IN-34

87652

P-17

NASA Technical Memorandum 105663
ICOMP-92-10; CMOTT-92-06

Kolmogorov Behavior of Near-Wall Turbulence and Its Application in Turbulence Modeling

Tsan-Hsing Shih
*Institute for Computational Mechanics in Propulsion
and Center for Modeling of Turbulence and Transition
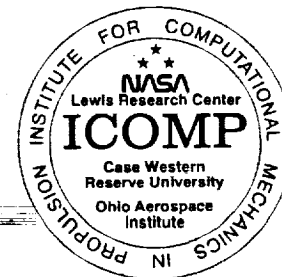Lewis Research Center
Cleveland, Ohio*

and

John L. Lumley
*Cornell University
Ithaca, New York*

INSTITUTE FOR COMPUTATIONAL MECHANICS IN PROPULSION
NASA
Lewis Research Center
ICOMP
Case Western
Reserve University
Ohio Aerospace
Institute

NASA

Kolmogorov Behavior of Near-Wall Turbulence and Its Application in Turbulence Modeling

Tsan-Hsing Shih*
Institute for Computational Mechanics in Propulsion
and Center for Modeling of Turbulence and Transition
Lewis Research Center
Cleveland, Ohio 44135

John L. Lumley
Cornell University
Ithaca, New York 14853

Abstract

The near-wall behavior of turbulence is re-examined in a way different from that proposed by Hanjalic and Launder[1] and followers[2],[3],[4],[5]. It is shown that at a certain distance from the wall, all energetic large eddies will reduce to *Kolmogorov* eddies (the smallest eddies in turbulence). All the important wall parameters, such as friction velocity, viscous length scale, and mean strain rate at the wall, are characterized by *Kolmogorov* microscales. According to this *Kolmogorov* behavior of near-wall turbulence, the turbulence quantities, such as turbulent kinetic energy, dissipation rate, etc. at the location where the large eddies become "*Kolmogorov*" eddies, can be estimated by using both direct numerical simulation (DNS) data and asymptotic analysis of near-wall turbulence. This information will provide useful boundary conditions for the turbulent transport equations. As an example, the concept is incorporated in the standard k-ε model which is then applied to channel and boundary layer flows. Using appropriate boundary conditions (based on *Kolmogorov* behavior of near-wall turbulence), there is no need for any wall-modification to the k-ε equations (including model constants). Results compare very well with the DNS and experimental data.

1. *Kolmogorov* behavior of near-wall turbulence

It is well known that at a sufficiently high Reynolds number, $R_{e\tau}$, there are following relations for the near-wall turbulence in a channel flow:

$$-\overline{uv}^+ + \frac{dU^+}{dy^+} = 1 \tag{1}$$

In the inertial sublayer ($y^+ > 30$),

$$U^+ = \frac{1}{k} \log y^+ + C \tag{2}$$

$$-\overline{uv}^+ \approx 1 \tag{3}$$

In the viscous sublayer ($y^+ < 5$),

$$U^+ = y^+ \tag{4}$$

$$-\overline{uv}^+ \approx 0 \tag{5}$$

1

where

$$U^+ \equiv \frac{U}{u_\tau}, \quad \overline{uv}^+ \equiv \frac{\overline{uv}}{u_\tau^2}$$

$$y^+ \equiv \frac{u_\tau y}{\nu} \quad R_{e\tau} \equiv \frac{u_\tau h}{\nu}$$

and $k = 0.4$ is von Kármán's constant, u_τ is the friction velocity, h is the channel half width, y is the distance from the wall, U and $\overline{uv}$ are the mean velocity and turbulent shear stress respectively. The *Kolmogorov length, velocity and time microscales* of turbulence are

$$\eta = (\frac{\nu^3}{\varepsilon})^{\frac{1}{4}}, \quad v = (\nu\varepsilon)^{\frac{1}{4}}, \quad \tau = (\frac{\nu}{\varepsilon})^{\frac{1}{2}} \tag{6}$$

where ν is the kinematic viscosity, and ε is the dissipation rate of turbulent kinetic energy. Equations (1)–(6) will be used intensively in analyzing the near-wall behavior of turbulence.

Now let us show that the important wall parameters, such as the friction velocity u_τ, the viscous length scale ν/u_τ, and the mean strain rate at the wall, are actually characterized by *Kolmogorov microscales*. We may use Eq.(6) to write,

$$\frac{u_\tau \eta}{\nu} = \frac{u_\tau}{\nu}(\frac{\nu^3}{\varepsilon})^{\frac{1}{4}} = (\frac{1}{\varepsilon^+})^{\frac{1}{4}}$$

where $\varepsilon^+ \equiv \varepsilon/\frac{u_\tau^4}{\nu}$. According to the direct numerical simulation data[6],[7],[8], the peak value of ε^+ near the wall increases slowly from 0.165 to 0.251 with increasing Reynolds number (based on the momentum thickness), and tends to an asymptotic value (see Fig.1). These DNS data show that $\frac{u_\tau \eta}{\nu}$ is of order one. If we take 0.251 as the asymptotic value of ε^+ at high Reynolds numbers, the value of $\frac{u_\tau \eta}{\nu}$ is about 1.413. This means that very near the wall

$$u_\tau \approx 1.413\, v \tag{7}$$

because $v\eta/\nu$ is always one. Equation (7) can be also shown in the following way:

$$v = (\nu\varepsilon)^{\frac{1}{4}} \approx (\nu\frac{u_\tau^3}{ky})^{\frac{1}{4}} = (\frac{1}{ky^+})^{\frac{1}{4}}u_\tau \tag{8}$$

where we have estimated ε with u_τ^3/ky, which is accurate in the inertial sublayer, and may still give the order of magnitude of ε further toward (but not at) the wall. Eq.(8) shows that v is indeed of order u_τ near the wall where ky^+ is of order 1.

