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FLUCTUATION DRIVEN ELECTROWEAK PHASE TRANSITION

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ABSTRACT

We examine the dynamics of the electroweak phase transition in the early Universe. For Higgs masses in the range $46 \leq M_H \leq 150$ GeV and top quark masses less than 200 GeV, regions of symmetric and asymmetric vacuum co-exist to below the critical temperature, with thermal equilibrium between the two phases maintained by fluctuations of both phases. We propose that the transition to the asymmetric vacuum is completed by percolation of these sub-critical fluctuations. Our results are relevant to scenarios of baryogenesis that invoke a weakly first-order phase transition at the electroweak scale.

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The realization that gauge symmetries can be restored at high temperatures, combined with the success of the big-bang model of cosmology, has generated a lot of interest in the study of cosmological phase transitions.[1] First-order phase transitions are characterized by an energy barrier separating the symmetric from the asymmetric phase at the critical temperature T_C when the two phases have equal free energy. First-order transitions may generate out-of-equilibrium conditions, which can have important effects upon the properties and evolution of the early Universe. Two well-known examples are models of inflationary cosmology that invoke a first-order transition at the grand-unified scale,[2] and the production of inhomogeneities at the quark-hadron transition.[3]

In this letter we study the electroweak phase transition in the minimal (i.e., one Higgs doublet) model. We will restrict our study to fairly light Higgs masses, ranging from 46 GeV up to 150 GeV, and top-quark masses in the range 100 to 200 GeV. For this range of parameters the phase transition is weakly first-order. The high temperature minimum of the potential is the symmetric state $\langle\phi\rangle = 0$. At some temperature $T_1 > T_C$ the potential develops a local asymmetric minimum at $\phi_+ > 0$. As the system cools, the difference in free energy between the symmetric and asymmetric state decreases; finally at the critical temperature T_C the asymmetric minimum is degenerate with the symmetric minimum. Below T_C the asymmetric minimum has the lower free energy. Eventually, at some temperature $T_2 < T_C$ the symmetric minimum becomes unstable.

Two scenarios have been proposed for the completion of such transitions. In the "standard" picture, the Universe remains in a homogeneous state of symmetric vacuum below T_C, until the symmetric state becomes unstable at T_2. Then the field evolves classically to the asymmetric minimum.[1] Recently a second scenario has been proposed where again the Universe remains in a homogeneous state of symmetric minimum to T_C, then between T_C and T_2 the homogeneous state is terminated by nucleation of bubbles of asymmetric (true-vacuum) phase which grow and eventually percolate the volume.[4]

We propose that the transition is completed by a new mechanism: percolation of subcritical fluctuations of the asymmetric phase. We argue that by the time the Universe has cooled to T_C, the vacuum is *not* a homogeneous state of symmetric vacuum, but rather an emulsion of symmetric and asymmetric vacua, each existing with equal probability. Below T_C the fraction of the Universe in the symmetric state gradually decreases, and the transition is completed by percolation of many regions of asymmetric phase.

We use a method developed by Gleiser, Kolb, and Watkins (GKW) designed to study the approach and maintenance of thermal equilibrium in phase transitions.[5] In this approach the thermal fluctuations of the field are modeled by the creation of regions (bubbles) of one phase inside of the other. These fluctuation regions are spherical and have a size of the thermal correlation length of the Higgs field, ℓ. GKW use detailed balance to find the rate of creation of fluctuation regions of false vacuum inside a true-vacuum region to be $\ell^{-1}\exp(-\Delta F/T)$ where ΔF is the difference in free energy of the region and the homogeneous state. If these rates are large compared to the expansion rate H, then the relative population of the phases should be distributed according to Boltzmann statistics. We find this to be the case for the electroweak transition with top and Higgs masses in the aforementioned ranges. Our results should be relevant to the recently proposed scenarios of baryogenesis at the electroweak scale, which naturally invoke out-of-equilibrium conditions during a first-order phase transition.[6]

In the study of phase transitions the Higgs field (or its equivalent) plays the role of the order parameter. In practice, when the system is initially in thermal equilibrium, the study of the phase transition reduces to the construction of the finite-temperature 1-loop effective potential, which incorporates the interactions of the Higgs field with itself and with other fields in the model at some temperature T.[7] The effective potential is equivalent to the homogeneous part of the free energy and its minima determine the equilibrium properties of the system. We neglect contributions of the Higgs field to the

