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Wormholes and Negative Energy from the Gravitationally Squeezed Vacuum

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ABSTRACT

Minkowski-signature wormhole solutions of the Einstein field equations require the existence of negative energy density in the vicinity of their throats. In this note, we point out that the gravitational interaction automatically generates squeezed vacuum states of matter, which by thier nature, entail negative energy and thus provide a natural source for maintaining this class of wormholes.

1. Introduction

Wormholes are handles in the spacetime topology linking widely seperated regions of the Universe. Major insights have been made in the past few years in understanding general properties and physical consequences of Minkowski-signature wormholes [1-3]. A key aspect of wormholes discovered in [1] has to do with the type of matter and energy needed to thread the wormhole throat: it must violate the weak energy hypothesis. Although no known form of classical matter violates this energy condition, the squeezed vacuum does, and moreover, the coupling of matter to gravity leads automatically to the production of squeezed vacuum states [4]. The negative energy of the squeezed vacuum can be understood in simple terms. Consider a single mode oscillator. Its vacuum state is represented in phase space (from the Wigner distribution) by a circle centered at the origin. The squeezed vacuum state, by contrast, leads to an elliptical region. As this ellipse rotates (with the angular frequency of the mode), its periodic profile exhibits quantum fluctuations both larger and smaller than the uniform profile characteristic of the unsqueezed vacuum state. In field theory, the energy of the unsqueezed vacuum gets renormalized to zero. Thus, any state having lower fluctuations than the ordinary vacuum must have a negative (renormalized) energy.

2. Quantized scalar in a uniform gravitational field

We make these concepts explicit by showing that the interaction between matter and gravity leads to a squeeze operator acting on the Fock space of particle states, which includes the vacuum. We consider a scalar under the influence of a uniform background gravitational field. From the equivalence principle, this can be handled by transforming to a uniformly accelerating frame (i.e., Rindler space). Take the background field pointing in the x-direction. The transformation from Minkowski (t, x) to Rindler (T, X) is $x = X\cosh(T)$ and $t = X\sinh(T)$, and the scalar equation to be solved is

$$\Box\Phi + m^2\Phi = 0. \tag{1}$$

The normalized solution is given by

$$\Phi_{\mathbf{k}_\perp, j}(T, X; \mathbf{x}_\perp) = \pi^{-1}[\sinh(\pi j)]^{1/2} K_{ij}((m^2 + \mathbf{k}_\perp^2)^{1/2} X) e^{-ijT} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}, \tag{2}$$

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where K is a modified Bessel function of imaginary order, m is the scalar mass, $\mathbf{k}_\perp = (k_y, k_z)$ refers to the transverse momentum and $j \geq 0$. Contrast these Rindler modes with the familiar plane wave solutions of (1) for Minkowski space:

$$U_{\mathbf{k}} = \frac{e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}}{(2\pi)^{3/2}\sqrt{2\omega_{\mathbf{k}}}}, \tag{3}$$

where $\omega_{\mathbf{k}} = (m^2 + \mathbf{k}^2)^{1/2}$. The complete solution of (1) in Rindler or Minkowski space can be expanded in terms of the sets (2) and (3), respectively. The expansion coefficients become, after canonical quantization, operators satisfying the algebras $[\bar{a}(j,\mathbf{k}_\perp), \bar{a}^\dagger(j',\mathbf{p}_\perp)] = \delta(j,j')\delta(\mathbf{k}_\perp - \mathbf{p}_\perp)$ and $[a(\mathbf{k}), a^\dagger(\mathbf{k})] = \delta(\mathbf{k}-\mathbf{p})$ The Rindler and Minkowski vacua are defined by $\bar{a}|\bar{0}>= 0$ and $a|0>= 0$. Completeness of the two sets of modes (2) and (3) leads to nontrivial relations among the Minkowski and Rindler creation/annihilation operators:

$$a(\mathbf{k}) = \int dj\, d^2\mathbf{p}_\perp\, \alpha(j,\mathbf{p}_\perp|\mathbf{k})\bar{a}(j,\mathbf{p}_\perp) + \beta^*(j,\mathbf{p}_\perp|\mathbf{k})\bar{a}^\dagger(j,\mathbf{p}_\perp), \tag{4}$$

together with the Hermitian conjugate. The Bogolyubov coefficients in (4) are computed from the inner product and measure the overlap between the Rindler and Minkowski modes: $\alpha = (\Phi_{\mathbf{k}_\perp,j}|U_{\mathbf{p}})$ and $\beta = -(\Phi_{\mathbf{k}_\perp,j}|U^*_{\mathbf{p}})$. A most important consequence of (4) is the *inequivalence* of the vacuum states $|\bar{0}>$ and $|0>$, and the Fock spaces built up from them. This inequivalence shows up physically as squeezing.

3. The Squeezed Vacuum

The free Hamiltonian for a massive scalar in Minkowski space is

$$H = \int d^3\mathbf{k}\, \omega_{\mathbf{k}}\, a^\dagger(\mathbf{k})a(\mathbf{k}), \tag{5}$$

where $\omega_{\mathbf{k}} = (m^2 + \mathbf{k}^2)^{1/2}$. From the point of view of the Rindler modes, (5) is a quasiparticle hamiltonian, and so the canonical transformation in (4) allows one to derive the exact interaction hamiltonian acting on Rindler states. One can also start from the exact Rindler hamiltonian, which has the same form as (5) when expressed in terms of the Rindler operators, and applying (4) leads to the exact Minkowski interaction hamiltonian. Since the intermediate momentum integrations are easier to carry out in the Minkowski picture, we derive the squeeze operator for Rindler states, but the equivalence principle guarantees the existence of an identical operator (expressed in Minkowski momenta) acting on the Minkowski modes. Using (4) and performing the intermediate integrations over the Minkowski momenta yields [4] $H = H_o + H'$ where

$$H_o = \int dj\, d^2\mathbf{p}_\perp\, \omega_{\mathbf{p}_\perp} h_1(j)\, \bar{a}^\dagger(j,\mathbf{p}_\perp)\bar{a}(j,\mathbf{p}_\perp), \tag{6}$$

and

$$H' = \int dj\, dj'\, d^2\mathbf{p}_\perp\, \omega_{\mathbf{p}_\perp} \Big(h_2(j,j')[\bar{a}(j',\mathbf{p}_\perp)\bar{a}(j,-\mathbf{p}_\perp) + \bar{a}^\dagger(j',\mathbf{p}_\perp)\bar{a}^\dagger(j,-\mathbf{p}_\perp)]$$

$$+h_1(j,j')\theta(j-j') + \theta(j'-j)\bar{a}^\dagger(j,\mathbf{p}_\perp)\bar{a}(j',\mathbf{p}_\perp)\Big), \tag{7}$$

where

$$h_1(j,j') = \frac{\cosh[\pi(j+j')/2](1+(j-j')^2)^{-1}}{2\pi[\sinh(\pi j)\sinh(\pi j')]^{1/2}}, \text{ and } h_2(j,j') = \frac{-e^{-pi(j-j')/2}(1+(j+j')^2)^{-1}}{4\pi[\sinh(\pi j)\sinh(\pi j')]^{1/2}}, \tag{8}$$

are functions computed from the Bogolyubov coefficients [4]. We begin to see the operator structure characteristic of squeezing. To make this precise, consider the Schrödinger equation for a state of the scalar formulated in terms of the interaction picture. The above splitting of the Hamiltonian suggests writing the full time evolution operator as $U = U^o U'$ where $U^o(T) = e^{-iH_oT}$. The interaction Hamiltonian in the interaction picture is computed from $H'_I(T) = U^{o\dagger}(T)\,H'\,U^o(T)$ [4]. Then, the state of the scalar at any time T is simply given by

$$|\Phi(T)>_I = \mathcal{T}exp\left(-i/\hbar\int_{T_o}^{T} H'_I(T')dT'\right)|\Phi(T_o)>, \tag{9}$$

where the time evolution operator is a (multi-mode) squeeze operator, by virtue of (6) and (7). This is the main result. If we now identify the initial state with the Rindler vacuum, $|\Phi(T_o)> = |\bar{0}>$, the final state is precisely the gravitationally squeezed vacuum. Since every quantum field is equivalent to an infinite collection of (coupled) harmonic oscillators, it should come as no surprise that the evolution operator for Φ is just a multi-mode generalization of the single mode squeeze operator. If we specialize to two scalar modes having equal but opposite values of the transverse momentum and with $j = j'$, then the evolution operator in (9) reduces to

$$S(z) = exp\left(\frac{-i}{2}[z\bar{a}(j,\mathbf{p}_\perp)\bar{a}(j,-\mathbf{p}_\perp) - z^*\bar{a}^\dagger(j,\mathbf{p}_\perp)\bar{a}^\dagger(j,-\mathbf{p}_\perp)]\right), \tag{10}$$

where $z = i\frac{h_2(j)}{h_1(j)}[e^{-2i\omega\mathbf{p}_\perp h_1(j)T} - 1]$ is the squeeze parameter for these modes. Apart from the bounded T-dependent factor, we see that appreciable squeezing obtains for $j \to 0$. Expressing j back in terms of physical quantities, we have $j = \frac{4\pi r_s}{\lambda}$, where λ is the mode wavlength and r_s is the Schwarzschild radius of the equivalent gravitational mass giving rise to the constant acceleration at the point r_s [4]. Thus, the interaction between matter and gravity leads to squeezed states of matter, and these provide a natural source of negative energy for supporting wormholes in Lorentzian spacetime.

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