The viscous length is defined as ν/u_τ. If u_τ is estimated with v, we obtain

$$\frac{\nu}{u_\tau} \approx \frac{\nu}{v} = \frac{\nu}{(\nu\varepsilon)^{\frac{1}{4}}} = \eta \tag{9}$$

The mean strain rate at the wall is of order $\partial U/\partial y$. Using Eq.(4), we obtain

$$\frac{\partial U}{\partial y} = \frac{u_\tau^2}{\nu} \approx \frac{v^2}{\nu} = \frac{(\nu\varepsilon)^{\frac{1}{2}}}{\nu} = \frac{1}{\tau} \tag{10}$$

2

Equations (7)–(10) show that near the wall all important physical parameters of turbulence are characterized by *Kolmogorov microscales*. We shall see later that there is a narrow region where the distance from the wall is smaller than the *Kolmogorov* length scale. In that region, the turbulence can not be self-sustained because of the overwhelming viscous action, and the flow is dominated by the viscous stress. Therefore, it is not important that what is the behavior of turbulence within that region ($y < \eta$). The important thing is the behavior of turbulence above that region. It is important to realize that approaching the wall, the turbulence is going to be described by *Kolmogorov microscales*, which are finite, and non-zero quantities. We will refer to this asymptotic property as *Kolmogorov behavior* of near-wall turbulence.

2. Near-wall asymptotic turbulence quantities

To study the near-wall behavior of turbulence, one needs first to find where (or at what distance from the wall) the size of energetic large eddies will reduce to *Kolmogorov microscale*, and second to estimate the turbulent quantities, such as the turbulent kinetic energy, the dissipation rate, etc., at that position.

Let y_η indicate the position where the integral scale of turbulence ℓ equals the *Kolmogorov* length scale η. We examine the variations of these two scales as the wall is approached from the inertial sublayer. The size of energetic large eddies, or the integral scale of turbulence is of order y. In the inertial sublayer, it can be estimated as (see Tennekes & Lumley[9])

$$\ell \approx ky \tag{11}$$

For the *Kolmogorov* length scale η, if we estimate ε with u_τ^3/ky (which is quite accurate in the inertial sublayer), we obtain

$$\eta \approx \left(\frac{\nu^3 ky}{u_\tau^3}\right)^{\frac{1}{4}} \tag{12}$$

Equations (11) and (12) show that both ℓ and η decrease toward the wall, but at different rates. The integral scale ℓ decreases much faster than η does. We may imagine that at some point near the wall, ℓ will approach η. We refer to this point as the limit point of turbulence, because passing beyond this point toward the wall, the turbulence is no longer important. Now if we use Eq.s (11) and (12), and let ℓ equal η, we obtain

$$\left(k\frac{u_\tau y}{\nu}\right)^{\frac{3}{4}} = 1 \tag{13}$$

Equation (13) indicates that the limit point of turbulence is

$$y_\eta^+ \approx \frac{1}{k} = 2.5$$

The numerical value of y_η^+ here should not be taken too seriously. It is an order of magnitude value, because we have assumed $\varepsilon = u_\tau^3/ky$ which is accurate only in the inertial

3

sublayer, and also this value can be changed by the point we pick for the viscous cutoff in the spectrum. It is reasonable to choose the value at the top of the viscous sublayer:

$$y_\eta^+ \approx 6 \qquad (14)$$

In the region where $y < y_\eta$, the self-sustaining turbulence cannot exist. Here we define turbulence to be a self-sustaining chaotic motion of the fluid, dominated by inertia, so that the scale of the dissipation is at least somewhat smaller than the scale of the energy containing eddies. As viscous effects become stronger, and the two scales approach each other, a point comes at which the chaotic motion can no longer sustain itself, but must depend on energy imported from neighboring regions. The chaotic disturbances of the viscous sublayer are of this nature; by our definition they are not turbulent, although surely chaotic, since they depend on the truly turbulent motions in the buffer layer and beyond for their energy.

The physical importance of Eq.(14) is that there exists a turbulence limit point y_η which is not zero. To study the asymptotic behavior of turbulence near the wall is to find the behavior of turbulence at this turbulence limit point.

Now let us estimate the turbulence quantities at the point y_η. First, let us look at the dissipation rate at that point, ε_η. In the inertial sublayer, ε is well represented by u_τ^3/ky or $(u_\tau^4/\nu)(1/ky^+)$ which shows that the dissipation rate increases toward the wall. If we extrapolate this relation to the limit point y_η, we obtain that ε_η is of order u_τ^4/ν. On the other hand, if we use Eq.(7) at y_η,

$$v_\eta = \frac{u_\tau}{1.413} = (\nu\varepsilon_\eta)^{\frac{1}{4}} \qquad (15)$$

we obtain

$$\varepsilon_\eta = 0.251 \frac{u_\tau^4}{\nu} \qquad (16)$$

This means that according to the DNS data, ε_η is indeed of order u_τ^4/ν. Eq.(16) gives the peak value of the dissipation rate near the wall.

At the turbulence limit point, the characteristic velocity is v_η, therefore the turbulent kinetic energy K at y_η can naturally be estimated as

$$K_\eta = \frac{1}{2}v_\eta^2 = \frac{1}{2}(\frac{u_\tau}{1.413})^2 = 0.250u_\tau^2 \qquad (17)$$

For the turbulent shear stress $-\overline{uv}$, if Eq.(1) and (4) are used at y_η, we obtain

$$-(\overline{uv})_\eta = 0 \qquad (18)$$

4

Other shear stresses should also vanish at y_η. Finally, the energy components $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ are not equal because of the anisotropy of turbulence near the wall. If we use the experimental data[9] in the inertial sublayer to estimate the anisotropy at y_η, i.e.

$$\frac{\overline{u^2}}{\overline{v^2}} = a \approx 6.250$$

$$\frac{\overline{w^2}}{\overline{v^2}} = b \approx 3.625$$

(19)

and $\overline{u^2} + \overline{v^2} + \overline{w^2} = 2K$, we obtain

$$\overline{u^2} = \frac{a}{a + b + 1} 2K$$

$$\overline{v^2} = \frac{1}{a + b + 1} 2K$$

$$\overline{w^2} = \frac{b}{a + b + 1} 2K$$

(20)

