite robust with respect to the randomness assumption. This fraction is calculated from Eqs. (6) and (8), which give the expected fraction supported by $r = 3$ antennas with nine spacecraft, $\beta = 1/3$, as $1 - F_r = 1 - E_r/(n\beta) = 1 - E_3/3 = 1 - 0.174 = 0.826$.

VI. Summary

The article has presented a model for the number of antennas needed to meet various fractions of full-coverage requirements for various numbers of spacecraft with view periods of random phase during the day. The trade-off between idle antennas and fractional tracking requirements met was clearly shown. More requirements met translates into more idle facilities. The model can be used to help calibrate the adequacy of facilities' plans in the longer term when mission sets are not so certain. It can also provide a zero calibration for the fraction of idle facilities in the existing Network. It seems robust with respect to the assumption of independent view periods.

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Table 1. Expected requirements not met and expected idle fraction versus expected number of antennas, 12-hour passes

No. of spacecraft, n	No. of antennas, r	$E_r/n\beta$	I_r/r
3	1	0.4167	0.1250
3	2	0.0833	0.3125
3	3	0	0.5000
4	1	0.5313	0.0625
4	2	0.1875	0.1875
4	3	0.0313	0.3490
4	4	0	0.5000
5	1	0.6125	0.0313
5	2	0.2875	0.1094
5	3	0.0875	0.2396
5	4	0.0125	0.3828
5	5	0	0.5000
6	1	0.6719	0.0156
6	2	0.3750	0.0625
6	3	0.1563	0.1563
6	4	0.0417	0.2813
6	5	0.0052	0.4031
6	6	0	0.5000
7	1	0.7165	0.0078
7	2	0.4487	0.0352
7	3	0.2277	0.0990
7	4	0.0848	0.1992
7	5	0.0201	0.3141
7	6	0.0022	0.4180
7	7	0	0.5000

Table 2. The fraction of time that zero through nine spacecraft are visible, 8-hour passes ($\beta = 1/3$)

Expected from random model		Observed on one particular day	
No. of spacecraft, k	Fraction α_k of time k visible	No. of spacecraft, k	Fraction of $\hat{\alpha}_k$ of time k visible
0	0.0260	0	0.1382
1	0.1171	1	0.0681
2	0.2341	2	0.1431
3	0.2731	3	0.1542
4	0.2048	4	0.2993
5	0.1024	5	0.1972
6	0.0341	6	0
7	0.0073	7	0
8	0.0009	8	0
9	5×10^{-5}	9	0
6–9	0.0424	6–9	0

Table 3. Nine view periods, July 10, 1990, Goldstone (8-hour passes centered on maximum elevation, DSS 14)

Pass durations

Spacecraft or planet	Start of pass, GMT	Midpoint, GMT	End of pass, GMT
Voyager 1	00:56	04:56	08:56
Pioneer 11	02:14	06:14	10:14
Voyager 2	03:29	07:29	11:29
Giotto	04:11	08:11	12:11
Saturn (planet)	04:12	08:12	12:12
Galileo	10:36	14:36	18:36
ICE	12:18	16:18	20:18
Pioneer 10	13:23	17:23	21:23
Magellan	13:37	17:37	21:37

Visibility durations and spacecraft tracked

Time period, GMT	Duration, hr	No. of spacecraft visible	No. of spacecraft tracked	No. of spacecraft not tracked
21:37–00:56	3:19	0	0	0
00:56–02:14	1:18	1	1	0
02:14–03:29	1:15	2	2	0
03:24–04:11	0:42	3	3	0
04:11–04:12	0:01	4	3	1
04:12–08:56	4:44	5	3	2
08:56–10:14	1:18	4	3	1
10:14–10:36	0:22	3	3	0
10:36–11:29	0:53	4	3	1
11:29–12:11	0:42	3	3	0
12:11–12:12	0:01	2	2	0
12:12–12:18	0:06	1	1	0
12:18–13:23	1:05	2	2	0
13:23–13:37	0:14	3	3	0
13:37–18:36	4:59	4	3	1
18:36–20:18	1:42	3	3	0
20:18–21:23	1:05	2	2	0
21:23–21:37	0:14	1	1	0

Recapitulation[a]

k	Total minutes	Fraction $\hat{\alpha}_k$[b]
0	199	0.1382
1	98	0.0681
2	206	0.1431
3	222	0.1542
4	431	0.2993
5	284	0.1972
6–9	0	0

[a] Average number of spacecraft not tracked = 111/160 = 0.694. Fraction of requirements not met = 111/(3 x 160) = 37/160 = 23.1 percent. Fraction of requirements met = 76.9 percent.

[b] Actual fraction with k spacecraft, α_k.

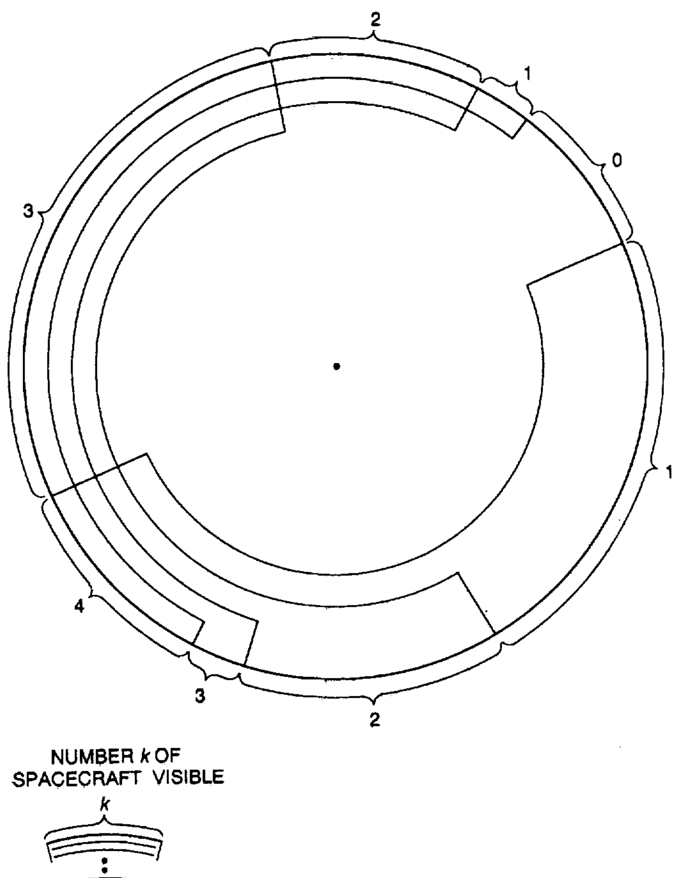

Fig. 1. Four spacecraft tracked around the clock, 12-hour passes.

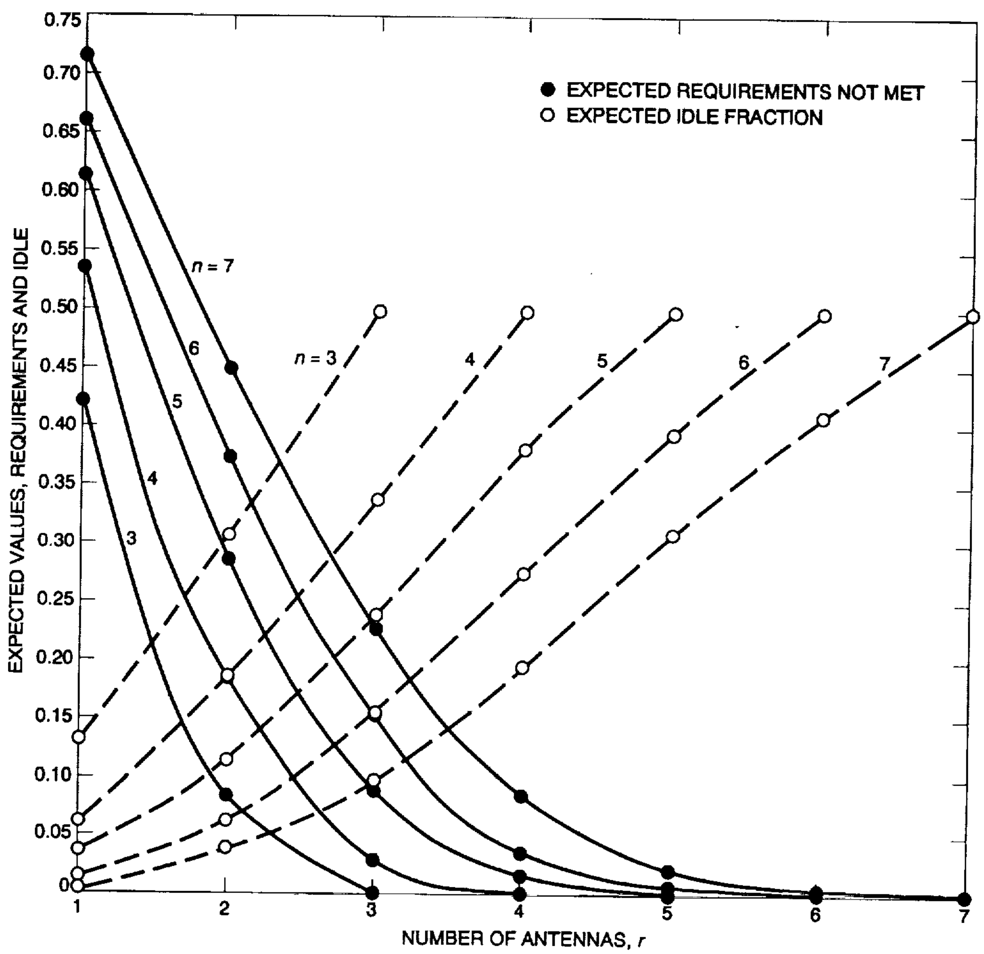

Fig. 2. Requirements not met and idle capacity versus number of antennas.