DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
COLLEGE OF ENGINEERING AND TECHNOLOGY
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23529-0247

PREDICTION OF RESPONSE OF AIRCRAFT PANELS SUBJECTED TO ACOUSTIC AND THERMAL LOADS

By
Chuh Mei, Principal Investigator

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Dr. Steve A. Rizzi, Technical Monitor
ACOD-Structural Acoustics Branch

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SUMMARY

A unique cooperative graduate studies in structural acoustics and fatigue has been supported for four years under NASA Grant NAG-1-838. The graduate educational program was jointly established by the Old Dominion University, NASA Langley Research Center and the Air Force Wright Laboratory in August 1989. Full and part-time graduate students earned masters and doctoral degrees while working in the laboratories at NASA Langley Research Center, and a spectrum of acoustic fatigue research was conducted. Research results documented in reports, papers and theses were produced in the following sonic fatigue activities:

1. Influence of Transverse Shear Deformation on Large Amplitude Random Response of Symmetric Composite Laminates,
2. Finite Element Free and Forced Vibrations of Rectangular Thin Composite Plates,
3. Finite Element Large Deflection Multi-Mode Random Response of Beam and Plate Structures at Elevated Temperature,
4. Numerical Simulation of Large Deflection Random Vibration of Laminated Composite Plates Using Finite Elements,
5. Finite Element Analysis of Thermal Buckling, Thermal Post-Buckling and Vibrations of Thermally Buckled Composite Plates,
6. Thermal Environment and Response of Quartz-Lamp Heating System, and
7. Prediction of the Three-Dimensional Rectangular Duct using the Boundary Element Method.

This report presents a brief summary of the accomplishments achieved during the four-year period. A list of publications, presentations and theses from this research grant is included.

TABLE OF CONTENTS

1. INTRODUCTION

This report summarizes the research accomplishments performed under the NASA Langley Research Center Grant No. NAG-1-838, entitled "Prediction of Response of Aircraft Panels Subjected to Acoustic and Thermal Loads," for the period December 16, 1987 to March 15, 1992. The primary effort of this research project has been focused on the development of analytical methods for the prediction of random response of structural panels subjected to combined and intense acoustic and thermal loads. The accomplishments on various acoustic fatigue research activities are described first, then followed by publications and theses.

2. INFLUENCE OF TRANSVERSE SHEAR ON NONLINEAR RANDOM VIBRATION

The effects of transverse shear deformation and large deflection on random response of symmetrically laminated composite rectangular plates were investigated the first time [1-3] in the literature. The first-order shear deformation plate theory for laminated composite materials by Yang, Norris and Stavsky [4] and the von Karman large deflection nonlinear strain-displacement relations are used in the development of governing differential equations of motion. The governing equations can be expressed in terms of the transverse deflection w and the Airy stress function F [1] as

$$N(I_1 + I_2) + U_1(w) = 0 \tag{2.1}$$

$$\begin{aligned} A^*_{22}F_{,xxxx} - 2A^*_{26}F_{,xxxy} &+ (2A^*_{12} + A^*_{66})F_{,xxyy} - 2A^*_{16}F_{,xyyy} + A^*_{11}F_{,yyyy} \\ &= w^2_{,xy} - w_{,xx}w_{,yy} \end{aligned} \tag{2.2}$$

where

$$I_1 = p(t) - \rho h \frac{\partial^2 w}{\partial t^2} \tag{2.3}$$

$$I_2 = F_{,yy}\, w_{,xx} + F_{,xx}\, w_{,yy} - 2F_{,xy}\, w_{,xy} \tag{2.4}$$

and N and U_1 are the mathematical operators defined as

$$\begin{aligned} N = k_1\frac{\partial^4}{\partial x^4} + k_2\frac{\partial^4}{\partial x^3 \partial y} + k_3\frac{\partial^4}{\partial x^2 \partial y^2} + k_4\frac{\partial^4}{\partial x \partial y^3} \\ + k_5\frac{\partial^4}{\partial y^4} + k_6\frac{\partial^2}{\partial x^2} + k_7\frac{\partial^2}{\partial x \partial y} + k_8\frac{\partial^2}{\partial y^2} - 1 \end{aligned} \tag{2.5}$$

$$U_1 = D_{11}\frac{\partial^4}{\partial x^4} + 4D_{16}\frac{\partial^4}{\partial x^3\partial y} + 2(D_{12}+2D_{66})\frac{\partial^4}{\partial x^2\partial y^2} + 4D_{26}\frac{\partial^4}{\partial x\partial y^3}$$
$$+ D_{22}\frac{\partial^4}{\partial y^4} + P_1\frac{\partial^6}{\partial x^6} + P_2\frac{\partial^6}{\partial x^5\partial y} + P_3\frac{\partial^6}{\partial x^4\partial y^2} + P_4\frac{\partial^6}{\partial x^3\partial y^3}$$
$$+ P_5\frac{\partial^6}{\partial x^2\partial y^4} + P_6\frac{\partial^6}{\partial x\partial y^5} + P_7\frac{\partial^6}{\partial y^6} \tag{2.6}$$

The excitation $p(t)$ is assumed to be stationary, ergodic and Gaussian with zero-mean, the coefficients k_i and P_i in Eqs. (2.5 and 2.6) are functions of the laminate stiffness A^*_{ij} and D_{ij} and are given in references [1-3]. The coefficients k_i and P_i are due to the transverse shear and they all go to zero if the shear deformation is neglected. The equations of motion are coupled, nonlinear and of order sixth.

The Galerkin's method with a single-mode approximation is applied to yield a Duffing-type forced modal equation. The equivalent linearization technique is then employed to obtain the root-mean-square (RMS) maximum deflection and RMS maximum strains at various sound spectrum levels (SSL). The transverse shearing strains or stresses, τ_{xz} and τ_{yz}, are obtained by integration of the equilibrium equations of three-dimensional elasticity. Four cases of random responses can be obtained from the analysis and they are: 1) linear, small deflection plate theory without transverse shear deformation, 2) linear theory with shear, 3) nonlinear, large deflection plate theory without shear and 4) nonlinear with shear. Boundary support conditions considered are all four edges simply supported, all clamped, and simply supported on two opposing edges and clamped on remaining two edges.

The nondimensional RMS maximum deflection and nondimensional RMS maximum strain are shown in Figs. 1 and 2, respectively, versus plate length-to-thickness ratio (a/h) for three-layer, cross-ply, square, graphite-epoxy laminate of $12\times 12\times$h in. at SSL of 130 dB (Re 2×10^{-5} N/m^2). It is clear from the figures that the effects of transverse shear are considerable for plate lengths are less than 30 times the thickness (a/h < 30). Linear small deflection theory with shear deformation would give accurate response predictions for moderately thick plates. For thin plates (a/h > 50), the linear and nonlinear solutions agree at low values of SSL, but disagree at high values. The nonlinear large deflection theory with shear deformation neglected will give accurate prediction for thin plates at high SSL values.

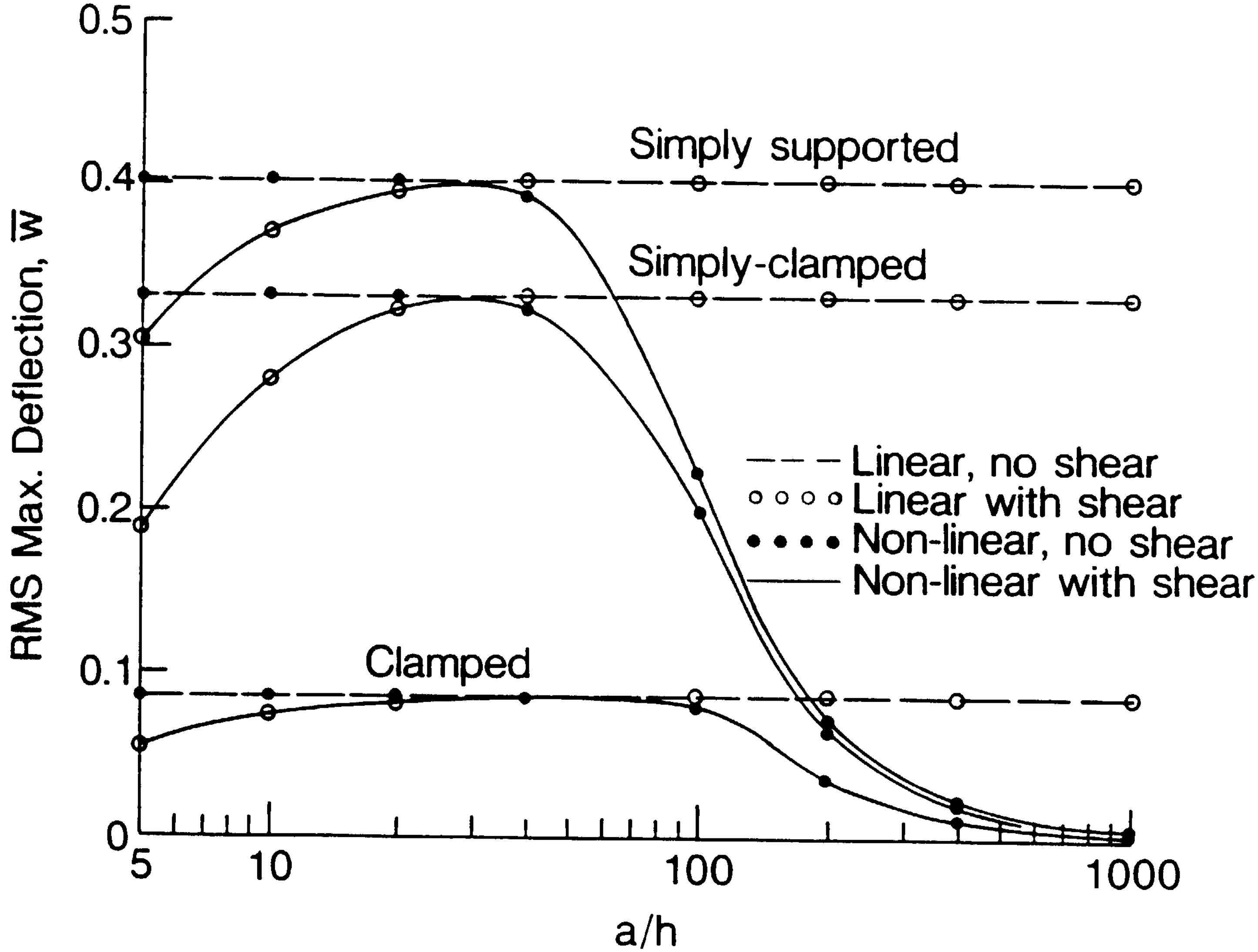

Figure 1. Nondimensional RMS maximum deflection of a three-layer cross-ply square plate with different boundary conditions at 130 dB SSL

