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Jer-Nan Juang, Chung-Wen Chen, and Minh Phan

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Estimation of Kalman Filter Gain from Output Residuals

Jer-Nan Juang*
NASA Langley Research Center, Hampton, VA 23665

Chung-Wen Chen**
North Carolina State University, Raleigh, NC 27695

Minh Phan +
Lockheed Engineering & Sciences Co., Hampton, VA 23665

Abstract

This paper presents a procedure for estimation of the Kalman filter gain from output residuals. The system state space model is assumed to be known, but the process and noise covariances are unknown. The proposed procedure consists of three basic steps. First, the output residuals are computed from the given model and a given set of input-output data. Second, a linear regression model for this part of the response is computed by a least-squares solution. Third, the Kalman filter gain is then estimated from the coefficients of this model. Numerical results using experimental data are presented to illustrate the validity of the developed procedure.

Introduction

A state space model of a linear system describes the system input and output via a quantity called the state vector. There is a class of controllers that uses the state information to compute the control input. However, the state vector itself is typically not accessible for direct measurement. A state estimator, also known as an observer, can be used to provide an estimate of the system state from input and output measurements.

In the presence of process and measurement noises, under ideal conditions, an optimal observer is the Kalman filter. The Kalman filtering problem has been studied for several decades. To compute the Kalman filter gain, the system model must be known, and the individual process and measurement noise covariances must also be known. In practice, these are somewhat restricted requirements, since neither the system nor the noise characteristics can be known exactly. Nevertheless, a mathematical model of the system can be derived analytically, or experimentally from input-output measurement data by a system identification method. An estimate of the measurement noise covariance may be obtained by examining the response of the sensor devices. The process noise covariances, however, is almost impossible to be obtained by direct measurement and some guess work is required.

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* Principal Scientist, Spacecraft Dynamics Branch, Fellow AIAA.

** Research Associate, Mars Mission Research Center, Member AIAA.

+ Engineer, Spacecraft Dynamics Branch, Member AIAA.

It is thus difficult to determine accurately the individual process and measurement noises characteristics. But, collectively, their information is present in the system input-output data. It is the purpose of this paper to formulate a procedure to estimate the Kalman filter gain directly from knowledge of the system and input-output data without the need to identify the process and measurement noise covariances individually. The problem of identifying both the system and the Kalman filter gain is studied in Refs. 1-5. In practice, due to the presence of other factors such as disturbances, non-linearities, non-whiteness of the process and measurement noises, etc... the resulting identified filter is not the Kalman filter. In such a case, the identified filter is simply an observer that is computed from input-output data that minimizes the filter residual in a least-squares sense.

The outline of this paper is the following. First, a mathematical statement of the problem is presented. The main section that follows formulates a least-squares solution for computing the coefficients of the model that describes the stochastic portion of the response. The Kalman filter gain is then estimated from these coefficients. The derivation is done in the time domain. For better understanding of the formulation, an interpretation in the z-domain is also provided. Numerical examples are given to illustrate the method proposed in this paper.

Statement of the Problem

Consider a linear discrete multivariable system in state space format

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w_1(k) \\ y(k) &= Cx(k) + w_2(k) \end{aligned} \tag{1}$$

The process noise $w_1(k)$ and measurement noise $w_2(k)$ are two statistically independent, zero-mean, stationary white noise processes. It is known from Kalman filter theory that there exists an optimal filter for the above system of the form

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + K\varepsilon(k) \\ \hat{y}(k) &= C\hat{x}(k) + Du(k) \end{aligned} \tag{2}$$

where $\varepsilon(k)$ is a white sequence of the Kalman filter residual,

$$\varepsilon(k) = y(k) - \hat{y}(k)$$

The filter gain K is a function of the system parameters and the process and measurement noise covariances. Suppose that a set of input-output data of the system given in Eq. (1) is available in $\mathbf{u}$ and $\mathbf{y}$ as

$$\begin{aligned} \mathbf{u} &= \begin{bmatrix} u(0) & u(1) & \cdots & u(N) \end{bmatrix} \\ \mathbf{y} &= \begin{bmatrix} y(0) & y(1) & \cdots & y(N) \end{bmatrix} \end{aligned} \tag{3}$$

The objective of the problem is to estimate the Kalman filter gain K from a given set of data. The system matrices A, B, C, D are assumed to be known. The statistical properties of the noises in the system, however, are assumed not known.

