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THREE-DIMENSIONAL SURFACE GRID GENERATION FOR CALCULATION OF THERMAL RADIATION SHAPE FACTORS

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SUMMARY

The present paper describes a technique to generate three-dimensional surface grids suitable for calculating shape factors for thermal radiative heat transfer. The surface under consideration is approximated by finite triangular elements generated in a special manner. The grid is generated by dividing the surface into a two-dimensional array of nodes. Each node is defined by its coordinates. Each set of four adjacent nodes is used to construct two triangular elements. Each triangular element is characterized by the vector representation of its vertices. Vector algebra is utilized to calculate all desired geometric properties of grid elements. The properties are used to determine the shape factors between the element and an area element in space. The generated grid can be graphically displayed using any software with 3-dimensional features. In the present paper, DISSPLA was used to view the grids

INTRODUCTION

The thermal radiation shape or configuration factor between two surfaces has been the subject of many investigations. The determination of this factor is important in thermal radiation heat transfer applications which are encountered in a wide variety of engineering systems. Analytical derivation of shape factors, even for simple configurations, is very complex. Therefore, numerous tables have been generated and tabulated in the literature for basic geometries [e.g., see Hamilton and Morgan (reference 1) and Howell (reference 2)]. Despite the existence of these tables, different configurations are often needed so that it is not practical to tabulate all possible geometries. Furthermore, cases involving complex geometries may result in analytically non-integrable expressions. Therefore, numerical methods become more attractive to use in such cases, especially when incorporated in a computer code which handles the heat transfer calculations.

This paper describes a grid generation technique which was developed to aid in the numerical calculation of the shape factor between an area element and an arbitrary three-dimensional surface in space. In the present scheme, a given three-dimensional area is discretized into finite triangular elements (FTE) using the surface nodes in a special manner. Each triangular element is characterized by the vector representation of its vertices. The differential shape factor between a differential area element whose location and orientation

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are fixed and each triangular element is calculated using the geometric properties of the triangle. The shape factor between an area element and the entire surface is obtained by summation of the differential shape factors for all triangles. This method is applicable to generalized three-dimensional areas without restrictions. However, blockage or shadowing effects due to other parts of the surface must be taken into account when the shape factor calculations are performed. Grids for selected sample cases were generated and the corresponding numerically calculated shape factors were compared with the analytical solutions in order to validate this technique.

SHAPE FACTOR EVALUATION

The shape factor between two differential planer area elements dA_1 and dA_2, shown in Figure 1, is defined (e.g., reference 1) as

$$F_{dA1-dA2} = \frac{\cos\theta_1 \cos\theta_2}{\pi S^2}\, dA_2 \tag{1}$$

where $F_{dA1\text{-}dA2}$ is the shape factor representing the fraction of the energy leaving the area element dA_1 that is arriving at the area element dA_2; θ_1 and θ_2 are the angles from normal for the surfaces 1 and 2, respectively; and S is the distance between the two surfaces.

In practical applications, it is usually desired to determine the radiant heat transfer between a planer differential area and a finite surface. Thus, the corresponding shape factor, $F_{dA1\text{-}A2}$, is obtained by integrating Equation 1 over A_2 as follows:

$$F_{dA1-dA2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2}\, dA_2 \tag{2}$$

Equations 1 and 2 are the basic equations governing the view of one surface relative to another in space. Equation 2 can be integrated to define an overall shape factor between two finite areas as outlined by Siegel and Howell (reference 3). The mathematical treatment of such equations is tedious. In addition, Equation 2 is integrable only for relatively simple geometries. Although numerous tables were generated and published in the literature (e.g., references 1 and 2), different geometries may be encountered in practical applications. Thus, approximate or numerical methods must be used in these cases.

A numerical integration of Equation 2 can be achieved for an arbitrary surface, A_2, by dividing the surface into discrete elements. An elemental shape factor for an element, ΔA_2, is expressed by replacing Equation 1 by its equivalent in finite form as

$$F_{dA1-\Delta A2} = \left(\frac{\cos\theta_1 \cos\theta_2}{\pi S^2}\right) \Delta A_2 \tag{3}$$

Then Equation 2 can be evaluated by the following summation:

$$F_{dA1-A2} = \frac{1}{\pi} \sum_{i=1}^{n} \left(\frac{\cos\theta_1 \cos\theta_2}{S^2}\right)_i \Delta A_i \tag{4}$$

where n is the total number of elements generated and i is an index representing the ith element. A special grid generation method has been developed to construct a mesh of finite triangular elements (FTE). The elements are used to calculate the shape factor expressed in Equation 4. Details of the calculations of the terms in Equation 4 are beyond the scope of this paper.

