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NASA Contractor Report 4430

Computer Code for Preliminary Sizing Analysis of Axial-Flow Turbines

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NASA

National Aeronautics and
Space Administration

Office of Management

Scientific and Technical
Information Program

1992

Summary

This report presents a computer program for the preliminary sizing analysis of axial-flow turbines. The computations are based on mean-diameter flow properties and a stage-average velocity diagram. Given as input to the program are power or pressure ratio, mass flow rate, inlet temperature and pressure, rotative speed, inlet and exit diameters (either hub, mean, or tip), exit radius ratio or stator exit angle, turbine loss coefficient, and gas properties. Computations are then performed for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse) or for any specified stage swirl split. Exit turning vanes can be included in the design. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, flow angles, and last-stage absolute and relative Mach numbers.

The analysis and code presented herein are modifications of those in reference 1. New features added to improve modeling rigor and extend code applicability include a generalized velocity diagram, a more flexible meanline path, a reheat model, a radial component of velocity, and a computation of free-vortex hub and tip velocity diagrams. Also, a loss-coefficient calibration was performed to provide recommended values for current technology airbreathing engine turbines.

The analysis method and a complete description of input and output are presented in this report. Sample cases are included to illustrate the use of the program.

Introduction

The preliminary analysis of a power or propulsion system involves many repetitive calculations to determine system performance, component performance, and component geometries over a range of conditions. This must be done in order to eventually determine the best system and operating conditions. For this type of screening analysis, complete design accuracy and detail for the components are not necessary. Approximate and rapid generalized procedures, rather than complex and time-consuming detailed design procedures, are sufficient to yield the desired component overall geometry and performance characteristics.

A computer program for the preliminary sizing analysis of axial-flow turbines was presented in reference 1. This meanline analysis uses the efficiency computation procedure of reference 2, which is based on a stage-average velocity diagram, and incorporates assumptions consistent with turbine technology from about 1970, when the program was developed. With turbines for some current and future propulsion systems exhibiting higher pressure ratios and more highly inclined flowpaths than in the past, it became apparent that modifications were required to improve modeling rigor and extend code applicability. New features added to accomplish this include a generalized velocity diagram, a more flexible meanline flowpath, a radial component of velocity, and the computation of free-vortex hub and tip velocity diagrams. Also, a loss-coefficient calibration was performed to provide recommended values for current technology airbreathing engine turbines.

The analysis method and a complete description of input and output are presented in this report. Sample cases are included to illustrate use of the program. Because reference 1 is no longer readily available, this report describes the entire analysis, not only the modifications.

Symbols

A	stage loss parameter
A_{an}	annulus area, m^2; ft^2
C	blade loss parameter
C_A	dimensional constant, 2π rad/rev; 60 sec/min
C_B	dimensional constant, 1; 550 ft $\cdot$ lb/sec $\cdot$ hp
c_x	axial chord, m; ft
c_p	heat capacity, joules/kg $\cdot$ K; Btu/lb $\cdot$ °R
D	diameter, m; ft
E	squared ratio of stage-exit axial velocity to stage-average axial velocity
F	blade loss weighting factor
g	dimensional constant, 1; 32.2 ft $\cdot$ lbm/sec^2 $\cdot$ lbf
Δh	specific work, J/kg; Btu/lb
i	stage number, $i = 1,2,\ldots,n$
J	dimensional constant, 1; 778 ft $\cdot$ lb/Btu
j	stage number for change in meanline slope
K	turbine loss coefficient
k	coefficients for axial chord correlation
M	Mach number

N	rotative speed, rad/sec; rpm
n	number of stages
P	shaft power, W; hp
p	pressure, N/m^2; lb/ft^2
R	gas constant, J/kg K; ft $\cdot$ lbf/lbm $°$R
Re	Reynolds number
r	radius, m; ft
T	temperature, K; $°$R
U	blade speed, m/sec; ft/sec
V	absolute gas velocity, m/sec; ft/sec
W	relative gas velocity, m/sec; ft/sec
w	mass flow rate, kg/sec; lb/sec
α	absolute-flow angle from axial direction, deg
β	relative-flow angle from axial direction, deg
γ	specific heat ratio
η	static efficiency
η'	total efficiency
θ	meanline angle from horizontal, deg
μ	viscosity, N $\cdot$ sec/m^2; lb/sec $\cdot$ ft
ρ	density, kg/m^3; lb/ft^3
ψ	work factor

Subscripts:

a	first stage
abs	absolute
cr	critical
ev	exit vane
ex	turbine exit
h	hub
i	stage i, $i = 1,2,\ldots,n$
id	ideal
in	turbine inlet
is	value at turbine inlet entropy
m	mean section
me	meridional component
n	last stage
rel	relative
ro	rotor
st	stator
t	tip
u	tangential component
x	axial component

0	stator inlet
1	stator exit
2	rotor exit

Superscripts:

| —— | turbine overall |
| $'$ | absolute total condition |
| $''$ | relative total condition |

Method of Analysis

The method is based upon an analysis of the flow at the turbine mean diameter. Radial gradients of the flow properties are not included in the flow and efficiency analyses. However, free-vortex hub and tip velocity-diagram parameters are computed in order to indicate the possible severity of flow conditions at the endwalls. The specific heat ratio is assumed constant throughout the turbine. For any given turbine, all stages, except the first, are specified to have the same shape velocity diagram. The first stage differs only in that the inlet flow can be at any specified flow angle. The velocity diagram shape depends upon the stage work factor value and the specified velocity diagram. Three specific types of velocity diagram (symmetrical, zero exit swirl, and impulse) or a diagram with a specified swirl split (i.e., distribution of swirl velocity between rotor inlet and rotor exit) can be used. Figure 1 illustrates the velocity diagram symbolism and presents the three specific types of velocity diagram for three values of stage work factor. The fundamentals of velocity diagrams and of the flow and loss modeling used herein can be found in reference 3.

Various input options dictate the exact nature of the calculation procedure. There is, however, one basic procedure that is direct and without iteration. This basic procedure will be presented, and then the alternative procedures required for the various input options will be discussed.

Basic Calculation Procedure

The required inputs for the basic procedure are shaft power P, mass flow rate w, inlet total temperature T'_{in}, inlet total pressure p'_{in}, rotative speed N, inlet mean diameter $D_{m,in}$, exit mean diameter $D_{m,ex}$, stator exit angle α_1, gas constant R, specific heat ratio γ, viscosity μ, loss coefficient K, and the squared ratio of stage-exit to stage-average (i.e., $(V_{me,0} + V_{me,1} + V_{me,2})/3$) meridional velocities E. Also specified for each calculation are the number of stages n and the type of velocity diagram. For a multistage turbine, the input variable specified as inlet diameter is used to calculate first rotor blade speed and annulus dimensions at the first rotor exit. Therefore, it is truly an inlet diameter only if the hub and tip diameters

(a)
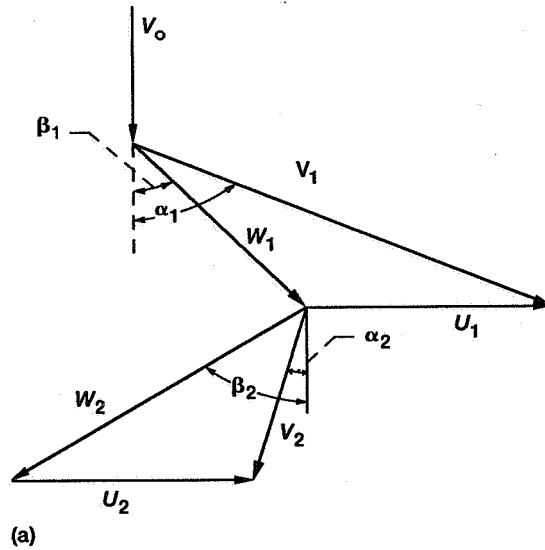
(b)

(a) Symbolism.
(b) Effect of diagram type and work factor on velocity diagram shape.

