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RADIATION FORCE ON A SINGLE ATOM IN A CAVITY

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ABSTRACT

We consider the radiation pressure microscopically. Two perfectly conducting plates are parallelly placed in vacuum. As the vacuum field hits the plates they get pressure from the vacuum. The excessive outside modes of the vacuum field push the plates together, which is known as the Casimir force. We investigate the quantization of the standing wave between the plates to study the interaction between this wave and the atoms on the plates or between the plates. We show that even the vacuum field pushes the atom to place it at nodes of the standing wave.

INTRODUCTION

Casimir showed that when two perfectly conducting plates are parallelly placed in the vacuum they attract each other [1]. Although there has been considerable interest in the Casimir effect within the field of quantum electrodynamics [2a], it was Milonni et al. who introduced the concept of the radiation pressure from the vacuum field to interpret the Casimir effect [3]. When there is a field of a mode separated by a conducting plate the radiation pressure exerted by the field is same both sides. When the distance between the plates is d, the mode separation is proportional to 1/d. Since less number of modes are accommodated between the plates than outside of the plates the excessive outside modes of the vacuum field push the plates together.

Milonni et al. calculate the vacuum radiation pressure intuitively. In this paper we study the vacuum radiation pressure in a microscopical view point. We briefly review previous works on the Casimir effect followed by a classical study of radiation force exerted by standing waves.

Quantum theory has given various applicabilities of the radiation pressure. Laser trapping and cooling and isotope separation have been realized based on radiation pressure. The radiation pressure can increase, decrease or deflect atomic velocities. Different isotopes of the same atom generally have slightly different electronic transition frequencies. The radiation pressure can be tuned to deflect only one kind of isotope in a mixture. Atoms can be slowed down by radiation pressure of counter propagating laser and the velocity distribution of the atoms is narrowed. This process cools down the atomic kinetic energy. Orthogonal pairs of counter–propagating laser beams are used to trap atoms for long periods of time [2b].

We investigate quantization of standing waves between the plates and find the quantum mechanical form of the Maxwell stress tensor. We then check the principle of momentum conservation in quantum physics. When the field interacts with a two–level atom in the cavity the atom is forced to be placed at nodes of the field. We calculated the size of the vacuum force exerted on the atom.

THE CASIMIR FORCE

This section reviews the work of Milonni et al with appropriate extensions. As shown in Fig. 1, the perfectly conducting plates are parallelly placed in the vacuum. We take the z–axis normal to the plates. When the radiation field strikes the plates with the angle of incidence θ, the pressure exerted by the radiation is the force, which is projected on the plate, divided by the area of incidence. Applying Gauss's law we find that the normal component of force per area is the energy per volume of the incident field. The radiation field is totally reflected by the perfectly conducting plates. The field between the plates propagates either to or from the plate so that the photon pressure on the plate is a half the photon energy. Thus the radiation pressure from a mode of the vacuum field is

$$P = \frac{\hbar\omega}{2V}\cos^2\theta \tag{1}$$

where ω is the frequency of the mode and V the quantization volume. When the plates are large enough x– and y–components of the wavevector k take continuous values while $k_z = n\pi/d$, where d is the separation of the plates and n integer. The total outward pressure is the sum of pressure exerted by each mode.

$$P_1 = \frac{\hbar c}{\pi^2 d}\sum\int_0^\infty dk_x \int_0^\infty dk_y \frac{(n\pi/d)^2}{k} \tag{2}$$

where the magnitude of the wavevector

$$k = [k_x^2 + k_y^2 + (n\pi/d)^2]^{\frac{1}{2}}. \tag{3}$$

Considering the modes outside the plates, all the components including the z–component of the wavevector take continuous values so that the total inward pressure is

$$P_2 = \frac{\hbar c}{\pi^3}\int_0^\infty dk_x \int_0^\infty dk_y \int_0^\infty dk_z \frac{k_z^2}{k^2}. \tag{4}$$

The wave vector is now $k = [k_x^2 + k_y^2 + k_z^2]^{\frac{1}{2}}$. The difference between the inward and outward pressure is physically meaningful:

$$P_2 - P_1 = \pi\hbar c/480d^4. \tag{5}$$

The attractive force is inversely–proportional to the fourth order of the separation of the plates. This is the Casimir force in agreement with Power [4].

