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NASA
Technical
Memorandum

NASA TM - 103581

# COUPLED LOADS ANALYSIS FOR SPACE SHUTTLE PAYLOADS

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May 1992



National Aeronautics and
Space Administration

George C. Marshall Space Flight Center

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TECHNICAL MEMORANDUM

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## I. INTRODUCTION

This report describes a method for determining the transient response of, and the resultant loads in, a system exposed to predicted external forces. For this example, the system consists of four racks mounted inside of a space station resource node module (SSRNMO) which is mounted in the payload bay of the space shuttle. The applied predicted external forces are Johnson Space Center (JSC) forcing functions which envelope worst-case forces applied to the shuttle during lift-off and landing. This analysis, called a coupled loads analysis, is used to:

1. Couple the payload and shuttle models together

2. Determine the transient response of the system

3. Recover payload loads, payload accelerations, and payload to shuttle interface forces.

## II. THEORY

### A. Craig-Bampton Reduction

#### 1. Models

A finite element model can be thought of as having both internal degrees of freedom (DOF's) and boundary DOF's where boundary DOF's connect the model with other models and internal DOF's do not (fig. 1). The models (mass and stiffness matrices) for the racks and the node were supplied by Boeing in the Craig-Bampton (CB) format, and the shuttle model was previously built and converted to the CB format. The CB format is the result of a CB reduction where the problem is reduced in size by ignoring the effects of some of the mode shapes. The size of the problem is reduced to the number of boundary DOF's plus the number of retained mode shapes from an eigenvalue problem. Because we retain only a portion of the modes for the substructures and for the entire shuttle system model, and since the reduced model is in a more diagonalized form, the problem is significantly reduced. The following is a discussion of the CB reduction method.

The motion of the internal DOF's $[X_I]$ can be described as the combination of (1) the internal motion with respect to a rigid boundary (all boundary DOF's are constrained), and (2) the internal motion due to the movement of the boundary and the relative deflection of the boundary DOF's (the boundary DOF's are subjected to different constraints).

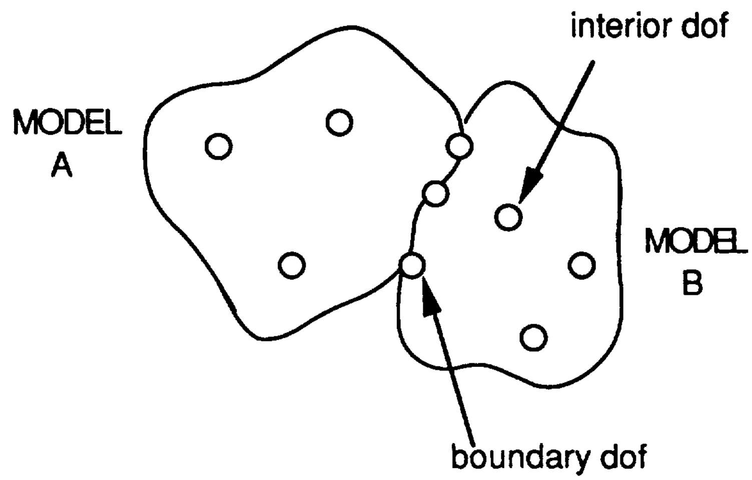


Figure 1. Generic coupled systems.

## 2. Constrained Modes

The first type of motion described above can be analyzed by constraining the boundary DOF's (making a rigid boundary) and then finding the normal modes or "constrained modes" as follows:

1. Constrain the boundary DOF by zeroing out the boundary DOF rows and columns in the mass (M) and stiffness (K) matrices.

2. Solve the eigenvalue problem $(K-w^2M)PHI_n = 0$ to get the constrained modes $PHI_n$ (eigenvectors), and system natural frequencies $w$ (square root of eigenvalues). Only the modes below a specified frequency, called a cutoff frequency, are considered significant to the problem and are retained in $PHI_n$. Reducing out these unnecessary mode shales reduces the size of the problem without losing much accuracy. The determination of the cutoff frequency will be discussed later.

The internal displacements $[X_I]$ with respect to a rigid boundary will now be denoted as $[X_{Icd}]$ for constrained. We can now write:

$$\{X_{Icd}\} = [PHI_n]\{q\} \ , \tag{1}$$

where $q$ is the modal displacement to be calculated later in the transient analysis.