GRID GENERATION METHOD

The present grid generation scheme was developed for an arbitrary three-dimensional surface in space. The surface under consideration can be defined by intersecting lines, as demonstrated by Figure 2. The points of intersection of these lines are the nodes used to construct the triangular elements as outlined below.

Consider a node b generated from the intersection of the ith α line with the jth β line as shown in Figure 2. This node is defined by its coordinates. For each node b (i,j), three additional neighboring points, c(i + 1,j), e(i,j + 1), and f(i + 1,j + 1), are generated in a similar manner. Two planer triangles are constructed using these four adjacent nodes. Each triangle is defined by the coordinates of its vertices. Using vector representation of the nodal points b, c, and f, triangle 1 is defined by vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, while triangle 2 is described by vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ for a typical (i,j) node, as illustrated in Figure 3.

For the purpose of shape factor calculations, the terms in Equation 4 require, for each element, the values of the area, location, and direction of its normal vector. The centroid of a triangular area was chosen to represent its location in space.

The scheme described above is general and is applicable to any arbitrary surface. For a given geometry, one must define the surface of interest in terms of its nodal points before using this scheme. Some simple configurations were included herein for demonstration purposes. The shape factors calculated using the present method were compared with tabulated formulas for examples discussed in the following section. The objective of the present paper is to test and demonstrate the grid generation method and, therefore, no attempt was made to optimize the grid or to perform an error analysis. For visualization purposes, the generated grids were plotted using the DISSPLA graphics system.

EXAMPLES

In order to test the present scheme. comparisons with tabulated shape factors were made for some example cases. In each case, the geometry for the area A_2 was used to generate the nodal points covering the surface under consideration. A general FTE grid generator using this nodal information was used to calculate the geometric properties of the triangular elements. These properties were used to calculate the shape factor defined in Equation 4. The following four configurations were considered to test the present scheme: (1) rectangular area parallel to an element adjacent to one corner (Figure 3), (2) circular area parallel to an element (Figure 4), (3) cylinder and an element parallel to its axis with its normal passing through one end (Figure 5), and (4) sphere and an element with its normal passing through the center (Figure 6). These test configurations, chosen for their simplicity, do not imply any limitation on the present scheme and were intended for validation only.

The results for Configuration 1 are summarized in Table I for various values of A/C and B/C (see Figure 3). Table I also includes the exact results obtained from the closed form solution reported by Hamilton and Morgan (reference 1) for comparison. All runs were made for a grid size of 25 x 20. Thus, the total number of triangular elements generated was 1000 in each run. The results are in excellent agreement with the exact solution for this grid. In general, the accuracy of the results depends on the grid size and the location of the element relative to the finite area. The grid can be plotted using any three-dimensional plot system to view the surface considered. Figure 7 is a computer-generated plot of the triangular elements constructed for a rectangle with A/B = 2. The plot shown was produced by the DISSPLA graphics system.

Table II summarizes the results for Configuration 2 (Figure 4) for various values of R/C and C/A. In this case, the nodal points were generated by choosing the α lines in the radial direction and the β lines in the tangential direction. The center of the circle can be avoided numerically by using a small concentric circle with a radius approaching zero. The results were compared with the exact solution reported by Hamilton and Morgan (reference 1) for a 25 x 25 grid. Figure 8 shows the plotted geometry.

Configuration 3 is an example of a three-dimensional surface. The surface nodes were generated from the intersection of the lines parallel to the cylinder axis (α lines) and circular lines in the tangential direct (β lines). The results summarized in Table III were obtained using a 20 x 50 grid and were compared with the exact solution reported by Hamilton and Morgan (reference 1). The grid is displayed in Figure 9 for A/R = 5.

The shape factor between a sphere and a differential element has been treated extensively in the literature, and formulas were derived for various cases involving the location and orientation of the element relative to the sphere. These formulas were summarized by Howell (reference 2). Configuration 4 is the simplest case in which the normal to the element passes through the center of the sphere. Table IV compares the results of the numerical calculations with the exact solution for various values of C/R and a 40 x 40 grid. Figure 10 is a DISSPLA output of the spherical grid.