Using Eq.(17), we may estimate these energy components at y_η:

$$(\overline{u^2})_\eta \approx 0.287 u_\tau^2$$

$$(\overline{v^2})_\eta \approx 0.046 u_\tau^2$$

$$(\overline{w^2})_\eta \approx 0.167 u_\tau^2$$

(21)

Equations (14), (16), (17), (18), and (21) represent the near-wall asymptotic behavior of turbulence. These equations provide useful information about the boundary conditions for turbulent transport equations such as K-ε equations, Reynolds stress equations, etc. These ideas can be used at any level of turbulence modeling. In the next two sections, we illustrate how they can be very easily incorporated into the eddy viscosity K-ε models.

3. Eddy viscosity

Eddy viscosity (for incompressible flows) is defined as

$$-\overline{u_i u_j} = \nu_T(U_{i,j} + U_{j,i}) - \frac{1}{3}\delta_{ij}\overline{u_k u_k}$$

(22)

For a two-dimensional channel flow, it becomes

$$-\overline{uv} = \nu_T \frac{dU}{dy}$$

(23)

Note that Equation (1) is valid for any finite y^+ (but not for $y^+ \to \infty$ at $R_{e\tau} \to \infty$), therefore we may use Equation (1) to study the behavior of the eddy viscosity ν_T. If we approximate dU/dy with u_τ/ky, and use Eqs. (1) and (23), we obtain

$$\nu_T = (ky^+ - 1)\,\nu$$

(24)

5

This equation shows that the eddy viscosity is $ku_\tau y$ for a large y^+. Now if we model ν_T as

$$\nu_T = C_\mu f_\mu \frac{K^2}{\varepsilon} \tag{25}$$

and use experimental data to estimate $K \approx 3.5u_\tau^2$ in the inertial sublayer, and $\varepsilon \approx u_\tau^3/ky$, we obtain

$$\nu_T = 12.25 \ C_\mu f_\mu ky^+\nu \tag{26}$$

Comparing (24) and (26), we obtain

$$f_\mu = 1 - \frac{1}{ky^+}$$
$$C_\mu = \frac{1}{12.25} \approx 0.082 \tag{27}$$

Eq.(27) shows that the damping function $f_\mu \approx 1$ when $y^+ \geq 30$, which means that the standard $K - \varepsilon$ eddy viscosity model is reasonable in the inertial sublayer, and a significant damping is needed only below $y^+ \leq 30$, which corresponds to the buffer layer. However, we do not expect that the form of (27) will be appropriate for the buffer layer region, because the assumptions made in these relations are restricted to a large y^+. Unfortunately, we are not able to find similar analytical expressions like Eqs.(24), (26) in the buffer layer to form the damping function like Eq.(27). We must then depend on the experimental or DNS data to propose appropriate form for C_μ and f_μ to fit Eqs. (23) and (25). Based on the work of Yang and Shih [10], we propose

$$C_\mu = 0.09$$
$$f_\mu = [1 - \exp(a_1 R_k + a_3 R_k^3 + a_5 R_k^5)]^{\frac{1}{2}}$$
$$a_1 = -1.5 * 10^{-4} \quad a_3 = -1.0 * 10^{-9} \quad a_5 = -5.0 * 10^{-10} \tag{28}$$
$$R_k = \frac{K^{1/2}y}{\nu}$$

This is no more than a curve fitting based on the solution of DNS of channel flow with $R_{e\tau} = 180$. Fortunately, it also works quite well together with the K-ε equations (described in the next section) for other cases (see section 5). We choose R_k in Eq.(28) instead of y^+ in order to avoid the unphysical behavior of f_μ near the flow separation or reattachment points where the friction velocity u_τ is zero.

4. K-ε equations

We may use standard k-ε model equations for the eddy viscosity model (25), and use Eq.(22) to calculate turbulent mean flows. The boundary conditions can be obtained from Eqs. (14), (16), and (17). The K-ε equations for incompressible flows can be in general

modeled as (see Appendix 1 for the derivation of ε equation):

$$\frac{DK}{Dt} = [(\nu + \frac{\nu_T}{\sigma_k})K_{,i}]_{,i} + \nu_T U_{i,j}(U_{i,j} + U_{j,i}) - \varepsilon \qquad (29)$$

$$\frac{D\varepsilon}{Dt} = [(\nu + \frac{\nu_T}{\sigma_\epsilon})\varepsilon_{,i}]_{,i} + C_1 f_1 \nu_T U_{i,j}(U_{i,j} + U_{j,i})\frac{\varepsilon}{K}$$

$$- C_2 f_2 \frac{\varepsilon^2}{K} + \nu \nu_T U_{i,jk}U_{i,jk} \qquad (30)$$

These equations are used only for the flow field outside of the turbulence limit point y_η, where K_η is non-zero. Therefore, Eq.(30) will not have singularity problems and will not need any near-wall modifications like other K-ε models do.[11],[12]

As examples, we write the equations and the boundary conditions for both the two-dimensional channel and boundary layer flows.

4.1 Channel flows

For channel flows, all the quantities are normalized by the friction velocity u_τ, and the half width of the channel h, and ν_T is normalized by ν.