1-loop potential. This should be valid for Higgs masses below about 150 GeV. In order for the potential to be stable with these small Higgs masses, the top quark must be less than about 200 GeV.[8] The transition can be studied using a high-temperature expansion of the effective potential which, as shown by Turok and Zadrozny[9] and by Anderson and Hall,[4] is very reliable in the relevant range of temperatures. They obtain for the potential (we will follow the notation of Ref. 4)

$$V(\phi,T) = D\left(T^2 - T_2^2\right)\phi^2 - ET\phi^3 + \frac{1}{4}\lambda_T\phi^4, \tag{1}$$

where the constants D and E are given by $D = \left[6(m_W/\sigma)^2 + 3(m_Z/\sigma)^2 + 6(m_T/\sigma)^2\right]/24$, and $E = \left[6(m_W/\sigma)^3 + 3(m_Z/\sigma)^3\right]/12\pi$. Here T_2 is the temperature at which the origin becomes an inflection point (*i.e.*, below T_2 the symmetric phase is unstable and the field can classically evolve to the asymmetric phase), and is given by $T_2 = \sqrt{(m_H^2 - 8B\sigma^2)/4D}$, where the physical Higgs mass is given in terms of the 1-loop corrected λ as $m_H^2 = (2\lambda + 12B)\,\sigma^2$, with $B = \left(6m_W^4 + 3m_Z^4 - 12m_T^4\right)/64\pi^2\sigma^4$. We use $m_W = 80.6$ GeV, $m_Z = 91.2$ GeV, and $\sigma = 246$ GeV. The temperature-corrected Higgs self-coupling is

$$\lambda_T = \lambda - \frac{1}{16\pi^2}\left[\sum_B g_B\left(\frac{m_B}{\sigma}\right)^4 \ln\left(m_B^2/c_BT^2\right) + \sum_F g_F\left(\frac{m_F}{\sigma}\right)^4 \ln\left(m_F^2/c_FT^2\right)\right], \tag{2}$$

where the sum is performed over bosons and fermions (in our case only the top quark) with their respective degrees of freedom $g_{B(F)}$, and $\ln c_B = 5.41$ and $\ln c_F = 2.64$.

Apart from T_2, there will be two temperatures of interest in the study of the phase transition. For high temperatures, the system will be in the symmetric phase with the potential only exhibiting one minimum at $\langle\phi\rangle = 0$. As the Universe expands and cools an inflection point will develop away from the origin at $\phi = 3ET_1/2\lambda_T$, where T_1 is given by $T_1 = T_2/\sqrt{1 - 9E^2/8\lambda_T D}$. For $T < T_1$, the inflection point separates into a local maximum at ϕ_- and a local minimum at ϕ_+, with $\phi_\pm = \{3ET \pm [9E^2T^2 - 8\lambda_T D(T^2 - T_2^2)]^{1/2}\}/2\lambda_T$. At the critical temperature $T_C = T_2/\sqrt{1 - E^2/\lambda_T D}$, the minima have the same free energy, $V(\phi_+) = V(0)$.

In the usual picture of a first-order transition, the field starts in thermal equilibrium in its symmetric minimum at $\langle\phi\rangle = 0$, and as the Universe cools below T_C the symmetric phase becomes metastable and decays by nucleation of bubbles of the asymmetric phase: bubbles of size greater than the critical size grow, converting the symmetric phase into the asymmetric phase. The success of this scenario depends crucially on the assumption that the field is in a homogeneous state of the symmetric minimum as the Universe cools below T_C. However, hot systems tend to fluctuate around their equilibrium states, and the probability to find the system in a state other than its ground state has a relative probability given by the Boltzmann factor, $\exp\left[-F(T)/T\right]$, where $F(T)$ is the free energy for the particular fluctuation. For high enough temperatures and slow enough cooling rates, the system will have a large probability to populate other accessible states. For a system with a metastable and a true-vacuum state the equilibrium probability is $\exp\left[-\left(\Delta F(T)\right)/T\right]$, with $\Delta F(T)$ being the free energy difference between the two states. For the electroweak model with the potential given by Eq. (1), as the temperature drops below T_1 thermal fluctuations may drive the system into equilibrium populating the new minimum at ϕ_+. If this is the case, as the temperature drops below T_C the Universe will be filled by a two-phase emulsion, and the kinetics of the transition will be quite different than the usual false vacuum decay scenario.