Mathematical Formulation

For simplicity of presentation, this section is divided into various sub-sections, each will deal with a specific aspect of the formulation. The problem is formulated in the following basic steps. First, since the system parameters are assumed known, it is possible to compute the deterministic and stochastic portions of the response for the given set of input data. This step will simplify the subsequent mathematics. Next, a least-squares solution for computing the coefficients of the model that describes the stochastic portion of the response is formulated. Finally, the Kalman filter gain is estimated from these coefficients.

1. Deterministic and Stochastic Portions of the Response

First, the mathematical problem defined in the previous section can be simplified considerably by observing that it is possible to partition the Kalman filter model into two parts as

$$\begin{aligned} \hat{x}_1(k+1) &= A\hat{x}_1(k) + Bu(k) \\ y_1(k) &= C\hat{x}_1(k) + Du(k) \end{aligned} \tag{4}$$

and

$$\begin{aligned} \hat{x}_2(k+1) &= A\hat{x}_2(k) + K\varepsilon(k) \\ y_2(k) &= C\hat{x}_2(k) + \varepsilon(k) \end{aligned} \tag{5}$$

where $\hat{x}(k) = \hat{x}_1(k) + \hat{x}_2(k)$ and $y(k) = y_1(k) + y_2(k)$. The quantities $\hat{x}_1(k)$ and $y_1(k)$ are portions of the state and output, respectively, caused by the known deterministic input $u(k)$. The quantities $\hat{x}_2(k)$ and $y_2(k)$ are portions of the state and output caused by the unknown stochastic input $\varepsilon(k)$.

Assuming zero initial conditions, $\hat{x}_1(0) = 0$, for the given set of input-output sequence, the deterministic and stochastic portions of the output can be easily computed. To see this, note that $y_1(k)$ can be easily found from the known values of A, B, C, D and $u(k)$ as

$$y_1(k) = \sum_{i=1}^{k} CA^{i-1}Bu(k-i) + Du(k) \tag{6}$$

The parameters $Y(i) = CA^{i-1}B$ in the above expression are known as the Markov parameters of the system. Knowing $y_1(k)$, the stochastic portion of the output, $y_2(k)$, can be computed from

$$y_2(k) = y(k) - y_1(k) \tag{7}$$

Hence, the mathematical problem reduces to that of finding K from Eq. (5), provided that A, C, and $y_2(k)$ are known. In this paper, the stochastic portion of the output, $y_2(k)$, is also referred to as output residuals.

2. Computation of a Model for the Stochastic Portion of the Response

A procedure to compute a model for the stochastic portion of the response given in Eq. (5) is presented in this section. Equation (5) can also be re-written as

$$
\begin{aligned}
\hat{x}_2(k+1) &= (A-KC)\hat{x}_2(k) + Ky_2(k) \\
y_2(k) &= C\hat{x}_2(k) + \varepsilon(k)
\end{aligned}
\tag{8}
$$

Assuming zero initial conditions, $\hat{x}_2(0)=0$, from Eq. (8),

$$
y_2(k) = \sum_{i=1}^{k} C(A-KC)^{i-1} K y_2(k-i) + \varepsilon(k)
\tag{9}
$$

For simplicity of notation, define the parameters

$$
\alpha_k = C(A-KC)^{k-1} K \ , \quad k = 1,\ 2,\ \dots,\ p
\tag{10}
$$

The matrix $A-KC$ is the Kalman filter system matrix, which is known to be asymptotically stable. Hence, for some sufficiently large value of p,