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial y}[\frac{1}{R_{e\tau}}(1 + \nu_T)\frac{\partial U}{\partial y}] + 1 \qquad (31)$$

$$\frac{\partial K}{\partial t} = \frac{\partial}{\partial y}[\frac{1}{R_{e\tau}}(1 + \frac{\nu_T}{\sigma_k})\frac{\partial K}{\partial y}] + \frac{1}{R_{e\tau}}\nu_T(\frac{\partial U}{\partial y})^2 - \varepsilon \qquad (32)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial y}[\frac{1}{R_{e\tau}}(1 + \frac{\nu_T}{\sigma_\epsilon})\frac{\partial \varepsilon}{\partial y}] + C_1 f_1 \frac{1}{R_{e\tau}}\nu_T(\frac{\partial U}{\partial y})^2\frac{\varepsilon}{K} - C_2 f_2 \frac{\varepsilon^2}{K} + \frac{1}{R_{e\tau}^2}\nu_T \frac{\partial^2 U}{\partial y^2}^2 \qquad (33)$$

where $R_{e\tau}$ is a known parameter for channel flows. The boundary conditions for the mean velocity are straightforward:

$$y = 0: \quad U = 0, \qquad y = 1: \quad \frac{\partial U}{\partial y} = 0 \qquad (34)$$

The boundary conditions for K and ε must be given at y_η. In the normalized form, $y = y^+/R_{e\tau}$. Using Eq.(14), (16) and (17) we obtain

$$y = y_\eta = \frac{6}{R_{e\tau}}: \quad \varepsilon_\eta = 0.251 R_{e\tau}, \ K_\eta = 0.25$$

$$y = 1: \quad \frac{\partial \varepsilon}{\partial y} = 0, \ \frac{\partial K}{\partial y} = 0 \qquad (35)$$

In solving Eq.(31), the eddy viscosity ν_T is negligible when $y \leq y_\eta$.

4.2 Boundary layer flows

For a turbulent boundary layer flow, all the quantities are normalized by the free stream velocity U_∞, and the length scale L. ν_T is normalized by ν, and the pressure P is normalized by ρU_∞^2.

$$\frac{DU}{Dt} = \frac{\partial}{\partial y}[\frac{1}{R_\infty}(1+\nu_T)\frac{\partial U}{\partial y}] - \frac{\partial P}{\partial x} \tag{36}$$

$$\frac{DK}{Dt} = \frac{\partial}{\partial y}[\frac{1}{R_\infty}(1+\frac{\nu_T}{\sigma_k})\frac{\partial K}{\partial y}] + \frac{1}{R_\infty}\nu_T(\frac{\partial U}{\partial y})^2 - \varepsilon \tag{37}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial y}[\frac{1}{R_\infty}(1+\frac{\nu_T}{\sigma_\epsilon})\frac{\partial \varepsilon}{\partial y}] + C_1 f_1 \frac{1}{R_\infty}\nu_T(\frac{\partial U}{\partial y})^2\frac{\varepsilon}{K} - C_2 f_2 \frac{\varepsilon^2}{K} + \frac{1}{R_\infty^2}\nu_T\frac{\partial^2 U}{\partial y^2}^2 \tag{38}$$

Boundary conditions for the mean velocity are:

$$y = 0: \quad U = 0, \qquad y = \infty: \quad \frac{\partial U}{\partial y} = 0 \tag{39}$$

The boundary conditions for K and ε are given at y_η. In the normalized form, $y = y^+/(R_\infty u_\tau)$. Using Eq.(14), (16) and (17) we obtain

$$y = y_\eta = \frac{6}{R_\infty u_\tau}: \quad \varepsilon_\eta = 0.251 R_\infty u_\tau^4, \; K_\eta = 0.25 u_\tau^2$$
$$y = \infty: \quad \frac{\partial \varepsilon}{\partial y} = 0, \; \frac{\partial K}{\partial y} = 0 \tag{40}$$

where the friction velocity is calculated from the solution of the mean velocity.

$$u_\tau = [\frac{1}{R_\infty}\frac{\partial U}{\partial y}]^{\frac{1}{2}}_{y=0} \tag{41}$$

In solving Eq.(36), the eddy viscosity ν_T is zero when $y \leq y_\eta$.

In practical applications, $R_{e\tau}$ and $R_{e\infty}$ are large numbers, hence y_η is usually very small. Therefore, as an approximation we may let $y_\eta = 0$, but ε_η and K_η must be given by Eqs.(35) and (40) respectively. These equations have been applied to the calculations shown in the next section.

5. Comparison of models

To compare the present model with the DNS data and other models (e.g. Jones and Launder[11], and Chien[12]), we have made calculations on two channel flows[6],[7] and two boundary layer flows[8],[15]. In the present model, all the model constants are the same as used in the standard K-ε model[13]. Therefore the present model will also be suitable for flows far from the wall. The other two models used here for comparison do not have

8

this property. Results are shown in figures 2 – 5. In figure 2 and figure 3, three models are compared with two DNS data for channel flows: one with $R_{e\tau} = 180$, the other with $R_{e\tau} = 395$. The profiles of mean velocity, Reynolds stress, turbulent kinetic energy and its dissipation rate are plotted in these figures. The present model is significantly better than the other two models. Figure 4 shows the similar comparison for a turbulent boundary with $R_{e\theta} = 1410$. The agreement between the present model and DNS data is excellent. Figure 5 shows the results compared with Klebanoff[15] and other boundary layer experiments. The skin friction from DNS data[8] is also shown in this figure. The results of present model are more consistent with the DNS data than the experiments.

It is also worthwhile to emphasize that the present model equations with the standard model coefficients have the simplest form among all two-equation models. Hence, we expect that they will have less numerical stiffness in complex turbulent flows.