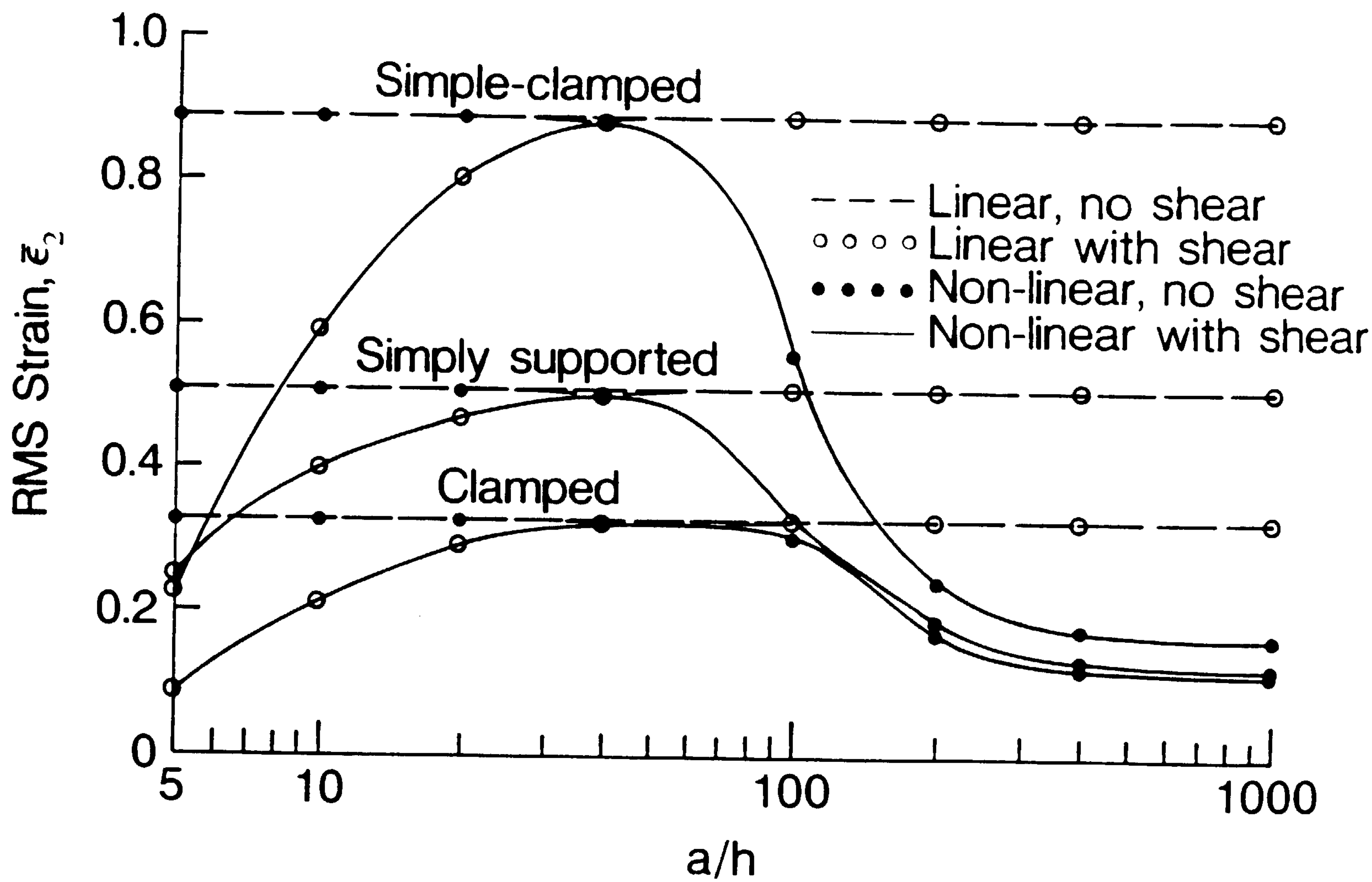

Figure 2. Nondimensional RMS maximum strain of a three-layer cross-ply square plate with different boundary conditions at 130 dB SSL

3. NONLINEAR FREE, FORCED AND RANDOM VIBRATIONS USING FINITE ELEMENTS

It is logical that nonlinear free and forced vibrations of complex structures using finite element will have to be first developed and well understood before any attempting on nonlinear random vibrations.

3.1 Free and Forced Vibrations

Extension of the finite element methods to nonlinear free [5] and forced [6] vibrations of beams was first presented by the principal investigator. Most recently, the finite element method has been extended to the analysis of free and forced vibrations of arbitrarily laminated composite plates [7, 8]. The system equation of motion for nonlinear forced vibrations of beam and plate structures can be written as [7, 8]

$$[M]\left\{\ddot{W}\right\} + ([K] + [K1] + [K2] - [P])\{W\} = 0 \tag{3.1}$$

where $[M]$, $[K]$ and $\{W\}$ are the mass, linear stiffness matrices and nodal displacement vector, respectively. The matrices $[K1]$ and $[K2]$ are the first and second order nonlinear stiffness matrices and depend linearly and quadratically upon nodal displacements, respectively. The nonlinear stiffness matrices are due to the large deflections. The matrix $[P]$ is the harmonic force matrix which depends on the amplitude of vibration $w_{\max}$ and the intensity of force excitation P_o. The nonlinear free vibration problem can be treated as a limiting case of the more general forced vibration with P_o or $[P] = 0$ in Eq. (3.1). The linearized updated-mode method with nonlinear time function (LUM/NTF) approximation, which was introduced in solving nonlinear structural-nonlinear aerodynamic panel flutter limit-cycle responses [9], is used for the free vibration (backbone curve) and forced vibration response from Eq. (3.1).

The frequency ratios $\omega/\omega_{\text{linear}}$ for S1 simply supported and C1 clamped [10] two-layered angle-ply (30/–30), graphite-epoxy laminates of $12 \times 12 \times 0.12$ in. subjected to a uniform harmonic force of $P_o = 0$ (free vibration), 0.1 and 0.2 are shown in Fig. 3. These results indicate that the simply supported plate yields a larger nonlinear response than the clamped case.

The harmonic force matrix is developed for a uniformly distributed harmonic load over the plate element. Application of the uniformly loaded finite element to the loading case for a concentrated force is to allow the area of the loaded element to become smaller and smaller. This is demonstrated for a concentrated force applied at the center of a simply supported square, three-layer, cross-ply plate with S1 [10] immovable inplane edges. The magnitude of the concentrated force is equal to the same force for a plate under a uniformly distributed harmonic loading of $P_o = 0.1$ acting over the total plate area. The nonlinear response for a concentrated force obtained with $(l/a)^2 = 1.0$ percent is shown in Fig. 4. The difference in the frequency ratios is less than 0.1% if a smaller loaded area of $(l/a)^2 = 0.25$ percent is used. The results show that concentrated force is approximately two to three times as severe as the uniformly distributed force for the case studied.

3.2 Random Vibrations

The finite element method is further extended to large deflection random response of beams and plates using single or multimode by Chiang [11-13]. Geometrical stiffness matrices have been developed for a beam and a rectangular plate element to account for the induced inplane forces due to large deflections. RMS maximum deflection for a rectangular ($15\times12\times0.040$ in.) aluminum plate using single mode at SSL from 90 to 130 dB is shown in Fig. 5. Results using classic equivalent linearization (EL) technique are also obtained to demonstrate the accuracy of the finite element (FL) results.

Multiple-mode RMS maximum deflections ($w_{\max}/\sqrt{I/A}$) at various SSL between 90 and 130 dB (Re. 2×10^{-5} N/m^2) for a simply supported and a clamped aluminum beam of damping ratio $\zeta = 0.01$ are shown in Table 1. To demonstrate the accuracy of the finite element (FE) results, classic EL method and the Fokker-Planck-Kolmogorov (FPK) equation approach (exact solution to the forced Duffing equation) are also given.

3.3 Random Vibrations of Thermally Buckled Structures

A limited amount of investigations on structural response subjected to intense thermal and acoustic loads exists in the literature [14]. Seide [15] was the first who studied large deflection random response of thermally buckled simply supported beam. The well known classical Woinowsky-Krieger large amplitude beam vibration equation was used. The Galerkin's method and time domain numerical simulation were then applied for predicting beam random responses. The Galerkin/numerical simulation approach was also applied recently by Vaicaitis [16, 17] to simply supported metal and orthotropic composite rectangular plates. The classical von Karman large deflection plate equations including the temperature and orthotropic property effects were employed. The classic analytical investigations have been limited to simple beam [15] and rectangular orthotropic plates [16, 17] with simply supported boundary conditions and uniform temperature distributions.

Locke and Mei [18, 19] extended the finite element method for the first time to structures subjected to thermal and acoustic loads applied in sequence. The thermal load considered is a steady-state temperature distribution $\Delta T(x, y)$. The thermal post-buckling structural problem is solved first to obtain the deflection and thermal stresses. The thermal deflection and stresses are then treated as initial deflection and initial stresses in the subsequent random vibration analysis. The Newton-Raphson iterative method is used in the thermal post-buckling analysis. For the problem of nonlinear random vibration, the linear vibration mode shapes of the thermally buckled structure are used to reduce the order of the system equations of motion to a set of nonlinear modal equations of a much smaller order. The equivalent linearization technique is then used to obtain iteratively the RMS response. Figures 6 and 7 show the excellent agreement between the finite element and the Galerkin/numerical simulation results by Seide [15] for a simply supported beam at a wide range of thermal post-buckling loads $\Delta T/\Delta T_{cr}$ = 1.25, 2.00 and 7.25 and intense SSL between 90 and 130 dB.

To demonstrate the versatility of the finite element method, a rectangular ($12\times 15\times 0.040$ in.) plate with clamped boundary conditions and subjected to a nonuniform temperature distribution

$$\Delta T(x,y) = T_o\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right)$$

and at SSL from 90 to 120 dB (Re. 2×10^{-5} N/m^2) is analyzed. The RMS μ-strain at the midpoint of the long-edge and in the direction perpendicular to the edge is shown in Fig. 8. At low SSL the maximum strain occurs at or near T/T_{cr} equal to one. However, at higher SSL, the maximum strain is seen to occur at higher values of T/T_{cr}.

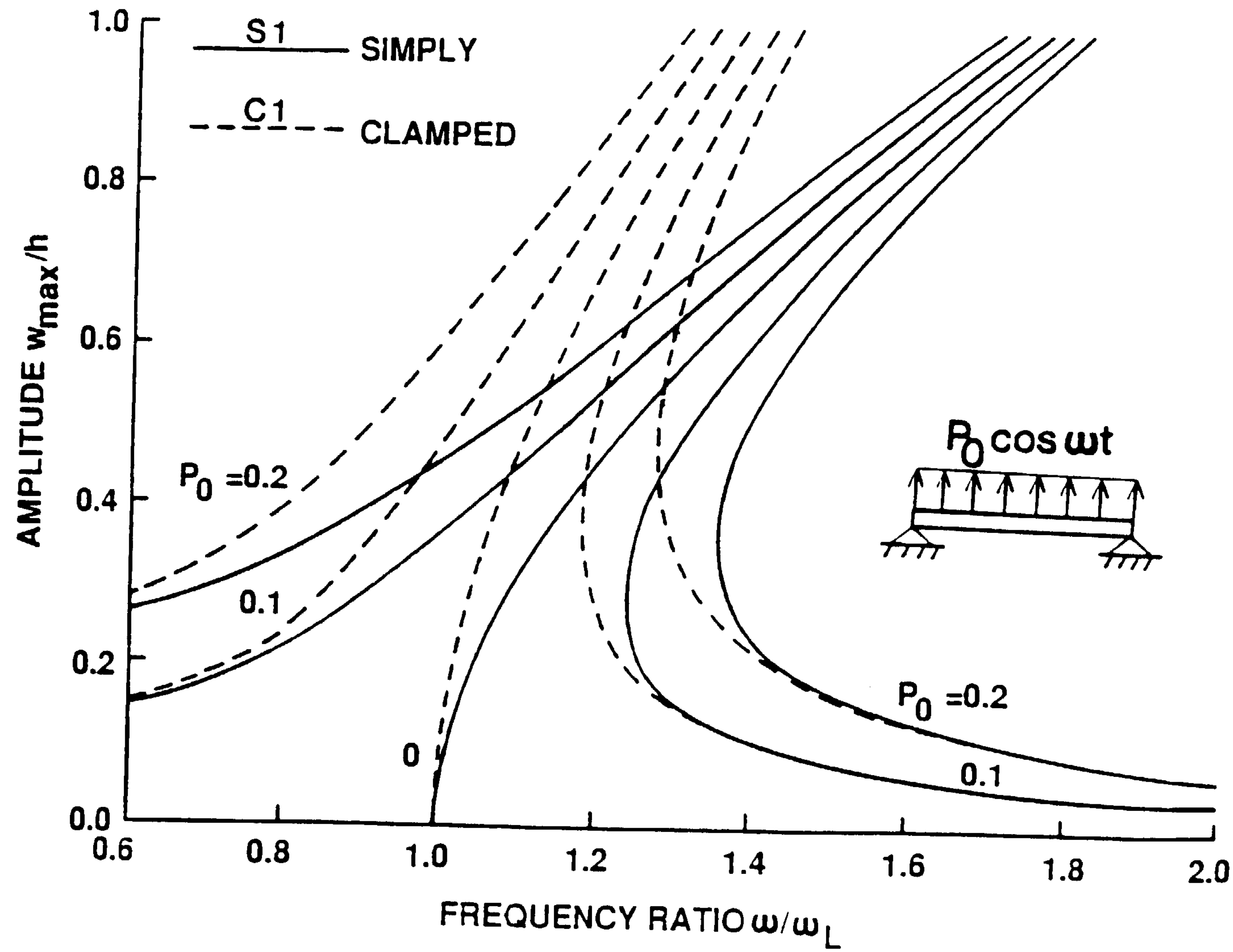

Figure 3. Amplitude versus frequency of a two-layer angle-ply (30/-30) square plate