$$
(A-KC)^{i-1} \approx 0 \ , \quad i > p
\tag{11}
$$

Equation (9) can then be approximated by a finite set of parameters $\alpha_1,\ \alpha_2,\ \dots,\ \alpha_p$ as

$$
y_2(k) = \sum_{i=1}^{p} \alpha_i y_2(k-i) + \varepsilon(k)
\tag{12}
$$

Writing Eq. (12) in matrix form for the given set of data

$$
\mathbf{y} = \boldsymbol{\alpha}\mathbf{Y} + \boldsymbol{\varepsilon}
\tag{13}
$$

where

$$
\mathbf{y} = \begin{bmatrix} y(0) & y(1) & \cdots & y(p) & y(p+1) & \cdots & y(N) \end{bmatrix}
$$

$$
\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_p \end{bmatrix}
$$

$$
\mathbf{Y} = \begin{bmatrix}
y_2(0) & y_2(1) & \cdots & y_2(p) & y_2(p+1) & \cdots & y_2(N) \\
 & y_2(0) & \cdots & y_2(p-1) & y_2(p-2) & \cdots & y_2(N-1) \\
 & & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & & & y_2(0) & y_2(1) & \cdots & y_2(N-p)
\end{bmatrix}
$$

$$
\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon(0) & \varepsilon(1) & \cdots & \varepsilon(p) & \varepsilon(p+1) & \cdots & \varepsilon(N) \end{bmatrix}
$$

The residual time history of the Kalman filter is known to be minimized and orthogonal to the data sequence. To obtain an estimation of the Kalman filter with a finite data record, we compute $\boldsymbol{\alpha}$ by the least-squares solution,

$$
\boldsymbol{\alpha} = \mathbf{y}\mathbf{Y}^{+}
\tag{14}
$$

where the superscript + denotes the pseudo-inverse. The parameter matrix $\boldsymbol{\alpha}$ contains the coefficients for a model of the stochastic portion of the response from which the Kalman filter gain K can be estimated. This is shown in the following section.

3. Estimation of the Kalman Filter Gain

Finally, the filter gain K can be computed from the above least-squares solution for $\boldsymbol{\alpha}$ by the following procedure. First define the Kalman filter gain Markov parameters

$$\beta_k = CA^{k-1}K \ , \quad k = 1,\ 2,\ \ldots,\ p \tag{15}$$

Making use of the definitions of α_k and β_k given in Eq. (10) and Eq. (15), the parameters $\beta_1,\ \beta_2,\ldots,\ \beta_p$ can be computed from $\alpha_1,\ \alpha_2,\ldots,\ \alpha_p$ by the recursive equation

$$\beta_k = \alpha_k + \sum_{i=1}^{k-1} \beta_{k-i}\alpha_i \ , \quad k = 1,\ 2,\ \ldots,\ p \tag{16}$$

Since A and C are known, the gain matrix K can now be solved from

$$K = \left(\mathbf{V}^T\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{Y}_K \tag{17}$$

where $\mathbf{V}$ is an observability matrix from A and C, and $\mathbf{Y}_K$ is a matrix formed by the parameters $\beta_k = CA^{k-1}K,\ k = 1,\ 2,\ \ldots,\ p$ computed in Eq. (16).

$$\mathbf{V} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix} , \quad \mathbf{Y}_K = \begin{bmatrix} CK \\ CAK \\ \vdots \\ CA^{p-1}K \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix}$$

Z-Domain Interpretation

This section provides an explanation of the above time-domain derivation by examining the equations in the z-domain. First, it is of interest to examine various input-output description in the z-domain. The results will then be specialized for the problem considered in this paper.