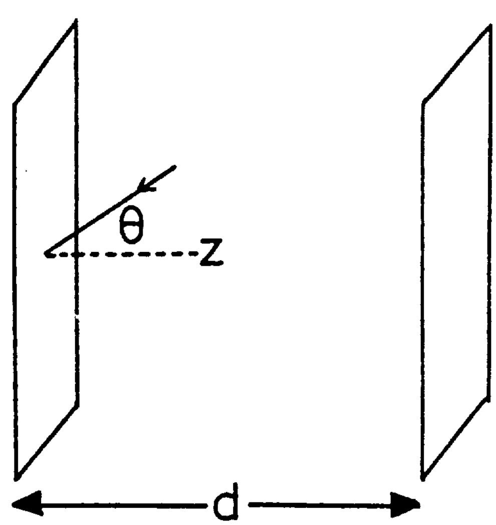

Fig.1 Electromagnetic field incident on one of two perfectly conducting plates separated by distance d.

RADIATION PRESSURE OF CLASSICAL FIELD

We consider the classical field for the quasimicroscopic study of the field–conductor interaction [5]. As shown in Fig.2 the classical electromagnetic field strikes the conducting plate with the angle of incidence θ. Taking the polarization of the electric field on the x–z plane. We write the incident electric field

$$\underline{E} = E_0(x,z,t)\cos\theta\,\underline{x} - E_0(x,z,t)\sin\theta\,\underline{z} \tag{6}$$

and the incident magnetic field

$$H = H_0(x,z,t)\,\underline{y}. \tag{7}$$

The electric field on the x–y plane of the conducting plate is zero thus the x and y components of the electric field is zero while the z–component of the total field on the plate

$$E_z = -2\,E_0(x,z{=}0,t)\sin\theta. \tag{8}$$

The total magnetic field is then

$$H_y = 2\,H_0(x,0,t). \tag{9}$$

When normal incidence is concerned, $E_z = 0$ and $H_y = 2H_0$. This is obvious from the property of the standing wave that the electric field meets the conducting plate at its node while the magnetic field sees the plate at its peak.

The z–component of the electric field attracts electric charges which causes the surface charge density

$$\epsilon_0 \cdot 2E_0 \sin\theta. \tag{10}$$

In the interior of the conductor there is certain charge flow due to the magnetic field. The current density is

$$\underline{\nabla} \times \underline{H}. \tag{11}$$

The charge and current cause the Lorentz force from the radiation field on the plate. The force per area on charges

$$P_e = -2\epsilon_0 E_0^2 \sin^2\theta \tag{12}$$

and that on current

$$P_j = \int_0^\infty dz[(\underline{\nabla} \times \underline{H}) \times \underline{B}]_z. \tag{13}$$

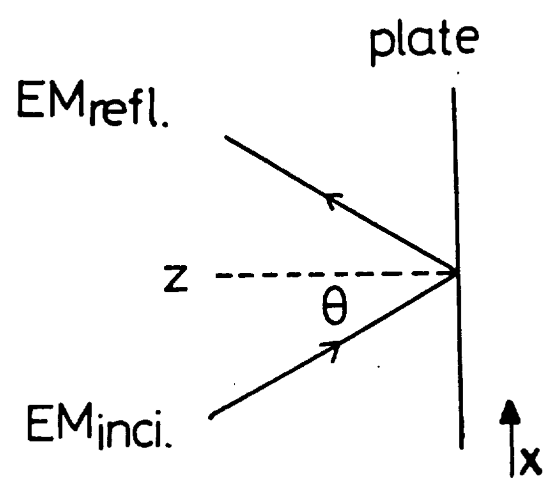

Fig.2 Electromagnetic field reflected by the conducting plate.

The intensity of the current decays exponentially so that

$$P_j = 2\mu_0 H_0^2 = 2\epsilon_0 E_0^2 \tag{14}$$

where we have used the relation between the electric and the magnetic field which is obtained from the Maxwell equations and $\epsilon_0\mu_0 = 1/c^2$. The total radiation pressure is then

$$P = P_e + P_j = 2\epsilon_0 E_0^2 \cos^2\theta. \tag{15}$$

For normal incidence, ie $\theta = 0$, the total radiation pressure is $2\epsilon_0 E_0^2$ which is solely from the magnetic field.