## 3. Constraint Modes

The second type of motion due to the displacement and deflection of the boundary DOF's (BDOF's) is analyzed as follows. The equation $F = KX$ can be written in partitioned form such that the elements of the matrices which pertain to internal DOF's can be segregated from the elements which pertain to BDOF's as shown in equation (2), where $I$ denotes internal DOF influence and $B$ denotes BDOF influence. Note that $F_I = 0$ in equation (2) because this equation by definition is written to analyze only the effects of movement and relative deflection of the BDOF's on the internal DOF's (the effect of the internal forces are taken care of in the first type of motion).

$$\begin{bmatrix} K_{II} & K_{IB} \\ K_{BI} & K_{BB} \end{bmatrix} \begin{Bmatrix} X_I \\ X_B \end{Bmatrix} = \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_B \end{Bmatrix} , \tag{2}$$

therefore,

$$[K_{II}]\{X_I\} + [K_{IB}]\{X_B\} = 0$$

$$\{X_I\} = -[K_{II}]^{-1}[K_{IB}]\{X_B\} ,$$

if we let

$$-[K_{II}]^{-1}[K_{IB}] = [PHI_C]$$

then

$$\{X_I\} = [PHI_C]\{X_B\} . \tag{3}$$

To gain more insight into what $PHI_C$ is, we can let $X_B$ be an identity matrix. In this case, each column of $PHI_C$ is a map of relative internal displacements (mode shape) resulting from a unit displacement of one BDOF while fixing (constraining) all the other BDOF's. For this reason, we call $PHI_C$ the constraint modes, and we will now refer to this type of $X_I$ as $X_{Ict}$. Rewriting equation (3) with this new notation we get:

$$\{X_{Ict}\} = [PHI_C]\{X_B\} . \tag{4}$$

$X_{Ict}$ is the matrix of internal displacements due to the relative displacements of the BDOF's found in $X_B$.

## 4. Equations of Motion

The complete internal motion is found by combining the constrained and the constraint types of motion.

$$\{X_I\} = \{X_{Icd}\} + \{X_{Ict}\} . \tag{5}$$

Substituting equations (1) and (4) into equation (5) we get:

$$\{X_I\} = [PHI_n]\{q\} + [PHI_C]\{X_B\} , \tag{6}$$

and since

$$\{X_B\} = \{X_B\} , \tag{7}$$

we can write:

$$\begin{Bmatrix} X_I \\ X_B \end{Bmatrix} = \begin{bmatrix} PHI_N & PHI_C \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} q \\ X_B \end{Bmatrix} . \tag{8}$$

We now define

$$\begin{bmatrix} PHI_N & PHI_C \\ 0 & 1 \end{bmatrix} = [TCB] \ , \tag{9}$$

where $[TCB]$ is the CB transformation matrix.

The number of columns in $PHI_N$ is equal to the number of retained modes, and the number of columns in $PHI_C$ is equal to the number of BDOF's. Therefore, the size of $[TCB]$ is equal to the number of retained modes plus the number of BDOF's. We will later see that the size of our reduced model is this same size.

The equation of motion for a system with forced vibration and no damping (damping will be taken care of later) is:

$$[M]\ddot{X}+[K]X = \{F\} \ . \tag{10}$$

If we write this equation in partitioned form with respect to internal and boundary DOF's, we can get:

$$\begin{bmatrix} M_{II} & M_{IB} \\ M_{BI} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{X}_I \\ \ddot{X}_B \end{Bmatrix} + \begin{bmatrix} K_{II} & K_{IB} \\ K_{BI} & K_{BB} \end{bmatrix} \begin{Bmatrix} X_I \\ X_B \end{Bmatrix} = \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} , \tag{11}$$

where $I$ = interior DOF's and $B$ = boundary DOF's. $M_{II}$ is a matrix of those mass matrix terms which represent only internal DOF's. $M_{BB}$ is a matrix of those mass matrix terms which represent only BDOF's. $M_{IB}$ and $M_{BI}$ have terms with a mixture of boundary and internal DOF influence. $K_{II}$, $K_{IB}$, $K_{BI}$, and $K_{BB}$ are defined the same. By substituting equations (8) and (9) into equation (11) we get,

$$\begin{bmatrix} M_{II} & M_{IB} \\ M_{BI} & M_{BB} \end{bmatrix} [TCB] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + \begin{bmatrix} K_{II} & K_{IB} \\ K_{BI} & K_{BB} \end{bmatrix} [TCB] \begin{Bmatrix} q \\ X_B \end{Bmatrix} = \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} . \tag{12}$$

To simplify and to make symmetric, we multiply through by $[TCB]^T$ and get, after several steps and substitutions,

$$\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + \begin{bmatrix} w^2 & 0 \\ 0 & K_{BB} \end{bmatrix} \begin{Bmatrix} q \\ X_B \end{Bmatrix} = [TCB]^T \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} , \tag{13}$$

where

$$M_{QB} = [PHI_N]^T[M_{II}][PHI_C] + [PHI_N]^T[M_{IB}]$$

$$M_{BQ} = [PHI_C]^T[M_{II}][PHI_N] + [M_{BI}][PHI_N]$$

$$w^2 = [PHI_N]^T[K_{II}][PHI_N] \text{ (squared constrained system natural frequencies)} \ .$$

If we let

$$\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix} = [MCB]$$

and

$$\begin{bmatrix} w^2 & 0 \\ 0 & K_{BB} \end{bmatrix} = [KCB] \ ,$$

then we can rewrite equation (13) to get the CB equation of motion:

$$[MCB]\begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + [KCB]\begin{Bmatrix} q \\ X_B \end{Bmatrix} = [TCB]^T\begin{Bmatrix} F_I \\ F_B \end{Bmatrix} . \tag{14}$$

## B. Coupling

To couple systems together, we put the models (mass and stiffness matrices) into a common coordinate system and then add the equations of motion.

As an example (which directly applies to the node problem), we will couple two subsystems called rack ($R$) and node ($N$) into a new system called the payload ($P$) system. Then we will couple this payload to the shuttle ($S$) system to get the complete lift-off ($LO$) system. The $R$ system attaches to the $N$ system, and the $P$ system attaches to the $S$ system at the node (fig. 2).

When $CB$ reductions are done, the BDOF's must include all boundaries to be used in future couplings. For example, the node attaches to the rack and to the shuttle. Therefore, the BDOF's for the node include the node to rack ($n/r$) and node to shuttle ($n/s$) BDOF's.

Since the models of the $R$, $N$, and $S$ systems are supplied in the $CB$ format, we are given the $CB$ transformations $[TCB_R]$, $[TCB_N]$, and $[TCB_S]$, where

$$\begin{Bmatrix} X_{I\,R} \\ X_{B\,r/n} \end{Bmatrix} = [TCB_R]\begin{Bmatrix} q_R \\ X_{B\,r/n} \end{Bmatrix} , \qquad \text{and} \quad \begin{Bmatrix} X_{I\,N} \\ X_{B\,n/r} \\ X_{B\,n/s} \end{Bmatrix} = [TCB_N]\begin{Bmatrix} q_N \\ X_{B\,n/r} \\ X_{B\,n/s} \end{Bmatrix} , \tag{15}$$

and

$$\begin{Bmatrix} X_{I\,S} \\ X_{B\,s/n} \end{Bmatrix} = [TCB_S]\begin{Bmatrix} q_S \\ X_{B\,s/n} \end{Bmatrix} .$$

Note that $n/s$ stands for node to shuttle coordinates and $s/n$ stands for shuttle to node coordinates. These $TCB$'s, which convert from modal coordinates to discrete coordinates, were generated as shown above during the initial $R$, $N$, and $S$ $CB$ reductions. To couple the $R$ system to the $N$ system, we first write the equations of motion from equation (14) for the rack and then the node systems.

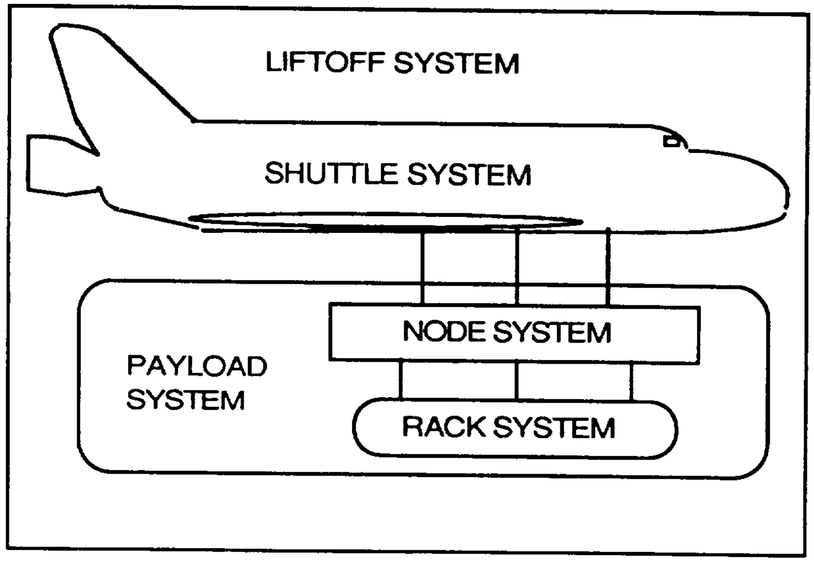


Figure 2. Lift-off coupled system.

$$[MCB]_R \begin{Bmatrix} \ddot{q}_R \\ \ddot{X}_{B\ r/n} \end{Bmatrix} + [KCB]_R \begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} = [TCB]^T{}_R \begin{Bmatrix} F_{I\ R} \\ F_{B\ r/n} \end{Bmatrix} , \tag{16a}$$

$$[MCB]_N \begin{Bmatrix} \ddot{q}_N \\ \ddot{X}_{B\ n/r} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KCB]_N \begin{Bmatrix} q_N \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix} = [TCB]^T{}_N \begin{Bmatrix} F_{I\ N} \\ F_{B\ n/r} \\ F_{B\ n/s} \end{Bmatrix} . \tag{16b}$$

To couple the rack and node systems together, we need to convert them to a common coordinate system which contains all coordinates found in the rack and node systems. Such a system, in this case, is the $\begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix}$ system. Since we are coupling the rack and node at the $r/n$ boundary, we are imposing the relation that $X_{B\ r/n} = X_{B\ n/r}$. Therefore, we will convert to a $\begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix}$ system, which also contains all coordinates found in the rack and node systems.

We can write:

$$\begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} = [TR] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} , \quad \text{and} \quad \begin{Bmatrix} q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TN] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} , \tag{17}$$

where $[TR]$ and $[TN]$ are the coordinate transformation matrices which convert from the new common (coupled) coordinate system to the rack and node systems, respectively. Unless a coordinate rotation is involved, these transformations are generally made up of 0's and 1's. From equations (17), we can solve for $[TR]$ and $[TN]$ which we will need later in this analysis to convert back to the component system coordinates ($R$ and $N$) from the common coupled system coordinates when recovering component loads. In this example,

$$[TR] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad [TN] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

Substituting equation (17) into equations (16a) and (16b), we get equations (18) and (19).

$$[MCB]_R[TR] \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KCB]_R[TR] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TCB]^T{}_R \begin{Bmatrix} F_{I\,R} \\ F_{B\ r/n} \end{Bmatrix} , \tag{18}$$

$$[MCB]_N[TN] \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KCB]_N[TN] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TCB]^T{}_N \begin{Bmatrix} F_{I\,N} \\ F_{B\ n/r} \\ F_{B\ n/s} \end{Bmatrix} . \tag{19}$$

Now we will multiply equations (18) and (19) by $[TR]^T$ and $[TN]^T$, respectively, to simplify.

$$[TR]^T[MCB]_R[TR] \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [TR]^T[KCB]_R[TR] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TR]^T[TCB]^T{}_R \begin{Bmatrix} F_{I\,R} \\ F_{B\ r/n} \end{Bmatrix} , \tag{20}$$

$$[TN]^T[MCB]_N[TN] \begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [TN]^T[KCB]_N[TN] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TN]^T[TCB]^T{}_N \begin{Bmatrix} F_{I\,N} \\ F_{B\ n/r} \\ F_{B\ n/s} \end{Bmatrix} . \tag{21}$$

Now, we can add equations (20) and (21) while letting

$$[TR]^T[MCB]_R[TR] + [TN]^T[MCB]_N[TN] = [MP] , \tag{22a}$$

and

$$[TR]^T[KCB]_R[TR] + [TN]^T[KCB]_N[TN] = [KP]\ , \tag{22b}$$

where $MP$ and $KP$ are the $P$ mass and stiffness matrices. The result is the general coupled payload equation of motion:

$$[MP]\begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KP]\begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TR]^T[TCB]^T_R\begin{Bmatrix} F_{I\ R} \\ F_{B\ r/n} \end{Bmatrix} + [TN]^T[TCB]^T_N\begin{Bmatrix} F_{I\ N} \\ F_{B\ n/r} \\ F_{B\ n/s} \end{Bmatrix}\ . \tag{23}$$

It is important to note that for shuttle payloads, there are generally no internal forces applied to the rack or the node, so $F_{I\ R} = F_{I\ N} = 0$. After making the substitutions for $[TR]^T$, $[TCB]^T_R$, $[TN]^T$, and $[TCB]^T_N$, the right hand side of equation (23) simplifies as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} PHI^T_N & 0 \\ PHI^T_C & I_R \end{bmatrix}\begin{Bmatrix} 0 \\ F_{B\ r/n} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} PHI^T_N & 0 \\ PHI^T_C & I_N \end{bmatrix}\begin{Bmatrix} 0 \\ F_{B\ n/r} \\ F_{B\ n/s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \{F_{B\ r/n}\} \\ 0 \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0 \\ 0 \\ \{F_{B\ n/r}\} \\ \{F_{B\ n/s}\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \{F_{B\ r/n}\} + \{F_{B\ n/r}\} \\ \{F_{B\ n/s}\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \{F_{B\ n/s}\} \end{Bmatrix}\ .$$

The last step in the simplification is due to the fact that the sum of the forces at the coupled boundary between the rack and the node are zero, so $F_{B\ n/r}+F_{B\ r/n} = 0$. This assumes that there are no externally applied forces at the boundary. Using the above simplifications, equation (23) becomes equation (24), the final payload equation of motion.

$$[MP]\begin{Bmatrix} \ddot{q}_R \\ \ddot{q}_N \\ \ddot{X}_{B\ r/n} \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KP]\begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F_{B\ n/s} \end{Bmatrix}\ . \tag{24}$$

To further illustrate that this method couples by simply adding at common BDOF's, we can expand equation (22a) to get equation (25) and then multiply to get equation (26).

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix}_R\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix}_N\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [MP]\ , \tag{25}$$

$$\begin{bmatrix} I & 0 & M_{QBr} \\ 0 & 0 & 0 \\ M_{BQr} & 0 & M_{BBr} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \begin{bmatrix} I & M_{QBn} \\ M_{BQn} & M_{BBn} \end{bmatrix} \\ 0 & & \end{bmatrix} = \begin{bmatrix} I & 0 & M_{QBr} \\ 0 & I & M_{QBn} \\ M_{BQr} & M_{BQn} & (M_{BBr}+M_{BBn}) \end{bmatrix} = [MP] \ . \tag{26}$$

Expanding equation (22b) would show the stiffness matrices coupling in the same way.

We will next couple the payload to the shuttle. To simplify the problem, we will do a *CB* reduction as before on the new payload system, saving only the boundaries needed for coupling; those common to the payload and shuttle systems. The *CB* reduction, performed as before, gives us a new *CB* transformation matrix $[TCB]_P$ which satisfies the following relations.

$$[TCB]_P = \begin{bmatrix} PHI_N & PHI_C \\ 0 & I \end{bmatrix}_P \ , \quad \text{and} \quad \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TCB]_P \begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} . \tag{27}$$

Again, we can get the *CB* mass and stiffness matrices.

$$[MCB]_P = [TCB]^T{}_P[MP][TCB]_P \ , \tag{28a}$$

$$[KCB]_P = [TCB]^T{}_P[KP][TCB]_P \ . \tag{28b}$$

The new *CB* equation of motion for the payload is:

$$[MCB]_P \begin{Bmatrix} \ddot{q}_P \\ \ddot{X}_{B\ n/s} \end{Bmatrix} + [KCB]_P \begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} = [TCB]^T{}_P\,[TN]^T \begin{Bmatrix} 0 \\ F_{B\ n/s} \end{Bmatrix} . \tag{29}$$

The CB equation of motion for the shuttle is derived the same way and is:

$$[MCB]_S \begin{Bmatrix} \ddot{q}_S \\ \ddot{X}_{B\ s/n} \end{Bmatrix} + [KCB]_S \begin{Bmatrix} q_S \\ X_{B\ s/n} \end{Bmatrix} = [TCB]^T{}_S \begin{Bmatrix} F_{I\,S} \\ F_{B\ s/n} \end{Bmatrix} . \tag{30}$$

Since there are externally applied forces on the shuttle internal DOF's, the $F_I$ term does not go to zero as for the payload system.

To couple the *P* and the *S* systems into the *LO* system, we must rearrange and add as we did for the rack and node subsystems. We can derive the coordinate transformation matrices $[TP]$ for the payload, and $[TS]$ for the shuttle from equations (31). The *LO* mass and stiffness matrices ($[MLO]$ and $[KLO]$) can then be found by using equations (32a) and (32b).

$$\begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} = [TP] \begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} \ , \quad \begin{Bmatrix} q_S \\ X_{B\ n/s} \end{Bmatrix} = [TS] \begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} , \tag{31}$$

$$[TP]^T[MCB]_P[TP]+[TS]^T[MCB]_S[TS]=[MLO] \ , \tag{32a}$$

and

$$[TP]^T[KCB]_P[TP]+[TS]^T[KCB]_S[TS]=[KLO] \ . \tag{32b}$$

As before for the payload system, we substitute equations (31) into equations (29) and (30). We multiply these modified versions of equations (29) and (30) through by $[TP]^T$ and $[TS]^T$, respectively, and then add them together. Substituting equations (32) into this new equation we get a coupled *LO* equation of motion.

$$[MLO]\begin{Bmatrix} \ddot{q}_P \\ \ddot{q}_S \\ \ddot{X}_{B\ n/s} \end{Bmatrix}+[KLO]\begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix}=[TS]^T[TCB]^T{}_S\begin{Bmatrix} F_{I\,S} \\ F_{B\ s/p} \end{Bmatrix}+[TP]^T[TCB]^T{}_P[TN]^T\begin{Bmatrix} 0 \\ F_{B\ n/s} \end{Bmatrix} . \tag{33}$$

Note that the payload attaches to the shuttle at the node (fig. 2). This means that $F_{B\ n/s}$ is the same as $F_{B\ p/s}$. Since the payload and shuttle are coupled at the common boundary, then $F_{B\ s/p}+F_{B\ p/s}=0$ (again assuming no externally applied boundary forces), and therefore $F_{B\ s/p}+F_{B\ n/s}=0$. Using this relationship after substituting all of the transformation matrices into equation (33) and then simplifying, we get equation (34), the complete *LO* equation of motion.

$$[MLO]\begin{Bmatrix} \ddot{q}_P \\ \ddot{q}_S \\ \ddot{X}_{B\ n/s} \end{Bmatrix}+[KLO]\begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix}=[TS]^T[TCB]^T{}_S\begin{Bmatrix} F_{I\,S} \\ 0 \end{Bmatrix} . \tag{34}$$

This is the basic equation used in the transient response analysis (see below) to get the modal responses ($q_S$) from the forcing function ($F_{I\,S}$).

## C. Transient Response

Equation (35), the eigenvalue problem, is solved for the eigenvalues ($w_{LO}{}^2$ where $w_{LO}$ are the system free-free natural frequencies), and the eigenvectors ($PHI_{LO}$) which are the system mode shapes.

$$([K_{LO}]-[w_{LO}{}^2][M_{LO}])[PHI_{LO}]=0 \ , \tag{35}$$

where

$$\begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix}=[PHI_{LO}]\{q_{LO}\} \ . \tag{36}$$

Since the mode shapes ($[PHI_{LO}]$) are just relative amplitudes, they can be multiplied by a constant and still retain the same shape. We can normalize the mode shapes by picking constants such that

$$[PHI_{LO}]^T [M_{LO}] [PHI_{LO}] = [I] \ . \tag{37}$$

When equation (37) is satisfied, then so is equation (38).

$$[PHI_{LO}]^T [K_{LO}] [PHI_{LO}] = [w_{LO}^2] \ . \tag{38}$$

Now equation (36) can be substituted into equation (34) and, after multiplying through by $[PHI_{LO}]^T$ to simplify, we can use equations (37) and (38) to get the normalized equation of motion with no damping.

$$[I] \{\ddot{q}_{LO}\} + [w_{LO}^2] \{q_{LO}\} = [PHI_{LO}]^T [TS]^T [TCB]^T{}_S \left\{ \begin{matrix} F_{Is} \\ 0 \end{matrix} \right\} . \tag{39}$$

At this time it is convenient to take damping into account. Damping is assumed to be proportional damping [2] as defined by equation (40).

$$[D] = [2zw] \ , \tag{40}$$

where $z$ is the damping ratio, which is assumed for lift-off to be 1 percent of critical damping for modes below 10 Hz and 2 percent of critical damping for modes above 10 Hz. For landing analysis, $z$ is assumed to be 1 percent of critical damping for all modes. Since damping force is proportional to velocity, our final equation of motion is then:

$$[I] \{\ddot{q}_{LO}\} + [2zw_{LO}] \{\dot{q}_{LO}\} + [w_{LO}^2] \{q_{LO}\} = [PHI_{LO}]^T [TS]^T [TCB]^T{}_S \left\{ \begin{matrix} F_{Is} \\ 0 \end{matrix} \right\} . \tag{41}$$

Note that we now know everything in this equation except the $q$'s and the external forces on the shuttle applied to the internal DOF's ($\{F_{IS}\}$). The forces are supplied by JSC in the form of forcing functions which are predictions of a certain shuttle environment during lift-off or landing. A forcing function is a matrix where each column represents specific forces on specific shuttle DOF's at one instant of time. The number of columns then is the number of time increments to be studied. Lift-off is usually run for about 10.5 s, and landing for about 2 s. The time step used can vary, but 0.005 s is typical.

By virtue of the modal analysis, equation (41) is a set of $n$ linearly independent differential equations with $n$ unknowns ($n$ = size of the model). We can now solve for the $q$'s for the first column (time step) of the forcing function. The results of this solution are used as the initial conditions for the solution of the next time step. The solutions keep "feeding" each other like this until all forcing function columns are used. This gives us the values of $q$ for all time steps during an event such as lift-off. The $q$'s represent the modal displacement, velocity, and acceleration of the system.

### D. Recovering System Response From Modal Response

Now that we have solved equation (41) for the modal displacement and acceleration responses of the system ($qs$), we need to determine what this means with respect to specific interface forces between subsystems and net center-of-gravity (CG) accelerations of subsystems. The hybrid modal acceleration method is frequently used to get interface forces and CG accelerations when the system is either determinate (six BDOF's) or slightly indeterminate (seven BDOF's). In some instances, even seven BDOF's require another method called the acceleration method, which is outside the scope of this report. The following makes use of the hybrid modal acceleration method.

The rigid body displacements (first six rows of $q$) are set equal to zero which forces the coordinate reference point to be the CG of the shuttle. Otherwise, the large movement of the entire shuttle system would be included in the displacement results. The modal displacements and accelerations are then transformed back to the point of interest, as shown below, using the transformation equations derived above and repeated here for clarity.

$$\begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} = [PHI_{LO}] \{q_{LO}\} \ . \tag{36}$$

$$\begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} = [TP] \begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} \ , \quad \begin{Bmatrix} q_S \\ X_{B\ n/s} \end{Bmatrix} = [TS] \begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} \ , \tag{31}$$

$$[TCB]_P = \begin{bmatrix} PHI_N & PHI_C \\ 0 & I \end{bmatrix}_P \ , \quad \text{and} \quad \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TCB]_P \begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} \ . \tag{27}$$

$$\begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} = [TR] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} \ , \quad \text{and} \quad \begin{Bmatrix} q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} = [TN] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} \ , \tag{17}$$

$$\begin{Bmatrix} X_{I\ R} \\ X_{B\ r/n} \end{Bmatrix} = [TCB_R] \begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} \ , \quad \text{and} \quad \begin{Bmatrix} X_{I\ N} \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix} = [TCB_N] \begin{Bmatrix} q_N \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix} \ , \tag{15}$$

and

$$\begin{Bmatrix} X_{I\ S} \\ X_{B\ s/n} \end{Bmatrix} = [TCB_S] \begin{Bmatrix} q_S \\ X_{B\ s/n} \end{Bmatrix} \ .$$

These equations can be used to find the response of any of the subsystems. For example, if we want the response of the rack during lift-off, we back into it from the system modal response as follows:

$$\begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} = [PHI_{LO}] \{q_{LO}\} \ ,$$

$$\begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} = [TP] \begin{Bmatrix} q_P \\ q_S \\ X_{B\ n/s} \end{Bmatrix} \ ,$$

$$\begin{Bmatrix} q_R \\ q_N \\ X_{B\ n/s} \\ X_{B\ r/n} \end{Bmatrix} = [TCB_P] \begin{Bmatrix} q_P \\ X_{B\ n/s} \end{Bmatrix} ,$$

$$\begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} = [TR] \begin{Bmatrix} q_R \\ q_N \\ X_{B\ r/n} \\ X_{B\ n/s} \end{Bmatrix} ,$$

$$\begin{Bmatrix} XI_R \\ X_{B\ r/n} \end{Bmatrix} = [TCB_R] \begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} .$$

Therefore, the modal response of the rack is:

$$\begin{Bmatrix} q_R \\ X_{B\ r/n} \end{Bmatrix} = [TR][TCB_P][TP][PHI_{LO}]\{q_{LO}\} , \tag{42a}$$

and the rack discrete response is:

$$\begin{Bmatrix} X_{I\ R} \\ X_{B\ r/n} \end{Bmatrix} = [TCB_R][TR][TCB_P][TP][PHI_{LO}]\{q_{LO}\} . \tag{42b}$$

Similarly, the modal response for the node is:

$$\begin{Bmatrix} q_N \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix} = [TN][TCB_P][TP][PHI_{LO}]\{q_{LO}\} , \tag{43a}$$

and the node discrete response is:

$$\begin{Bmatrix} X_{I\ N} \\ X_{B\ n/r} \\ X_{B\ n/s} \end{Bmatrix} = [TCB_N][TN][TCB_P][TP][PHI_{LO}]\{q_{LO}\} . \tag{43b}$$

### E. Recovering Loads Using Loads Transformation Matrices

Now that we have the response, we need to convert this information into loads through the use of loads transformation matrices (LTM's). The LTM's for finding interface forces (IFF's) and center-of-gravity accelerations (CGA's) are derived in the following.

### 1. Interface Forces

IFF's are simply the boundary forces ($F_B$) between two subsystems. Recalling equation (13):

$$\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + \begin{bmatrix} w^2 & 0 \\ 0 & K_{BB} \end{bmatrix} \begin{Bmatrix} q \\ X_B \end{Bmatrix} = [TCB]^T \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} . \tag{13}$$

There are no applied internal forces in the rack or node ($F_I = 0$) and since

$$[TCB]^T = \begin{bmatrix} PHI_N & 0 \\ PHI_C & I \end{bmatrix} ,$$

equation (13) simplifies to:

$$\begin{bmatrix} I & M_{QB} \\ M_{BQ} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + \begin{bmatrix} w^2 & 0 \\ 0 & K_{BB} \end{bmatrix} \begin{Bmatrix} q \\ X_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_B \end{Bmatrix} , \tag{44}$$

from which we see that the boundary forces (same as IFF's) are:

$$L_{IFF} = [F_B] = [M_{BQ}\ M_{BB}] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + [0\ K_{BB}] \begin{Bmatrix} q \\ X_B \end{Bmatrix} . \tag{45}$$

$[M_{BQ}\ M_{BB}]$ is sometimes called LTMA, the acceleration LTM, and $[0\ K_{BB}]$ is called LTMD, the displacement LTM. Rewriting equation (45) we get equation (46):

$$L_{IFF} = [LTMA] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + [LTMD] \begin{Bmatrix} q \\ X_B \end{Bmatrix} . \tag{46}$$

Substituting equation (42a) from the example above into equation (46), we get equation (47) which gives the interface forces between the rack and the node:

$$\begin{aligned} L_{IFF\ R} &= [LTMA]_R[TR][TCB_P][TP][PHI_{LO}]\{\ddot{q}_{LO}\} \\ &+ [LTMD]_R[TR][TCB_P][TP][PHI_{LO}]\{q_{LO}\} . \end{aligned} \tag{47}$$

### 2. Net CGA's

A net CGA is the average acceleration of a subsystem CG assuming the subsystem to be a rigid body. This should not be confused with the acceleration of a point at the CG. We need to know the forces at the CG of the subsystems. Since we now know $F_B$, the forces at the boundary (equation (45)), we need to transform them to the CG. To do this we solve for the rigid body transformation (RBT) such that:

$$\{X_B\} = [RBT]\ \{X_{CG}\} . \tag{48}$$

Since $[RBT]$ is orthogonal, then $[RBT]^T = [RBT]^{-1}$, and it follows that

$$\{X_{CG}\} = [RBT]^T \{X_B\} \quad , \tag{49}$$

and that

$$[F_{CG}] = [RBT]^T [F_B] \quad . \tag{50}$$

To convert equation (45) to the CG coordinate system, we multiply through by $[RBT]^T$ and get:

$$[RBT]^T [F_B] = [RBT]^T [M_{BQ}\ M_{BB}] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} + [RBT]^T [0\ K_{BB}] \begin{Bmatrix} q \\ X_B \end{Bmatrix} \tag{51}$$

Since converting to the CG coordinate system is an RBT, all coordinates retain the same relative position. Therefore, for CGA's we assume a rigid model which means there are no forces due to deflections and so:

$$[RBT]^T [0\ \ K_{BB}] = 0 \quad . \tag{52}$$

Substituting equations (50) and (52) into equation (51) we get:

$$[F_{CG}] = [RBT]^T [M_{BQ}\ \ M_{BB}] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} . \tag{53}$$

Since

$$[F_{CG}] = [M_{CG}] \{\ddot{X}_{CG}\} \quad , \tag{54}$$

then

$$\{\ddot{X}_{CG}\} = [M_{CG}]^{-1} [F_{CG}] \quad . \tag{55}$$

Substituting equation (53) into equation (55) we get:

$$\{\ddot{X}_{CG}\} = [M_{CG}]^{-1} [RBT]^T [M_{BQ}\ \ M_{BB}] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} . \tag{56}$$

We can get $[M_{CG}]$ by performing the transformation:

$$[M_{CG}] = [RBT]^T [M_{BB}] [RBT] \quad . \tag{57}$$

Substituting equation (57) into equation (56) we get:

$$\{\ddot{X}_{CG}\} = ([RBT]^T [M_{BB}][RBT])^{-1} [RBT]^T [M_{BQ}\ \ M_{BB}] \begin{Bmatrix} \ddot{q} \\ \ddot{X}_B \end{Bmatrix} . \tag{58}$$

If we let

$$[ACG] = ([RBT]^T[M_{BB}][RBT])^{-1}[RBT]^T[M_{BQ} \;\; M_{BB}] \; , \tag{59}$$

then from equation (58),

$$\{\ddot{X}_{CG}\} = [ACG]\left\{ \begin{array}{c} \ddot{q} \\ \ddot{X}_B \end{array} \right\} . \tag{60}$$

Substituting equation (42a), from the example above, into equation (60) we get equation (61) which gives the CGA's for the rack during lift-off.

$$\{\ddot{X}_{CG}\}_R = [ACG]_R[TR][TCB_P][TP][PHI_{LO}] \; \{\ddot{q}_{LO}\} \; . \tag{61}$$

## III. SUMMARY

Coupled loads analyses require a lot of bookkeeping techniques. First the models are coupled while creating a trail of transformation equations in order to later back out to the subsystem level. The equations of motion are then written for the coupled system. Next, the transient analysis is performed by applying the forcing functions to the equations of motion to get the system displacements and accelerations during the dynamic event. Finally, the system transient results are used along with the transformation equations to back into specific subsystem responses. At this time, it is important to point out that we can calculate many more different kinds of component responses other than interface forces and CGA's. These responses were needed for the SSRNMO and therefore were used in this report. The topic of a future report might be the usage of LTM's to recover other types of responses.

## APPROVAL

## COUPLED LOADS ANALYSIS FOR SPACE SHUTTLE PAYLOADS

By J. Eldridge

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

for J.C. BLAIR
Director, Structures and Dynamics Laboratory

☆ U S GOVERNMENT PRINTING OFFICE 1992—631-060/60112

| REPORT DOCUMENTATION PAGE | Form Approved OMB No. 0704-0188 |
|---|---|



| 1. AGENCY USE ONLY *(Leave blank)* | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | May 1992 | Technical Memorandum |

**4. TITLE AND SUBTITLE**
Coupled Loads Analysis for Space Shuttle Payloads

**5. FUNDING NUMBERS**

**6. AUTHOR(S)**
J. Eldridge

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
National Aeronautics and Space Administration
Washington, DC 20546

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
NASA TM-103581

**11. SUPPLEMENTARY NOTES**
Prepared by Structures and Dynamics Laboratory, Science and Engineering Directorate.

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Unclassified — Unlimited

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*

This report describes a method for determining the transient response of, and the resultant loads in, a system exposed to predicted external forces. In this case, the system consists of four racks mounted on the inside of a space station resource node module (SSRNMO) which is mounted in the payload bay of the space shuttle. The predicted external forces are forcing functions which envelope worst case forces applied to the shuttle during lift-off and landing. This analysis, called a coupled loads analysis, is used to ; (a) couple the payload and shuttle models together, (b) determine the transient response of the system, and then (c) recover payload loads, payload accelerations, and payload to shuttle interface forces.

**14. SUBJECT TERMS**
Transient Response, Space Shuttle, Coupled Loads, Payloads

**15. NUMBER OF PAGES**
20

**16. PRICE CODE**
NTIS

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | Unlimited |