Figure 1.—Velocity diagrams (constant axial velocity).

are assumed constant across the first stage. For a one-stage turbine, the specified exit diameter is used for the calculations, and the inlet diameter is of no significance.

The first- and last-stage blade speeds are

$$U_a = \frac{\pi N D_{m,in}}{C_A} \qquad (1)$$

$$U_n = \frac{\pi N D_{m,ex}}{C_A} \qquad (2)$$

For more than two stages, there are three options for stage blade-speed variation (i.e., mean flowpath):

(1) Linear variation between first and last stages

$$U_i = \frac{U_n - U_a}{n - 1} (i - 1) + U_a \qquad (3a)$$

(2) Constant from first stage to j^{th} stage ($i = 1$ to j)

$$U_i = U_a \tag{3b-1}$$

and then linear to last stage ($i = j$ to n)

$$U_i = \frac{U_n - U_a}{n - j} (i - j) + U_a \tag{3b-2}$$

(3) Linear from first stage to j^{th} stage ($i = 1$ to j)

$$U_i = \frac{U_n - U_a}{j - 1} (i - 1) + U_a \tag{3c-1}$$

and then constant to last stage ($i = j$ to n)

$$U_i = U_n \tag{3c-2}$$

Turbine specific work is

$$\overline{\Delta h'} = \frac{C_B}{J} \frac{P}{w} \tag{4}$$

and is equal to the sum of the specific work of the stages:

$$\overline{\Delta h'} = \sum_{i=1}^{n} \Delta h_i' \tag{5}$$

Expressing stage specific work in terms of stage work factor as

$$\Delta h_i' = \frac{U_i^2}{gJ} \psi \tag{6}$$

and substituting equation (6) into equation (5) yield

$$\overline{\Delta h'} = \sum_{i=1}^{n} \frac{U_i^2}{gJ} \psi \tag{7}$$

Since the velocity diagram shape is specified to be the same for all stages, the stage work factor is the same for all stages and is computed by rearranging equation (7) as

$$\psi = \frac{gJ \overline{\Delta h'}}{\sum_{i=1}^{n} U_i^2} \tag{8}$$

The value of ψ is the primary factor determining turbine efficiency.

The method used for computing turbine efficiency is basically similar to that presented in reference 2, but has the following
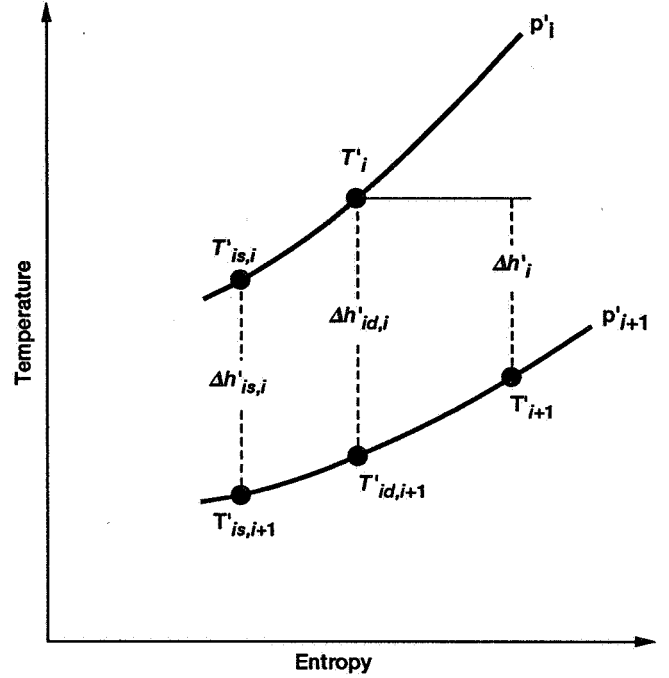

Figure 2.—Temperature-entropy diagram for i^{th} stage.

additional features: (1) the turbines considered in this report are not restricted to a constant mean-section diameter, (2) exit vanes to provide axial flow leaving the turbine can be included in the design, (3) the velocity diagrams also can be specified as symmetrical or by an input swirl split, and (4) the effect of turbine reheat is included.

Referring to figure 2, turbine overall total efficiency is defined as

$$\overline{\eta}' = \frac{\overline{\Delta h'}}{\overline{\Delta h_{id}'}} = \frac{\overline{\Delta h'}}{\sum_{1}^{n} \Delta h_{is,i}'} = \frac{\overline{\Delta h'}}{\sum_{1}^{n} \left(\frac{T_{is,i}'}{T_i'}\right) \Delta h_{id,i}'} \tag{9a}$$

or alternately expressed as

$$\overline{\eta}' = \frac{1}{\sum_{1}^{n} \left(\frac{T_{is,i}'}{T_i'}\right) \left(\frac{\Delta h_{id,i}'}{\Delta h_i'}\right) \left(\frac{\Delta h_i'}{\overline{\Delta h'}}\right)} \tag{9b}$$

Combining equations (6) and (7) yields

$$\frac{\Delta h_i'}{\overline{\Delta h'}} = \frac{U_i^2}{\sum_{1}^{n} U_i^2} \tag{10}$$

and, by definition

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$$\eta_i' = \frac{\Delta h_i'}{\Delta h_{id,i}'} \tag{11}$$

Substituting equations (10) and (11) into equation (9b) yields

$$\overline{\eta}' = \frac{1}{\sum\limits_1^n \left(\dfrac{T_{is,i}'}{T_i'}\right)\left(\dfrac{1}{\eta_i'}\right)\left(\dfrac{U_i^2}{\sum\limits_1^n U_i^2}\right)} \tag{12}$$

The terms T_i' and $T_{is,i}'$, using equation (6), can be written as

$$T_i' = T_{i-1} - \frac{\Delta h_{i-1}'}{c_p} = T_{i-1} - \frac{U_{i-1}^2}{gJc_p}\psi \tag{13}$$

$$T_{is,i}' = T_{is,i-1}' - \frac{\Delta h_{is,i-1}'}{c_p} = T_{is,i-1}' - \left(\frac{T_{is,i-1}'}{T_{i-1}'}\right)\left(\frac{1}{\eta_{i-1}'}\right)\frac{U_{i-1}^2}{gJc_p}\psi \tag{14}$$

where

$$c_p = \frac{\gamma}{\gamma-1}\frac{R}{J} \tag{15}$$

Since both T_i' and $T_{i_{sp}}'$ are equal to T_{in} at the turbine inlet (i.e., at $i = 1$), equations (13) and (14) can be evaluated by recursion once the η_i' are known.

The stage efficiency computation method is explained fully in reference 2, and only the key equations are presented here. Stage total efficiency can be expressed as

$$\eta' = \frac{1}{1 + \dfrac{A}{2}\psi} \tag{16}$$

where the stage loss parameter A is

$$A = \frac{K\mathrm{Re}^{-0.2}}{\cot \alpha_1}(F_{st}C_{st} + F_{ro}C_{ro} + C_{ev}) \tag{17}$$

The constant of proportionality K, called the turbine loss coefficient in this report, must be determined empirically, and recommended values based on experimental efficiencies are presented later in this report. The Reynolds number used in this calculation is defined as

$$\mathrm{Re} = \frac{2w}{\mu D_{m,in}} \tag{18}$$

Some of the terms within the parentheses in equation (17) are the same for all cases, while others depend on stage location, velocity diagram type, and use of exit vanes. The rotor weighting factor F_{ro} and rotor loss parameter C_{ro} are the same for all cases:

$$F_{ro} = 2 \tag{19}$$

$$C_{ro} = 2\cot^2\alpha_1\left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \frac{1}{\psi}\right)^2 + \left(\frac{V_{u,2}}{\Delta V_u} - \frac{1}{\psi}\right)^2$$
$$\tag{20}$$

For last stages of turbines having exit vanes,

$$C_{ev} = 2\cot^2\alpha_1\left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,2}}{\Delta V_u}\right)^2 \tag{21}$$