1. Input-Output Models in the Z-Domain

Assuming that the system in Eq. (2) is asymptotically stable, the system input-output relationship can be written in terms of the Kalman filter as

$$y(k) = \sum_{i=1}^{q} CA^{i-1}Bu(k-i) + Du(k) + \sum_{i=1}^{q} CA^{i-1}K\varepsilon(k-i) + \varepsilon(k) \tag{18}$$

where the order of the linear difference equation, q, is sufficiently large such that A^k is negligibly small for $k \geq q$. Taking the z-transforms of both sides of Eq. (18) yields

$$\mathbf{Y}(z) = \mathbf{M}(z)\mathbf{U}(z) + \mathbf{N}(z)\mathbf{E}(z) \tag{19}$$

where the transfer functions $\mathbf{M}(z)$ and $\mathbf{N}(z)$ are given as

$$\mathbf{M}(z) = \sum_{i=1}^{q} CA^{i-1}Bz^{-i} + D \quad , \quad \mathbf{N}(z) = \sum_{i=1}^{q} CA^{i-1}Kz^{-i} + I$$

Note that the coefficients in $\mathbf{M}(z)$ are the Markov parameters of the system, and the coefficients in $\mathbf{N}(z)$ are the Kalman filter gain Markov parameters discussed previously. Together they contain information about the system and the Kalman filter. The z-transfer functions of the deterministic and stochastic parts of the response are $\mathbf{Y}_1(z)$ and $\mathbf{Y}_2(z)$, respectively

$$\mathbf{Y}_1(z) = \mathbf{M}(z)\mathbf{U}(z) \quad , \quad \mathbf{Y}_2(z) = \mathbf{N}(z)\mathbf{E}(z) \tag{20}$$

$$\mathbf{Y}(z) = \mathbf{Y}_1(z) + \mathbf{Y}_2(z)$$

On the other hand, Eq. (2) can also be re-written as

$$\begin{aligned} \hat{x}(k+1) &= (A - KC)\hat{x}(k) + (B - KD)u(k) + Ky(k) \\ y(k) &= C\hat{x}(k) + Du(k) + \varepsilon(k) \end{aligned} \tag{21}$$

which yields the following input-output relationship

$$y(k) - \sum_{i=1}^{r} C(A - KC)^{i-1} Ky(k-i) = Du(k) + \sum_{i=1}^{r} C(A - KC)^{i-1}(B - KD)u(k-i) + \varepsilon(k) \tag{22}$$

where the order of the linear difference equation, r, is sufficiently large such that $(A - KC)^k$ is negligibly small for $k \geq r$. Taking the z-transforms of both sides of Eq. (22) yields

$$\mathbf{P}(z)\mathbf{Y}(z) = \mathbf{Q}(z)\mathbf{U}(z) + \mathbf{E}(z) \tag{23}$$

where

$$\mathbf{P}(z) = I - \sum_{i=1}^{r} C(A - KC)^{i-1} Kz^{-i} = I - \sum_{i=1}^{r} \alpha_i z^{-i}$$

$$\mathbf{Q}(z) = D + \sum_{i=1}^{r} C(A - KC)^{i-1}(B - KD)z^{-i}$$

Comparing Eq. (19) to Eq. (23) suggests that

$$\mathbf{M}(z) = \mathbf{P}(z)^{-1}\mathbf{Q}(z) \quad , \quad \mathbf{N}(z) = \mathbf{P}(z)^{-1} \tag{24}$$

A proof of the above relation is provided in the appendix. The significance of the above relations is that if the coefficients of $\mathbf{P}(z)$ and $\mathbf{Q}(z)$ are known, then $\mathbf{M}(z)$ and $\mathbf{N}(z)$, which characterize the system and the Kalman filter, can be computed. In fact, knowledge of the coefficients of $\mathbf{M}(z)$ and $\mathbf{N}(z)$ is sufficient to compute a state space model of the system (A, B, C, D) and the Kalman filter gain K. The problem of identification of the coefficients of $\mathbf{P}(z)$ and $\mathbf{Q}(z)$ for the purpose of identifying a state space model and the Kalman filter gain is treated in Refs. 1-3.