The radiation pressure on the conducting plate is the normal component of the Maxwell stress tensor [6]. The normal component of the Maxwell stress tensor is

$$T_{zz} = \frac{1}{2}\epsilon_0(E_x^2+E_y^2-E_z^2) + \frac{1}{2}\mu_0(H_x^2+H_y^2-H_z^2). \tag{16}$$

Substituting the total field we have in eqs.(8, 9) we find

$$T_{zz} = 2\epsilon_0 E_0^2 \cos^2\theta \tag{17}$$

which agrees with the result (15) obtained using the quasimicroscopical interaction theory.

QUANTUM MAXWELL STRESS TENSOR

To simplify the problem we consider normal incidence on the plate. The electric field polarized along the x–axis propagates through the z–axis in fig.2. If the field is a travelling wave the vector potential operator for a mode $\underline{k}$ of frequency ω is known as [7a]

$$\hat{\underline{A}} = \left[\frac{\hbar}{2\epsilon_0 V\omega}\right]^{\frac{1}{2}} \underline{i}\,(\hat{a}e^{-i\omega t+ikz} + \hat{a}^{\dagger}e^{i\omega t-ikz}) \tag{18}$$

where $\underline{i}$ is the unit vector along the x–axis and the caret is to denote the operator. The operator $\hat{a}$ and $\hat{a}^{\dagger}$ are respectively the field annihilation and creation operators. The cavity composed of the two parallel conducting plates accommodate standing waves rather than travelling waves. With a similar analogy to find the vector potential in eq.(18) we obtain the vector potential for the standing wave [7b]

$$\hat{\underline{A}} = \left[\frac{\hbar}{\epsilon_0 V\omega}\right]^{\frac{1}{2}} \underline{i}\cos kz(\hat{a}e^{-i\omega t} + \hat{a}^{\dagger}e^{i\omega t}) \tag{19}$$

where $k = n\pi/d$, $n = 1, 3, 5, \cdots$ and d is the length of the cavity. From the one–dimensional potential vector we obtain the electric $\hat{\underline{E}}$ and magnetic $\hat{\underline{B}}$ field operators as

$$\hat{\underline{E}} = -\frac{\partial A}{\partial t}\,\underline{i} = i\left[\frac{\hbar\omega}{\epsilon_0 V}\right]^{\frac{1}{2}} \underline{i}\cos kz(\hat{a}e^{-i\omega t} - \hat{a}^{\dagger}e^{i\omega t}) \tag{20}$$

and

$$\hat{\underline{B}} = \frac{\partial A}{\partial z}\,\underline{j} = -\frac{1}{c}\left[\frac{\hbar\omega}{\epsilon_0 V}\right]^{\frac{1}{2}} \underline{j}\sin kz(\hat{a}e^{-i\omega t} + \hat{a}^{\dagger}e^{i\omega t}) \tag{21}$$

From eq.(16) we may write the Maxwell stress tensor for the quantized field in the cavity.

$$\hat{T}_{zz} = \frac{1}{2}(\epsilon_0\hat{E}_x^2 + \frac{1}{\mu_0}\hat{B}_y^2). \tag{22}$$

With the use of eqs.(20, 21)

$$\hat{T}_{zz} = \frac{\hbar\omega}{2V}\{\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - (\hat{a}^\dagger\hat{a}^\dagger e^{2i\omega t} + \hat{a}\hat{a}\, e^{-2i\omega t})\cos 2kz\}. \tag{23}$$

If the number state $|n>$ resides in the cavity the radiation pressure on the conducting plates is

$$P = <\hat{T}_{zz}> = \frac{\hbar\omega}{2V}(n+1). \tag{24}$$

For the vacuum state, ie $n = 0$, the radiation pressure in eq.(24) becomes $P = \hbar\omega/2V$ which is in agreement with eq.(1) for normal incidence. When the cavity is prepared with the squeezed vacuum the radiation pressure on the cavity wall is

$$P = \frac{\hbar\omega}{2V}(\cosh 2r + \sinh 2r \cos kz \cos 2\omega t) \tag{25}$$

where r is the real squeeze parameter [8]. The second term in the curly bracket fast–oscillates around zero. Taking the time average of the radiation pressure we get

$$P = \frac{\hbar\omega}{2V}\cosh 2r \tag{26}$$

As squeezing gets severe, the radiation pressure increases exponentially. The time average radiation pressure for the coherent state of amplitude α is

$$P = \frac{\hbar\omega}{2V}(\alpha^2 + 1). \tag{27}$$

The momentum density of the electromagnetic field is proportional to the Poynting vector $\hat{\underline{S}}$. While the Poynting vector is clearly defined in classical theory as $\underline{E} \times \underline{H}$, that is not the case in quantum theory because the operator–ordering problem arises. When the Poynting vector is concerned in calculation of the intensity of a field we have the normal ordering of the field operators, ie $\hat{S} \propto \hat{a}^\dagger\hat{a}$ [7]. For the problem in hand we define the Poynting vector with the usual quantum mechanical symmetrization:

$$\hat{S}_z = \frac{1}{2}(\hat{\underline{E}} \times \hat{\underline{H}} - \hat{\underline{H}} \times \hat{\underline{E}}). \tag{28}$$

RADIATION FORCE ON AN ATOM

If a two–level atom is isolated in the cavity composed of two parallel conducting plates the atom is subject to the radiation pressure from the standing wave. To simplify the problem we assume that the field propagation is along the z–axis in Fig.2. We consider the two–level atom with ground state $|g>$ and excited state $|e>$ in interaction with the radiation field of frequency ω. The atom of total mass M has the atomic centre–of mass momentum

$$\hat{\underline{P}} = -i\hbar\underline{\nabla}. \tag{29}$$

The total Hamiltonian of the coupled system is[9]

$$\hat{H} = \hat{H}_{atom}(t) + \hat{H}_{field}(t) + \hat{\underline{P}}(t)^2/2M + \hat{H}_i(t) \tag{30}$$

where under the electric–dipole approximation the interaction Hamiltonian

$$\hat{H}_i(t) = i\hbar g \cos kz\, \{\hat{a}(t) - \hat{a}^\dagger(t)\}\, \{\hat{\pi}(t) + \hat{\pi}^\dagger(t)\} \tag{31}$$

with the coupling constant

$$g = e\left[\frac{\omega}{\hbar\epsilon_0 V}\right]^{\frac{1}{2}} \underline{\epsilon}\cdot\underline{D} \tag{32}$$

The operators $\hat{\pi}$, $\hat{\pi}^\dagger$ are the atomic transition operators.

The radiation force is $d\hat{P}/dt$. Using the Heisenberg equation of motion for the momentum operator

$$\hat{F}(t) = \frac{\partial \hat{P}(t)}{\partial t} = -\frac{i}{\hbar}[\hat{P}(t), \hat{H}] = -\frac{\partial \hat{H}_i(t)}{\partial z}. \quad (33)$$

To calculate the time–dependent interaction Hamiltonian we need find the time–dependence of operators. With the use of the Heisenberg equation of motion

$$-i\hbar\frac{\partial \hat{a}(t)}{\partial t} = -\hbar\omega g\hat{a}(t) + i\hbar g\cos kz\,\{\hat{\pi}(t) + \hat{\pi}^{\dagger}(t)\}. \quad (34)$$

The equation is formally integrated to give [7]

$$\hat{a}(t) = e^{-i\omega t}\left[\hat{a} - g\cos kz\int_0^t\{\hat{\pi}(t') + \hat{\pi}^{\dagger}(t')\}\,e^{i\omega t'}dt'\right] \quad (35)$$

where $\hat{a} = \hat{a}(0)$. Similarly for the transition operator

$$\hat{\pi}(t)=e^{-i\omega_0 t}\left[\hat{\pi}-g\cos kz\int_0^t[2\hat{\pi}^{\dagger}(t')\hat{\pi}(t')-1][\hat{a}(t')-\hat{a}^{\dagger}(t')]e^{i\omega t'}dt'\right] \quad (36)$$

where $\hat{\pi} = \hat{\pi}(0)$ and $\hbar\omega_0$ is the energy difference between the excited and the ground states. The operators $\hat{a}(t)$ and $\hat{\pi}(t)$ in eqs.(35, 36) still have integration to carry out. It is not simple to solve the iterative integral equations. For the weak coupling between the field and the atom we calculate the first–order perturbative solution