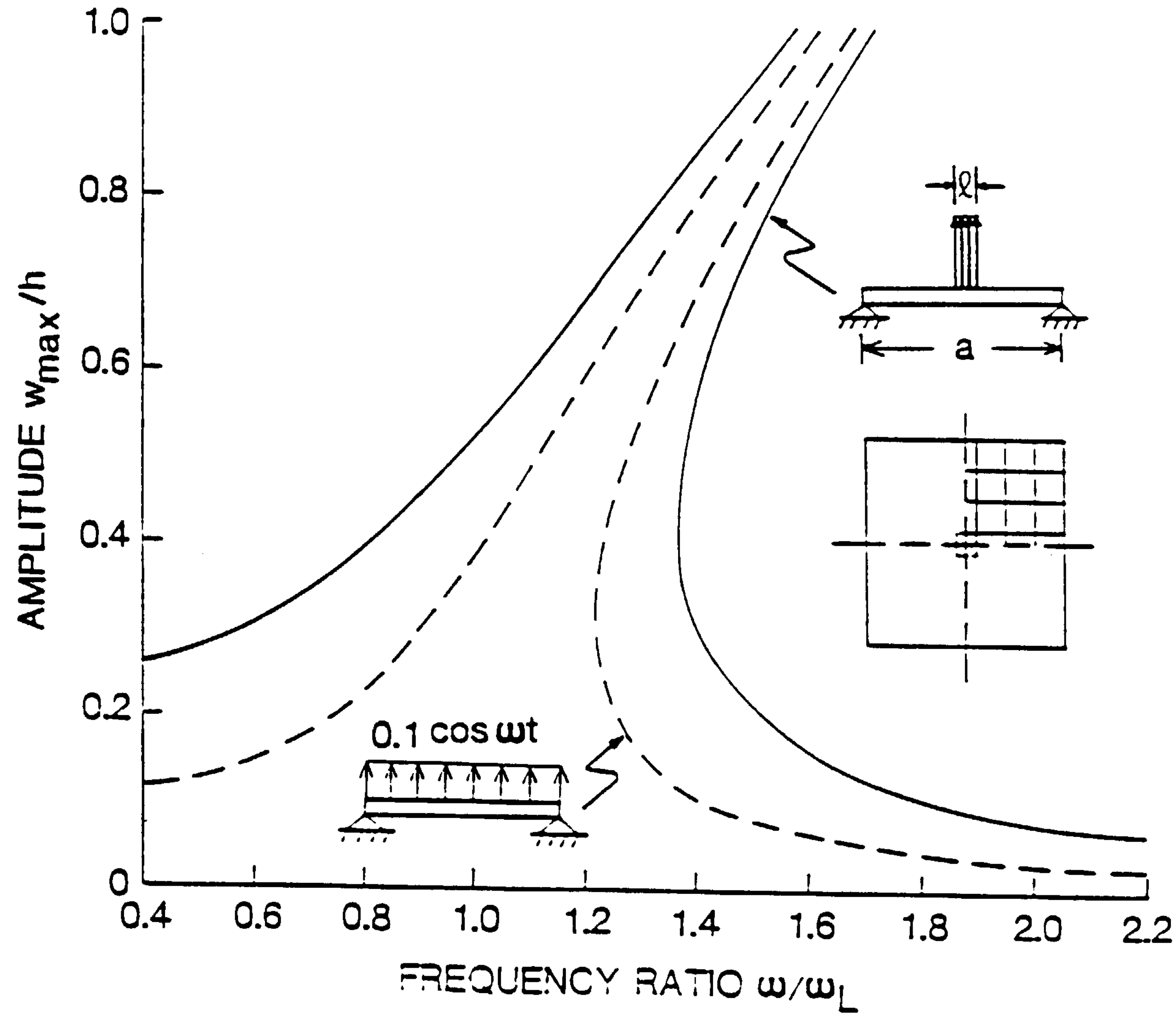

Figure 4. Amplitude versus frequency of a three-layer cross-ply square simply supported (S1) plate

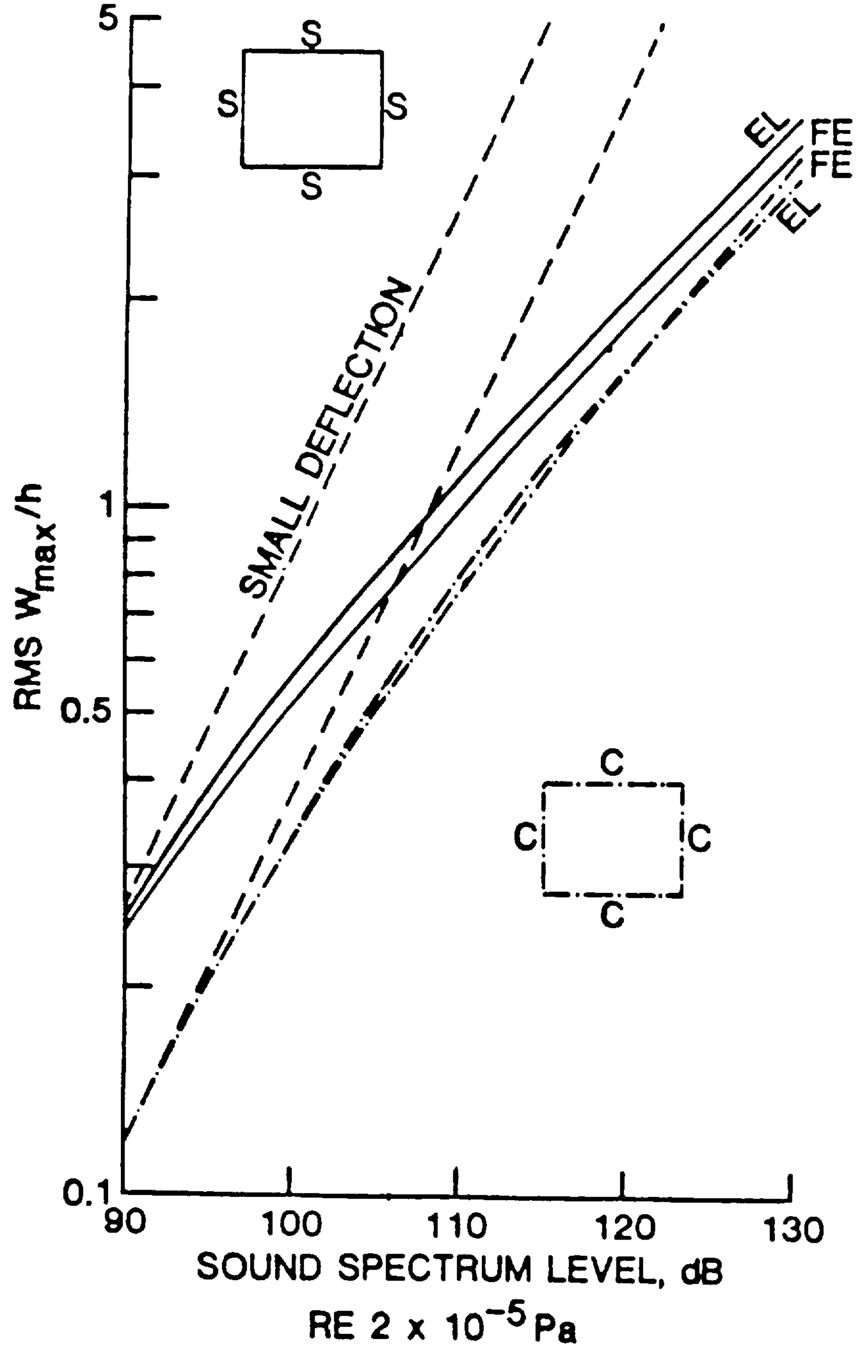

Figure 5. RMS maximum plate deflection vs sound spectrum level using one mode

Table 1 Comparison of RMS maximum deflections for simply supported beam and clamped beam using different methods

SSL	FPK method	EL method		FE method		
	single mode	single mode	three modes	single mode	three modes	five modes
			Simply Supported			
90	0.3198	0.3194	0.3194	0.3212	0.3175	0.3174
100	0.8615	0.8456	0.8456	0.8648	0.8522	0.8520
110	1.8867	1.7938	1.7937	1.8645	1.8231	1.8222
120	3.6248	3.3843	3.3834	3.5569	3.4663	3.4637
130	6.6134	6.1325	6.1289	6.5217	6.4275	6.4174
			Clamped			
90	0.1005	0.1005	0.1006	0.1006	0.0980	0.0979
100	0.3155	0.3154	0.3156	0.3159	0.3078	0.3076
110	0.9392	0.9354	0.9357	0.9449	0.9231	0.9223
120	2.3549	2.2850	2.2845	2.3591	2.3113	2.3098
130	4.8720	4.5952	4.5883	4.8742	4.7361	4.7335

Three modes — mode 1, 3 and 5.

Five modes — mode 1, 3, 5, 7 and 9.

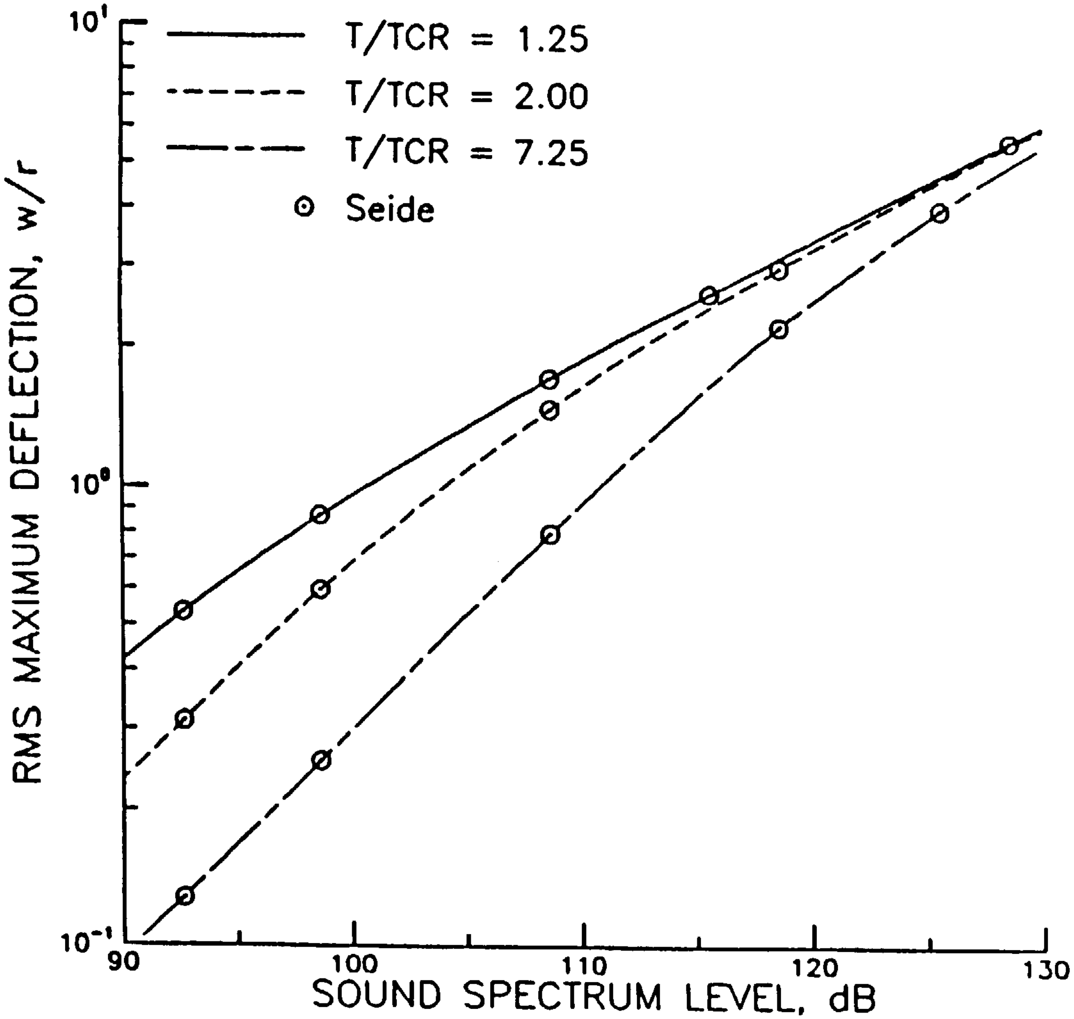

Figure 6. RMS maximum deflection versus sound spectrum level for a simply supported beam subjected to uniform temperature distributions

