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GENERALIZED PARAMETRIC DOWN CONVERSION, MANY PARTICLE INTERFEROMETRY, AND BELL'S THEOREM

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ABSTRACT

A new field of multi-particle interferometry is introduced using a nonlinear optical spontaneous parametric down conversion (SPDC) of a photon into more than two photons. The study of SPDC using a realistic Hamiltonian in a multi-mode shows that at least low conversion rate limit is possible. The down converted field exhibits many stronger nonclassical phenomena than the usual two photon parametric down conversion. Application of the multi-particle interferometry to a recently proposed many particle Bell's theorem on Einstein-Podolsky-Rosen problem is given.

INTRODUCTION

A two photon *spontaneous* parametric down conversion (SPDC)[1] has been known to be an effective source of highly correlated photon pairs that exhibit many interesting nonclassical properties, such as squeezed states, antibunching, violation of classical inequalities, etc. Our study, which starts with a realistic Hamiltonian not only shows that the divergence problem[2,3], which occured in the usual parametric approximation, does not occur when the pump is quantized, but also shows that the phase matching problem, in principle, doesn't prohibit the phenomena to occur.

It is possible[4] to have the phenomenon at least in the low conversion rate limit. Since we know that quantum interferometers do not require a high coversion rate (indeed we like to have only one set of photons in the entire setup at any time), we can introduce a multi-particle quantum optical interferometry in which one measures the quantum correlation properties among more than two particles. One can construct three-photon coherent state interferometers in the form of a generalized momentum-position interferometry, a generalized form of a Franson-type position-time interferometry, and a generalized polarization correlation experiments, and look for their nonclassical behaviors.

I. GENERALIZED PARAMETRIC DOWN CONVERSION

Starting with an interaction Hamiltonian for three photon SPDC in the parametric approximation which allows multiple mode down conversion from the pump with wavevector k_o and frequency ω_o:

$$H_I = \int dv \; \Sigma\Sigma\Sigma \; K \; \{a_1 a_2 a_3 e^{-i\Lambda k\cdot r + i\omega_o t} + a_1^+ a_2^+ a_3^+ e^{i\Lambda k\cdot r - i\omega_o t}\}, \qquad (1)$$

we obtain the expressions for the time development of the operators a, a^+ :

$$\dot{a}_{ka} = -i\omega_k a_{ka} - i \; \Sigma\Sigma K \; a_1^+ a_2^+ e^{-i\omega_o t} \cdot \delta(k_0 - k - k_1 - k_2) \qquad (2a)$$

$$\dot{a}^+_{ka} = i\omega_k a^+_{ka} + i \; \Sigma\Sigma K \; a_1 a_2 e^{i\omega_o t} \cdot \delta(k_0 - k - k_1 - k_2) \qquad (2b)$$

A major difference between Eqs.(2) and the equivalent two photon case is that in this case the δ function at the end of Eqs.(2) cannot eliminate the summations (or integrals, for a continuum) over k_1, k_2 unless we have a special selection mechanism such as ideal phase matching, or photon resonances, for the specific down converted frequencies.

But in any case, the equations can be solved and yield the same type of curves for the photon number, although in non-ideal phase matched cases, we have much smaller values. For example, for the 3 photon degenerate case we have

$$\ddot{N} = 18K^2 \cdot (3N^2 + 3N + 2) \quad , \text{etc.} \qquad (3)$$

Except for the two photon case, which has a well known analytic solution $N = \sinh^2 Kt$ that diverges

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at infinity, it can be shown that in all higher order cases the photon number diverges at a finite time. On the other hand, if we quantize the pump field, the interaction Hamiltonian H_{IQ} becomes

$$H_{IQ} = \int dv \sum\sum\sum\sum K_Q \{a_1 a_2 a_3 a_o^+ e^{-i\Lambda k\cdot r} + a_1^+ a_2^+ a_3^+ a_o e^{i\Lambda k\cdot r}\}, \quad (4)$$

where K_Q is a quantum pump equivalent to K in parametric approximation.

From this we have a time development of the down converted photon number for the three photon degenerate SPDC:

$$N = 18K_Q^2\{(3N^2+3N+2)N_0(t)-N^3+5N+2)\}, \quad (5)$$

where $N_0(t)$ gives the expression for the depleted pump beam and is related to the down converted beam as $\langle N_0(t)\rangle=\langle N_0(0)\rangle-\langle N(t)\rangle$. The extra term with a negative sign in Eq.(5) will slow down the change of the slope of the curve when the pump depletion becomes significant. Notice that the expression in the quantum pump reduces to the parametric approximation for $N_0(t) \gg N$. The photon number will eventually oscillate greatly for large Kt. This is true even in the case when we don't have an ideal phase matching.

II. MULTIPARTICLE INTERFEROMETRY

Two fundamental relations for a multi-photon spontaneous parametric down conversion, i.e.

$$k_0 = k_1+k_2+ \cdots +k_n, \quad (6)$$

$$\omega_0 = \omega_1+\omega_2+ \cdots +\omega_n, \quad (7)$$

along with the facts that the pump beam with k_o and ω_o is a coherent one and that the n-photon down converted state is represented by the *product* of the individual photon states, tell us that each individual down converted photon doesn't have a definite phase, while the total system carries the phase information. This n-photon correlation property opens a new field of multi-photon interferometry in which one measures the joint detection probability of n photons.

