Photon Polarization Version of the GHZ-Mermin Gedanken

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ABSTRACT

We have defined a photon polarization analog of the GHZ experiment that was initially proposed for spin-1/2 quanta. Analogs of the ket states and Pauli spin matrix operators are presented.

DISCUSSION

We have developed an explicit photon polarization version of the three-quanta GHZ experiment (Ref. 1) discussed by Mermin (Ref. 2). We define operators, eigenkets, and measurements in the two-dimensional space of photon polarization that map directly onto the Pauli spin matrix representation of a spin-1/2 system. This construction enables us to represent the GHZ experiment in terms of photon polarization measurements on a three-photon quantum state.

This photon analog of the spin-1/2 counterparts is developed by using retardation plates, which rotate the polarization of an incident photon by producing an associated change of phase. A $\lambda/4$ retardation plate with optic axis at $45°$ to an imposed X-Y coordinate system will act analogously to σ_x, up to a phase representing a polarization-independent translation. A $\lambda/4$ plate with optic axis parallel to one of the X-Y axes produces the σ_z operation, again up to an inessential phase factor. The product operation defined by both retardation plates accomplishes the operation $\sigma_y = i\sigma_z \cdot \sigma_x$.

The polarization states that represent eigenkets of σ_z are $|X\rangle$ and $|Y\rangle$, measured by using a birefringent polarization analyzer (e. g. a Wollaston prism) oriented to send each of these polarizations into a distinct direction, en route to one of two separate photomultiplier tube detectors. The polarization states that represent eigenkets of σ_x are defined similarly, to be light linearly polarized at $45°$ and $135°$ to the X-axis. These polarizations are measured by rotating the analyzer at $45°$ to its σ_z orientation, and recording which of the two phototubes generated an output pulse. Finally, the eigenkets of σ_y are found to be left- and right-circularly polarized light in this representation. These are measured (i. e. ±1 is determined) by inserting a quarter-wave plate in front of the previously-defined analyzer, to generate in one output direction light that was originally left circular polarized, and in the other direction the originally right circular polarization.

The experiment consists of first verifying that the three-photon state being studied is a +1 eigenstate of each of the operators A= $\sigma_y^1 \cdot \sigma_x^2 \cdot \sigma_x^3$, B= $\sigma_x^1 \cdot \sigma_y^2 \cdot \sigma_x^3$, and C= $\sigma_x^1 \cdot \sigma_x^2 \cdot \sigma_y^3$. A measurement of

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operator $O = A \cdot B \cdot C$ represents the decisive test between quantum mechanics and local reality theories. The quantum mechanical prediction for the observed eigenvalue of O is -1, whereas the local reality prediction can be easily shown to be $+1$.

The most difficult element of any non-gedanken experiment similar to the one presented here is the construction of the three-photon quantum state. The requisite state vector is a perfectly anti-ferromagnetic "entangled" ket proportional to
$[|X\rangle_1 |X\rangle_2 |X\rangle_3 - |Y\rangle_1 |Y\rangle_2 |Y\rangle_3]$ which is difficult to manufacture (Ref. 3) by any physical process, such as a three-photon emission of an excited species. We have proposed a method of creating this state by hand, a task not obviously possible because most operators on optical photons do not restrict themselves to the Hilbert space of any one photon exclusive of the others.

We assemble the needed three-photon ket by sending a product $|X\rangle_1 |X\rangle_2 |X\rangle_3$ of three one-photon number states (obtainable by using an attenuated Glauber state for each one) onto an array of beam splitters and interferometers (either Mach-Zehnder or Michelson will work here) to achieve a summation of correlated amplitudes at the end of the apparatus (see Figure). The total apparatus is a three-tiered structure, with one tier for each photon. For clarity, only one tier is shown in the figure. The boxed interferometers are specific to each tier; all other optical components (beamsplitters and mirrors) are common to all three tiers, a requirement that can be satisfied for sufficiently large components. The entangled state is obtained by using phases in the apparatus to cause the amplitudes of some components of the total ket to sum to zero. This cancellation of undesired "cross terms" produces the anti-ferromagnetic state needed in the GHZ test.

REFERENCES

1. Greenberger, D. M., M. Horne, and A. Zeilinger, 1989, "Going Beyond Bell's Theorem," in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, M. Kafatos, ed., Dordrecht, The Netherlands, p. 73.

2. Mermin, N. D., 1990, "What's Wrong with These Elements of Reality?," *Physics Today* (June 1990), p. 9.

3. Greenberger, D. M., M. A. Horne, and A. Zeilinger, 1990, "Bell's Theorem Without Inequalities," *Am. J. Phys.*, 58(12), p. 1131.

pump

❙ = beamsplitter

❘ = half wave plate

❘ = mirror

D2 = phototube

KDP crystal
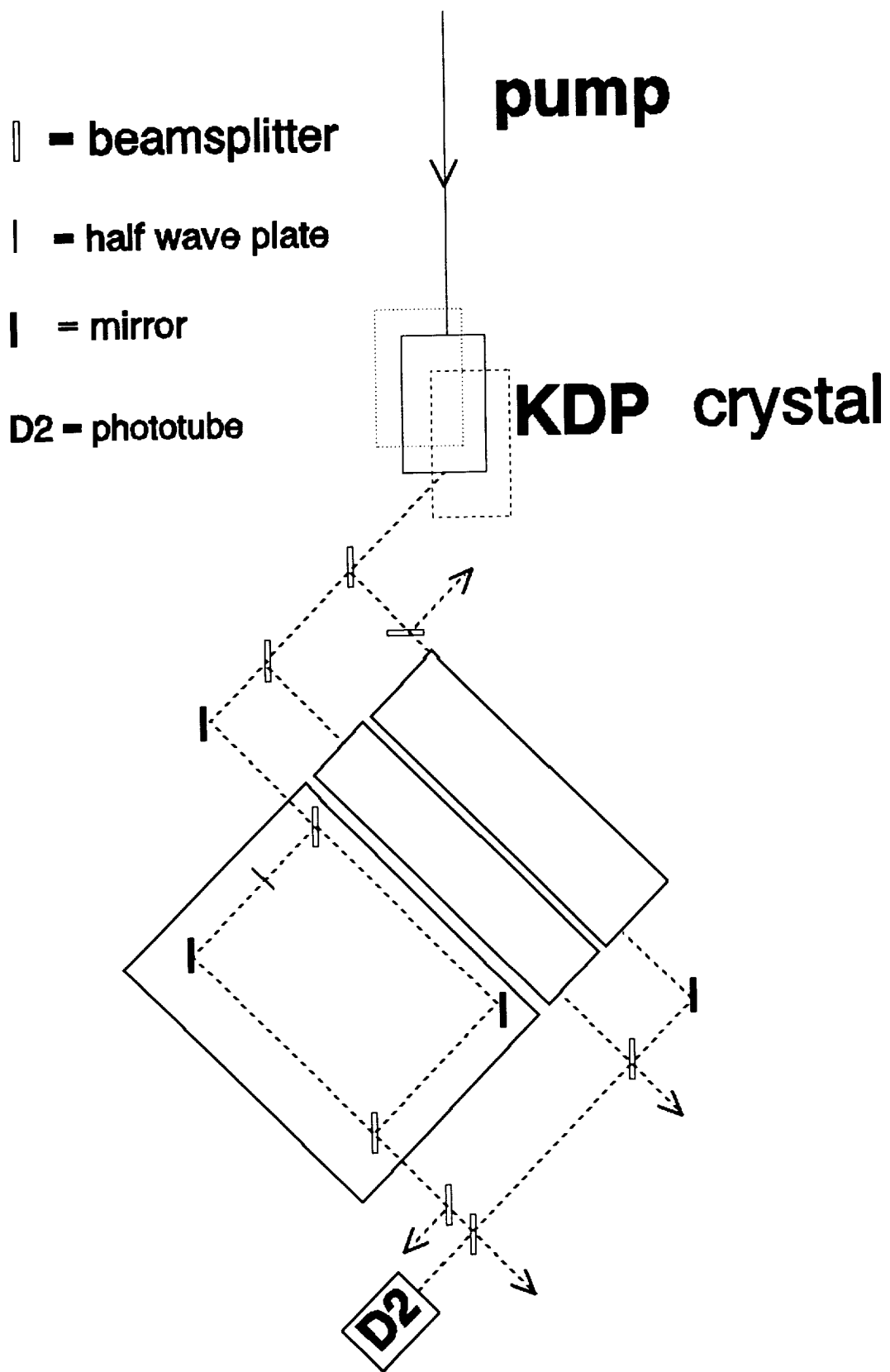
Apparatus Defining the Three-Photon State