For all other stages

$$C_{ev} = 0 \tag{22}$$

For first-stage stators having a specified inlet angle α_0, the stator loss parameter is expressed

$$C_{st} = \left[1 + \cot^2\alpha_1\left(2 + \tan^2\alpha_0\right)\right]\left(\frac{V_{u,1}}{\Delta V_u}\right)^2 \tag{23}$$

For all other stators

$$C_{st} = (1 + 2\cot^2\alpha_1)\left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,2}}{\Delta V_u}\right)^2 \tag{24}$$

The stator weighting factor F_{st} also depends on whether or not the stator is a first-stage stator. For a first-stage stator having a specified inlet angle α_0,

$$F_{st} = \frac{1 - \dfrac{3\tan\alpha_0}{\tan\alpha_1}}{1 - \dfrac{\tan\alpha_0}{\tan\alpha_1}} \tag{25}$$

For all other stators

$$F_{st} = \frac{1 - \dfrac{3(V_{u,2}/\Delta V_u)}{V_{u,1}/\Delta V_u}}{1 - \dfrac{V_{u,2}/\Delta V_u}{V_{u,1}/\Delta V_u}} \tag{26}$$

The rotor inlet and exit swirl parameters $V_{u,1}/\Delta V_u$ and $V_{u,2}/\Delta V_u$ of equations (19) to (26) depend on the specific type of velocity diagram or are input for the general diagram. The following table presents the relations for evaluating the swirl parameters:

Velocity	Inlet swirl parameter, $V_{u,1}/\Delta V_u$	Exit swirl parameter, $V_{u,2}/\Delta V_u$
Symmetrical	$\dfrac{1+\psi}{2\psi}$	$\dfrac{1-\psi}{2\psi}$
Zero exit swirl	1	0
Impulse	$\dfrac{2+\psi}{2\psi}$	$\dfrac{2-\psi}{2\psi}$
General	Input	$\dfrac{V_{u,1}}{\Delta V_u}-1$

With turbine total efficiency obtained from the foregoing equations, the turbine exit velocities and state conditions are computed as follows:

$$\Delta V_{u,n} = \psi U_n \tag{27}$$

$$V_{u,1,n} = \frac{V_{u,1}}{\Delta V_u}\,\Delta V_{u,n} \tag{28}$$

$$V_{u,2,n} = \frac{V_{u,2}}{\Delta V_u}\,\Delta V_{u,n} \tag{29}$$

$$V_{me,n} = V_{u,1,n}\cot\alpha_1 \tag{30}$$

$$V_{me,ex} = \sqrt{E}\,V_{me,n} \tag{31}$$

$$V_{u,ex} = \begin{cases} V_{u,2,n} & \text{no exit vanes} \\ 0 & \text{exit vanes} \end{cases} \tag{32}$$

$$V_{ex} = \sqrt{V_{me,ex}^2 + V_{u,ex}^2} \tag{33}$$

$$p'_{ex} = p'_{in}\left(1 - \frac{\overline{\Delta h'}}{c_p T'_{in}\overline{\eta}'}\right)^{\gamma/(\gamma-1)} \tag{34}$$

$$T'_{ex} = T'_{in} - \frac{\overline{\Delta h'}}{c_p} \tag{35}$$

$$T_{ex} = T'_{ex} - \frac{V_{ex}^2}{2gJc_p} \tag{36}$$

$$p_{ex} = p'_{ex}\left(\frac{T_{ex}}{T'_{ex}}\right)^{\gamma/(\gamma-1)} \tag{37}$$

In order that the turbine total and static efficiencies be consistent with the computed exit velocity, the static efficiency is computed as

$$\overline{\eta} = \frac{\overline{\Delta h'}}{c_p T'_{in}\left[1 - \left(\dfrac{p_{ex}}{p'_{in}}\right)^{(\gamma-1)/\gamma}\right]} \tag{38}$$

In order to calculate continuity for a flowpath having significant inclination, it is necessary to determine a flowpath slope. Consistent with the preliminary nature of this analysis, a correlation relating axial chord to mean diameter was used to determine axial length. The correlation, based on the geometry of existing turbines, is of the form

$$c_x = k_1 D_m + k_2 D_m^2 + k_3 + k_4 D_m^{-1} \tag{39}$$

and was established over a range of mean diameters from 0.10 to 1.25 m (0.33 to 4.17 ft). Because of the scatter in the available data, correlations were determined for low, mean, and high aspect ratios. The correlation coefficients are presented in table I. The three correlation curves for ratio of axial chord to mean diameter are plotted in figure 3. Using one-third of axial chord length for spacing between blade rows, the slope for a linear meanline between stage 1 exit and stage n exit is

$$\tan\theta = \frac{0.5(D_{m,ex} - D_{m,in})}{2(4/3)\displaystyle\sum_2^n c_{x,i}} = \frac{3(D_{m,ex} - D_{m,in})}{16\displaystyle\sum_2^n c_{x,i}} \tag{40}$$

For two-segment meanlines, the lower or upper limit for the summation is changed as appropriate for the linear section, and the slope for the constant section is set equal to zero.

TABLE I.—AXIAL CHORD CORRELATION COEFFICIENTS

$$\left| c_x = k_1 D_m + k_2 D_m^2 + k_3 + k_4 D_m^{-1}. \right|$$

Aspect ratio	$k_1 \times 10^2$	$k_2 \times 10^3$		$k_3 \times 10^2$		$k_4 \times 10^3$	
		m^{-1}	ft^{-1}	m	ft	m^2	ft^2
High	0.5716	7.8984	2.4074	1.4911	4.8921	−0.7498	−8.0708
Mid	3.0740	9.9323	3.0274	1.4714	4.8275	−.7411	−7.9771
Low	6.6559	8.7134	2.6558	1.1949	3.9203	−.5580	−6.0063

Exit annulus area, radius ratio, and hub and tip diameters are obtained as follows:

$$\rho_{ex} = \frac{p_{ex}}{RT_{ex}} \tag{41}$$

$$V_{x,ex} = V_{me,ex} \cos \theta_n \tag{42}$$

$$A_{an,ex} = \frac{w}{\rho_{ex} V_{x,ex}} \tag{43}$$

$$\left(\frac{r_h}{r_t}\right)_{ex} = \frac{1 - \dfrac{A_{an,ex}}{\pi D_{m,ex}^2}}{1 + \dfrac{A_{an,ex}}{\pi D_{m,ex}^2}} \tag{44}$$

$$D_{t,ex} = \frac{2D_{m,ex}}{1 + \left(\dfrac{r_h}{r_t}\right)_{ex}} \tag{45}$$

$$D_{h,ex} = D_{t,ex} \left(\frac{r_h}{r_t}\right)_{ex} \tag{46}$$

Absolute and relative flow angles, which are the same for each stage, are computed from the last-stage velocities:

$$\alpha_2 = \tan^{-1} \frac{V_{u,2,n}}{V_{me,n}} \tag{47}$$

$$W_{u,1,n} = V_{u,1,n} - U_n \tag{48}$$

$$W_{u,2,n} = V_{u,2,n} - U_n \tag{49}$$

$$\beta_1 = \tan^{-1} \frac{W_{u,1,n}}{V_{me,n}} \tag{50}$$

$$\beta_2 = \tan^{-1} \frac{W_{u,2,n}}{V_{me,n}} \tag{51}$$

Figure 3.—Axial chord correlation.
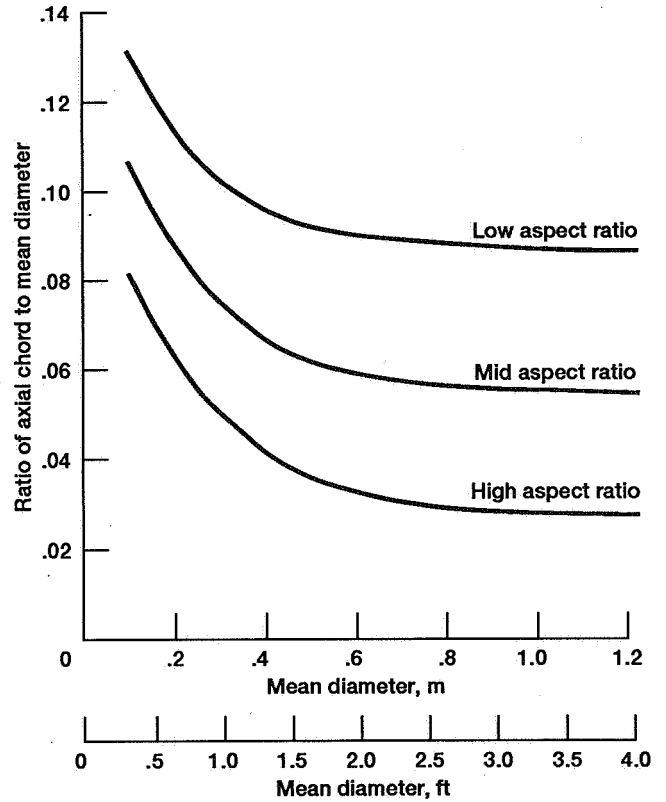
Absolute and relative Mach numbers are computed for the last stage, where velocities are highest and temperatures are lowest, thus making the Mach numbers most severe:

$$V_{2,n} = \sqrt{V_{u,2,n}^2 + V_{me,ex}^2} \tag{52}$$

$$W_{2,n} = \sqrt{W_{u,2,n}^2 + V_{me,ex}^2} \tag{53}$$

$$W_{1,n} = \sqrt{W_{u,1,n}^2 + V_{me,n}^2} \tag{54}$$

$$V_{1,n} = \sqrt{V_{u,1,n}^2 + V_{me,n}^2} \tag{55}$$

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$$T''_{1,n} = T''_{2,n} = T'_{ex} - \frac{V^2_{2,n} - W^2_{2,n}}{2gJc_p} \qquad (56)$$

$$T_{1,n} = T''_{1,n} - \frac{W^2_{1,n}}{2gJc_p} \qquad (57)$$

$$M_{abs,2} = \frac{V_{2,n}}{\sqrt{\gamma g R T_{ex}}} \qquad (58)$$

$$M_{rel,2} = \frac{W_{1,n}}{\sqrt{\gamma g R T_{ex}}} \qquad (59)$$

$$M_{rel,1} = \frac{W_{1,n}}{\sqrt{\gamma g R T_{1,n}}} \qquad (60)$$

$$M_{abs,1} = \frac{V_{1,n}}{\sqrt{\gamma g R T_{1,n}}} \qquad (61)$$

Efficiency and continuity computations are based solely on the meanline flow parameters. Although radial variations of velocity in a turbine need not be of the free-vortex type, the free-vortex hub and tip velocity-diagram parameters can give a qualitative indication of the possible severity of end wall flow conditions. With blade speed directly proportional to diameter

$$U \propto D \qquad (62)$$

and applying the free-vortex conditions for radial variations of the velocity components

$$V_u \propto \frac{1}{D} \qquad (63)$$

and

$$V_{me} = \text{Constant} \qquad (64)$$

the hub and tip velocity diagrams are determined from equations (47) to (61).

In order to establish the flow annulus geometry near the turbine inlet, a flow analysis is made at the first-stage exit as follows:

$$\Delta h'_a = \frac{U^2_a}{gJ} \psi \qquad (65)$$

$$T'_{2,a} = T'_{in} - \frac{\Delta h'_a}{c_p} \qquad (66)$$

$$p'_{2,a} = p'_{in} \left(1 - \frac{\Delta h'_a}{c_p T'_{in} \eta'_a} \right)^{\gamma/(\gamma-1)} \qquad (67)$$

$$\Delta V_{u,a} = \psi U_a \qquad (68)$$

$$V_{u,2,a} = \frac{V_{u,2}}{\Delta V_u} \Delta V_{u,a} \qquad (69)$$

$$V_{me,2,a} = \sqrt{E} \frac{V_{u,1}}{\Delta V_u} \Delta V_{u,a} \cot \alpha_1 \qquad (70)$$

$$V_{2,a} = \sqrt{V^2_{u,2,a} + V^2_{me,2,a}} \qquad (71)$$

$$T_{2,a} = T'_{2,a} - \frac{V^2_{2,a}}{2gJc_p} \qquad (72)$$

$$p_{2,a} = p'_{2,a} \left(\frac{T_{2,a}}{T'_{2,a}} \right)^{\gamma/(\gamma-1)} \qquad (73)$$

$$V_{x,2,a} = V_{me,2,a} \cos \theta_a \qquad (74)$$

$$\rho_{2,a} = \frac{p_{2,a}}{R T_{2,a}} \qquad (75)$$

$$A_{an,2,a} = \frac{w}{\rho_{2,a} V_{x,2,a}} \qquad (76)$$

$$\left(\frac{r_h}{r_t} \right)_{2,a} = \frac{1 - \dfrac{A_{an,2,a}}{\pi D^2_{m,in}}}{1 + \dfrac{A_{an,2,a}}{\pi D^2_{m,in}}} \qquad (77)$$

$$D_{t,2,a} = \frac{2 D_{m,in}}{1 + \left(\dfrac{r_h}{r_t} \right)_{2,a}} \qquad (78)$$

$$D_{h,2,a} = D_{t,2,a} \left(\frac{r_h}{r_t} \right)_{2,a} \qquad (79)$$

When a constant annulus is assumed for the first stage, the first-stage exit dimensions become the inlet dimensions for that turbine.

Alternative Calculation Procedure

The basic calculation procedure described in the previous section requires as inputs the inlet and exit mean diameters, stator exit angle, and shaft power. Alternatively, the hub or tip diameters could be specified as input, and the mean diameters computed; the exit radius ratio could be specified as input, and the stator exit angle computed; and/or the turbine pressure ratio could be specified as input, and the shaft power computed. These alternative input options require iterative calculation procedures such as described in this section.

With hub or tip diameters rather than mean diameters specified at the inlet and exit, it is necessary to assume initial values for the inlet and exit radius ratios. Initial values for inlet and exit mean diameters are then obtained as

$$D_m = \frac{\left(1 + \dfrac{r_h}{r_t}\right)D_h}{2\,\dfrac{r_h}{r_t}} \tag{80}$$

when hub diameter is input and

$$D_m = \frac{\left(1 + \dfrac{r_h}{r_t}\right)D_t}{2} \tag{81}$$

when tip diameter is input. The computation then proceeds from equation (1) to equation (44), and the computed exit radius ratio is compared with the assumed value. If they are not the same (within a given tolerance), then the computed value of exit radius ratio is used to calculate a new value for exit mean diameter (from eq. (80) or (81)), and the computation procedure is repeated until convergence is obtained. Then, computation proceeds through equation (77), and the computed inlet radius ratio is compared with the assumed value. If they are not within tolerance, the computed value of inlet radius ratio is used to calculate a new value for inlet mean diameter, and the computation procedure is repeated from equation (1). This entire procedure is repeated until both inlet and exit radius ratios converge in the same calculation pass. When hub or tip diameter rather than mean diameter is used as input, a two-segment meanline cannot be specified.

With exit radius ratio rather than stator exit angle specified, a value of stator exit angle is assumed for the evaluation of equation (17). The computation proceeds through equation (41). Equation (44) is then used to compute the exit annulus area from the input value of radius ratio, and the exit axial velocity is then obtained from equation (43). The density used in equation (43), however, is not consistent with the exit area, and equations (43), (42), (33), (36), and (41) must be iterated until convergence is obtained. Then, the stator exit angle is computed as

$$\alpha_1 = \tan^{-1}\frac{V_{u,1,n}}{V_{me,n}} \tag{82}$$

and compared with the assumed value. If they are not within tolerance, the computed value of stator exit angle is used for the evaluation of equation (17), and the computation procedure just given is repeated until stator exit angle converges. The remainder of the computation is then completed.