$$\hat{a}(t) = e^{-i\omega t}(\hat{a} + \alpha\hat{\pi} - \beta\hat{\pi}^{\dagger}) \quad (37)$$

$$\hat{\pi}(t)=e^{-i\omega_0 t}[\hat{\pi} + (2\hat{\pi}^{\dagger}\hat{\pi} - 1)(\xi\hat{a} - \zeta\hat{a}^{\dagger})] \quad (38)$$

where

$$\alpha = \frac{g\cos kz}{i(\omega_0 - \omega)}\{\exp[-i(\omega_0 - \omega)t] - 1\}$$

$$\beta = \frac{g\cos kz}{i(\omega_0 + \omega)}\{\exp[i(\omega_0 + \omega)t] - 1\}$$

$$\xi = -\frac{g\cos kz}{i(\omega_0 - \omega)}\{\exp[i(\omega_0 - \omega)t] - 1\}$$

$$\zeta = \frac{g\cos kz}{i(\omega_0 + \omega)}\{\exp[i(\omega_0 + \omega)t] - 1\}.$$

We are interested in the radiation force on the atom when the cavity is initially prepared with the vacuum state which is a limiting case of number states. We substitute eqs.(37, 38) and their Hermitian conjugates into eq.(31) to find the radiation force on the atom with the use of eq.(33)

$$\hat{F}(t) = -2\hbar k g^2\sin 2kz\,\{[(2\hat{\pi}^{\dagger}\hat{\pi} - 1)\hat{a}\hat{a}^{\dagger} + \hat{\pi}\hat{\pi}^{\dagger}]A + [(2\hat{\pi}^{\dagger}\hat{\pi} - 1)\hat{a}^{\dagger}\hat{a} - \hat{\pi}\hat{\pi}^{\dagger}]B\} \quad (39)$$

where the first term in the curly bracket varies slowly with the parameter

$$A \equiv \frac{1 - \cos(\omega_0 - \omega)t}{\omega_0 - \omega} \quad (40)$$

while the second term varies fast with the parameter

$$B \equiv \frac{1 - \cos(\omega_0 + \omega)t}{\omega_0 + \omega} \quad (41)$$

The second term is so called the counter–rotating term which is often neglected when the field frequency is near resonant with the atomic frequency.

If the field is initially prepared with the number state $|n\rangle$ and the atom in its ground state the radiation force is

$$\hat{F}(t) = 2\hbar k g^2 \sin 2kz \, \{nA + (n+1)B\}. \tag{42}$$

Neglecting the fast oscillating term $(n+1)B$, which is so called the rotating wave approximation, we write

$$< \hat{F}(t) > = 2n\hbar k g^2 \sin 2kz \, \frac{1 - \cos(\omega_0 - \omega)t}{\omega_0 - \omega}. \tag{43}$$

For the vacuum field, $n = 0$. While the slowly varying contribution is zero, we have the radiation force from the counter–rotating term

$$< \hat{F}(t) > = 2\hbar k g^2 \sin 2kz \, \frac{1 - \cos(\omega_0 + \omega)t}{\omega_0 + \omega}. \tag{14}$$

The time–average force is thus

$$< \hat{F} > = 2\hbar k g^2 \sin 2kz \, \frac{1}{\omega_0 + \omega.} \tag{45}$$

Eq.(47) shows that the time–average force is zero when the electric field is not only at its nodes but also at its peaks. This reflects the fact that the radiation field exerts the force proportional to the gradient of the intensity.

CONCLUSIONS

We have studied the radiation pressure from the field between the conducting plates. We quantized the standing wave and introduced the Maxwell stress tensor in the quantum mechanical form. The Maxwell stress tensor is calculated for the radiation pressure on the plates. For the special case of the vacuum field the radiation pressure shows the Casimir force which pushes the plates together.

When the atom interacts with the travelling wave the time–average radiation force on the atom is zero [10]. The standing wave on the other hand exerts force on the atom to push the atom to a node of the field. Since the force depends on the gradient of the intensity, that is the electric field squared, the force is zero at the nodes and peaks of the field.

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