GKW assume that the dominant statistical fluctuations are sub-critical bubbles of roughly a correlation volume which interpolate between the two minima of the free energy. Denoting the minima for the electroweak model ϕ_0 and ϕ_+, for the symmetric and asymmetric states, the rates for fluctuations between the two states are

$$\Gamma(T)_{[0\to+]} \simeq m_0(T)\exp\left[-F_+(T)/T\right]; \quad \Gamma(T)_{[+\to0]} \simeq m_0(T)\exp\left[-F_0(T)/T\right], \tag{3}$$

for a fluctuation of the asymmetric (symmetric) phase within a region of the symmetric (asymmetric) phase. $F_+(T)$ is the free energy of a fluctuation of the asymmetric phase and $F_0(T)$ is the free energy of a fluctuation of the symmetric phase. For simplicity, we assumed the same correlation length $[\ell(T)^{-1} = m_0(T) = \sqrt{V''(0,T)}]$ around the two minima. Now we must estimate the free-energies $F_+(T)$ and $F_0(T)$. The free energy of a fluctuation in the order parameter is given by (for details see GKW)

$$F(T) = \int d^3x \left[\frac{1}{2}(\nabla\phi)^2 + V(\phi,T)\right], \tag{4}$$

where $V(\phi,T)$ is given by Eq. (1) and the order parameter ϕ is the amplitude of the Higgs field. We are interested in fluctuations of roughly a correlation volume that convert regions of symmetric phase into regions of asymmetric phase and vice-versa, which will give the dominant contribution to the transition amplitude. Since these field configurations are not solutions of the euclidean equations of motion, we adopt a variational approach to determine the dominant configurations with minimal free energy. Thus, we take for the sub-critical bubbles,

$$\phi_+(r) = \phi_+ \exp\left(-r^2/\ell^2\right); \qquad \phi_0(r) = \phi_+\left[1 - \exp\left(-r^2/\ell^2\right)\right], \tag{5}$$

where $\phi_{+(0)}(r)$ is an $O(3)$-symmetric bubble of asymmetric (symmetric) phase nucleated in the symmetric (asymmetric) phase. Introducing the dimensionless variables $X(\rho) \equiv \phi(r)/\sigma$, $\tilde{\ell}(T) = \ell(T)\sigma$, $\theta = T/\sigma$, and $\rho = r\sigma$, the free-energies are given by

$$F_+(\theta) = \pi^{3/2} X_+^2 \tilde{\ell}\sigma \left[\frac{3\sqrt{2}}{8} + \tilde{\ell}^2 \left(\frac{D\sqrt{2}}{4}\left(\theta^2 - \theta_2^2\right) - \frac{E\theta\sqrt{3}}{9} X_+ + \frac{\lambda_T}{32} X_+^2\right)\right] \tag{6}$$

and

$$\begin{aligned} F_0(\theta) \;=\; & \pi^{3/2} X_+^2 \tilde{\ell}\sigma \left[\frac{3\sqrt{2}}{8} + \tilde{\ell}^2 \left(\frac{D}{4}(\theta^2 - \theta_2^2)(\sqrt{2} - 8)\right.\right. \\ & \left.\left. + \lambda_T X_+^2 \left(-1 + \frac{3\sqrt{2}}{8} - \frac{\sqrt{3}}{9} + \frac{1}{32}\right) + E\theta\left(3 - \frac{3\sqrt{2}}{4} + \frac{\sqrt{3}}{9}\right) X_+\right)\right]. \end{aligned} \tag{7}$$

The free energy $F_+(T)/T$ for $T = T_C$ is shown in Fig. 1 as a function of the Higgs mass for several values of the top mass. This free energy will determine the equilibration properties of the system as the temperature drops below T_1. Note that $F_+(T)$ increases as the temperature drops. This is a consequence of the fact that the free energy is dominated by the gradient energy, and as the temperature decreases the asymmetric minimum moves away from the origin. In order to establish thermal equilibrium by overcoming the energy barrier, the thermal fluctuation rate in going from $\phi = 0$ to $\phi = \phi_+$ must be large compared to the expansion rate of the Universe: $\Gamma_{[0\to+]}/H \gtrsim 1$, with $H \simeq 1.66 g_*^{1/2} T^2/M_{Pl}$, and $g_* \simeq 110$ is the number of effective relativistic degrees of freedom at the electroweak scale. Negleting pre-factors, this condition can be easily seen to lead to the inequality $F_+(T)/T \lesssim 34$.