Appendix 1. K-ε equations

The equation for the dissipation rate of turbulent kinetic energy ε is proposed following Lumley[14]:

$$\frac{D\varepsilon}{Dt} = [(\nu + \frac{\nu_T}{\sigma_\epsilon})\varepsilon_{,i}]_{,i} - \frac{\varepsilon^2}{K}\Psi \qquad (A1)$$

where Ψ stands for the entire mechanism of the production and destruction of the dissipation rate ε. At the level of the K-ε model, we assume that Ψ is a function of $\nu, \nu_T, K, \varepsilon, U_{i,j}$, and $U_{i,jk}$. Because Ψ is an invariant, it must be a function of the invariants that can be constructed from these quantities: $R_t, \nu_T U_{i,j} U_{i,j}/\varepsilon$, and $\nu \nu_T U_{i,jk} U_{i,jk}/\varepsilon^2$, where R_t is the turbulent Reynolds number $\frac{K^2}{\nu\varepsilon}$. We now expand Ψ in a Taylor series about these invariants and keep only the linear terms to obtain

$$\Psi = \psi_0 + \psi_1 \frac{\nu_T U_{i,j} U_{i,j}}{\varepsilon} + \psi_2 \nu \nu_T U_{i,jk} U_{i,jk} \frac{K}{\varepsilon^2} \qquad (A2)$$

where the coefficients ψ_0, ψ_1 and ψ_2 are in general function of R_t. Inserting (A2) into (A1), we obtain

$$\frac{D\varepsilon}{Dt} = [(\nu + \frac{\nu_T}{\sigma_\epsilon})\varepsilon_{,i}]_{,i} + C_1 f_1 \nu_T U_{i,j}(U_{i,j} + U_{j,i})\frac{\varepsilon}{K}$$
$$- C_2 f_2 \frac{\varepsilon^2}{K} + \nu \nu_T U_{i,jk} U_{i,jk} \qquad (A3)$$

where C_1, C_2 are model constants, and f_1, f_2 are in general functions of R_t, and $\psi_2 = -1$. The forms of C_1, C_2 and f_1, f_2 are chosen to be the same forms as used in the standard K-ε model:

$$C_1 = 1.44, \quad C_2 = 1.92$$

$$f_1 = 1, \quad f_2 = 1 - 0.22\exp{(-\frac{R_t^2}{36})} \qquad (A4)$$

$$\sigma_k = 1, \quad \sigma_\epsilon = 1.3$$

References

[1] Hanjalic, K. and Launder, B.E., "Contribution Towards a Reynolds-stress Closure for Low–Reynolds–Number Turbulence," *J. Fluid Mech.* 74. April 1976, pp. 593–619.

[2] Patel, V.C., Rodi, W. and Scheuerer, G., "Turbulence models for near–wall and low–Reynolds–number flows: A review," *AIAA Journal*, 23, 1985, pp. 1308–1319.

[3] Mansour, N.N., Kim,J. and Moin. P., "Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow," *J. Fluid Mech.*, Vol. 194, 1988, pp. 15–44.

[4] Lai, Y.G. and So, R.M.C., "On near-wall turbulent flow modeling," *J. Fluid Mech.* (1990), vol.221, pp.641-673.

[5] Shih, T. -H., "An Improved $k - \epsilon$ Model for Near-Wall Turbulence and Comparison with Direct Numerical Simulation," NASA TM-103221, August 1990.

[6] Kim, J., Moin, P. and Moser, R., "Turbulent Statistics in fully Developed Channel Flow at Low Reynolds Number," Journal of Fluid Mechanics, 177, pp. 133-166, 1987.

[7] Kim, J., Personal communication, 1992.

[8] Spalart, P. R., "Direct Simulation of a Turbulent Boundary Layer up to $Re_\theta = 1410$," Journal of Fluid Mechanics, Vol. 187, pp. 61-98, 1988.

[9] Tennekes, H. and Lumley, J.L., A First Course in Turbulence, *The MIT Press.*

[10] Yang, Z. and Shih, T. -H., "A $k - \epsilon$ Modeling of Near Wall Turbulence,"Proceedings of 4th International Symposium on Computational Fluid Dynamics, UC Davis, 1991.

[11] Jones, W. P. and Launder, B. E., "The Calculation of Low-Reynolds Number Phenomena with a Two-Equation Model of Turbulence, "International Journal of Heat and Mass Transfer, Vol. 16, pp. 1119-1130, 1973.

[12] Chien, K.-Y., "Predictions of Channel and Boundary-Layer Flows with a Low Reynolds Number Turbulence Model," AIAA Journal, Vol. 20, pp. 33-38, Jan 1982.

[13] Launder, B.E. and Spalding, D.B., "Computer Methods in Applied Mechanics and Engineering, **3**, 269 (1974).

[14] Lumley, J.L., "Computational modeling of turbulent flows," *Adv. Appl. Mech.* 1978, 18, 123.

