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Artificial neural networks offer some interesting possibilities for use in control. Our current research is on the use of neural networks as generalized splines for identification of aircraft aerodynamics. A system identification model is outlined which can be used to train neural networks on an aircraft model. The model can then be used in a nonlinear control scheme. The effectiveness of network training is demonstrated.

Nonlinear Control Law for Aircraft

The nonlinear dynamic equations of motion of an aircraft can be separated into two parts. The \mathbf{f} term is a function of the state vector, \mathbf{x} , only. It represents the well known dynamics and kinematics of the aircraft and is a function of the mass and inertias of the vehicle. The \mathbf{g} term is a function of both the state and the controls, \mathbf{u} . This term represents the aerodynamic and thrust interactions of the aircraft. These effects are less well known and are not as accurately modeled.

There are several methods for developing nonlinear control laws for systems. A Nonlinear-Inverse-Dynamic control law takes on the given form. The development of this control law requires that the **f** and **g** terms be differentiable with respect to the state and control vectors. This imposes limitations on the methods of approximation of the **g** vector.

· Nonlinear dynamic equations of an aircraft

$$\dot{x} = f(x) + g(x, u) + w$$

f – dynamics and kinematics g – aerodynamics and thrust

· Nonlinear Inverse Dynamics control law

$$\mathbf{u} = \mathbf{c}(\mathbf{x}) + \mathbf{C}(\mathbf{x})\mathbf{v}$$

· Requires differentiability and invertibility

System Identification Task

The system identification problem that we are investigating has additional constraints imposed on it due to the nonlinear control schemes selected. The identifier must make a functional approximation of the unknown portions of the model. It must also be a sufficiently smooth approximation so that the desired number of differentiations (typically 2 or 3 for aircraft applications) can be made in the nonlinear-inverse-dynamic type control law. We would also like to be able to update the approximation concurrent to operation, so an on-line model is desirable.

- Functional Approximation
- Smooth Fit
- · On-line updates

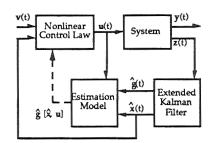
On-line Model Estimation

Based on the Estimation-Before-Modeling paradigm previously investigated at Princeton, an on-line estimation structure is developed. An Extended Kalman Filter is implemented which estimates an augmented state vector. This augmented vector includes both the original states and new states which represent the unknown portion of the original dynamic system, g. (Through non-dimensionalization, g is recognized as the six basic aerodynamic force and moment coefficients of an aircraft.) These unknown functions are modeled as random walk (Gauss-Markov) processes. estimated values of x and g, along with u, are then given to an Estimation Model. The estimation model then has sufficient information to generate a functional form for g. As a side benefit, the Extended Kalman Filter also generates the estimate of the original state vector which is needed by most nonlinear control schemes.

Our interests are in artificial neural networks and they will be used as the estimation model in what is to follow.



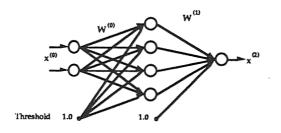
$$\mathbf{x}_{a}(t) = \left\{\mathbf{x}(t)^{T} \ \mathbf{g}[\mathbf{x}(t), \mathbf{u}(t)]^{T}\right\}^{T}$$



Feedforward Neural Networks

One of the most common artificial neural networks is the feedforward network. It uses a weighted interconnection of nonlinear nodes often separated into distinct layers. The nonlinear function of the network nodes is usually a squashing function like a sigmoid. When all of the weights and connections are determined, the network defines a (possibly) multi-input multi-output function.

The power of neural networks is the ability to learn from examples. Using the on-line estimate model previously described, a neural network can be trained to approximate any continuous function. Although many training algorithms are possible, the standard Back-Propagation algorithm was used for this research.



$$\mathbf{x}^{(k)} = \mathbf{s}^{(k)} \left[\mathbf{W}^{(k-1)} \ \mathbf{x}^{(k-1)} \right]$$
$$\mathbf{x}^{(N)} = \hat{\mathbf{g}} \left[\mathbf{x}^{(0)} \right]$$

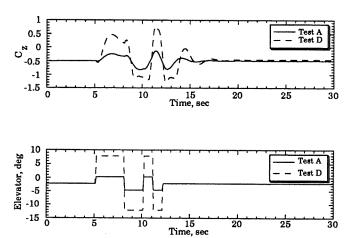
Feedforward Network Properties

Feedforward networks have many properties which are very useful in function approximation. The networks, as defined, perform generalized spline interpolation. The network nodes are the basis functions for this interpolation. Feedforward networks easily handle multivariate function which is a major difficulty for standard interpolation schemes. The networks have good localization properties, i.e. nearby inputs cause nearby outputs. This localization also has implications in training the networks. Finally, since the feedforward networks are weighted interconnections of nonlinear elements, the differentiability of the entire network depends on the differentiability of the individual nodes. Using the standard sigmoid as the nonlinear function in the nodes results in a network that is infinitely differentiable.

- Generalized spline interpolation
- Multivariate
- Input localization
- Differentiable

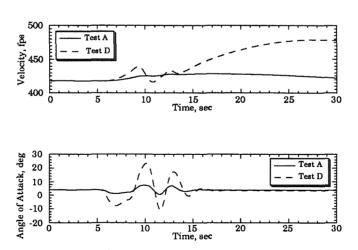
Network Training Input

To test the effectiveness of a neural network in learning the types of functions desired, an example is developed. First, a "3211" elevator input is put into a nonlinear simulation of a 737 aircraft. The amplitude of the input is varied from +/- 2.5 degrees to +/- 10 degrees in Tests A through D. The measurements of the aircraft are processed by an Extended Kalman Filter as detailed in the on-line estimation model. Included in the output is an estimate of the aircraft normal force coefficient, Cz. Since the normal force coefficient is essentially the negative of the lift coefficient, the stall of the aircraft can be seen in the Test D output at a normal force coefficient of approximately -1.2.



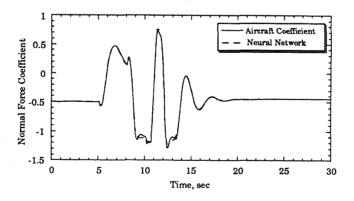
Network Training Input (cont.)

The velocity and angle of attack encountered during the training input is given here. The severest case, Test D, causes a 60 feet/second speed change and angles of attack ranging from -10 degrees to over 20 degrees. Since stall is in the 15 degree range for this aircraft, this test puts the aircraft through extensive maneuvers.



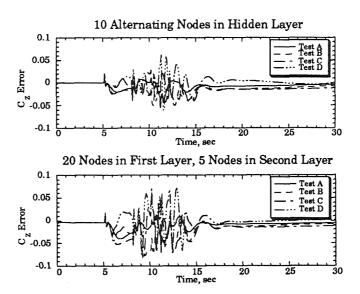
Neural Network Output

Using a single hidden layer network with 10 nodes, the network can accurately model the time evolution of the normal force coefficient of the aircraft. The expanded vertical scale allows the effects of stall in the 10 and 13 second periods to be viewed. About 800 presentations of test data A through D were used to train the network.



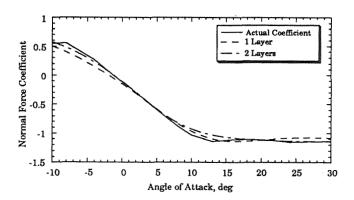
Network Coefficient Error

To better demonstrate how well the networks learn, the difference between the neural network output and the actual normal force coefficient are plotted for two different neural networks. While very busy, the bounds of the errors indicate that the networks have learned the coefficients to within about 5% during the maneuvers and much better at other times.



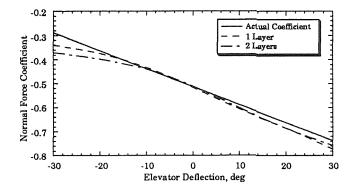
Network Aerodynamic Derivatives

For control of an aircraft, the derivatives of the aerodynamic coefficients are of possibly more importance than the value of the coefficient themselves. This slide shows the variation of the normal force coefficient with respect to the angle of attack of the aircraft. Once again, since the normal force coefficient is related to the lift coefficient, this curve is essentially the negative of the lift curve of the aircraft except at high angles of attack. The fit of the two different neural networks is very good especially near the trim angle of attack at about 4 degrees. The fit is reasonable at very high and low angles of attack too.



Network Aerodynamic Derivatives (cont.)

Looking at the variation of the normal force coefficient with the elevator deflection is also informative. Once again, the neural networks are able to reasonably accurately fit the desired curve, especially in the region of the trim position of about -2 degrees. This figure has an expanded abscissa. The network is only trained with elevator data between +/- 10 degrees, but does a reasonable job in the region outside of this. Care must be taken when the generalization ability of the networks are used like this.



Future Considerations

Future investigations of neural networks for system identification will take several directions. The networks here were trained starting from random initial weights. increases the burden of learning by requiring the network both to find the form of the function and also to accurately match the function. By using a pre-training algorithm, possibly based on a least-squares fit, initial weights may be chosen which give a reasonable initial fit for the function and may indicate the number of nodes and layers that are needed for such fits. A second consideration is the conflicting demands of aircraft system identification and neural network training. The system identification needs inputs which will excite the aircraft while the network needs input information which will cover the input space sufficiently to allow good function fits everywhere. Since the aircraft is reasonably well understood, extra information, such as aircraft derivatives, may be beneficially used in the training process. Also, there are many other network training algorithms that may be advantageous to use.

- · Initial Weight Selection
- Input Space Coverage
- Extra Training Information
- Network Training Algorithms

Conclusions

The system identification model method is an effective method to provide the information necessary to train neural networks on-line. Feedforward networks can be used to model the aircraft aerodynamic coefficients although there is room for improvement in both the fit of the functions and the methods used to train them.

- System identification model based on Estimation-Before-Modeling is novel and effective
- Feedforward neural networks can model aerodynamic coefficients
- · Room for improvement in training and modeling