From Fig. 1 we see that for most of the parameter space studied $F_+(T_C)/T_C$ is comfortably less than the critical value of 34, so equilibrium should be established at T_C by fluctuations going from $\langle\phi\rangle = 0$ to $\langle\phi\rangle = \phi_+$.[10] (Obviously, fluctuations in the opposite direction will have smaller free energy until $T = T_C$ when the two free-energies are the same.[5]) Thus, we conclude that at T_C the Universe is *not* in a homogeneous state of symmetric vacuum as assumed in all previous works on the subject. Another indication that large fluctuations in the Higgs field will be important is to note that at $T = T_C$, $\phi_+/T = 2E/\lambda_T \sim 2 \times 10^{-2}/\lambda_T$.[4] Of course λ_T depends upon m_T and T, but using its tree-level value of $\lambda_0 = m_H^2/2\sigma^2 = 0.04(m_H/100\ \text{GeV})^2$ is a good approximation[4] and shows that ϕ_+/T at T_C is never much greater than unity, and typically is less than unity. Since T sets the scale for thermal fluctuations, the system should "feel" both minima.[11]

So far we have established that (for Higgs and top-quark masses we considered), it is quite easy for equilibrium of the two vacuum states to be achieved. In this case as T drops below T_C, the Universe will be filled by a two-phase emulsion, with rapidly fluctuating regions of symmetric and asymmetric phases, separated by roughly a correlation volume. As T drops below T_C fluctuations from the asymmetric phase back to the symmetric phase become more and more suppressed, and the asymmetric phase will occupy more than 50%

of the Universe. The mechanism by which the transition is completed is complicated and will depend on the temperature at which the fluctuation rate freezes out, T_F. If $T_F > T_2$, the symmetric phase is still locally stable, and correlation volume regions of this phase will shrink under surface tension, while regions of the asymmetric phase, having lower free energy will percolate. In Fig. 2 we show the ratio of both rates to the expansion rate as a function of the temperature for $m_H = 60$ GeV and $m_T = 130$ GeV. Only for fairly light Higgs will T_F be larger than T_2. For $T_F < T_2$, the symmetric phase becomes unstable and fluctuations to the symmetric phase can classically roll back down to the asymmetric phase. The Universe will be quickly permeated by the asymmetric phase, since any interface region is energetically disfavored and will move toward the symmetric phase converting it into the true vacuum.

We have shown that for the minimal standard model, with $46 < m_H \leq 150$ GeV and $m_T < 200$ GeV thermal equilibrium will be maintained during the electroweak phase transition despite the fact that there is an energy barrier between the two phases. We assumed that equilibrium is maintained by the thermal nucleation of sub-critical field configurations of roughly a correlation volume, since these are the statistically dominant fluctuations at temperature T. The free energy of these configurations was estimated by assuming they are $O(3)$ symmetric and that they interpolate between the two phases. Preliminary numerical simulations indicate that the ansatz used here is correct within 10%, although a more detailed analysis is necessary. In any case, the lesson is clear; given enough time and heat, a system will thermalize due to the nucleation of field configurations that convert one vacuum into another. This result can be easily extended to the recently proposed scenarios of baryogenesis at the electroweak scale. Of course, we must go beyond the standard model since it does not have a source of CP violation, and extensions keeping only a Higgs doublet[12] and with two Higgs doublets[13] have been proposed. A successful baryogenesis scenario cannot assume a metastable symmetric phase below T_C only because there is a barrier between the two phases.