[15] Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," NACA-TN-3178, 1954.
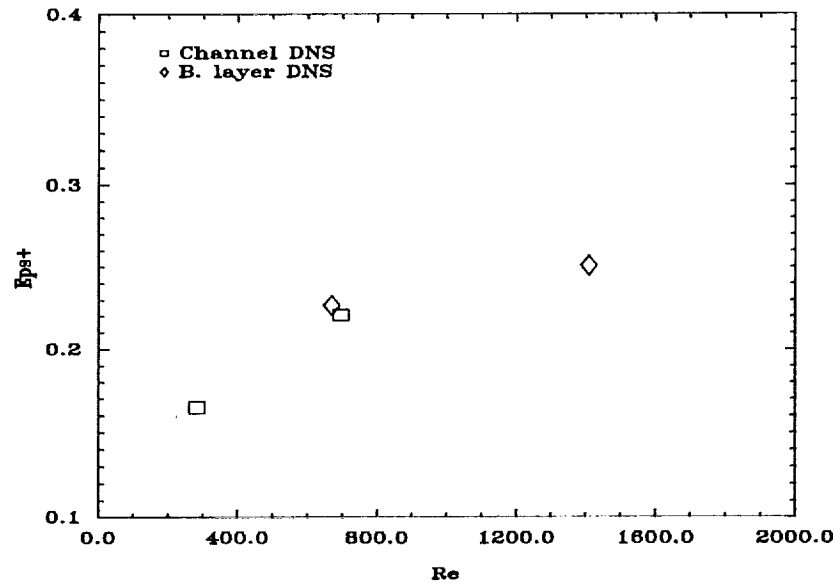
Figure 1. The asymptotic value of the dissipation rate at high $R_{e\theta}$.
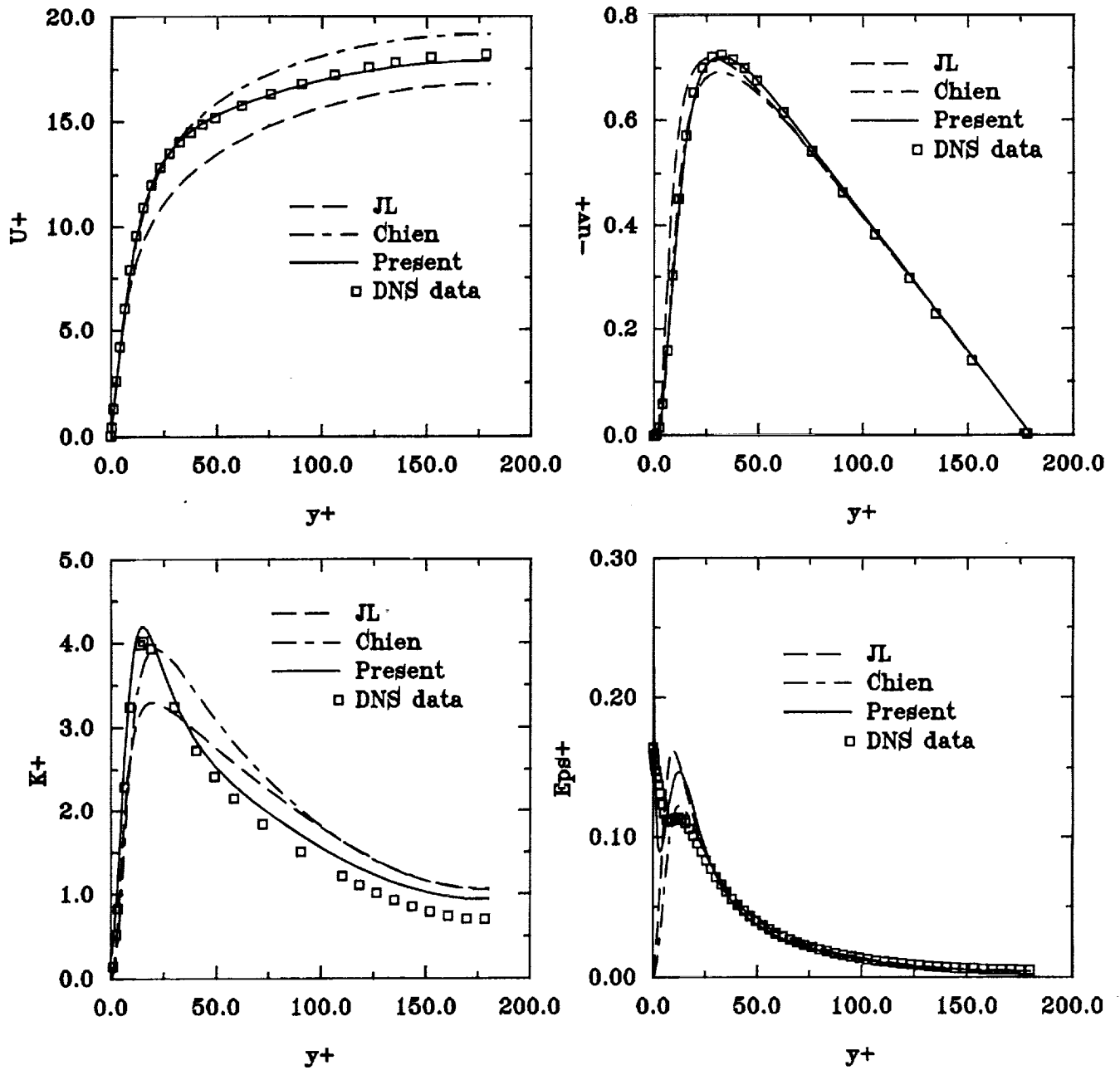
Figure 2. Comparison of models with DNS of 2-D channel with $R_{e\tau} = 180$.
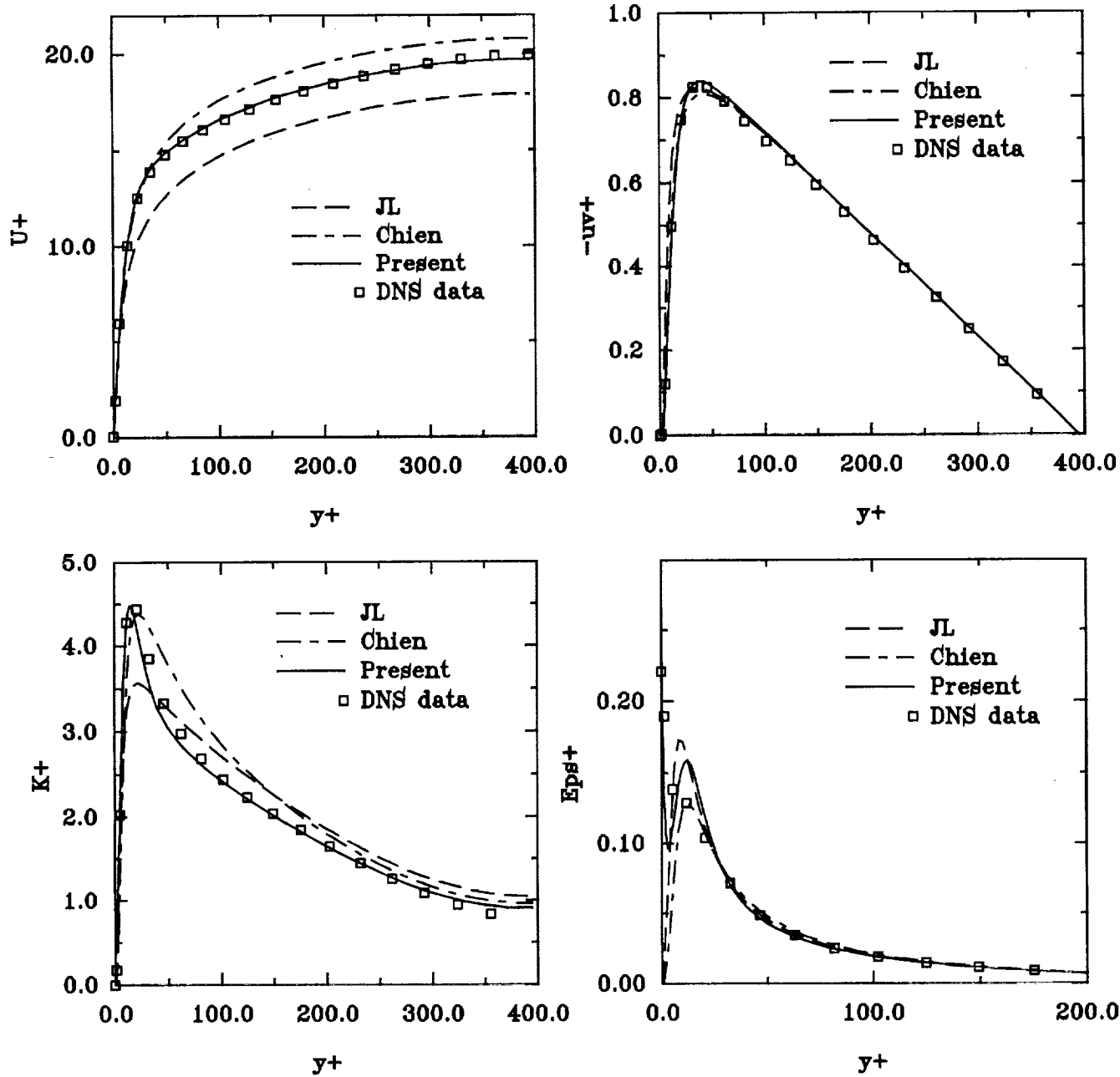
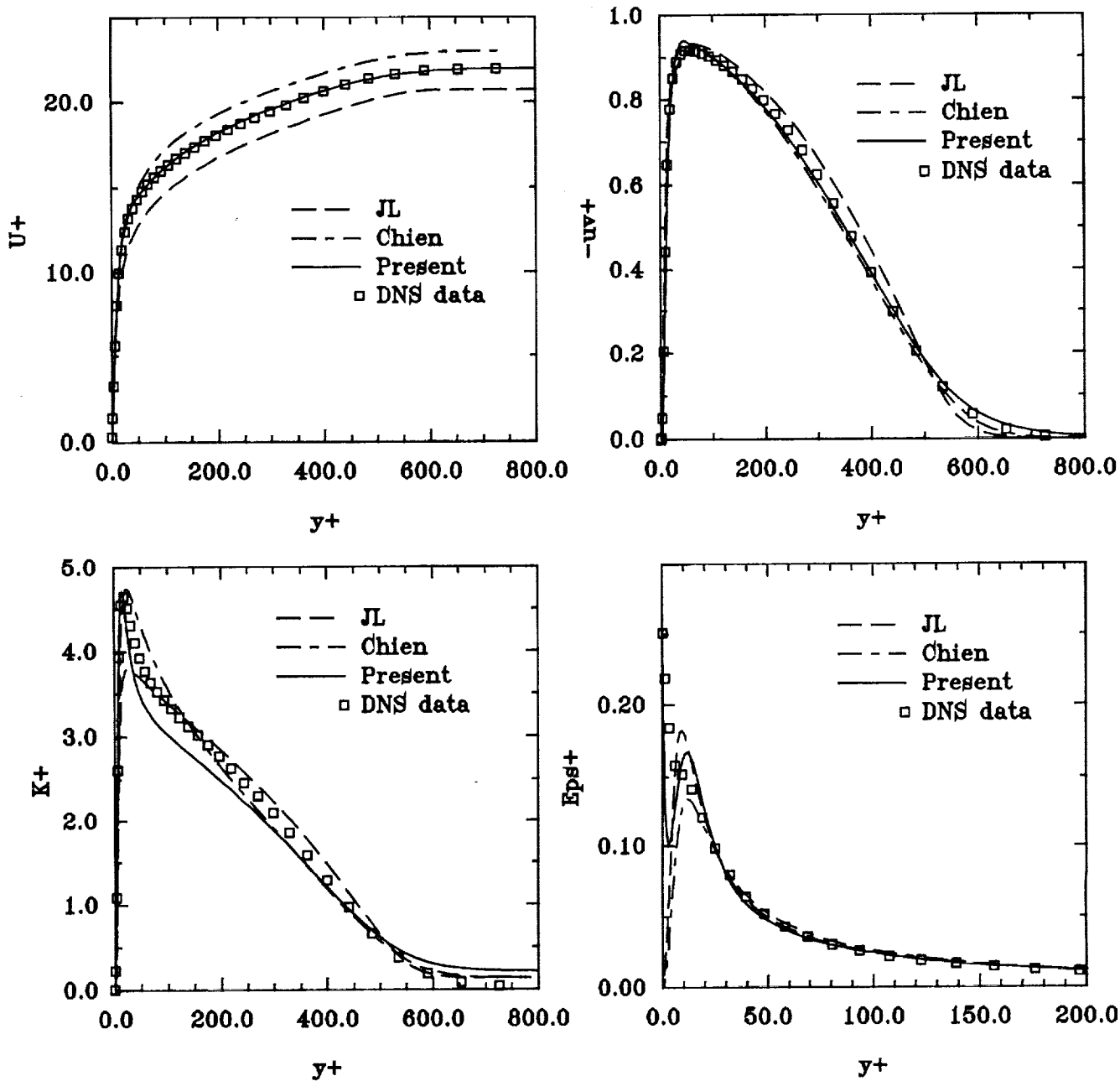Figure 3. Comparison of models with DNS of 2-D channel with $R_{e\tau} = 395$.