CONCLUSION

This paper demonstrates the use of a special grid generator for calculating thermal radiation shape factors. The present method is based on finite triangular elements which are used to integrate the shape factor equation numerically. It has the advantage of being applicable to arbitrary three-dimensional surfaces without restrictions in addition to simplicity. Comparisons were made with tabulated results to validate the method.

REFERENCES

1. Hamilton, D. C.; and Morgan, W. R.: "Radiant-Interchange Configuration Factors," NASA TN-2836, 1952.

2. Howell, John R.: *A Catalog of Radiation Configuration Factors*, McGraw-Hill, New York, 1982.

3. Siegel, Robert; and Howell, John R.: *Thermal Radiation Heat Transfer*, Second ed, McGraw-Hill, New York, 1981.

Table I. Results of Configuration 1 for Various Values of A/C and B/C

A/C	B/C	EXACT SOLUTION	PRESENT METHOD
0.1	0.1	0.003141	0.003141
0.1	10	0.024865	0.024864
2	1	0.167375	0.167361
2	10	0.223400	0.223046
10	10	0.247971	0.246073

Table II. Results of Configuration 2 for Various Values of R/C and C/A

R/C	C/A	EXACT SOLUTION	PRESENT METHOD
0.5	1	0.06588	0.06515
0.5	10	0.1975	0.1959
1	1	0.2764	0.2736
1	10	0.4975	0.4957
5	1	0.9585	0.9608
5	10	0.9615	0.9622

Table III. Results of Configuration 3 for Various Values of A/R and C/R

A/R	C/R	EXACT SOLUTION	PRESENT METHOD
0.1	5	0.003050	0.003046
2	5	0.053239	0.053162
2	10	0.013430	0.013423
5	5	0.086983	0.086870
5	10	0.029211	0.029199
10	10	0.042189	0.042174

Table IV. Results of Configuration 4 for Various Values of C/R

C/R	EXACT SOLUTION	NUMERICAL
1.5	0.4444	0.4427
2	0.2500	0.2490
10	0.0100	0.00996
100	0.0001	0.0001

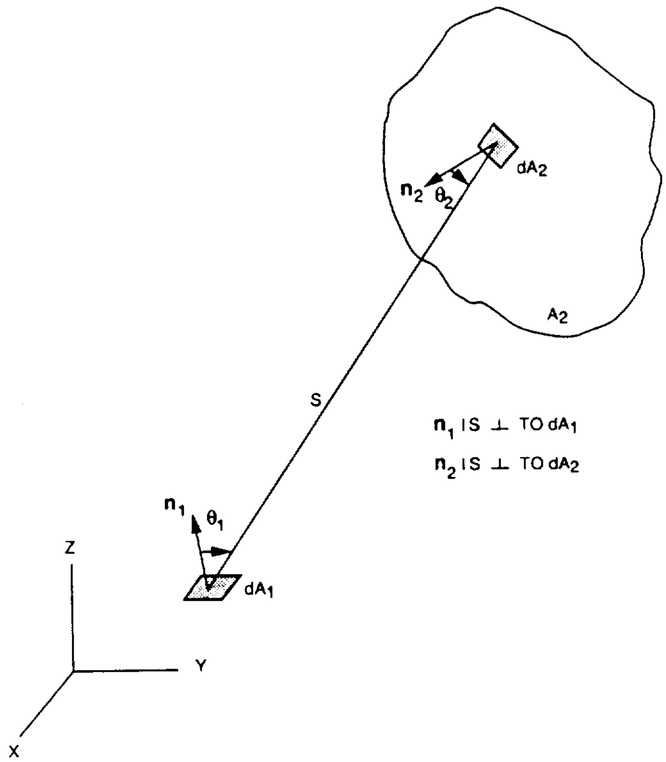

Figure 1. Geometric representation of shape factor between two differential elements.