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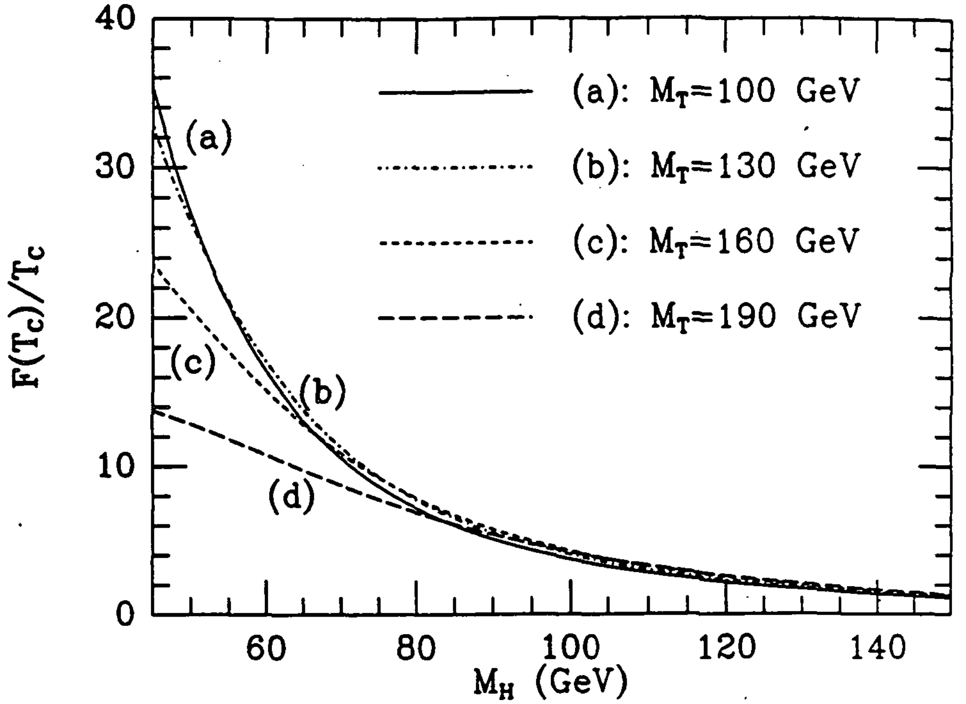

FIG. 1. The free energy of the sub-critical fluctuation at the critical temperature as a function of the Higgs mass for several values of the top-quark mass.

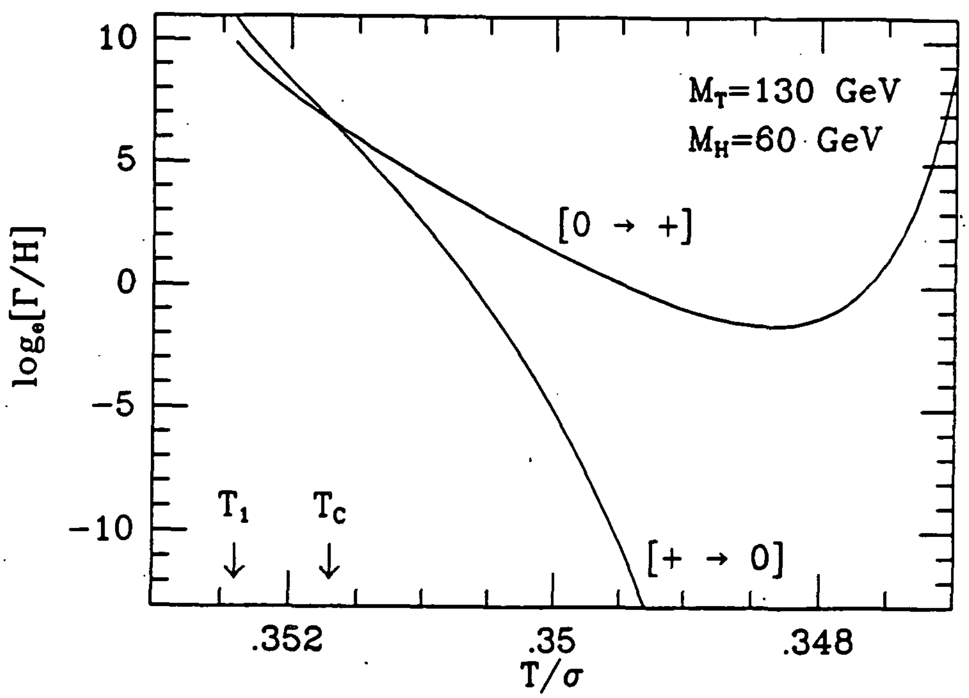

FIG. 2. The ratio of the fluctuation rate to the expansion rate as a function of temperature.