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Solution of Geometrically Nonlinear Statics Problems

by the p-Version of the Finite Element Method

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ABSTRACT

This report is submitted in accordance with instructions given in NASA Form 1463A(TEMP), which is part of the Research Grant Award document. The report contains an executive summary and bibliographical references to all publications issued in the course of research. The abstracts of doctoral dissertations are given in the Appendix. This research has motivated a closely related investigation at the University of Maryland. Bibliographical references to this related investigation are included also.

EXECUTIVE SUMMARY

This project has been concerned with the following fundamental question: *Is it possible to use computers for the simulation of structural systems with the same degree of reliability as if full scale physical experiments were performed?* Finding an answer to this question is very important: Full scale experiments are generally very time consuming, very expensive, and do not yield all data of interest directly. For example, load and displacement data, surface strain, etc., can be measured directly but strains in the interior regions, stress intensity factors, cannot. Numerical simulation does not have such restrictions. Reliable numerical simulation will make it possible to reduce the costs of engineering and improve the quality of engineering decisions based on computed information very substantially.

A high degree of reliability can be achieved in numerical simulation only if both the errors of idealization and errors of discretization can be shown to be small.

The error of idealization is the error between the actual physical quantities on which engineering decisions are based (e.g., maximum principal stress, first natural frequency, etc.) and the same data corresponding to the exact solution of the mathematical model. The error of discretization is the error between the quantities of interest corresponding to the exact and approximate solutions of a mathematical model.

A great deal of attention is being given to the problem of controlling errors of discretization by researchers and code developers but very little attention has been given to the problem of controlling errors of idealization. Yet it is not sensible, and can be very misleading, to provide information about discretization errors but say nothing about idealization errors.

The key to controlling discretization errors it to create some sequence of discretizations such that the corresponding approximating solutions clearly converge to a limiting solution, which is the exact solution of the mathematical model. Important results were achieved in the mid-1980's in clarifying how optimal convergence rates can be achieved for the class of problems to which most structural and mechanical engineering problems belong.

Similar principles apply to controlling errors of idealization. Any mathematical model is in fact a special case of a more general model. For example, a model based on the linear theory of elasticity is a special case of a model based on the large displacement - linear strain theory of elasticity. It must be possible to show that the quantities of interest are substantially independent of the choice of model.

This project was concerned with controlling errors of idealization for structural plates and shells. Although very large amounts of effort were spent on the development of reliable models for structural plates and shells by many investigators over a span of nearly 30 years, the quality of software in current engineering practice is considered unsatisfactory by most users. The main source of difficulty is that classical formulations have been implemented, with a number of ad hoc modifications to overcome various restrictions in data structure and element type. The nature of the problem is such, however, that selection of the proper formulation is model-dependent. Certain data, such as displacements, the low natural frequencies, buckling loads can be solved accurately with classical formulations in general, on the other hand boundaries, plate intersections, structural connections, etc., which are often the regions of greatest practical interest, are beyond the scope of classical formulations. Important problems, such as the problem of delamination, cannot be treated by classical models. In order to be able to simulate both the structural and strength responses, it is essential to have a converging sequence of models having the property that the exact solution of each member of the sequence converges to the exact solution of the corresponding fully three-dimensional problem. Such a sequence of models has been developed under this project for isotropic and laminated plates. The main accomplishments of this project were as follows:

(1) A hierarchic system of mathematical models was developed and demonstrated for homogeneous elastic plates and shells. The exact solutions of the hierarchic system of models converge to the exact solution of the fully three-dimensional problem at an optimal rate. It was shown that it is very advantageous to implement hierarchic models such that the Cartesian components rather than the curvilinear components of the displacement vector field are approximated. This very substantially simplifies the modelling a large variety of practical problems which typically involve thin and thick plates and shells, stiffeners, and regions where fully three-dimensional representation is required. The hierarchic models are not susceptible to *shear locking*, a common problem in conventional models, when the thickness of plates and shells is small.

(2) Application of the hierarchic system of models to the problem of structural stability of orthotropic shells was investigated and documented. This class of problems can be treated well with the lowest member of the hierarchy, however tight control of discretization errors is essential. Model problems were solved.

(3) The hierarchic system of models developed for homogeneous plates and shells has been generalized for plates made of laminated composites. As a result, the basic principles which govern systematic dimensional reduction for laminated composites have been clarified and demonstrated on model problems. The principles which govern the construction of hierarchic models for homogeneous plates and shells are now understood as special cases of the comprehensive theory developed and demonstrated in the course of this project. Furthermore, it has been shown in a related investigation that the sequence of hierarchic models converges at an optimal rate.

LIST OF PUBLICATIONS RESULTING FROM THE PROJECT

[1] Szabó, B. A. and Sahrmann, G. J., "Hierarchic Plate and Shell Models Based on p-Extension" *International Journal for Numerical Methods in Engineering*, Vol. 26, pp. 1855-1881 (1988).

[2] Sahrmann, G. J., "Hierarchic Plate and Shell Models Based on p-Extension", D. Sc. Dissertation, Washington University, St. Louis, Missouri, May, 1988.

[3] Schiermeier, J. E., "Geometrically Nonlinear Analysis with the p-Version of the Finite Element Method", D. Sc. Dissertation, Washington University, St. Louis, Missouri, May, 1990.

[4] Szabó, B. A., "Hierarchic Plate and Shell Models Based on p-Extension", *Proc., Symposium on Analytical and Computational Models for Shells*, ASME Winter Annual Meeting, Edited by A. K. Noor, T. Belytschko and J. C. Simo San Francisco CA, Dec. 10-15, pp. 317-331, 1989.

[5] Babuška, I., Szabó, B. and Actis, R., "Hierarchic Models for Laminated Composites" Report WU/CCM-90/4, Center for Computational Mechanics Washington University, St. Louis, (1990). Accepted for publication in the *International Journal for Numerical Methods in Engineering*.

[6] Actis, R., "Hierarchic Models for Laminated Plates" D. Sc. Dissertation, Washington University, St. Louis, Missouri, December 1991.

LIST OF RELATED PUBLICATIONS

[1] Schwab, C., "Dimensional Reduction for Elliptic Boundary value Problems", PH.D. Thesis, Department of Mathematics, University of Maryland, College Park, 1989.

[2] Babuška, I. and Li, L., "The Problem of Plate Modeling. Theoretical and Computational Results", Technical Report BN-1116, Institute for Physical Science and Technology, University of Maryland, College Park, December, 1990.

[3] Babuška, I. and Li, L., "Hierarchic Modeling of Plates" *Computers and Structures*, Vol. 40, pp. 419-430 (1991).

APPENDIX

ABSTRACTS OF DOCTORAL DISSERTATIONS

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HIERARCHIC PLATE AND SHELL MODELS BASED ON p-EXTENSION

Glenn J. Sahrmann, D.Sc. 1988

A hierarchic sequence of plate and shell models, based on the p-version of the finite element method, is proposed in this thesis. The hierarchic sequence of models allows the implementation of a hierarchic sequence of plate and shell theories, beginning with the Reissner-Mindlin theory and ending with three dimensional elasticity, in the finite element analysis of plates and shells. In the p-version of the finite element method, the polynomial basis over each finite element domain is increased in a hierarchic manner. Here p refers to the degree of the highest complete polynomial present in the finite element approximation.

A hierarchic sequence of both two-dimensional, (beam-columns and arches), and three dimensional plate and shell models, based on the principle of virtual work, was implemented in a computer program. Quality control and error analysis procedures were also implemented. These procedures are based on: (1) the estimation of the error in the energy norm, (2) the performance of equilibrium tests, and (3) the observation of the convergence of quantities of interest.

GEOMETRICALLY NONLINEAR ANALYSIS WITH THE p-VERSION OF THE FINITE ELEMENT METHOD

John E. Schiermeier, D.Sc. 1990

The purpose of this research was to develop methods of using p-version elements in geometrically nonlinear problems involving small strains and large displacements. A hierarchic sequence of shell models is presented, and is first explained in relation to classical shell models. The three-dimensional elasticity equations are presented, and then the principle of virtual work is used to formulate the

finite element solution with the assumptions of linear strain-displacement relations and a linear constitutive law. The mapping and basis functions are described in detail for the hexahedral and quadrilateral elements. These basis functions are the key to the hierarchic sequence. Construction of the element stiffness matrices and load vectors and computation of the post-processing quantities are explained. In addition, a method of computing the stress resultants based on the principle of virtual work is developed. The lowest level of the hierarchic model is used to solve the linear problem of a spherical shell with edge ring, for which the stress resultants are computed using two different methods and compared to the highest level of the hierarchic model.

Several basic nonlinear solution methods are presented, and general issues relating to those methods are discussed. A Newton method is described in detail and applied to a two-dimensional circular arch with uniform pressure loading. Symmetric buckling modes, non-symmetric buckling modes, and multiple control variables are investigated with several different meshes for the arch. Two alternate nonlinear methods are also described and applied to the arch. A linearization method is then described in detail and applied to the cylindrical panel with a circular hole and axial compression, which is the main focus problem of the research. This method exploits the hierarchic structure of the finite element spaces to accelerate convergence. Several studies are performed to examine the characteristics of this nonlinear solver, and several different types of mesh refinement are attempted, which show that the limit loads are substantially independent of the finite element spaces. A study is also performed to compare the relative efficiencies in the choice of the initial deformation vector. An extrapolation technique using theoretical convergence rates for the p-extension is applied to the limit loads, which provides an error estimation procedure. An alternate nonlinear method is described and applied to the panel.

HIERARCHIC MODELS FOR LAMINATED PLATES

Ricardo Luis Actis, D.Sc. 1991

Structural plates and shells are three-dimensional bodies, one dimension of which happens to be much smaller than the other two. Thus the quality of a plate or shell model must be judged on the basis of how well its exact solution approximates the corresponding three-dimensional problem. Of course, the exact solution depends not only on the choice of the model but also on the topology, material properties, loading and constraints. The desired degree of approximation depends on the analyst's goals in performing the analysis. For these reasons models have to be chosen adaptively. Hierarchic sequences of models make adaptive selection of the model which is best suited for the purposes of a particular analysis possible.

The principles governing the formulation of hierarchic models for laminated plates are presented. The essential features of the hierarchic models described herein are: (a) The exact solutions corresponding to the hierarchic sequence of models converge to the exact solution of the corresponding problem of elasticity for a fixed laminate thickness, and (b) the exact solution of each model converges to the same limit as the exact solution of the corresponding problem of elasticity with respect to the laminate thickness approaching zero.

The formulation is based on one parameter (β) which characterizes the hierarchic sequence of models, and a set of constants whose influence has been assessed by a numerical sensitivity study. The recommended selection of these constants results in the number of fields increasing by three for each increment in the power of β.

Numerical examples analyzed with the proposed sequence of models are included and good correlation with the reference solutions was found. Results were obtained for laminated strips (plates in cylindrical bending) and for square and rectangular plates with uniform loading and with homogeneous boundary conditions. Cross-ply and angle-ply laminates were evaluated and the results compared with those of MSC/PROBE.

Hierarchic models make the computation of any engineering data possible to an arbitrary level of precision within the framework of the theory of elasticity.

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