With the turbine inlet-total- to exit-static-pressure ratio rather than shaft power specified as input, an initial value of turbine static efficiency is assumed. Turbine work is then computed from

$$\overline{\Delta h'} = \overline{\eta}c_p T'_{in}\left[1 - \left(\frac{p_{ex}}{p'_{in}}\right)^{(\gamma-1)/\gamma}\right] \tag{83}$$

instead of from equation (4). The computation then proceeds through equation (38) to compute a static efficiency. If the computed and the assumed values are not within tolerance, a new value of static efficiency is assumed, and the computation is repeated until convergence. The remainder of the computation is then completed.

Loss-Coefficient Evaluation

Ten turbines that were designed for airbreathing engine applications and performance tested in component facilities were selected to serve as the basis for the loss coefficient evaluation. These turbines represent the output of several different design systems and cover a wide range of design characteristics. Six of the turbines were designed and tested by NASA, two were designed and tested by GE Aircraft Engines Co., and two were designed by Pratt & Whitney, one of which was tested by Pratt & Whitney and the other by NASA. The characteristics of these turbines are presented in table II. The number of stages varies from 1 to 5, stage work factors (ψ) from 1.2 to 4.7, stator angles from 51° to 80°, exit mean diameters, except for one, from 47 to 69 cm (18 to 27 in.), and radius ratios from 0.50 to 0.86. All the listed total efficiencies except for two were as measured. In those two cases, corrections were made to account for the effects of cooling air (ref. 4) and a large discrepancy in flow rate from design intent (ref. 5).

These turbines were modeled as closely as possible for the subject computer program, and design performance was computed over a range of loss coefficients. The variation of efficiency with loss coefficient was a direct proportion between the quantity $(1 - \eta)$ and loss coefficient, K. Calculated efficiencies are plotted against measured efficiencies in figure 4 for a constant loss coefficient of 0.3. As seen, for all cases except one, the calculated efficiency was within one point of the measured efficiency. In view of the simplistic modeling used for this computer program, this agreement is admittedly fortuitous. It does illustrate, however, that this program is capable of predicting reasonable efficiencies over a wide range of airbreathing engine turbine conditions. A loss coefficient value of 0.3 is recommended for use in the absence of additional information.

TABLE II.—TURBINES USED FOR EVALUATION

| Number of stages	Average stage work factor, ψ	Average stator angle, deg	Exit mean diameter		Exit radius ratio	Measured efficiency	Reference
			cm	in.			
1	1.7	67.0	66.0	26.0	0.73	0.923	6
2	1.7	64.5	66.0	26.0	.63	.932	7
1	1.2	63.4	20.3	8.0	.68	.917	8
1	1.9	73.1	47.0	18.5	.85	.886	9
3.5	4.0	51.4	49.8	19.6	.50	.880	5
4.5	4.7	66.6	49.8	19.6	.57	.855	10
1	1.7	75.0	46.7	18.4	.86	.895	11
1	1.6	79.6	56.4	22.2	.85	.901	12
2	1.3	71.6	69.3	27.3	.82	.898	4
5	2.6	61.0	63.5	25.0	.62	.920	13
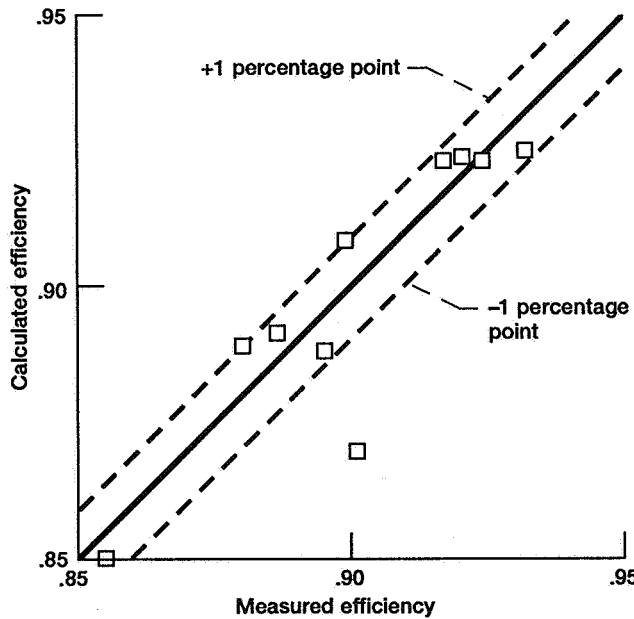
Figure 4.—Comparison of measured and calculated total efficiencies. Loss coefficient, 0.3.

Description of Input and Output

This section presents a detailed description of the program input, normal output, and error messages. Included in the input and output sections are several example cases illustrating the use of the program and the various options.

Input

The program input, a sample of which is presented in table III, consists of a title record and the required physical data and option indicators in NAMELIST form. The title, which is printed as a heading on the output listing, can contain up to 77 characters located anywhere in columns 2 to 78 on the title record. A title, even if it is left blank, must be the first record of the input data. Additional titles can be used to identify different cases being run in the same data file. This is done by placing a title in front of the data for the particular case and using the option indicator ITIT as subsequently described.

The physical data and option indicators are input in data records having the NAMELIST name INPUT. The variables and indicators that compose INPUT and the proper units are as follows. These must be input for all cases except where otherwise indicated. Either the SI units or the U.S. customary units shown below may be used.

PTIN   inlet total pressure, N/cm^2; $lb/in.^2$

TTIN   inlet total temperature, K; $°R$

MU   gas viscosity, $N \cdot sec/m^2$; $lb/sec \cdot ft$

R   gas constant, $J/kg \cdot K$; $ft \cdot lbf/lbm \cdot °R$

GAM   specific heat ratio

DIN   inlet diameter—hub or mean or tip value as specified by the indicator IDIAM, cm; in.

DEX   exit diameter—hub or mean or tip value as specified by the indicator IDIAM, cm; in.

RREX   exit radius ratio; RREX may be omitted in the case where both IDIAM = 2 and IALPH = 0; RREX is used as first trial value when IALPH = 0 and IDIAM = 1 or 3

RPM   rotative speed, rad/sec; rpm

POW   shaft power—omit when IPR = 1, kW; hp

W   mass flow rate, kg/sec; lb/sec

TABLE III.—SAMPLE INPUT

```
THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNIT ARE USED.
&INPUT PTIN=113.1,TTIN=2660.,MU=.376E-04,R=53.37,GAM=1.302,DIN=22.,DEX=24.,
RPM=11400.,POW=12900.,W=53.5,ALPHA=65.,KLOSS=.35,NMIN=2,NMAX=2,E=1.2,
IALPH=0,IDIAM=2,IVD=1,IEV=0,IPR=0,IU=2 &END
&INPUT IDIAM=1,RREX=.8,DIN=20.,DEX=21., ITIT=1 &END
      THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA
&INPUT IALPH=1,IEV=1, &END
&INPUT IALPH=0,IEV=0,IVD=2 &END
&INPUT IVD=3 &END
&INPUT IPR=1,IVD=1,PRTS=3.66 &END
```

ALPHA   stator exit angle from axial direction; ALPHA is used as first trial value when IALPH = 1, deg

ALPHA0   turbine inlet flow angle; required only when KALPH0 = 2, deg

VU1DVU   ratio of rotor inlet swirl to total change in swirl; input only when IVD = 5

KLOSS   turbine loss coefficient; a value of 0.3 is recommended in the absence of additional information

NMIN   minimum number of stages for which the calculations are performed

NMAX   maximum number of stages for which the calculations are performed; results are obtained for all stage numbers between NMIN and NMAX

NMID   stage number at which meanline changes slope; may be omitted when IMID = 0

E   squared ratio of stage-exit to stage-average meridional velocities

PRTS   turbine inlet-total to exit-static-pressure ratio; omit when IPR = 0

IALPH   indicates whether stator exit angle or turbine exit radius ratio is specified:

     IALPH = 0—turbine is designed for specified ALPHA

     IALPH = 1—turbine is designed for specified RREX

IDIAM   indicates whether input diameters are hub, mean, or tip values:

     IDIAM = 1—input diameters are hub values
     IDIAM = 2—input diameters are mean values
     IDIAM = 3—input diameters are tip values

IVD   indicates type of velocity diagram used:

     IVD = 1—symmetrical diagrams

     IVD = 2—zero exit swirl diagrams

     IVD = 3—impulse diagrams

     IVD = 4—zero exit swirl diagrams if $\psi \le 2.0$ and impulse diagrams if $\psi \ge 2.0$

     IVD = 5—ratio of rotor inlet swirl to total change in swirl is input as VU1DVU

ITIT   indicates use of titles in addition to that required as first line of data package:

     ITIT = 1—title line precedes next data set; must be input for each additional title because ITIT is automatically restored to zero after each title is read

IEV   indicates use of exit vanes:

     IEV = 0—no exit vanes

     IEV = 1—exit vanes are used to turn turbine exit flow to axial direction

IPR   indicates whether shaft power or pressure ratio is specified:

     IPR = 0—shaft power is input

     IPR = 1—turbine inlet-total- to exit-static-pressure ratio is input

IU   indicates type of units used for input and output:

     IU = 1—SI units

     IU = 2—U.S. customary units

KALPH0   indicates turbine-inlet flow angle option:

     KALPH0 = 0—turbine-inlet flow is axial (default)

     KALPH0 = 1—turbine-inlet flow angle equals stage-exit flow angle

     KALPH0 = 2—turbine-inlet flow angle is input as ALPHA0

IAR   indicates blading aspect ratio:

     IAR = 1—high aspect-ratio blading

     IAR = 2—mid aspect-ratio blading (default)

     IAR = 3—low aspect-ratio blading

IMID   indicates meanline shape:

     IMID = 0—meanline linear from stage 1 to stage N (default)

     IMID = 1—meanline constant from stage 1 to stage NMID and then linear to stage N

     IMID = 2—meanline linear from stage 1 to stage NMID and then constant to stage N

The first line of the input file shown in table III is the mandatory title card, which can contain any desired message. The next three lines are the first data set, which contains all required inputs. This first case represents computation in accordance with the basic calculation procedure described previously. Data input for subsequent cases need only include those values that differ from previous case data. The fifth line is the second data set and represents the option where hub diameter is input. Also, the second case data specify that a title, which is the sixth data line, precede the third case data. Cards 7 to 10 represent four additional cases illustrating the use of different input options. The output corresponding to this sample input is described in the following section.

Output

The program output consists of title headings, the input variables, and computed results. This section presents normal output. Error message output is described in the next section.

Table IV presents the output that corresponds to the sample input shown in table III. The top line of output is a program identification title that is automatically printed. The second line is the title card message. The next four lines are the input variables and their associated values for the first data set. The input variable names are spelled out. The units for the input variable values are as described in the "Input" section. The zeros

printed under EXIT RADIUS RATIO and T-S PRESSURE RATIO indicate that these are computed for this case and not specified by the input. The fact that the input diameters are mean diameters is indicated by the MN in the variable name. Hub and tip diameters would be indicated by HB and TP, respectively. These four lines of output are printed for each new data set.

The next group of eight lines is the computation results satisfying the input requirements. The output parameters are spelled out and are self-explanatory. These temperatures, pressures, velocities, and angles are meanline values. On the first line are the number of stages, the stage work factor, the Reynolds number, and the diagram type, which is symmetrical for this first case. The remainder of the output includes exit and inlet tip and hub diameters in the first column, exit total and static temperatures and pressures in the second column, total and static efficiencies and velocity diagram angles in the third column, and first- and last-stage blade speeds and last-stage absolute velocity components in the last column. The last line of this output group presents the last-stage absolute and relative Mach numbers.

The next group of four lines is the hub and tip free-vortex values of Mach numbers and angles. The four columns present the stator-exit absolute, rotor-inlet relative, rotor-exit relative, and stage-exit absolute values. The last line of output for each case is the meanline slope based on the specified (mid in this case) aspect-ratio blading.

After the computations for each input case are completed, the input data for the next case are printed. The second case presented here is that where the specified diameters are hub values. The third input case in table IV is preceded by an additional title line. This causes the next output to begin at the top of a new page with the program identification and the title message.

The third case is for a specified exit radius ratio rather than for a specified stator exit angle. This is indicated by a zero appearing under STATOR EX ANG in the row of input variables. Exit vanes are included in this case, as indicated by WITH EXIT VANES printed after number of stages on the first line of result output. The fourth and fifth cases, as indicated by the top line of the fourth column of result output, are for zero exit swirl and impulse diagrams, respectively. Turbine pressure ratio rather than shaft power is specified in the sixth case. For this case, the input pressure ratio was chosen to be the same as the computed value obtained for the second case. As should be expected, the program converges to identical solutions for both cases. An extra line of output consisting of shaft power and specific work is printed for the case where pressure ratio is specified as input.

Error Messages

The program contains seven output messages. Five of these indicate the nonexistence of a meanline solution satisfying

TABLE IV.—SAMPLE OUTPUT

TURBINE VELOCITY DIAGRAM ANALYSIS

THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNIT ARE USED.

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET MN DIA	EXIT MN DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRES RATI
12900.0	53.50	2660.00	113.10	11400.00	22.00	24.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.

STAGES= 2			
EXIT TIP DIAMETER = 27.18	STAGE WORK FACTOR= 1.63	REYNOLDS NO.=0.1552E+07	DIAGRAMS ARE SYMMETRICAL
EXIT HUB DIAMETER = 20.82	EXIT TOTAL TEMP =2083.64	STATOR EXIT ANGLE= 65.00	FIRST STAGE MEAN SPEED= 1094.32
EXIT RADIUS RATIO =0.7658	EXIT STATIC TEMP =2030.85	STAGE EXIT ANGLE =-27.10	LAST STAGE MEAN SPEED = 1193.81
INLET TIP DIAMETER= 24.16	EXIT TOTAL PRESS = 34.85	ROTOR INLET ANGLE= 27.10	LAST STAGE INLET SWIRL= 1567.87
INLET HUB DIAMETER= 19.84	EXIT STATIC PRESS= 31.20	ROTOR EXIT ANGLE =-65.00	LAST STAGE EXIT SWIRL = -374.06
INLET RADIUS RATIO=0.8213	T-T PRESS RATIO = 3.245	TOTAL EFFICIENCY =0.907	LAST STAGE MERID VELOC= 800.93
LAST STG M1 ABS =0.7810	T-S PRESS RATIO = 3.625	STATIC EFFICIENCY=0.839	EXIT MERID MACH NUMBER= 0.3759
	LAST STG M1 REL = 0.3708	LAST STG M2 REL =0.8263	LAST STG M2 ABS = 0.4149

HUB: LAST STG M1 ABS =0.8915	LAST STG M1 REL = 0.4862	LAST STG M2 REL =0.7684	LAST STG M2 ABS = 0.3980
STATOR EXIT ANGLE = 67.98	ROTOR INLET ANGLE = 46.57	ROTOR EXIT ANGLE =-63.51	STAGE EXIT ANGLE =-30.53
TIP: LAST STG M1 ABS =0.7010	LAST STG M1 REL = 0.3277	LAST STG M2 REL =0.8590	LAST STG M2 ABS = 0.3757
STATOR EXIT ANGLE = 62.16	ROTOR INLET ANGLE = 2.51	ROTOR EXIT ANGLE =-66.51	STAGE EXIT ANGLE =-24.31

STAGE 1- 2 MEANLINE SLOPE = 14.85 DEG BASED ON MID ASPECT-RATIO BLADING

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRES RATI
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.