13

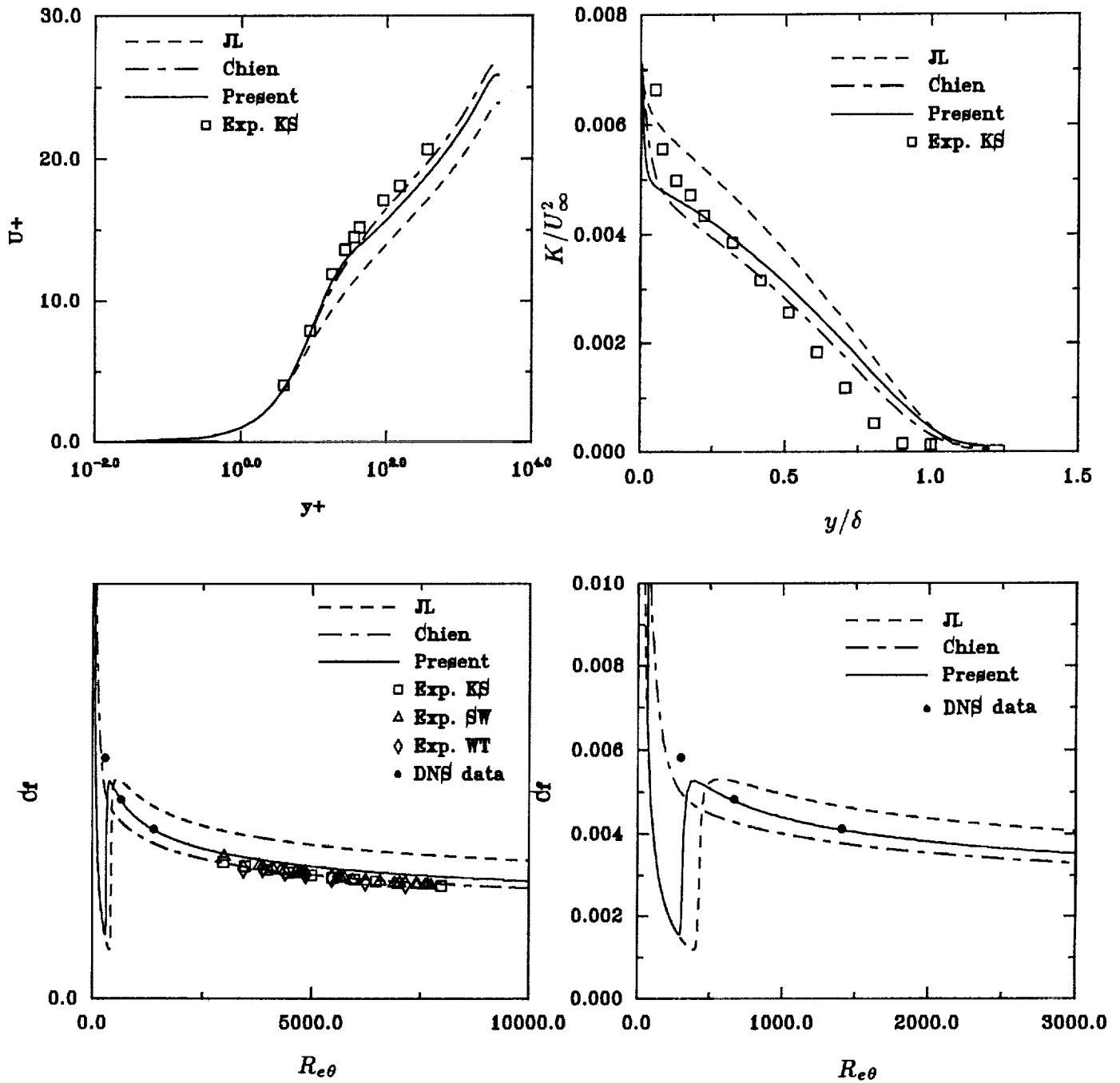Figure 4. Comparison of models with DNS of 2-D boundary layer flow with $R_{e\theta} = 1410$.

Figure 5. Comparison of models with the experiments of 2-D boundary layer
flows (Klebanoff[13] and others).

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 1992	3. REPORT TYPE AND DATES COVERED Technical Memorandum

4. TITLE AND SUBTITLE

Kolmogorov Behavior of Near-Wall Turbulence and Its Application in Turbulence Modeling

5. FUNDING NUMBERS

WU–505–62–21

6. AUTHOR(S)

Tsan-Hsing Shih and John L. Lumley

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135–3191

8. PERFORMING ORGANIZATION REPORT NUMBER

E–7025

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, D.C. 20546–0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM–105663
ICOMP–92–10
CMOTT–92–06

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified - Unlimited
Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The near-wall behavior of turbulence is re-examined in a way different from that proposed by Hanjalic and Launder[1] and followers[2],[3],[4],[5]. It is shown that at a certain distance from the wall, all energetic large eddies will reduce to *Kolmogorov* eddies (the smallest eddies in turbulence). All the important wall parameters, such as friction velocity, viscous length scale, and mean strain rate at the wall, are characterized by *Kolmogorov* microscales. According to this *Kolmogorov* behavior of near-wall turbulence, the turbulence quantities, such as turbulent kinetic energy, dissipation rate, etc. at the location where the large eddies become *"Kolmogorov"* eddies, can be estimated by using both direct numerical simulation (DNS) data and asymptotic analysis of near-wall turbulence. This information will provide useful boundary conditions for the turbulent transport equations. As an example, the concept is incorporated in the standard k-ε model which is then applied to channel and boundary layer flows. Using appropriate boundary conditions (based on *Kolmogorov* behavior of near-wall turbulence), there is no need for any wall-modification to the k-ε equations (including model constants). Results compare very well with the DNS and experimental data.

14. SUBJECT TERMS

Turbulence modeling

15. NUMBER OF PAGES
16

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	