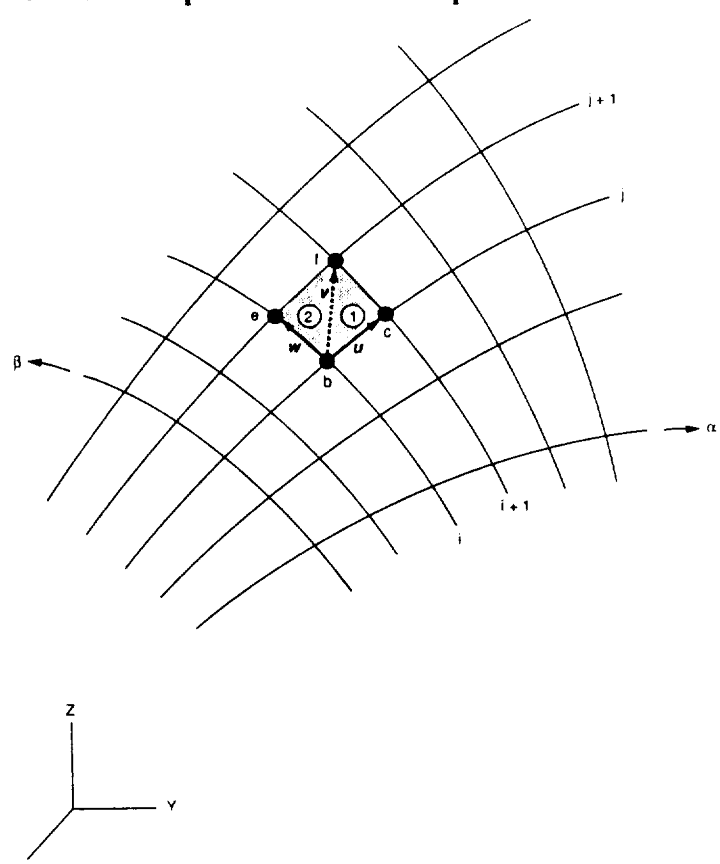

Figure 2. Illustration of FTE grid generation.

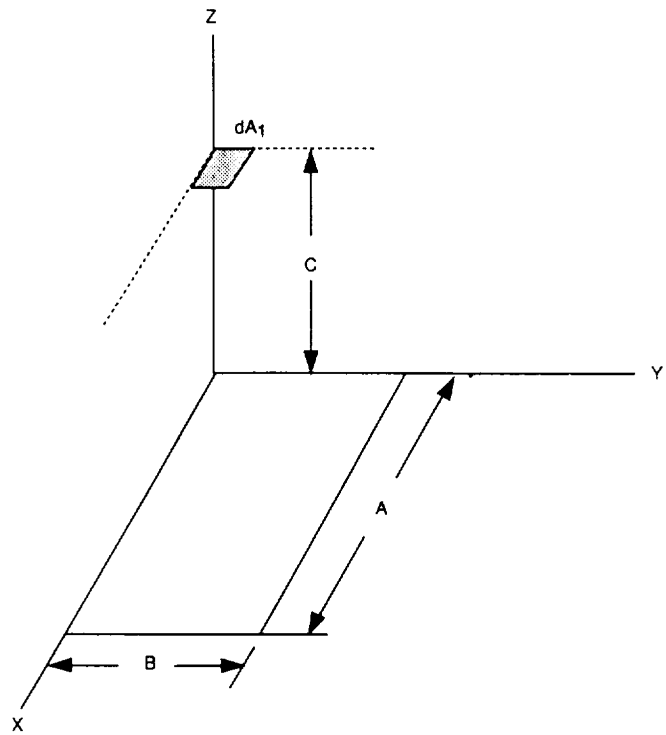

Figure 3. Geometry and coordinate system for Configuration 1.

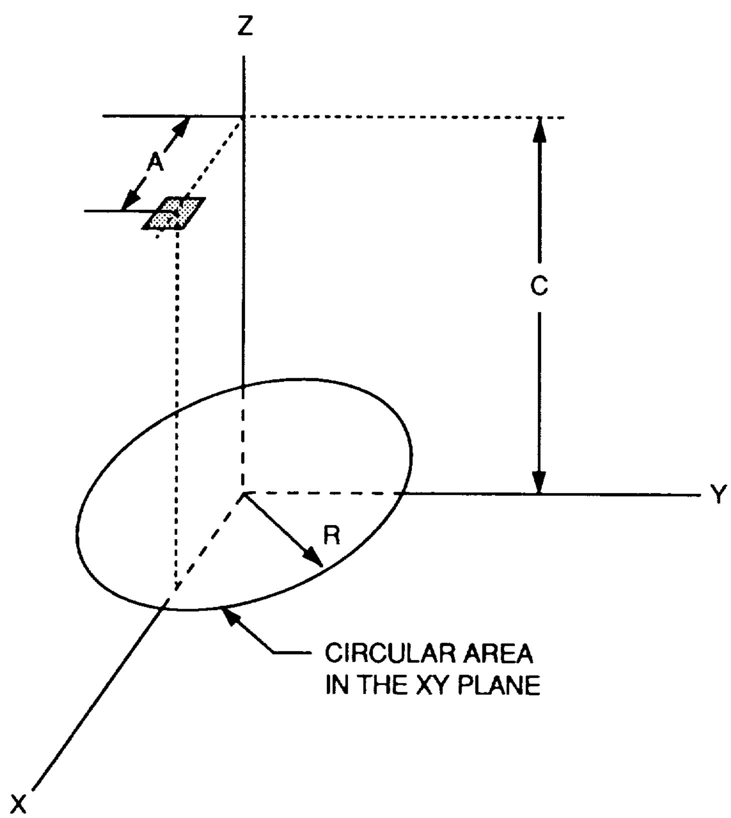

Figure 4. Geometry and coordinate system for Configuration 2.

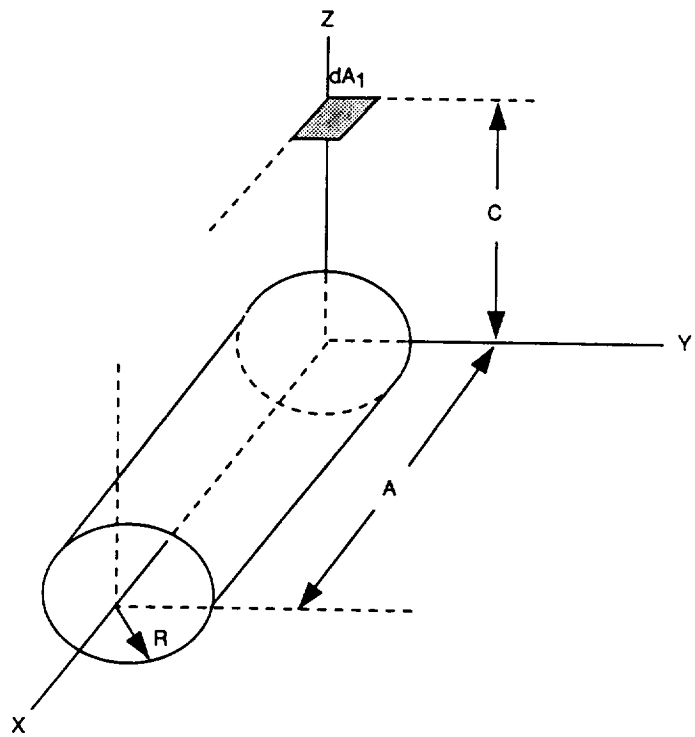

Figure 5. Geometry and coordinate system for Configuration 3.

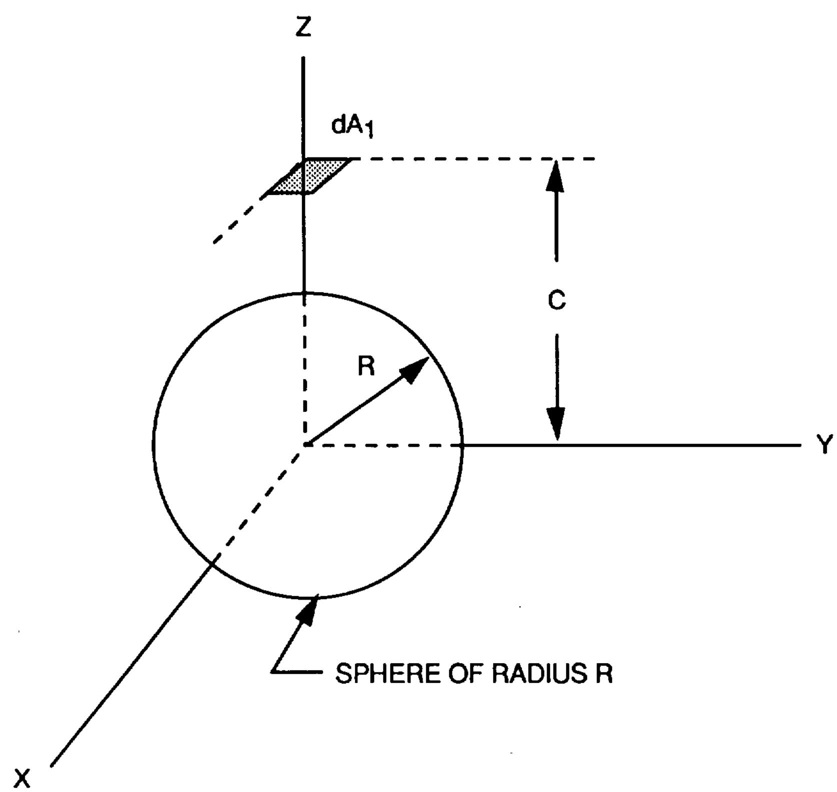

Figure 6. Geometry and coordinate system for Configuration 4.

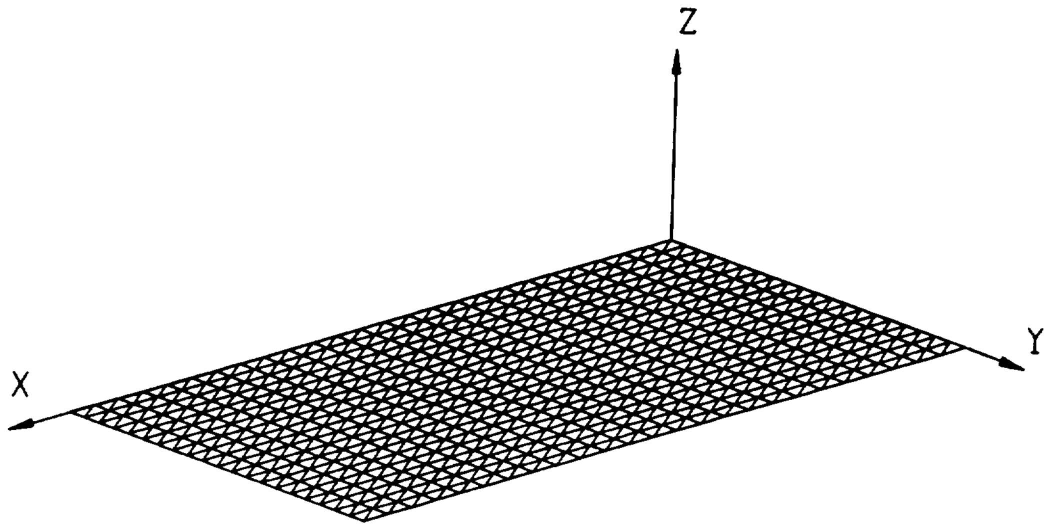

Figure 7. FTE grid for a rectangle (A/B = 2).

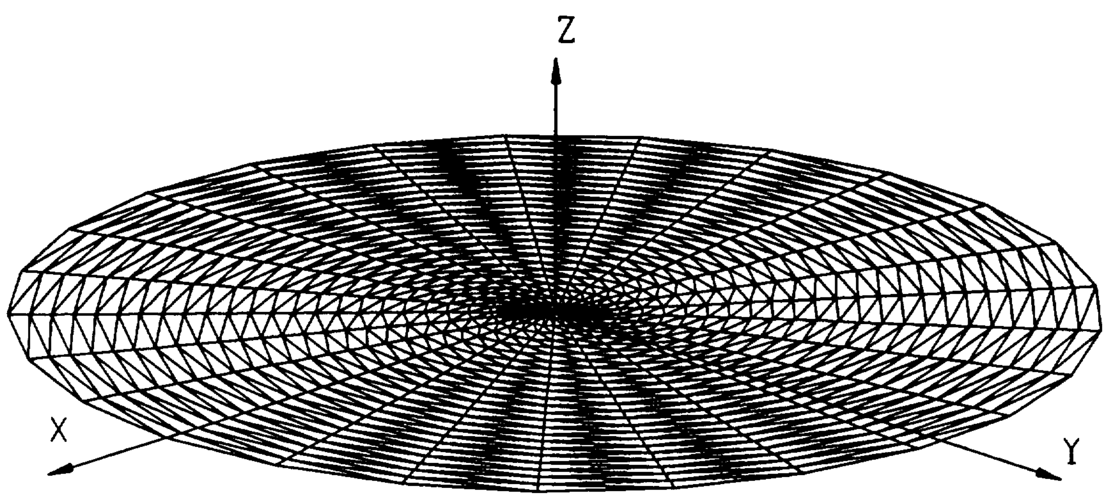

Figure 8. FTE grid for a circular area.

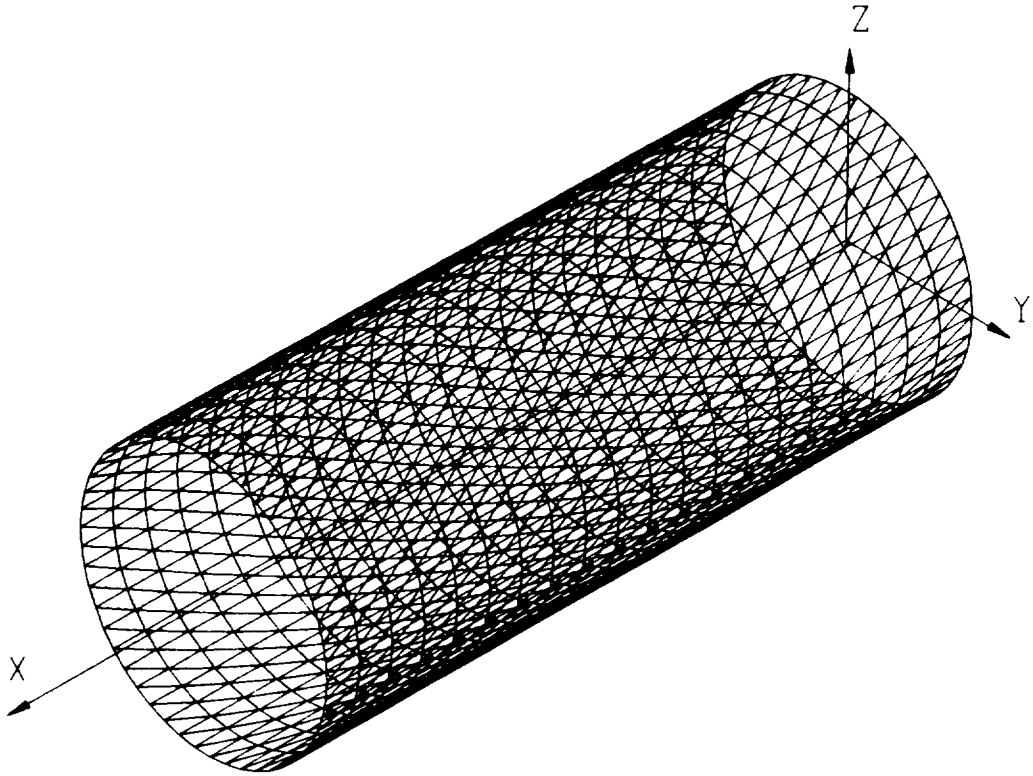

Figure 9. FTE grid for a cylinder (A/R = 5).

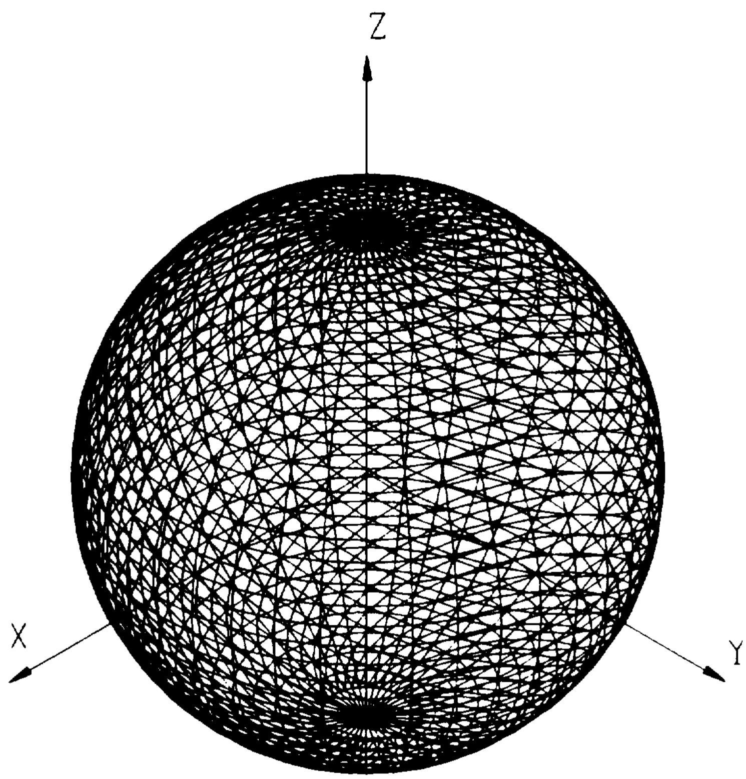

Figure 10. FTE grid for a sphere.