Our scheme for multi-photon interferometry starts with forming a quantum mechanically entangled state.

$$|\Psi\rangle = 2^{-1/2}\{ |+_1+_2\cdots+_n\rangle + e^{i(\Phi_1+\Phi_2+\cdots+\Phi_n)} |-_1-_2\cdots-_n\rangle \}, \quad (8)$$

where $|+_i\rangle$ and $|-_i\rangle$ refer to the two different possible states of photon i and Φ_i represents the phase difference between those two states. It is a matter of indifference whether $|+_i\rangle$ and $|-_i\rangle$ states are switched for any particle(s) i.

Now if our measurement M on the system involves off-diagonal matrix elements, i.e., a mixing of the two possible states, then the quantum mechanical expectation value $\langle\Psi| M |\Psi\rangle$ for the measurement will generally contain terms that oscillate sinusoidally with $\Phi_1+\Phi_2+\cdots+\Phi_n$. These off diagonal elements or the mixing of the states may be achieved by making use of beam splitters or a polarization analyzer whose axis lies in between the two orthogonal polarization axes. We stay with three particle systems because we would have an extremely small chance of getting a right set of correlated photons in higher order.

(1) Generalized Horne-Shimony-Zeilinger[5] interferometer: This two-photon momentum-position interferometer was implemented by Rarity and Tabster[6]. Recently, a three-particle version of the experiment was proposed by Greenberger et.al[7] to test against a family of local realism. Their gedanken three particle setup can be realized through the three photon SPDC which we described in the previous section. One would have an expectation value for the three photon joint detection that oscillates sinusoidally with $\Phi_1+\Phi_2+\Phi_3$.

(2) Generalized Franson Interferometer:

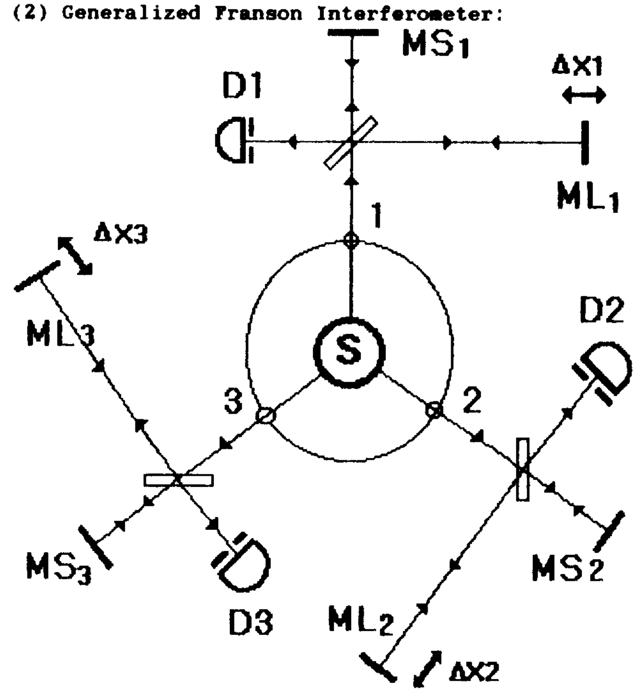

Fig.1 Three arm Franson interferometer

Franson[8] devised a two particle gedanken interferometor that uses the interference between two possible states each of which belongs to a different emission time.

The experiment was implemented by Ou et al.[9] and by Kwiat et al.[10] A generalized three-photon Franson interferometry may use the Ou et al.-type of setup with three Michelson interferometers as in Fig.1. The same analysis should go through as in the two photon case and the expectation value for the coincidence counting rate of three photons will exhibit a sinusoidal oscillation that depends upon the accumulated optical path differences between two possible paths.

(3) Polarization Interferometer: Finally, we construct a third type of entangled state formed by two orthogonal polarizations of photons for a three photon polarization interferometer. Suppose all three down converted photons are x-polarized. (one can in principle enforce this by placing x-filters after the apertures) In one set of paths (primed ones) we place half wave plates and in the other set of paths (unprimed ones) we place compensators and the variable phase shifters Φ_i. Then we combine the beams on the beam splitters so that the polarization states may be mixed before they are registered by two channel linear polarization analyzers. If we count a detection of an x-polarized photon as +1 and a y-polarized photon as -1, using two channel analyzers, then we would have a three-photon joint detection probability:[11]

$$E(\Phi_1,\Phi_2,\Phi_3) = \eta^3 \cos(\Phi_1+\Phi_2+\Phi_3). \quad (9)$$

III. BELL'S THEOREM AND MORE

In general, the many particle correlated system we discussed here is not a mere generalization of two particle correlated system. It exhibits much stronger nonclassical effects than the usual two particle correlated system through its additional degree of freedom. Some found a stronger squeezing[3] and a more prominent antibunching[12]. We found a stronger violation of classical generalized Cauchy-Schwartz inequality by a factor of (n-1)/n in a simple higher order system which can be easily generalized to other systems. We also found that in a Franson-type time-energy interferometer classical stochastic electrodynamics fails rapidly to reproduce quantum mechanical result in visibility by a factor of 1/2 for each additional order.

Finally, we saw the dramatic breakdown of local realism in many particle system due to Greenberger et.al (GHZ)[7]. It has shown that any local theories that is based on EPR type realism faces contradiction as it tries to immitate quantum mechanical results in a many particle correlated system. This theorem can be implemented by multiphoton interferometries which we discribed in Section II. Mermin[13] also has shown that the violation of Bell type inequality in a many particle system increases exponentially as it goes to a higher order. This is just an another example of a strong violation of classical limits by a many particle system through its additional quantum degree of freedom.

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