STAGES= 2			
EXIT TIP DIAMETER = 27.32	STAGE WORK FACTOR= 1.61	REYNOLDS NO.=0.1542E+07	DIAGRAMS ARE SYMMETRICAL
EXIT HUB DIAMETER = 21.00	EXIT TOTAL TEMP =2083.64	STATOR EXIT ANGLE= 65.00	FIRST STAGE MEAN SPEED= 1101.56
EXIT RADIUS RATIO =0.7685	EXIT STATIC TEMP =2031.50	STAGE EXIT ANGLE =-26.48	LAST STAGE MEAN SPEED = 1201.87
INLET TIP DIAMETER= 24.29	EXIT TOTAL PRESS = 34.87	ROTOR INLET ANGLE= 26.48	LAST STAGE INLET SWIRL= 1565.51
INLET HUB DIAMETER= 20.00	EXIT STATIC PRESS= 31.27	ROTOR EXIT ANGLE =-65.00	LAST STAGE EXIT SWIRL = -363.64
INLET RADIUS RATIO=0.8233	T-T PRESS RATIO = 3.243	TOTAL EFFICIENCY =0.907	LAST STAGE MERID VELOC= 799.73
LAST STG M1 ABS =0.7798	T-S PRESS RATIO = 3.617	STATIC EFFICIENCY=0.840	EXIT MERID MACH NUMBER= 0.3753
	LAST STG M1 REL = 0.3682	LAST STG M2 REL =0.8249	LAST STG M2 ABS = 0.4123

HUB: LAST STG M1 ABS =0.8883	LAST STG M1 REL = 0.4805	LAST STG M2 REL =0.7664	LAST STG M2 ABS = 0.3944
STATOR EXIT ANGLE = 67.94	ROTOR INLET ANGLE = 46.03	ROTOR EXIT ANGLE =-63.48	STAGE EXIT ANGLE =-29.82
TIP: LAST STG M1 ABS =0.7007	LAST STG M1 REL = 0.3271	LAST STG M2 REL =0.8580	LAST STG M2 ABS = 0.3735
STATOR EXIT ANGLE = 62.20	ROTOR INLET ANGLE = 1.97	ROTOR EXIT ANGLE =-66.52	STAGE EXIT ANGLE =-23.77

STAGE 1- 2 MEANLINE SLOPE = 14.89 DEG BASED ON MID ASPECT-RATIO BLADING

TABLE IV.—Concluded.

TURBINE VELOCITY DIAGRAM ANALYSIS

THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.8000	0.00	53.37	1.302	0.376E-04	0.350	1.200	0.000

```
        STAGES= 2 W/EXT VANES           STAGE WORK FACTOR= 1.67          REYNOLDS NO.=0.1569E+07          DIAGRAMS ARE SYMMETRICAL
        EXIT TIP DIAMETER = 26.25       EXIT TOTAL TEMP  =2083.64        STATOR EXIT ANGLE= 59.54         FIRST STAGE MEAN SPEED= 1082.83
        EXIT HUB DIAMETER = 21.00       EXIT STATIC TEMP =2014.59        STAGE EXIT ANGLE =-23.12         LAST STAGE MEAN SPEED = 1175.15
        EXIT RADIUS RATIO =0.8000       EXIT TOTAL PRESS =  34.80        ROTOR INLET ANGLE= 23.12         LAST STAGE INLET SWIRL= 1569.27
        INLET TIP DIAMETER= 23.54       EXIT STATIC PRESS=  30.10        ROTOR EXIT ANGLE =-59.54         LAST STAGE EXIT SWIRL = -394.12
        INLET HUB DIAMETER= 20.00       T-T PRESS RATIO  =   3.250       TOTAL EFFICIENCY =0.906          LAST STAGE MERID VELOC= 1011.01
        INLET RADIUS RATIO=0.8497       T-S PRESS RATIO  =   3.758       STATIC EFFICIENCY=0.819          EXIT MERID MACH NUMBER= 0.4764
        LAST STG M1 ABS   =0.8263       LAST STG M1 REL  = 0.4555        LAST STG M2 REL  =0.8797         LAST STG M2 ABS       = 0.5113

HUB: LAST STG M1 ABS   =0.9135          LAST STG M1 REL  = 0.5370        LAST STG M2 REL  =0.8255         LAST STG M2 ABS       = 0.4827
     STATOR EXIT ANGLE = 62.40          ROTOR INLET ANGLE = 37.99        ROTOR EXIT ANGLE =-58.19         STAGE EXIT ANGLE      =-25.66
TIP: LAST STG M1 ABS   =0.7603          LAST STG M1 REL  = 0.4186        LAST STG M2 REL  =0.8945         LAST STG M2 ABS       = 0.4656
     STATOR EXIT ANGLE = 56.84          ROTOR INLET ANGLE =  6.59        ROTOR EXIT ANGLE =-60.93         STAGE EXIT ANGLE      =-21.02

        STAGE 1- 2 MEANLINE SLOPE =  13.98 DEG BASED ON MID ASPECT-RATIO BLADING
```

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.000

```
        STAGES= 2                       STAGE WORK FACTOR= 1.67          REYNOLDS NO.=0.1568E+07          DIAGRAMS ARE ZERO EX SWRL
        EXIT TIP DIAMETER = 26.34       EXIT TOTAL TEMP  =2083.64        STATOR EXIT ANGLE= 65.00         FIRST STAGE MEAN SPEED= 1083.42
        EXIT HUB DIAMETER = 21.00       EXIT STATIC TEMP =2015.77        STAGE EXIT ANGLE =  0.00         LAST STAGE MEAN SPEED = 1177.29
        EXIT RADIUS RATIO =0.7974       EXIT TOTAL PRESS =  34.43        ROTOR INLET ANGLE= 40.62         LAST STAGE INLET SWIRL= 1962.12
        INLET TIP DIAMETER= 23.56       EXIT STATIC PRESS=  29.85        ROTOR EXIT ANGLE =-52.15         LAST STAGE EXIT SWIRL =    0.00
        INLET HUB DIAMETER= 20.00       T-T PRESS RATIO  =   3.285       TOTAL EFFICIENCY =0.899          LAST STAGE MERID VELOC= 1002.33
        INLET RADIUS RATIO=0.8488       T-S PRESS RATIO  =   3.789       STATIC EFFICIENCY=0.815          EXIT MERID MACH NUMBER= 0.4722
        LAST STG M1 ABS   =1.0042       LAST STG M1 REL  = 0.5592        LAST STG M2 REL  =0.7284         LAST STG M2 ABS       = 0.4722

HUB: LAST STG M1 ABS   =1.1294          LAST STG M1 REL  = 0.6998        LAST STG M2 REL  =0.6523         LAST STG M2 ABS       = 0.4298
     STATOR EXIT ANGLE = 67.52          ROTOR INLET ANGLE = 51.90        ROTOR EXIT ANGLE =-48.78         STAGE EXIT ANGLE      = 0.00
TIP: LAST STG M1 ABS   =0.9106          LAST STG M1 REL  = 0.4681        LAST STG M2 REL  =0.7507         LAST STG M2 ABS       = 0.4298
     STATOR EXIT ANGLE = 62.58          ROTOR INLET ANGLE = 26.36        ROTOR EXIT ANGLE =-55.07         STAGE EXIT ANGLE      = 0.00

        STAGE 1- 2 MEANLINE SLOPE =  14.19 DEG BASED ON MID ASPECT-RATIO BLADING
```