2. A Model for the Stochastic Portion of the Response in the Z-Domain

In the present case, since the system model is assumed to be known, the problem is considerably simplified. The coefficients of $\mathbf{P}(z)$ can be determined directly from $\mathbf{A}(z)$ which is the linear difference model for the stochastic part of the output is given in Eq. (8).

$$\mathbf{A}(z)\mathbf{Y}_2(z) = \mathbf{E}(z) \tag{25}$$

where

$$\mathbf{A}(z) = I - \sum_{i=1}^{p} C(A-KC)^{i-1} K z^{-i} = I - \sum_{i=1}^{p} \alpha_i z^{-i}$$

Compare $\mathbf{A}(z)$ with $\mathbf{P}(z)$ in Eq. (23) immediately yields $\mathbf{A}(z) = \mathbf{P}(z)$, hence

$$\mathbf{N}(z) = \mathbf{P}(z)^{-1} = \mathbf{A}(z)^{-1} \tag{26}$$

Equation (26) states that $\mathbf{N}(z)$, from which the Kalman filter gain K can be computed, can be obtained by inverting the transfer function $\mathbf{A}(z)$ or $\mathbf{P}(z)$. Instead of direct inversion of the polynomial $\mathbf{A}(z)$, the coefficients of $\mathbf{N}(z)$ can be computed from the relation $\mathbf{A}(z)\mathbf{N}(z) = I$, which is simply

$$\left(I - \alpha_1 z^{-1} - \alpha_2 z^{-2} - \cdots\right)\left(I + \beta_1 z^{-1} + \beta_2 z^{-2} - \cdots\right) = I$$

Equating like powers of z^{-i}, $i = 1, 2, \ldots$ and solving for the coefficients β_i, $i = 1, 2, \ldots$ immediately yields the same result as given in Eq. (16), e.g.,

$$\begin{aligned}
\beta_1 &= \alpha_1 \\
\beta_2 &= \alpha_2 + \alpha_1\beta_1 \\
\beta_2 &= \alpha_3 + \alpha_2\beta_1 + \alpha_1\beta_2 \\
&\vdots
\end{aligned}$$

Experimental Results

The developed procedure is applied to experimental data of a truss structure at NASA Langley Research Center partially shown in Fig. 1. The L-shaped structure consists of nine bays on its vertical section, and one bay on its horizontal section, extending 90 inches and 20 inches, respectively. The shorter section is clamped to a steel plate which is rigidly attached to the wall. The system has two air jet thruster inputs, and two accelerometer outputs. For identification, the structure is excited using random inputs to both thrusters. The input-output signals are sampled at 250 Hz. A data record of 2,000 points is used for identification.

First, a 26-th order state space model of the system and its associated observer/filter gain are identified by the Observer/Kalman Filter Identification algorithm (OKID)[3] with an assumed order of 40. Since the retained system and filter model is only of 26-th order, the resultant residual of the reduced order filter is not necessarily minimized or white. This is due to the singular value truncation step during realization to obtain a reduced system and filter model of 26-th order. The procedure developed in this paper is then used to identify an improved filter gain, keeping the OKID-identified system model the same.

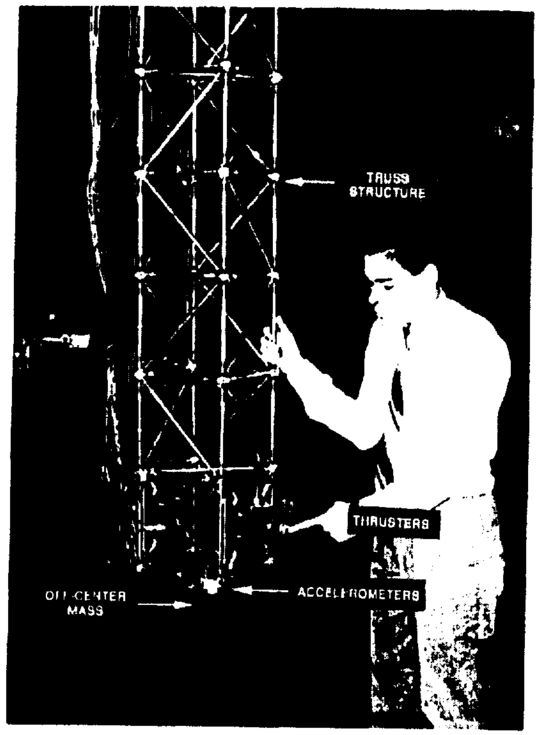

Figure 1: Truss structure test configuration

To verify that the new observer gain is in fact an improved one, the auto-correlation of the filter residuals at different time shifts for the first and second outputs are shown in Figs. 2 and 3, respectively. First, when compared with the original residuals, the norms of the filter residuals are reduced with the improved filter gain. This is indicated by the value of the auto-correlation corresponding to the number of time shift being equal to zero. Second, observe that in both cases, there is an general improvement in the whiteness of the filter residual with the improved gain, where the auto-correlation function is used as a whiteness measurement.

It is important to view the results in light of the limitations in the application of the method. First, only for a linear system with white process and measurement noises, the Kalman filter residual is minimized and white. In practice, however, this is not the case as any deviations from the ideal assumptions will reflect in the identification results. Second, the computation procedure requires that the true system model be known. This is a rather ideal assumption that is not satisfied in practice. Third, because the Kalman filter describes a stochastic process, even when the conditions outlined above are met, one can only identify the true Kalman filter gain with an infinite set of data samples. In the example shown, the system considered here is an actual truss structure, the data in the computation is actual test data, the state space model used is an experimentally identified one, and the data record is only 2000 samples long. The example illustrates the use of the method in providing an improved estimation of the Kalman filter gain from experimental data, assuming that an experimentally identified state space model is in fact a valid representation of the system.

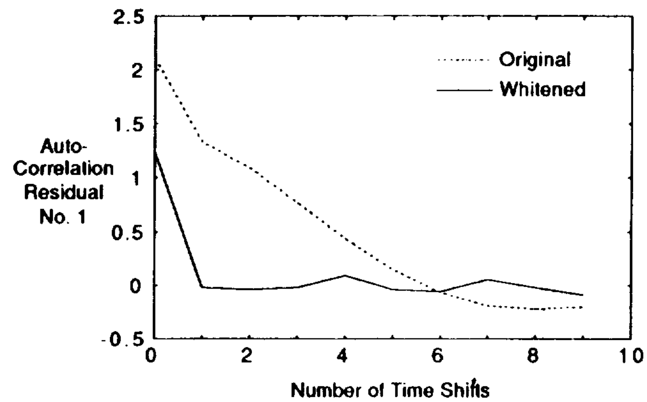

Figure 2: Auto-correlation of filter residual for the first output using original and improved filter gains.

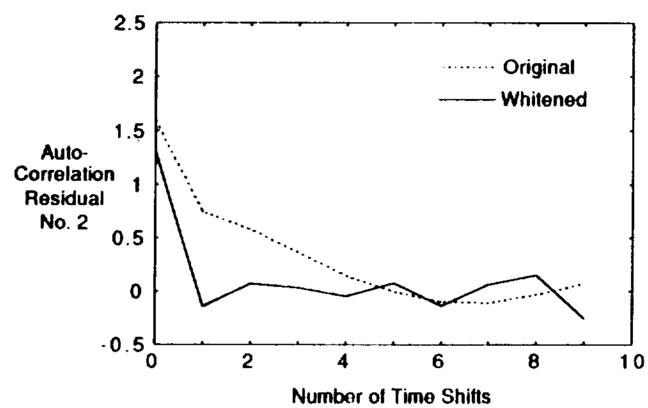

Figure 3: Auto-correlation of filter residual for the second output using original and improved filter gains.

The identified filter can be used to describe the system input-output map. An overlay of the actual responses of the test structure, and the estimated responses provided by the filter is shown in Figs. 4 and 5 for each of the two outputs, respectively. The solid curves represent the actual measured responses of the truss structure whereas dashed curves represent the estimated responses. The two curves in each figure practically overlap. This reveals that the identified observer adequately describes the input-output relationship of the system. In fact, the difference between the actual response and the estimated response is precisely the filter residual which is discussed earlier in Figs. 2 and 3 for each of the two outputs, respectively.

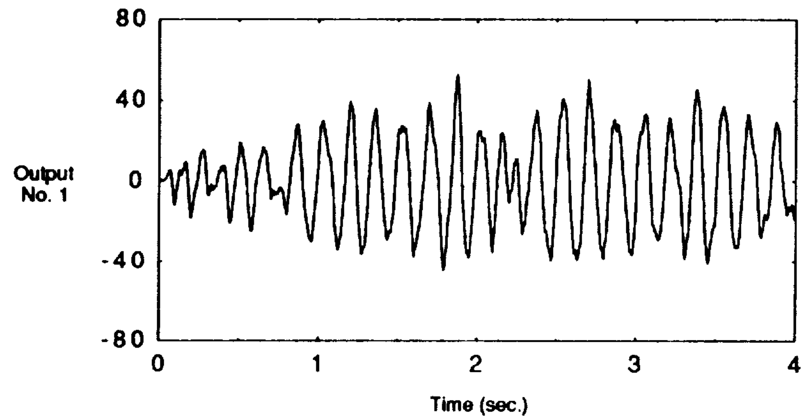

Figure 4: Actual and reconstructed response using identified filter for the first output.

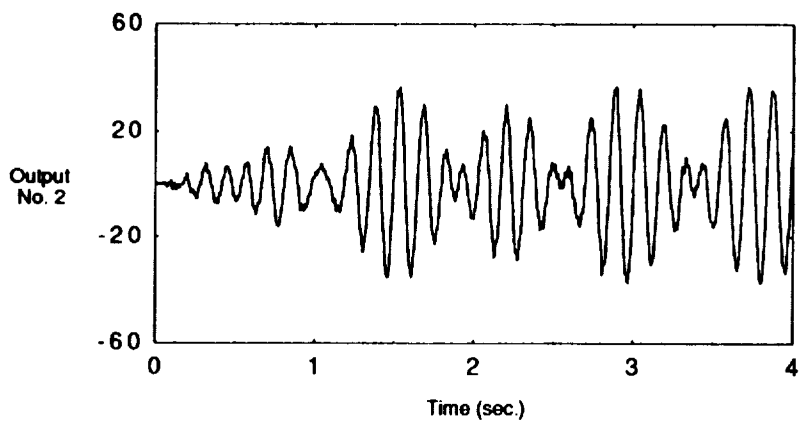

Figure 5: Actual and reconstructed response using identified filter for the second output.

Concluding Remarks

This paper presents a procedure to estimate the Kalman filter gain from input-output measurement data, assuming that the system parameters are known. The procedure consists of three basic steps. First, the stochastic portion of the response is computed. Second, the coefficients of a linear difference model for this portion of the response are estimated by a least squares solution that minimizes the filter residual. Third, the Kalman filter gain is computed from these model coefficients. In practice, due to the presence of other factors such as non-linearities, unmodelled dynamics, etc... the estimated filter gain is not necessarily the Kalman filter gain. Nevertheless, the identified filter gain is optimal in that the resultant filter residual is minimized in the least-squares sense for the given system and the given set of input-output data.

Appendix

Statement of Proof:

Given the following polynomials

$$\mathbf{P}(z) = I - \sum_{i=1}^{r} C\bar{A}^{i-1}Kz^{-i} \ , \quad \mathbf{Q}(z) = D + \sum_{i=1}^{r} C\bar{A}^{i-1}(B-KD)z^{-i} \ , \quad \bar{A} = A - KC$$

$$\mathbf{M}(z) = \sum_{i=1}^{q} CA^{i-1}Bz^{-i} + D \ , \quad \mathbf{N}(z) = \sum_{i=1}^{q} CA^{i-1}Kz^{-i} + I \tag{A.1}$$

where r and q are sufficiently large such that the truncation error is negligible,

$$\bar{A} = (A-KC)^{i} \approx 0 \ , \quad i \geq r \ , \qquad A^{i} \approx 0 \ , \quad i \geq q \tag{A.2}$$

It will be shown in the following that

$$\mathbf{M}(z) = \mathbf{P}(z)^{-1}\mathbf{Q}(z) \tag{A.3}$$

$$\mathbf{N}(z) = \mathbf{P}(z)^{-1} \tag{A.4}$$

Proof:

First, for a stable system matrix A

$$\sum_{i=1}^{\infty} CA^{i-1}Bz^{-i} = C\left(Iz^{-1} + Az^{-2} + A^{2}z^{-3} + \cdots\right)B$$
$$= C(Iz-A)^{-1}B$$

Similarly,

$$\sum_{i=1}^{\infty} C\bar{A}^{i-1}Kz^{-i} = C\left(Iz-\bar{A}\right)^{-1}K$$

$$\sum_{i=1}^{\infty} CA^{i-1}Kz^{-i} = C(Iz-A)^{-1}K$$

$$\sum_{i=1}^{\infty} C\bar{A}^{i-1}(B-KD)z^{-i} = C\left(Iz-\bar{A}\right)^{-1}(B-KD)$$

Relation (A.4) can be shown first by making use of the initial assumption regarding the stability of A and $\bar{A}$ in (A.2), the matrix inversion lemma, and the above equations,

$$
\begin{aligned}
\mathbf{P}(z)^{-1} &= \left(I - \sum_{i=1}^{\infty} C\bar{A}^{i-1} K z^{-i}\right)^{-1} = \left[I - C\left(Iz - \bar{A}\right)^{-1} K\right]^{-1} \\
&= I + C\left(Iz - \bar{A} - KC\right)^{-1} K \\
&= I + C(Iz - A)^{-1} K \\
&= I + \sum_{i=1}^{\infty} C A^{i-1} K z^{-i} = \mathbf{N}(z)
\end{aligned}
\tag{A.5}
$$

To show the relation (A.3), consider the product $\mathbf{P}(z)^{-1}\mathbf{Q}(z)$,

$$
\mathbf{P}(z)^{-1}\mathbf{Q}(z) = \left[I + C(Iz - A)^{-1} K\right]\left[D + C\left(Iz - \bar{A}\right)^{-1}(B - KD)\right]
$$

Hence,

$$
\begin{aligned}
\mathbf{P}(z)^{-1}\mathbf{Q}(z) &= D + C\left(Iz - \bar{A}\right)^{-1}(B - KD) + C(Iz - A)^{-1} KD + C(Iz - A)^{-1} KC\left(Iz - \bar{A}\right)^{-1}(B - KD) \\
&= D + C(Iz - A)^{-1} KD + C\left[I + (Iz - A)^{-1} KC\right]\left(Iz - \bar{A}\right)^{-1}(B - KD) \\
&= D + C(Iz - A)^{-1} KD + C(Iz - A)^{-1}(Iz - A + KC)\left(Iz - \bar{A}\right)^{-1}(B - KD) \\
&= D + C(Iz - A)^{-1} KD + C(Iz - A)^{-1}(B - KD) \\
&= D + C(Iz - A)^{-1} B = \mathbf{M}(z)
\end{aligned}
\tag{A.6}
$$

Equation (A.3) is thus proved.

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Jer-Nan Juang, Chung-Wen Chen, and Minh Phan

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This paper presents a procedure for estimation of the Kalman filter gain from output residuals. The system state space model is assumed to be known, but the process and noise covariances are unknown. The proposed procedure consists of three basic steps. First, the output residuals are computed from the given model and a given set of input-output data. Second, a linear regression model for this part of the response is computed by a least-squares solution. Third, the Kalman filter gain is then estimated from the coefficients of this model. Numerical results using experimental data are presented to illustrate the validity of the developed procedure.

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