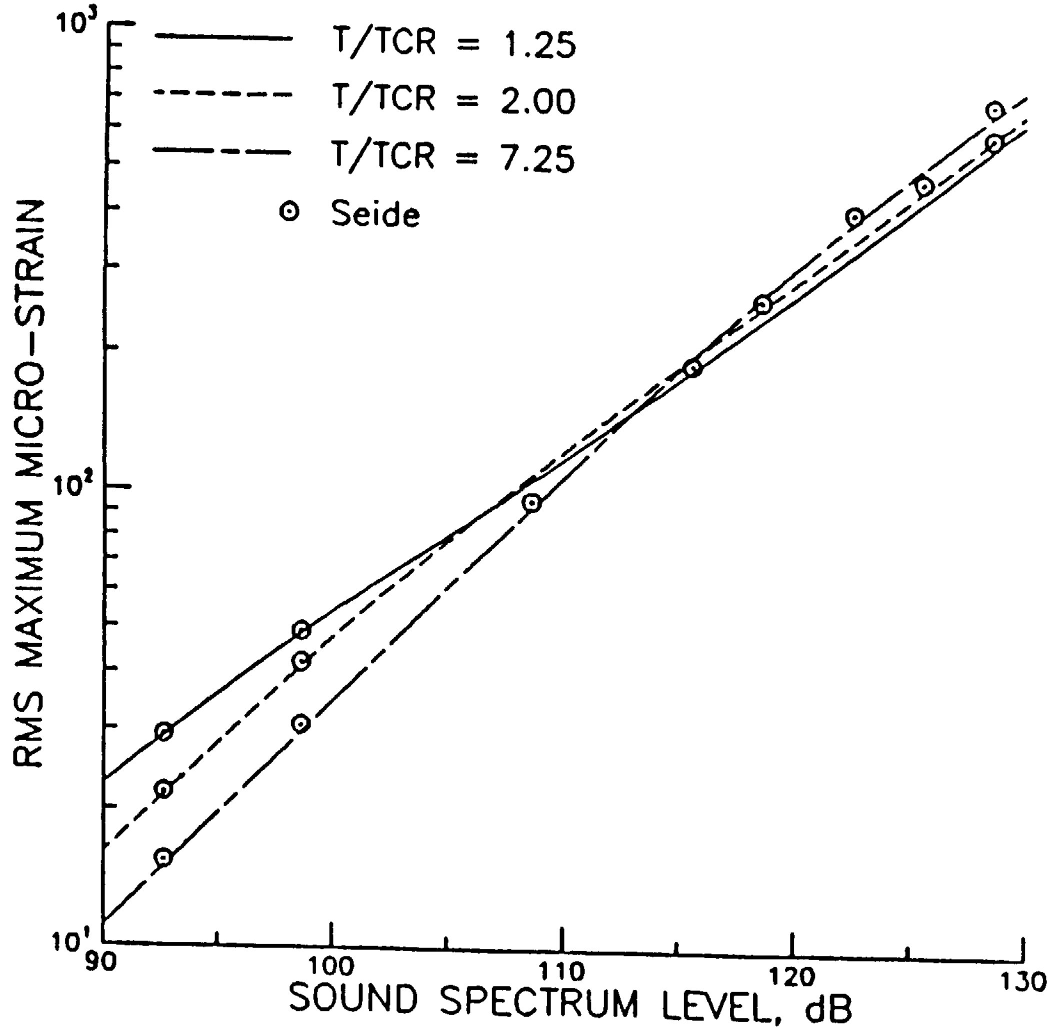

Figure 7. RMS maximum strain versus sound spectrum level for a simply supported beam subjected to uniform temperature distributions

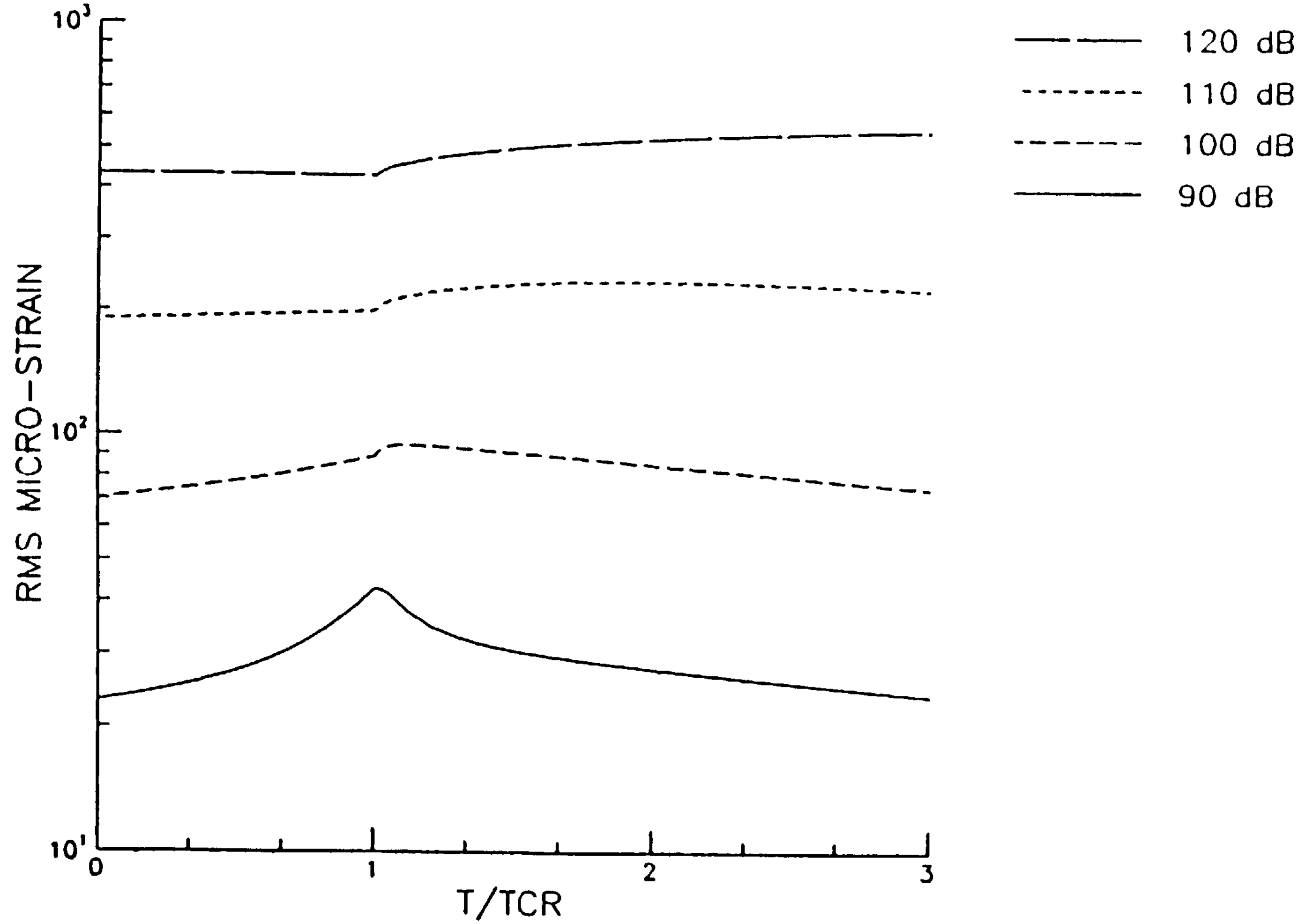

Figure 8. RMS micro-strian, long edge midpoint, versus temperature for a clamped plate subjected to a nonuniform temperature distribution

4. FINITE ELEMENT/NUMERICAL SIMULATION OF NONLINEAR RANDOM RESPONSE OF COMPOSITE PLATES

Large deflection random responses of composite plates using the finite element method and time domain numerical simulation [20-22] have been investigated by Robinson. The finite element nonlinear equations of motion for a composite plate subjected to random excitation are derived by application of the principle of virtual work. A set of nonlinear algebraic equations is then obtained with a weighted-satisfaction of the governing equations of motion over a time interval. The input Gaussian white excitation is generated by filtering a white noise process through a linear filter with the desired cut-off frequency (6 kHz), drop-offs and amplifications. The Wilson-θ parameter iteration algorithm is employed for longer time interval step. Five types of response statistics are computed, they are the moments, spectra, auto-correlation, probability distributions and peak probability distributions. Figures 9-11 show the maximum deflection and strain spectra, and strain probability distributions of a clamped $12 \times 15 \times 0.048$ in., graphite-epoxy (E_1 = 30 Mpsi, E_2 = 1.2 Mpsi, G = 0.65 Mpsi, ν_{12} = 0.25 and $\rho = 1.449 \times 10^{-3}$ lb-s/in.^{4}) with stacking sequence $(90/45/-45/0)_s$ at 123 and 153 dB. The spectra show the peak broadening behavior of the nonlinear stiffness and the non-zero mean strain due to large deflection. The strain spectrum at 123 dB also indicates a second peak due to membrane straining.

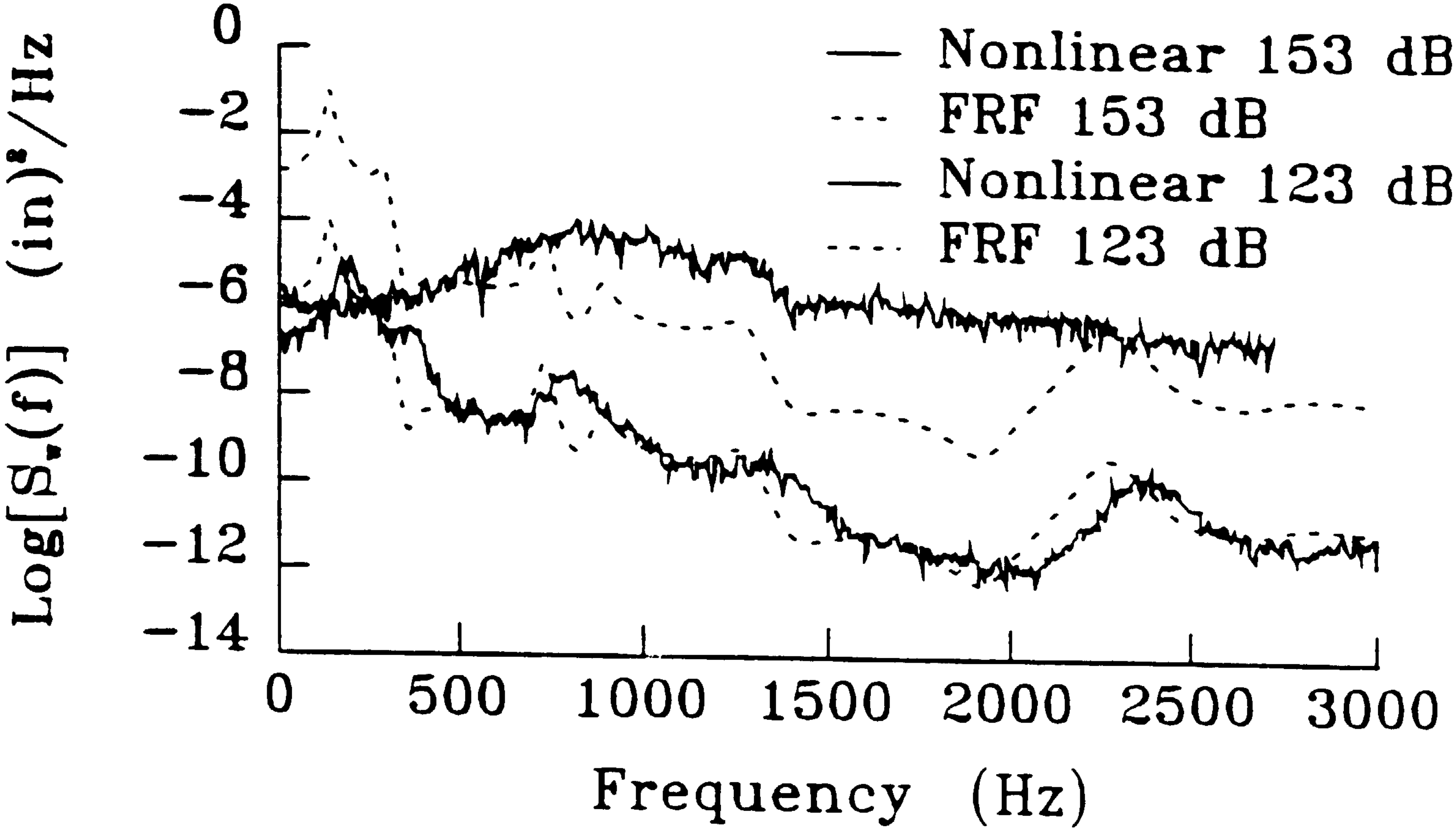

Figure 9. Frequency response functions and nonlinear maximum deflection spectra of a $(90/45/-45/0)_s$ rectangular clamped plate

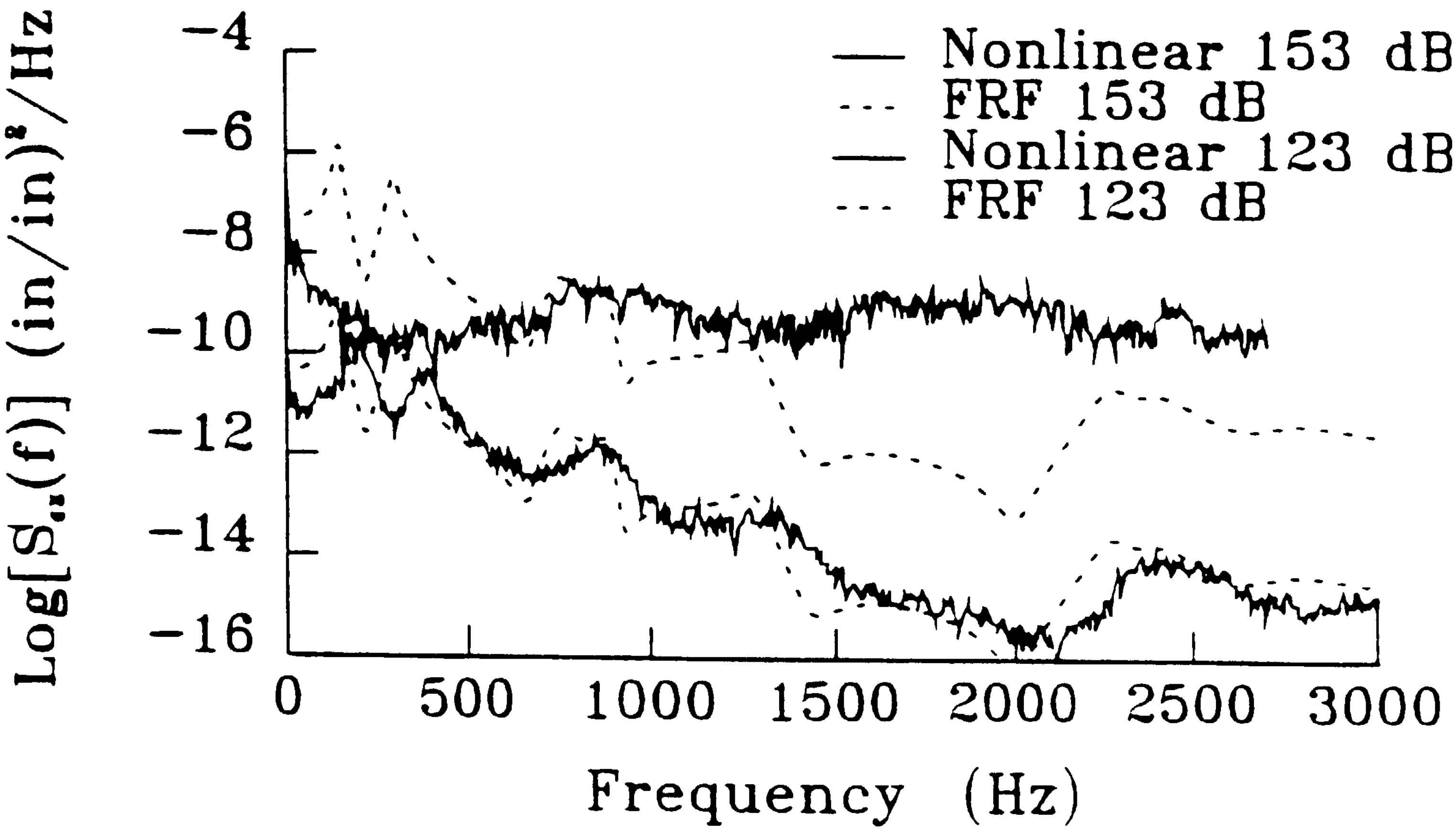

Figure 10. Frequency response-functions and nonlinear strain $(\epsilon)_x$ spectra of a $(90/45/-45/0)_s$ rectangular clamped plate

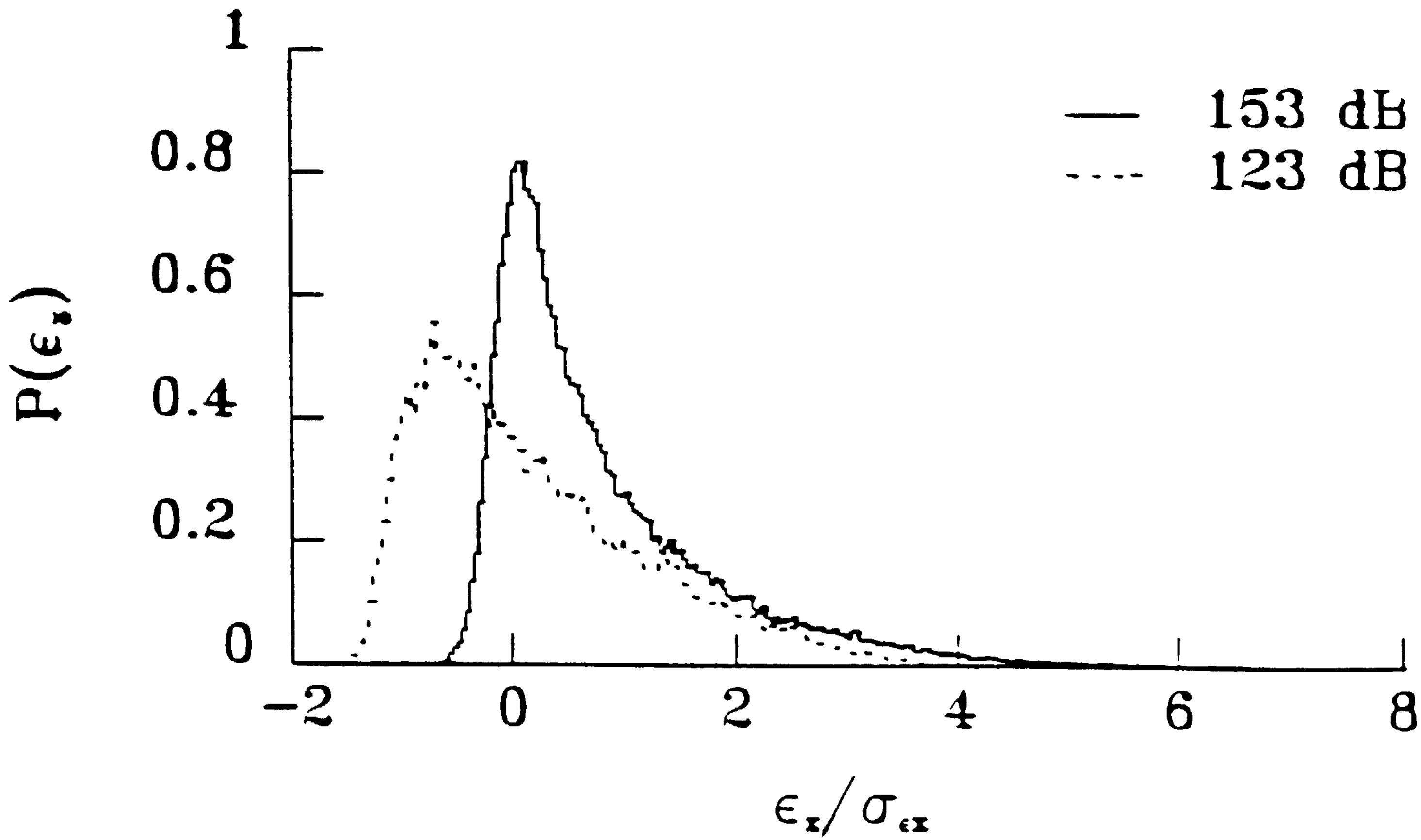

Figure 11. Probability density for nonlinear strain $(\epsilon)_x$ of a $(90/45/-45/0)_s$ rectangular clamped plate

5. THERMAL BUCKLING AND POST-BUCKLING, AND VIBRATIONS OF THERMALLY BUCKLED COMPOSITE PLATES

The finite element system equations of motion for an arbitrarily laminated composite plate undergoing small amplitude vibrations and subjected to temperature change $\Delta T(x,y)$ are derived [23, 24] and they are of the form as:

$$[M]\left\{\ddot{W}\right\} + \left([K] - [K_{N\Delta T}] + \frac{1}{2}[N1] + \frac{1}{3}[N2]\right)\{W\} = \{P_{\Delta T}\} \tag{5.1}$$

where $[M]$ and $[K]$ are the system mass and linear stiffness matrices, $[K_{N\Delta T}]$ is the geometric stiffness matrix due to thermal membrane force vector $\{N_{\Delta T}\}$, and $[N1]$ and $[N2]$ are the first-order and second-order nonlinear stiffness matrices due to large deflections. The matrices $[N1]$ and $[N2]$ are linearly and quadratically dependent on the system displacement vector $\{W\}$, respectively. The $\{P_{\Delta T}\}$ is a steady-state thermal load vector due to temperature distribution $\Delta T(x,y)$. A novel solution procedure has been developed in [23, 24] by considering the system displacement $\{W\}$ as the sum of a time-independent particular solution $\{W\}_s$ and a time-dependent homogeneous solution $\{W(t)\}_t$ with the assumption that the vibration amplitude $\{W\}_t$ is small; that is

$$\{W\} = \{W\}_s + \{W(t)\}_t \tag{5.2}$$

Substituting this into the system equations of motion, Eq. (5.1) can be resulted into two sets of equations in the form as

$$\left([K] - [K_{N\Delta T}] + \frac{1}{2}[N1]_s + \frac{1}{3}[N2]_s\right)\{W\}_s = \{P_{\Delta T}\} \tag{5.3}$$

and

$$[M]\left\{\ddot{W}\right\}_t + ([K] - [K_{N\Delta T}] + [N1]_s + [N2]_s)\{W\}_t = 0 \tag{5.4}$$

where $[N1]_s$ and $[N2]_s$ denote the first and second-order stiffness matrices that are linearly and quadratically dependent on $\{W\}_s$. Equation (5.3) is set of nonlinear algebraic equations which yields the thermal post-buckling deflection under thermal load $\{P_{\Delta T}\}$. It is solved using incremental temperature steps and a Newton-Raphson iterative scheme to arrive at the converged post-buckling deflection $\{W\}_s$. Equation (5.4) is an basic eigenvalue problem, and it is evident that the temperature ($[K_{N\Delta T}]$) and thermally buckled equilibrium position ($[N1]_s$ and $[N2]_s$) effects on the vibration characteristics have been both considered.

Figures 12 and 13 compare the thermal post-buckling deflections between the finite element and classic analytical solutions. Figure 12 is for a clamped square aluminium plate and the classical solution by Paul [25] is chosen because it uses 25 modal functions in the post-buckling analysis and should be very accurate. Uniform and nonuniform temperature distributions

$$\Delta T(x,y) = T_o\left(1 - \cos\frac{2\pi x}{a}\right)\left(1 - \cos\frac{2\pi y}{b}\right) \tag{5.5}$$

are considered. The finite element results compare very favorably with Paul's.

The post-buckling behavior of a simply supported, $(\pm 45/0/90)_s$, square (10 in.), graphite-reinforced composite plate under uniform temperature is compared with Meyers and Hyer's classic result [26]. As the plate lamination is symmetric, the critical temperature will be a bifurcation point, at which the plate will buckle in either in the positive or negative direction, and it will then follow one the deflection curves in the post-buckling range as the temperature increases.

The influence of moderately large initial imperfections in deflection on thermal post-buckling behavior can be easily considered in the finite element method. Figure 14 shows the maximum thermal deflection of a simply supported, square (12 in.) graphite-epoxy $(-30/30)_2$ laminate with three different initial maximum deflections w_i/h = 0.25, 0.50 and 0.75. The stiffness of the plate is increased as the initial deflection is increased.

The change in the fundamental frequency ω/ω_o versus temperature rise is shown in Fig. 15. As expected, the fundamental frequency decreases to zero at the buckling temperature. The effects that the initial imperfections in deflection has on the fundamental frequency is also shown in Fig. 15. Obviously, with any initial deflection, the plate will no longer buckle and the frequency will not decrease to zero with increasing temperature.

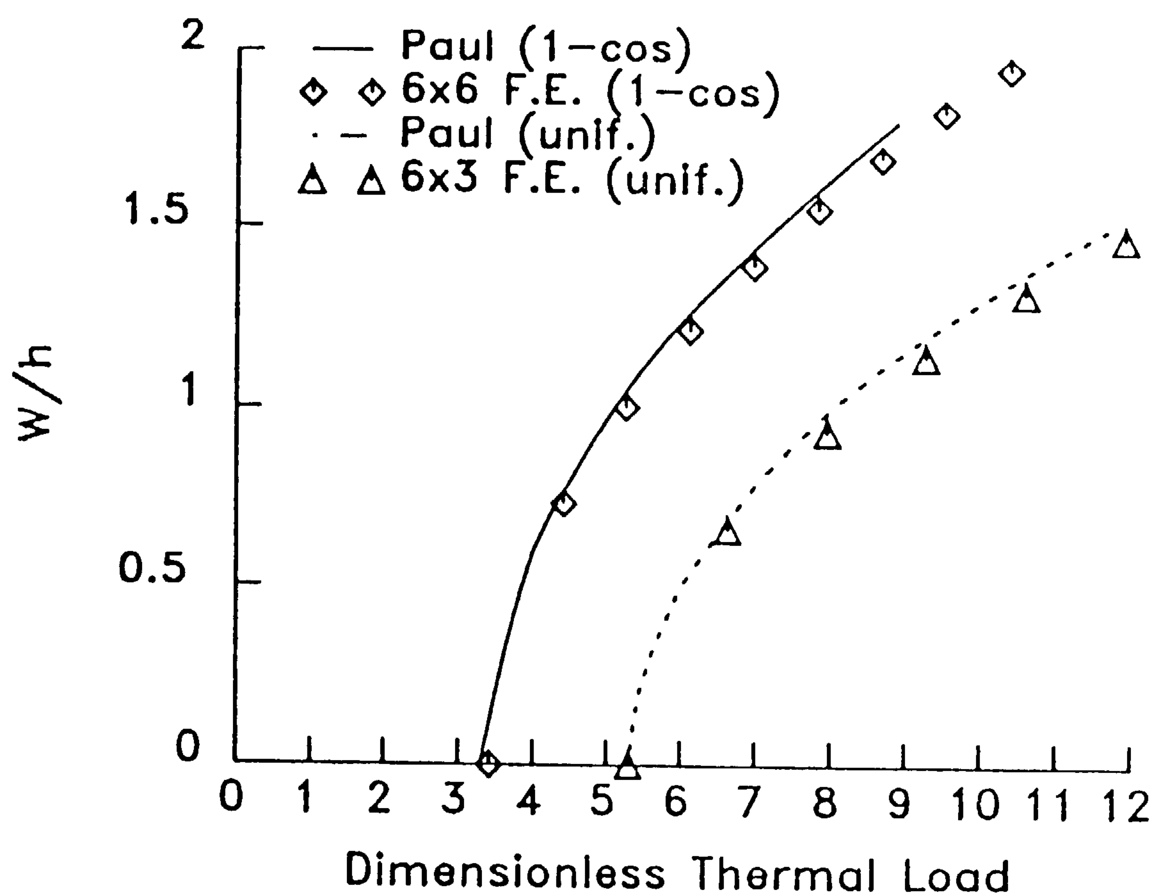

Figure 12. Comparison of finite element with Paul's 25-mode post-buckling results for clamped square aluminum plate subjected to uniform and nonuniform temperature distributions

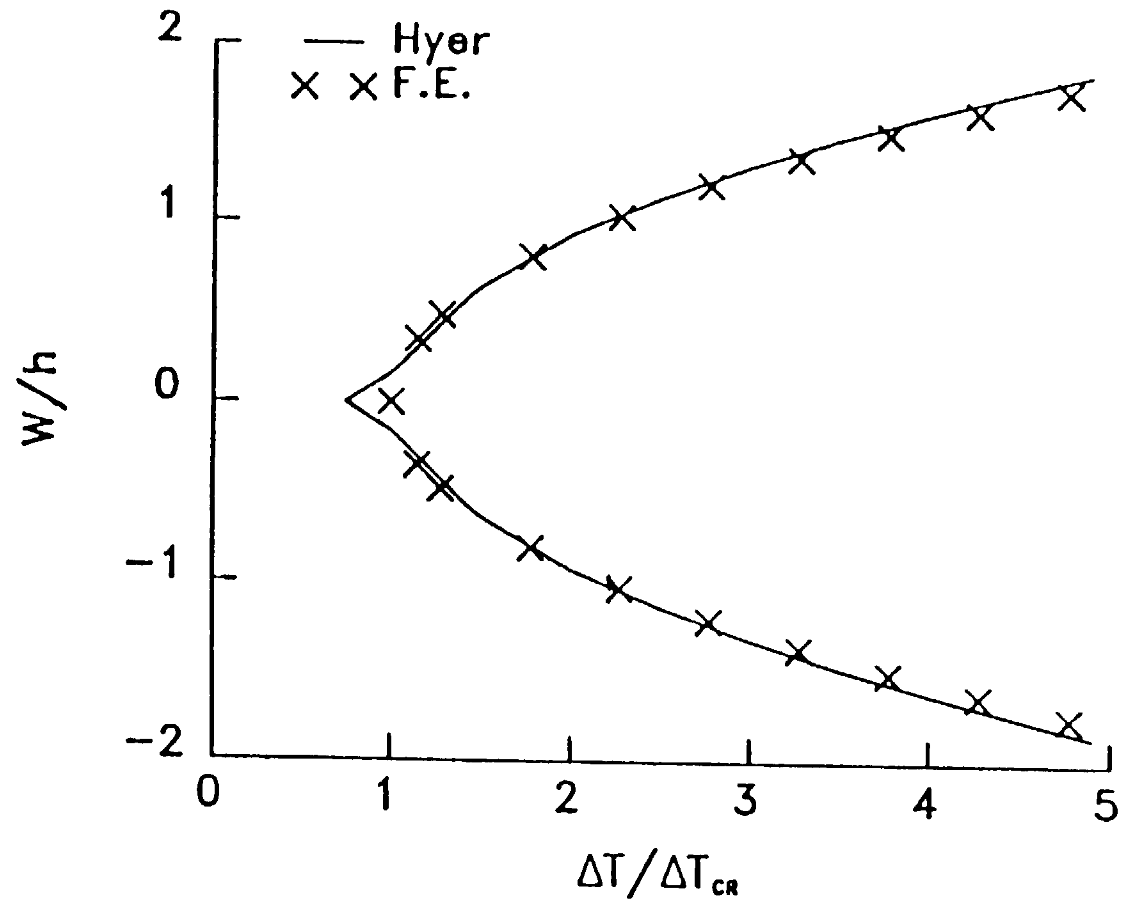

Figure 13. Comparison of finite element with Meyers and Heyer's post-buckling results for a simply supported square graphite-reinforced $(\pm45/0/90)_s$ laminate

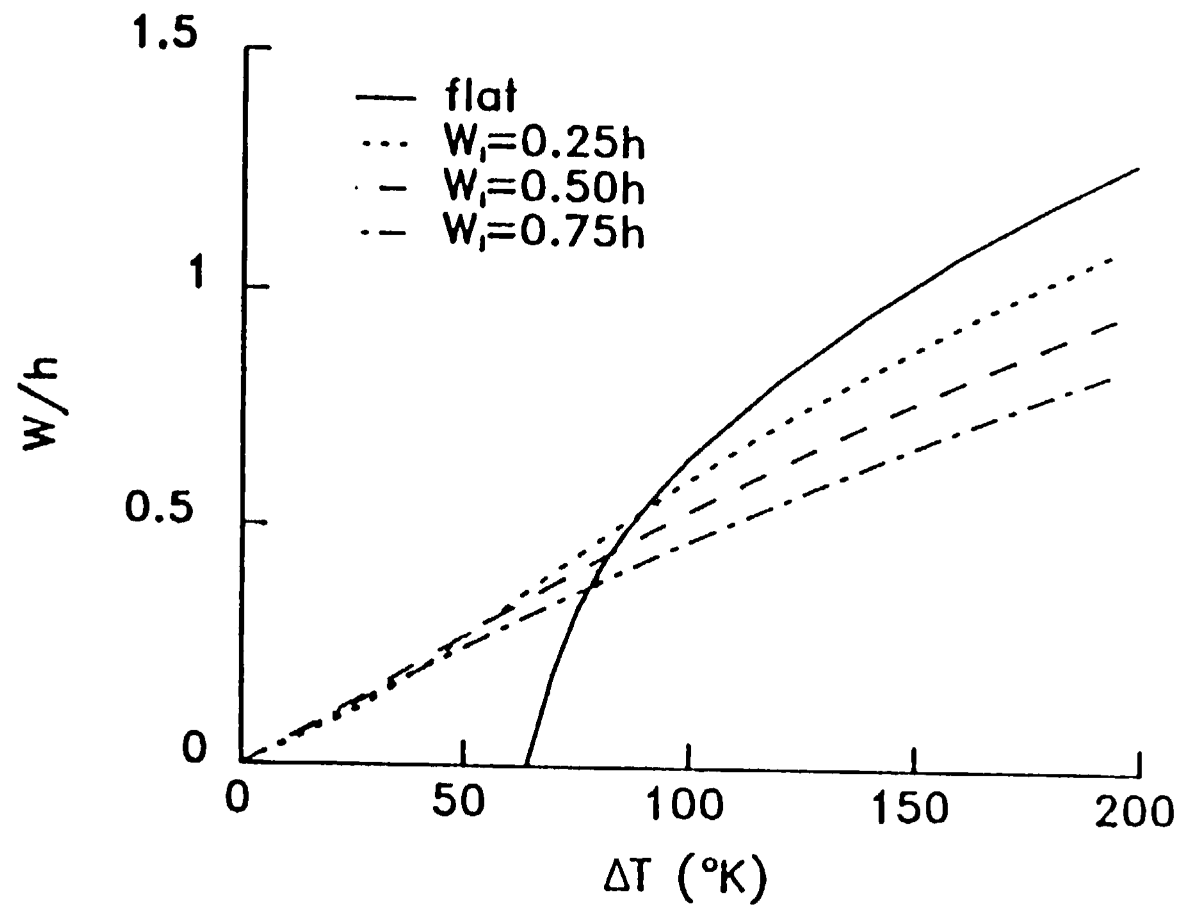

Figure 14. Influence of initial imperfections in deflection on thermal post-buckling behavior for a simply supported square graphite-epoxy $(-30/30)_2$ laminate.

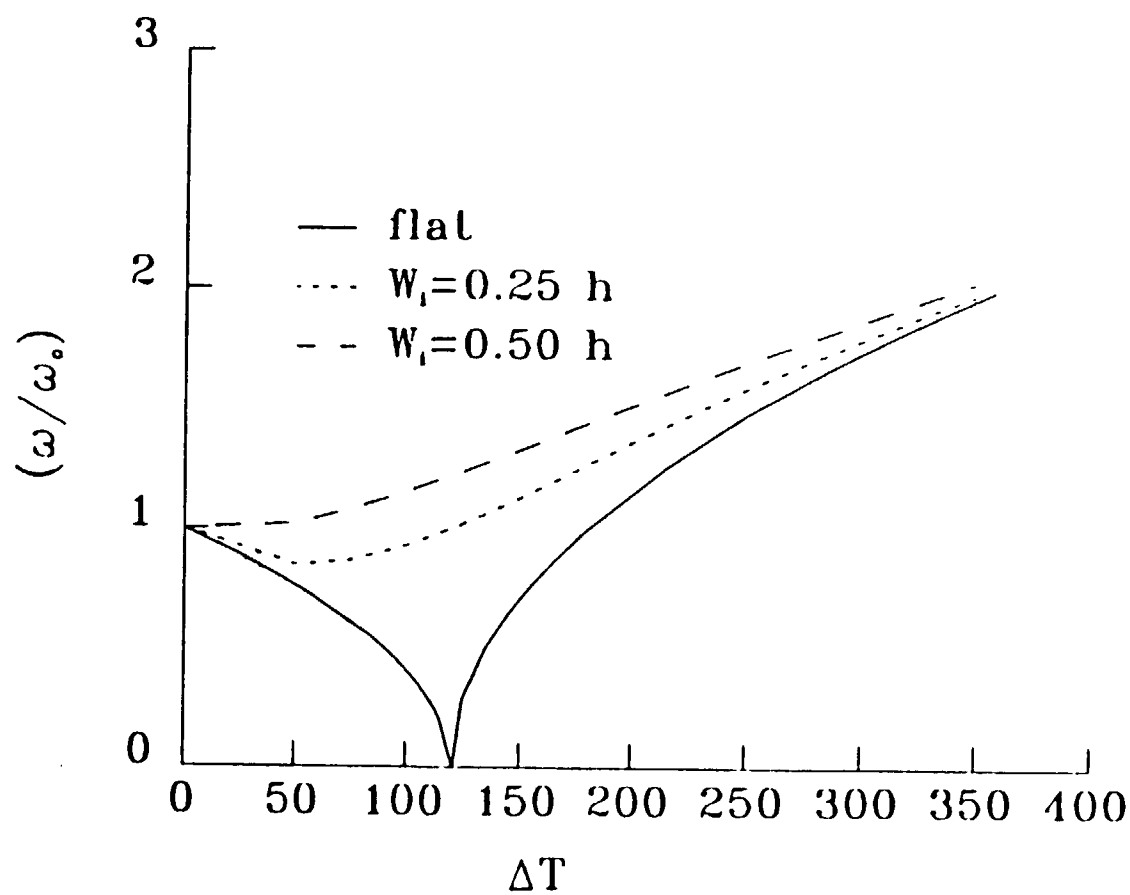

Figure 15. Fundamental frequency ratio versus temperature for a simply supported square graphite-epoxy $(-30/30)_2$ laminate with initial imperfections in deflection

6. PREDICTION OF THE RADIANT FIELD PRODUCED BY QUARTZ HEATING SYSTEMS

The ability to predict the radiant heat flux distribution produced by a reflected quartz heating system is desirable for use in thermal-structural analyses. Therefore, a classical analysis technique is developed for this purpose and proved very effective in the analysis of a single unreflected quartz heater [27]. This classical analysis was extended to incorporate the effects of reflector. Through this expansion process, serious limitations of the classical analysis were discovered [28]. Therefore, it was decided that a more versatile and robust technique must be applied to these more complicated heating systems.

A Monte Carlo simulation method [29] is thus developed for predicting the radiant heat flux from a heating system. The heating system consists of tungsten-filament tubular quartz lamps and individual or common (multi-lamp) reflectors. The simulation method allows for a multitude of model refinements not amenable to the classical analysis developed earlier. Attempts at correlating with experimental results revealed that a lamp must be treated as combination of two sources, where the quartz envelope gains a majority of its energy from the emissions of the corresponding filament. The source augmentation due to reflection and contributions from other sources is modeled in the simulation.

The simulated results compare well with experimental measurements from the three systems tests. The first system consists of single lamp of 25.4 cm lighted length. The lamp is placed 3.33 cm beneath a highly polished titanium reflector and 15.24 cm above the Gardon-type heat flux sensor plane. The success of the Monte Carlo simulation shown in Fig. 16 is primarily due to the treatment of the lamp as two interacting sources. It also indicates that the inclusion of those higher order effects is very important even for the simple cases where reflector and source interaction is minimal.

Experimental radiant heat flux were measured for a second system consisting of a single (63.5 cm) lamp and a parabolic reflector and 30.48 cm above the sensor plane. The test data are shown in Fig. 17 in comparison to the simulation predictions.

Figures 18 and 19 show the radiant heat flux results of the third system, a single lamp or six-lamp with a multiple six-lamp reflector, respectively.

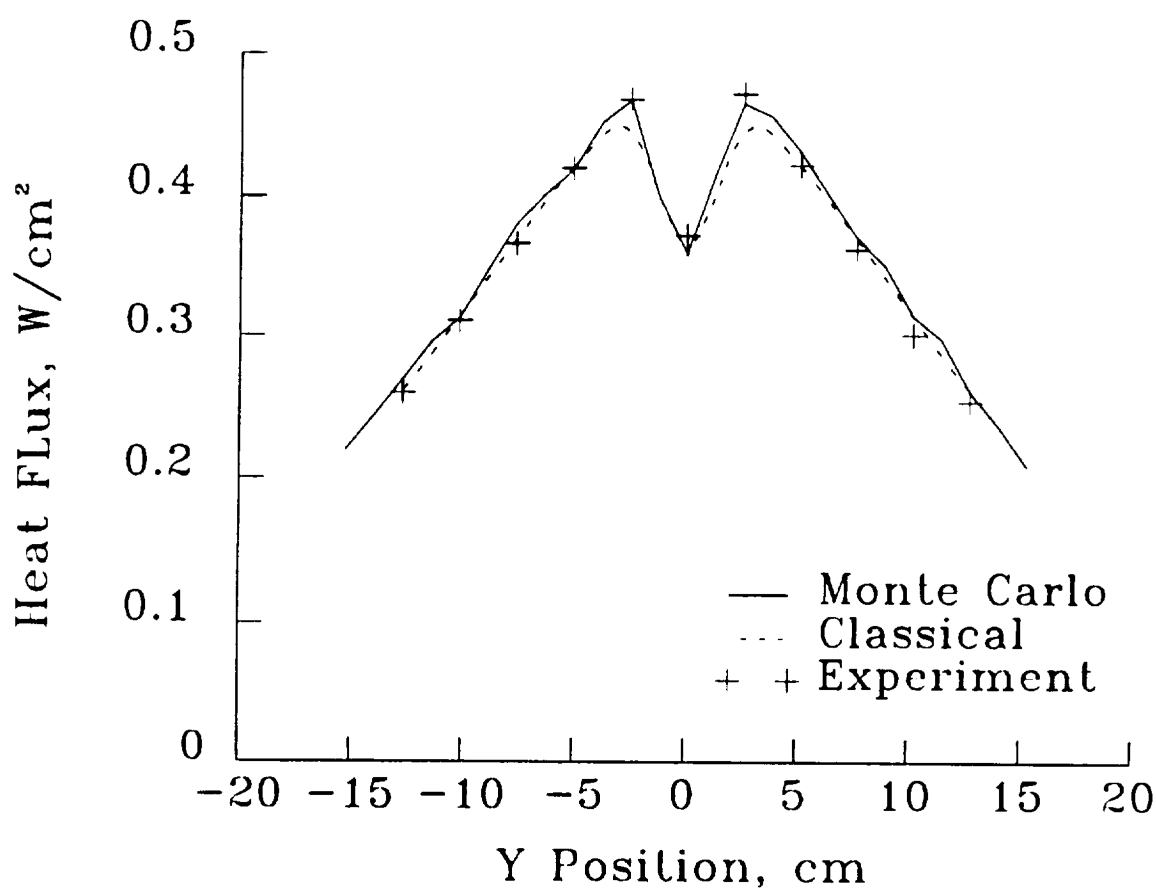

Figure 16. Transverse heat flux distribution for the single lamp with flat reflector

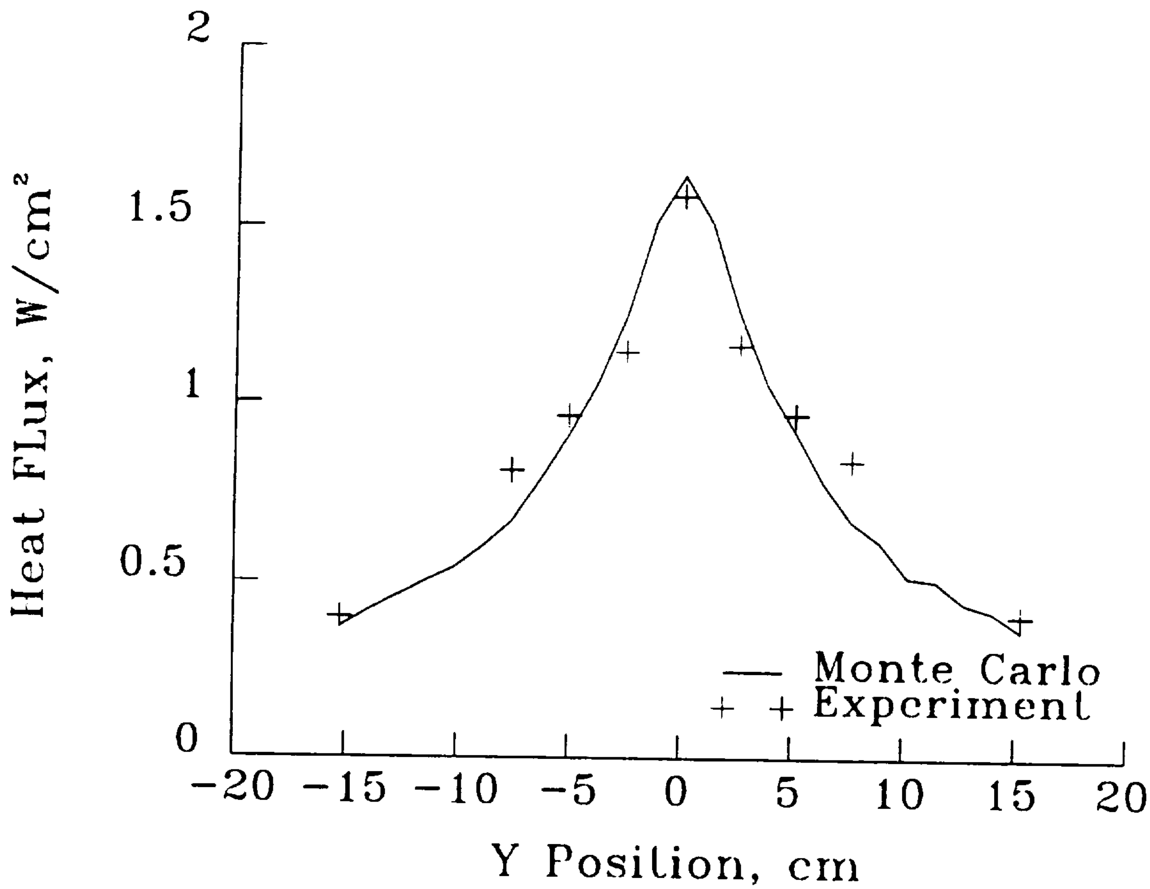

Figure 17. Transverse heat flux distribution for the single lamp with parabolic reflector

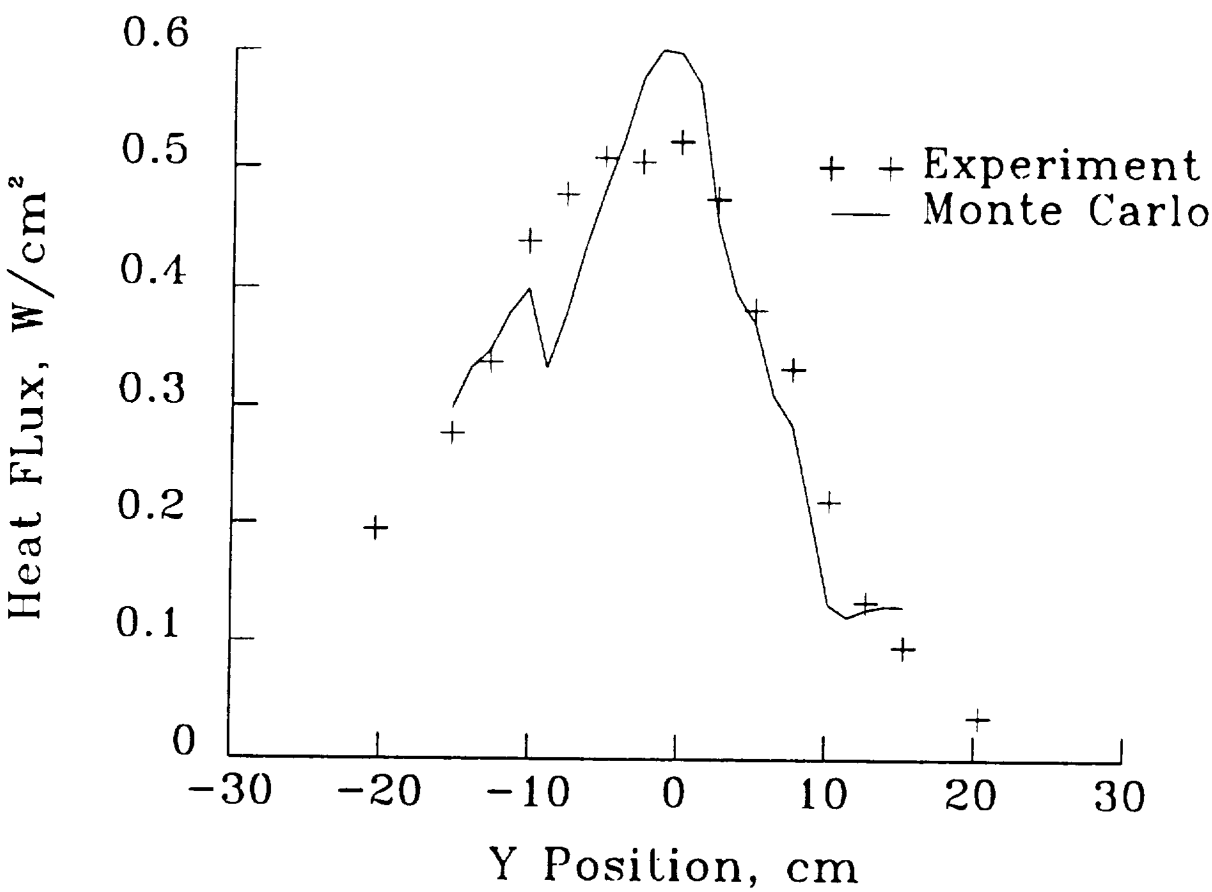

Figure 18. Transverse heat flux distribution for the single lamp with multi-lamp reflector

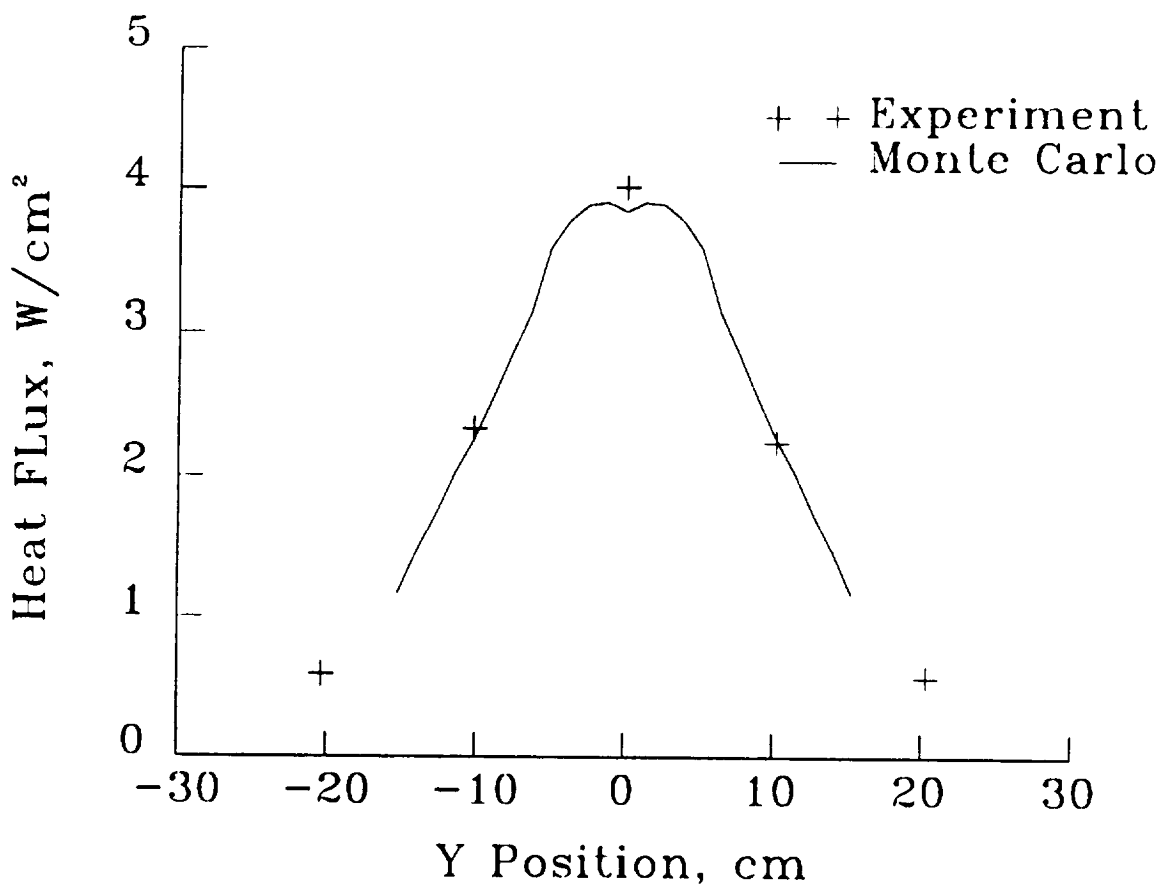

Figure 19. Transverse heat flux distribution for the six-lamp heating system

7. BOUNDARY ELEMENT RESEARCH

The Boundary Element Method (BEM) is a good approximating technique that is widely used in the solution of acoustic field problems. BEM needs less grid generation and computational time, and is perfectly suited to study far-field acoustics. Over the past ten years, interest in boundary element techniques has been very popular.

In the development of the newer supersonic and hypersonic flight vehicles, an increased need for the analysis of acoustic fields and the response of structures to acoustic loads has become necessary. Research in the analysis of these acoustic fields in ducts has dominated much of the boundary element research. The first objective was to study the boundary element method for acoustic problems in a two-dimensional field. Many types of problems with complicated boundary conditions were investigated. All of the selected problems had exact solutions. These exact solutions were used to validate the boundary element as an accurate approximating method. Two types of elements were used: constant elements and linear elements. Both types of elements gave very good approximations for all selected problems. As expected, the linear elements produced less error than the constant elements for all of the problems For one problem, a frequency graph was plotted for linear elements and compared with the exact solution shown in Fig. 20 [30, 31]. The linear elements exactly predicted the pressure for wavenumbers less than 2. For higher wavenumbers, the pressure values begin to deviate from the exact solution slightly. From running different cases of varying wavenumbers, a very important idea was learned about element discretization. There must be at least four elements per wavelength to give accurate solutions.

The next step in research was to expand the domain problem to ducts with sudden area changes [32]. Two basic muffler type problems were selected for comparison with the boundary element method. In each case, constant and linear element results were compared with the exact solutions. For both problems, the constant and linear results gave very good approximations for low wavenumbers. Since the number of elements did not change, it was expected that the results would deviate from the exact solution as the wavenumber increased. This was evident in the frequency plots as the wavenumber increased. After validating the boundary element method for sudden area changes in ducts, another duct problem was selected where no exact solutions exist, Fig. 21. The transmitted pressure was studied as the angle of the mid-section was rotated. As the angle increased from 0 to 45 degree, it was expected that the transmitted pressure would decrease exponentially. As seen in Fig. 21, the pressure did decrease very rapidly as the angle increased. This type of duct problem was selected to show that the boundary element method can handle a wide range of very complicated problems.

The next research step was to study three-dimensional duct problems with exact solutions. This has been done for very simple problems. The computer code is now being updated to handle more complicated three-dimensional problems. Research is also being done in the area of combining finite and boundary element methods, finite element for modeling structures and boundary element for modeling acoustics. This seems to be a very fast growing concept because boundary elements and finite elements both offer special features that compliment each other. By combining the two methods, more realistic and complicated problems can be handled.

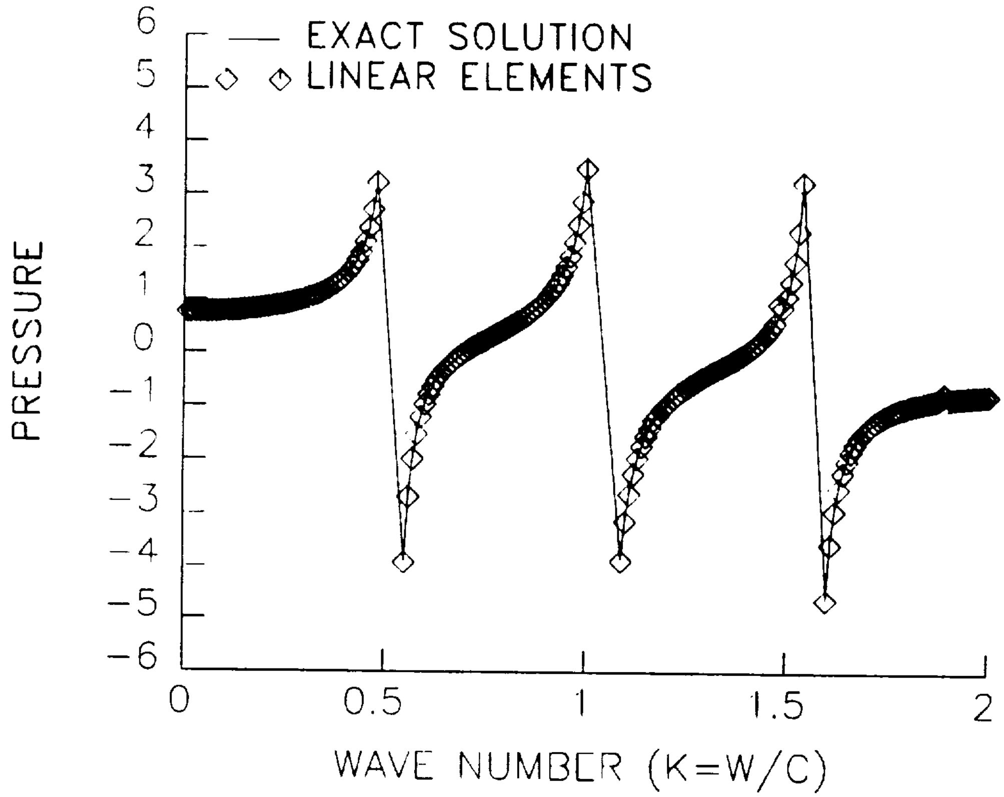

Figure 20. Comparison of boundary element and exact solution of a two-dimensional duct

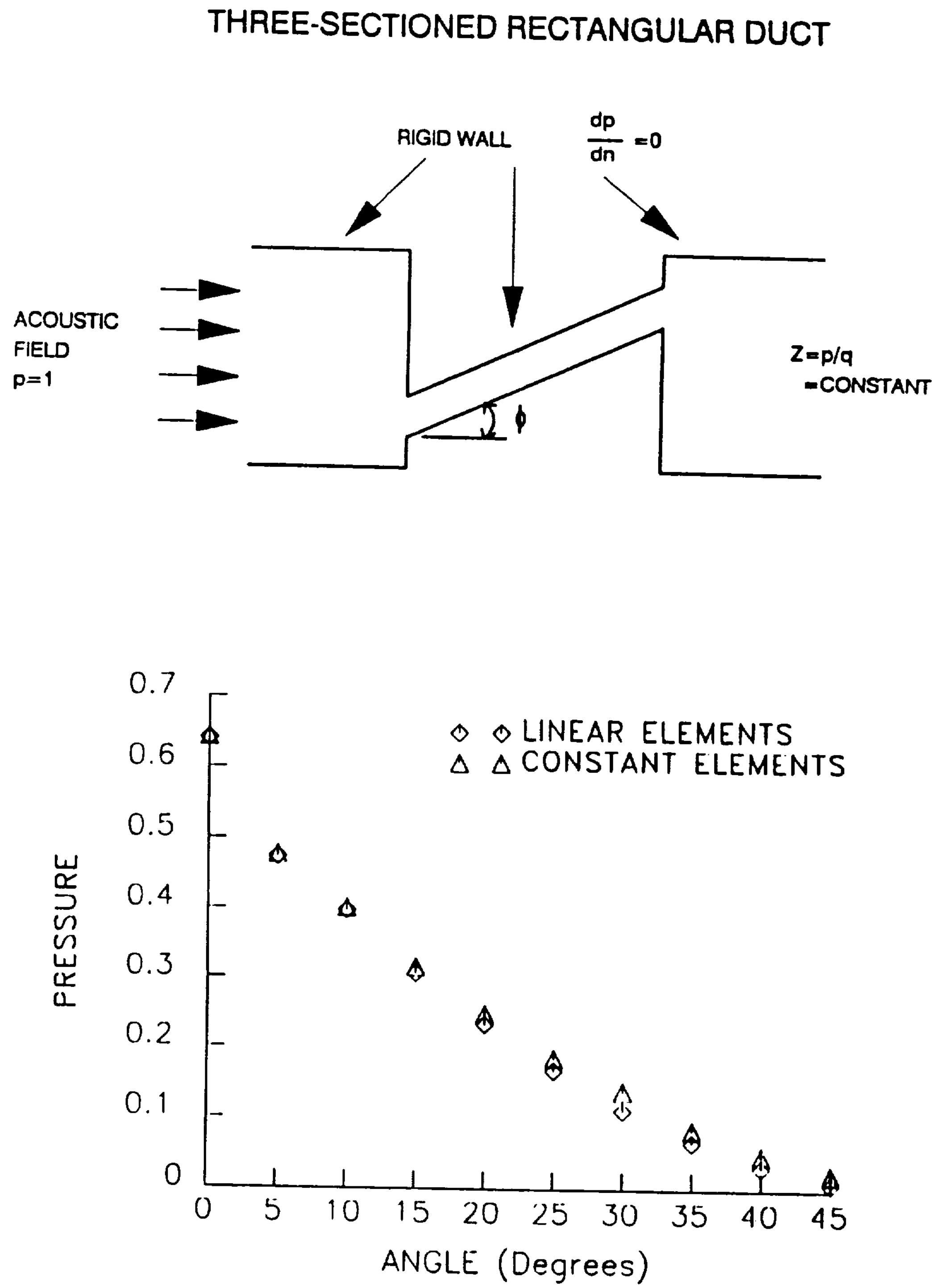

Figure 21. Transmitted pressure versus angle

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