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	0.000

```
        STAGES= 2                       STAGE WORK FACTOR= 1.68          REYNOLDS NO.=0.1576E+07          DIAGRAMS ARE IMPULSE
        EXIT TIP DIAMETER = 26.11       EXIT TOTAL TEMP  =2083.64        STATOR EXIT ANGLE= 65.00         FIRST STAGE MEAN SPEED= 1078.13
        EXIT HUB DIAMETER = 21.00       EXIT STATIC TEMP =1999.26        STAGE EXIT ANGLE = 10.46         LAST STAGE MEAN SPEED = 1171.57
        EXIT RADIUS RATIO =0.8044       EXIT TOTAL PRESS =  33.80        ROTOR INLET ANGLE= 44.42         LAST STAGE INLET SWIRL= 2157.44
        INLET TIP DIAMETER= 23.35       EXIT STATIC PRESS=  28.28        ROTOR EXIT ANGLE =-44.42         LAST STAGE EXIT SWIRL =  185.70
        INLET HUB DIAMETER= 20.00       T-T PRESS RATIO  =   3.346       TOTAL EFFICIENCY =0.887          LAST STAGE MERID VELOC= 1102.11
        INLET RADIUS RATIO=0.8566       T-S PRESS RATIO  =   3.999       STATIC EFFICIENCY=0.788          EXIT MERID MACH NUMBER= 0.5213
        LAST STG M1 ABS   =1.1222       LAST STG M1 REL  = 0.6640        LAST STG M2 REL  =0.6995         LAST STG M2 ABS       = 0.5287

HUB: LAST STG M1 ABS   =1.2610          LAST STG M1 REL  = 0.8199        LAST STG M2 REL  =0.6168         LAST STG M2 ABS       = 0.4844
     STATOR EXIT ANGLE = 67.42          ROTOR INLET ANGLE = 53.81        ROTOR EXIT ANGLE =-39.74         STAGE EXIT ANGLE      = 11.70
TIP: LAST STG M1 ABS   =1.0182          LAST STG M1 REL  = 0.5561        LAST STG M2 REL  =0.7135         LAST STG M2 ABS       = 0.4808
     STATOR EXIT ANGLE = 62.67          ROTOR INLET ANGLE = 32.78        ROTOR EXIT ANGLE =-48.35         STAGE EXIT ANGLE      =  9.45

        STAGE 1- 2 MEANLINE SLOPE =  14.18 DEG BASED ON MID ASPECT-RATIO BLADING
```

SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	EXIT HB DIA	EXIT RADIUS RATIO	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS COEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
0.0	53.50	2660.00	113.10	11400.00	20.00	21.00	0.0000	65.00	53.37	1.302	0.376E-04	0.350	1.200	3.660

```
        STAGES= 2                       STAGE WORK FACTOR= 1.62          REYNOLDS NO.=0.1542E+07          DIAGRAMS ARE SYMMETRICAL
        EXIT TIP DIAMETER = 27.35       EXIT TOTAL TEMP  =2079.52        STATOR EXIT ANGLE= 65.00         FIRST STAGE MEAN SPEED= 1101.57
        EXIT HUB DIAMETER = 21.00       EXIT STATIC TEMP =2026.66        STAGE EXIT ANGLE =-26.78         LAST STAGE MEAN SPEED = 1202.59
        EXIT RADIUS RATIO =0.7677       EXIT TOTAL PRESS =  34.53        ROTOR INLET ANGLE= 26.78         LAST STAGE INLET SWIRL= 1572.70
        INLET TIP DIAMETER= 24.29       EXIT STATIC PRESS=  30.90        ROTOR EXIT ANGLE =-65.00         LAST STAGE EXIT SWIRL = -370.11
        INLET HUB DIAMETER= 20.00       T-T PRESS RATIO  =   3.276       TOTAL EFFICIENCY =0.907          LAST STAGE MERID VELOC=  803.40
        INLET RADIUS RATIO=0.8233       T-S PRESS RATIO  =   3.660       STATIC EFFICIENCY=0.840          EXIT MERID MACH NUMBER= 0.3775
        SHAFT POWER       = 12992.2     SPECIFIC WORK    =  171.68
        LAST STG M1 ABS   =0.7840       LAST STG M1 REL  = 0.3711        LAST STG M2 REL  =0.8297         LAST STG M2 ABS       = 0.4156

HUB: LAST STG M1 ABS   =0.8937          LAST STG M1 REL  = 0.4852        LAST STG M2 REL  =0.7713         LAST STG M2 ABS       = 0.3981
     STATOR EXIT ANGLE = 67.95          ROTOR INLET ANGLE = 46.25        ROTOR EXIT ANGLE =-63.50         STAGE EXIT ANGLE      =-30.16
TIP: LAST STG M1 ABS   =0.7042          LAST STG M1 REL  = 0.3289        LAST STG M2 REL  =0.8626         LAST STG M2 ABS       = 0.3764
     STATOR EXIT ANGLE = 62.18          ROTOR INLET ANGLE =  2.30        ROTOR EXIT ANGLE =-66.51         STAGE EXIT ANGLE      =-24.04

        STAGE 1- 2 MEANLINE SLOPE =  14.99 DEG BASED ON MID ASPECT-RATIO BLADING
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the specified input requirements, while the other two indicate the nonexistence of a hub or tip solution satisfying free-vortex conditions. These messages are presented in this section, and their causes are discussed.

(1) INSUFFICIENT ENERGY—This message is caused by the computed turbine exit total temperature being less than zero.

It indicates that the turbine specific work requirement is greater than the energy available in the gas. Therefore, either the specified shaft power must be decreased or the specified flow must be increased.

(2) INSUFFICIENT IDEAL ENERGY—This message is caused by the computed ideal energy being more than that available

from an infinite expansion of the gas. It indicates that the computed efficiency is too low to yield a valid solution. Corrective action includes decreasing power, increasing flow, using more stages, or perhaps using a different velocity diagram.

(3) NEGATIVE TEX—This message is caused by the computed turbine exit static temperature being less than zero. It indicates a low value of turbine exit total temperature and/or a high value of turbine exit velocity. Corrective action could be the same as for message (2) or decreasing a too high value of stage exit to average meridional kinetic energy ratio (E).

(4) INSUFFICIENT EXIT AREA—This message is caused by the computed exit area being larger than that available in the turbine. Such a situation can be remedied in many ways, including increasing exit diameter, decreasing stator exit angle, increasing inlet pressure, decreasing mass flow, and increasing stage exit to average meridional kinetic energy ratio.

(5) INSUFFICIENT INLET AREA—This message is caused by the computed inlet area being larger than that available in the turbine. Corrective measures are the same as for message (4).

(6) NEGATIVE T AT STATOR HUB EXIT—This message is caused by the stator-exit hub static temperature being less than zero as a result of a very high absolute velocity. Although a non-free-vortex design (not available from this program) might not result in a similar condition, it is extreme enough to indicate that the mean-section design may require change such as a larger diameter, smaller stator angle, or more stages. This condition does not interrupt the mean section computations.

(7) NEGATIVE T AT ROTOR TIP EXIT—This message is caused by the rotor-exit tip static temperature being less than zero as a result of a very high absolute velocity. Corrective measures are the same as for message (6) above.

National Aeronautics and Space Administration
Lewis Research Center, Cleveland, Ohio 44135
August 12, 1991

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REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE February 1992	3. REPORT TYPE AND DATES COVERED Contractor Report

4. TITLE AND SUBTITLE

Computer Code for Preliminary Sizing Analysis of Axial-Flow Turbines

6. AUTHOR(S)

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5. FUNDING NUMBERS

WU–505–69–50

G NAG3-1165

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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8. PERFORMING ORGANIZATION REPORT NUMBER

E-6150

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA CR-4430

11. SUPPLEMENTARY NOTES

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12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified - Unlimited
Subject Category 07

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This mean-diameter flow analysis uses a stage-average velocity diagram as the basis for the computation of efficiency. Input design requirements include power or pressure ratio, flow rate, temperature, pressure, and rotative speed. Turbine designs are generated for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse) or for any specified stage swirl split. Exit turning vanes can be included in the design. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, flow angles, and last-stage absolute and relative Mach numbers. This report presents the analysis method and a description of the computer program input and output with sample cases. The analysis and code presented herein are modifications of those described in NASA TN-D-6702. These modifications improve modeling rigor and extend code applicability.

14. SUBJECT TERMS	15. NUMBER OF PAGES 16
Turbines; Axial flow	16. PRICE CODE A03

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT