IN 34
89331

NASA Contractor Report 189624
ICASE Report No. 92-11

P.32

ICASE

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Contract No. NAS1-18605
March 1992

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Operated by the Universities Space Research Association

National Aeronautics and
Space Administration

Langley Research Center
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(NASA-CR-189624) INTERACTION OF DISTURBANCES WITH AN OBLIQUE DETONATION WAVE ATTACHED TO A WEDGE Final Report (ICASE) 32 p CSCL 20D

N92-24709

Unclas
G3/34 0089331

INTERACTION OF DISTURBANCES WITH AN OBLIQUE DETONATION WAVE ATTACHED TO A WEDGE[1]

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ABSTRACT

The linear response of an oblique overdriven detonation to imposed free stream disturbances or to periodic movements of the wedge is examined. The freestream disturbances are assumed to be steady vorticity waves and the wedge motions are considered to be time periodic oscillations either about a fixed pivot point or along the plane of symmetry of the wedge aligned with the incoming stream. The detonation is considered to be a region of infinitesmal thickness in which a finite amount of heat is released. The response to the imposed disturbances is a function of the Mach number of the incoming flow, the wedge angle, and the exothermicity of the reaction within the detonation. It is shown that as the degree of overdrive increases, the amplitude of the response increases significantly; furthermore, a fundamental difference in the dependence of the response on the parameters of the problem is found between the response to a free stream disturbance and to a disturbance emanating from the wedge surface.

[1]Research was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18605 while the authors were in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665.

1. Introduction.

The research and development of supersonic propulsion devices for aerospace planes that will cruise at hypersonic speeds has renewed interest in the interaction of freestream disturbances and shock waves and the interaction of such disturbances with shock induced detonation waves. The latter interaction is of particular importance in the theoretical propulsion device known as the Oblique Detonation Wave Engine (ODWE) in which all burning takes place in a thin overdriven detonation attached to a wedgelike surface. Of course, the stability of such a detonation structure has never been demonstrated; however, numerical calculations (Fujiwara, Matsuo, and Nomoto[1]) of a supersonic flow past a wedge in a reacting gas have indicated that an oblique detonation might be possible as the degree of overdrive is increased. The former interaction is of importance to both the ODWE and the SCRAMJET concept. In the ODWE, inert shock structures are utilized to precondition the incoming stream such that an oblique, overdriven detonation is possible at the predetermined location. In the SCRAMJET, mixing of fuel and oxidizer in high speed flows is of major concern, and the enhancement of turbulent mixing by oscillating shock waves has been proposed (Kumar, Bushnell, & Hussaini[2]). In addition to the freestream disturbances which are inherently present in practically all high-speed flows of technological importance, the oscillation of the surfaces to which the shock/detonation wave are attached must be considered. These oscillations will induce curvature into the shock/detonation, and furthermore, induce the generation of pressure, vorticity and entropy waves. It is the purpose of this study to investigate the interaction of freestream disturbances and an oblique overdriven detonation (the shock wave is considered as the zero heat release limit of this problem) and to investigate the generation of pressure, entropy and vorticity waves by the small movements of the structure to which the oblique detonation wave is attached.

The model problem, assumes a wedge of half angle θ in a uniform supersonic stream. In the presence of the wedge, the supersonic flow abruptly changes direction through a thin layer at an angle χ to the axis of symmetry of the wedge in which all of the macroscopic quantities such as density, pressure, temperature and velocity change in an order one sense (see Fig. 1). The angle χ is a function of the flow speed, wedge angle, and the exothermicity and is found by assuming the layer to be of infinitesmal thickness and applying the generalized Rankine-Hugoniot conditions to relate the velocities, pressure, temperature and density ahead of and behind the discontinuity. In an inert flow, the discontinuity is just a regular shock wave; whereas in a reacting flow, the thin layer is considered as a detonation with a finite amount of heat release. The base flow was examined by Gross[3] and later by Pratt, Humphreys, & Glenn[4] in the context of its application to the ODWE. It is clear that when heat release is allowed for, there is both a maximum and a minimum wedge angle for which the weak oblique detonation could exist. The maximum wedge angle corresponds to detachment while the minimum wedge angle corresponds to the Chapman-Jouget conditions in which the normal component of the flow behind the detonation is sonic.

The equations governing the gas flow both ahead of and behind the shock/detonation are the Euler equations along with the ideal gas law. The no mass flux condition is imposed on the wedge surface, so that the normal component of the flow must have the same velocity as the velocity of the wedge normal to its surface. The analysis is accomplished by linearizing the Euler equations about the base state and applying the Rankine-Hugoniot conditions at the mean position of the detonation.

Two distinct problems are considered: an oscillating wedge in an otherwise uniform supersonic stream of a reactive mixture hereafter referred to as the Oscillating Wedge Problem, and a stationary wedge in a non-uniform flow consisting of a uniform stream and a superimposed disturbance hereafter referred to as the Stationary Wedge Problem. The former was first proposed by Carrier[5] and Van Dyke[6] for the nonreacting flow. Their interest was in the surface pressure distribution and the resultant forces and moments and their relevance to oscillating airfoils. This problem was reconsidered by Hussaini, Collier and Bushnell[7] from a different point of view. They were interested in vorticity generation owing to shock motion in non-reacting flows. The latter problem of a shear wave interaction with a detonation wave induced by a wedge in a supersonic flow is an extension of the study of Jackson, Kapila, and Hussaini[8] in which a weak sinusoidal vorticity wave is obliquely convected through an overdriven detonation with the angle of incidence taken as the angle that the upstream flow makes with the normal to the detonation. In that study, the flow field behind the detonation wave is unobstructed, and two possible disturbance patterns exist depending upon the angle of incidence. For constant upstream Mach number and constant heat release, there exists a critical angle of incidence below which the flow downstream is subsonic and above which the flow downstream is supersonic. For supersonic flow behind the detonation (*i.e.* for large angles of incidence), the acoustic disturbance generated by the incident vorticity wave is also sinusoidal in nature; for subsonic flow behind the detonation, the acoustic disturbance is exponentially damped. Non-linear numerical calculations by Lasseigne, Jackson and Hussaini[9] confirmed this behaviour and provided some limits on the applicability of the linear results. In particular, for flows not near the critical angle and with small vorticity disturbance amplitudes up to 10%, the linear theory quite accurately predicts the response. For flow angles near critical or for larger disturbance amplitudes, a steady state is not achieved numerically and perhaps may not exist, and therefore, comparisons with the linear theory could not be made. In the present study, only overdriven detonations in which the flow behind the detonation is supersonic are considered.

In each problem considered, the focus is on (*i*) the deviation of the detonation position from its constant undisturbed position, (*ii*) the vorticity and pressure disturbances generated or transmitted at the detonation, and (*iii*) the variation of these disturbances with increasing exothermicity of the reaction and inflow Mach number.

2. Model Problems and Governing Equations

In this section and the rest of the paper, the following nomenclature is used (see Figure 1 for details): (i) $(x,\, y)$ are coordinates along and normal to the wedge centerline with the origin at the wedge apex, and $(u,\, v)$ are the corresponding velocity components; (ii) $(X,\, Y)$ are the coordinates along and normal to the wedge surface, and $(U,\, V)$ are the velocity components in these directions respectively; (iii) (ζ, η) are the coordinates normal and tangential to the detonation and $(M,\, N)$ are velocity components normal and tangential to the detonation; (iv) an overbar indicates a dimensional variable, a tilde indicates a nondimensional variable in the freestream, $()_0$ denotes a base state quantity, and a prime designates a disturbance quantity.

The problem of a steady plane detonation attached to a wedge in a uniform flow field has been given in Gross[3]. The turning angle χ of the detonation is a function of the flow velocity, the exothermicity, and the wedge angle. In solving the generalized Rankine Hugoniot equations, a diagram similar to Figure 2a is generated for each incoming Mach number or a diagram similar to Figure 2b is generated for each value of the exothermicity parameter. The physically relevant portion of the curves for the applications considered is the positively sloped portion. This corresponds to a weak overdriven detonation (as classified by Pratt[4]) in which the normal velocity behind the detonation is subsonic, but the overall flow behind the detonation is supersonic.

The equations governing the flow on each side of the detonation are the compressible Euler equations for an ideal gas. The dimensional form of the conservation of mass equation, the momentum equations and energy equation for an inviscid ideal gas in the coordinate system tangential and normal to the wedge surface is

$$\bar{\rho}_{\bar{t}} + \bar{U}\bar{\rho}_{\bar{X}} + \bar{V}\bar{\rho}_{\bar{Y}} + \bar{\rho}(\bar{U}_{\bar{X}} + \bar{V}_{\bar{Y}}) = 0 \tag{1a}$$

$$\bar{\rho}(\bar{U}_{\bar{t}} + \bar{U}\bar{U}_{\bar{X}} + \bar{V}\bar{U}_{\bar{Y}}) + \bar{p}_{\bar{X}} = 0 \tag{1b}$$

$$\bar{\rho}(\bar{V}_{\bar{t}} + \bar{U}\bar{V}_{\bar{X}} + \bar{V}\bar{V}_{\bar{Y}}) + \bar{p}_{\bar{Y}} = 0 \tag{1c}$$

$$(\bar{p}/\bar{\rho}^{\gamma})_{\bar{t}} + \bar{U}(\bar{p}/\bar{\rho}^{\gamma})_{\bar{X}} + \bar{V}(\bar{p}/\bar{\rho}^{\gamma})_{\bar{Y}} = 0 \tag{1d}$$

The exothermic reaction is assumed to be concentrated in the detonation of infinitesmal thickness, and therefore, the Rankine-Hugoniot conditions with heat release provide the appropriate conditions for the flow immediately behind the detonation. For an ideal gas, the dimensional Rankine-Hugoniot conditions are

$$\{\bar{\rho}\bar{M}\}|_f = \{\bar{\rho}\bar{M}\}|_b \tag{2a}$$

$$\{\bar{p} + \bar{\rho}\bar{M}^2\}|_f = \{\bar{p} + \bar{\rho}\bar{M}^2\}|_b \tag{2b}$$

$$\{\frac{\gamma}{\gamma-1}\frac{\bar{p}}{\bar{\rho}} + \frac{1}{2}\bar{M}^2 + \bar{q}\}|_f = \{\frac{\gamma}{\gamma-1}\frac{\bar{p}}{\bar{\rho}} + \frac{1}{2}\bar{M}^2\}|_b \tag{2c}$$

$$\bar{N}|_f = \bar{N}|_b \tag{2d}$$

where $|_f$ and $|_b$ indicate that the quantities should be evaluated on the fresh side and burnt side of the detonation. The uniform state of the burnt gas in the absence of disturbances has been chosen to nondimensionalize the flow variables. Hence, the characteristic scale for velocity is the sound speed a_b^*; for pressure, density and temperature, the scales are p_b^*, ρ_b^* and T_b^*. For the Stationary Wedge Problem, the characteristic length scale L^* is chosen such that the wavenumber of the incoming vorticity disturbance is unity; for the Oscillating Wedge Problem two types of motion are considered: a pivot about a fixed point along the centerline of the wedge and a lateral periodic motion along the centerline. For the pivoting motion of the wedge, the length scale is chosen as the distance from the apex to the pivot point; and for the lateral motion of the wedge, the length scale is chosen as a_b^*/ω^* where ω^* is the dimensional frequency of the wedge oscillations (in nondimensional units the frequency of the wedge oscillations will be unity). All length scales are assumed to be very large compared to the detonation thickness.

According to the variable designations, $\tilde{M}_0 + \tilde{M}'$ and $\tilde{N}_0 + \tilde{N}'$ are the nondimensional velocities of the gas in the fresh mixture normal and tangential to the undisturbed detonation position, and $M_0 + M'$ and $N_0 + N'$ are the same quantities in the burnt mixture. Furthermore, $\tilde{p}_0 + \tilde{p}'$, $\tilde{\rho}_0 + \tilde{\rho}'$, and $\tilde{T}_0 + \tilde{T}'$ are the nondimensional pressure, density and temperature in the fresh mixture with $p_0 + p'$, $\rho_0 + \rho'$ and $T_0 + T'$ being the same quantities in the burnt mixture. Because of the variables chosen for nondimensionalization, $p_0 = \rho_0 = T_0 = 1$. If the small deflection in the detonation position is given by $\zeta = h'(\eta, t)$ (see Figure 1), the constant leading order quantities at the detonation are:

$$\tilde{M}_0 = \frac{(1+\gamma M_0^2) + \sqrt{(1-M_0^2)^2 + 2M_0^2(\gamma+1)Q}}{M_0(\gamma+1)}, \tag{3a}$$

$$\tilde{N}_0 = N_0, \tag{3b}$$

$$\tilde{\rho}_0 = \frac{M_0}{\tilde{M}_0}, \tag{3c}$$

$$\tilde{T}_0 = 1 - Q + \frac{1}{2}(\gamma-1)(M_0^2 - \tilde{M}_0^2), \tag{3d}$$

$$\tilde{p}_0 = \frac{M_0}{\tilde{M}_0}\left(1 - Q + \frac{1}{2}(\gamma-1)(M_0^2 - \tilde{M}_0^2)\right), \tag{3e}$$

and the disturbance quantities at the detonation are:

$$M' = \alpha_1(N_0 h'_\eta + h'_t) + A_1\tilde{M}', \tag{4a}$$

$$N' = \alpha_2 h'_\eta + A_2\tilde{N}', \tag{4b}$$

$$p' = \gamma\alpha_3(N_0 h'_\eta + h'_t) + \gamma A_3 \tilde{M}', \tag{4c}$$

where

$$\alpha_1 = \frac{1-(\gamma-1)\tilde{M}_0 M_0 + \gamma M_0^2}{(1-M_0^2)}(1-\tilde{\rho}_0),$$
$$A_1 = \frac{\tilde{\rho}_0(1+\gamma M_0^2) - 2\gamma M_0^2 + (\gamma-1)\tilde{M}_0 M_0}{(1-M_0^2)},$$
$$\alpha_2 = (\tilde{M}_0 - M_0),$$
$$A_2 = 1,$$
$$\alpha_3 = -\frac{M_0}{(1-M_0^2)}\left\{2-(\gamma-1)\tilde{M}_0 M_0 + (\gamma-1)M_0^2\right\}(1-\tilde{\rho}_0),$$
$$A_3 = \frac{M_0}{(1-M_0^2)}\left\{2+2(\gamma-1)M_0^2 - (\gamma-1)\tilde{M}_0 M_0 - \tilde{\rho}_0(2+(\gamma-1)M_0^2)\right\}.$$

In the above disturbance relations, $\tilde{p}' = \tilde{\rho}' = \tilde{T}' = 0$ which is appropriate for the three problems considered. The exothermicity parameter is $Q = (\gamma-1)\bar{q}/a_b^{*2}$. For convenience, a second exothermicity parameter, an inflow Mach number, and the inflow Mach number normal to the detonation are defined as $q = (\gamma-1)\bar{q}/a_f^{*2}$, $U_{in} = \bar{U}_{0\,f}/a_f^*$, and $M_{in} = k\sin(\chi)\bar{U}_{0\,f}/a_f^*$, respectively.

For the Stationary Wedge Problem, the velocity field for the weak sinusoidal vorticity disturbance in the fresh incoming gas is assumed to be

$$\tilde{U} = \tilde{U}_0 + \epsilon\tilde{U}_0\cos(y+\delta), \quad \tilde{V} = 0, \tag{5}$$

where ϵ is small; the normal and tangential components are

$$\tilde{M}_0 + \tilde{M}' = \tilde{U}_0\sin(\chi) + \epsilon\tilde{U}_0\sin(\chi)\cos(y+\delta), \tag{6a}$$
$$\tilde{N}_0 + \tilde{N}' = \tilde{U}_0\cos(\chi) + \epsilon\tilde{U}_0\cos(\chi)\cos(y+\delta). \tag{6b}$$

For the Oscillating Wedge Problem, the free stream is assumed to be undisturbed $\tilde{U} = \tilde{U}_0$; therefore

$$\tilde{M}_0 = \tilde{U}_0\sin(\chi), \qquad \tilde{M}' = 0, \tag{7a}$$
$$\tilde{N}_0 = \tilde{U}_0\cos(\chi), \qquad \tilde{N}' = 0. \tag{7b}$$

The small disturbance in the incoming flow or the oscillation of the wedge produces a small deflection of the detonation from its otherwise planar state. This deflection generates a small amplitude acoustic, vorticity and entropy disturbance in the gas behind the detonation. Therefore, the following forms are introduced

$$U = U_0 + \epsilon U', \tag{8a}$$

$$V = \epsilon V', \tag{8b}$$

$$p = 1 + \epsilon p', \tag{8c}$$

$$\rho = 1 + \epsilon \rho'. \tag{8d}$$

The disturbance quantities satisfy the linearized Euler equations

$$U'_t + U_0 U'_X + \frac{1}{\gamma} p'_X = 0, \tag{9a}$$

$$V'_t + U_0 V'_X + \frac{1}{\gamma} p'_Y = 0, \tag{9b}$$

$$p'_t + U_0 p'_X + \gamma(U'_X + V'_Y) = 0, \tag{9c}$$

$$(p' - \gamma\rho')_t + U_0(p' - \gamma\rho')_X = 0. \tag{9d}$$

In addition, the zero flux condition must be imposed at the wedge surface. For the Stationary Wedge Problem where the wedge is considered stationary this condition is simply

$$V'|_{Y=0} = 0. \tag{10}$$

For the Oscillating Wedge Problem where the periodic motion is a pivoting motion about the point $(x, y) = (b, 0)$ with angle of rotation given by $\Gamma = \epsilon \exp(i\omega t)$, the condition becomes

$$V'|_{Y=0} = -[U_0 + i\omega(X - b\cos(\theta)]e^{i\omega t}, \tag{11}$$

where the pivot point is $b = 1$ as a result of the nondimensionalization. For lateral motion of the wedge where the position of the apex is given by $(x_a, y_a) = (\epsilon e^{i\omega t}, 0)$, the condition is

$$V'|_{Y=0} = -i\omega \sin\theta \, e^{i\omega t}. \tag{12}$$

with $\omega = 1$ as a result of the nondimensionalization. Finally, the conditions at the detonation (4), where $M' = \sin(\chi-\theta)\epsilon U' - \cos(\chi-\theta)\epsilon V'$ and $N' = \cos(\chi-\theta)\epsilon U' + \sin(\chi-\theta)\epsilon V'$, completely determine the solution and also determine the unknown detonation position $h'(\eta, t)$.

3. Analytic Solutions

A. Stationary Wedge Problem. In this problem where the wedge is stationary and the free stream is convecting a steady vorticity wave given by (5), the entire solution, including the detonation position h', is steady. The governing equations are separable, and the functions which satisfy the no flux condition at the wedge surface are:

$$\begin{aligned} p' &= p_i \cos(\mu_i X + \Delta_i)\cos(\mu_i \beta Y), \\ V' &= \frac{\beta}{\gamma U_0} p_i \sin(\mu_i X + \Delta_i)\sin(\mu_i \beta Y), \\ U' &= -\frac{1}{\gamma U_0} p_i \cos(\mu_i X + \Delta_i)\cos(\mu_i \beta Y); \end{aligned}$$

and

$$p' = 0, \qquad V' = 0, \qquad U' = S(Y).$$

The constants p_i, μ_i and Δ_i are arbitrary, $S(Y)$ is an arbitrary function which represents the vorticity disturbance, and $\beta^2 = U_0^2 - 1$. A general steady disturbance is represented by a linear combination of the above functions or by:

$$p' = -\gamma U_0 \{f(X+\beta Y) + f(X-\beta Y)\} \tag{13a}$$

$$V' = \beta\{f(X+\beta Y) - f(X-\beta Y)\} \tag{13b}$$

$$U' = S(Y) + \{f(X+\beta Y) + f(X-\beta Y)\} \tag{13c}$$

where f is an arbitrary function. The conditions across the detonation $X = \lambda Y$ ($y = Y(\cos\theta + \lambda\sin\theta)$ and $\eta = \sqrt{1+\lambda^2}Y$ along the detonation) can be written as an overdetermined linear system for two of the unknown functions $S(Y)$ and $\epsilon h_Y = \sqrt{1+\lambda^2}h'_\eta$

$$
\begin{pmatrix} 1 & -\alpha_1 N_0 \\ \lambda & -\alpha_2 \\ 0 & -\alpha_3 N_0 \end{pmatrix} \begin{pmatrix} S(Y) \\ h_Y \end{pmatrix} \tag{14}
$$

$$
= \begin{pmatrix} \hat{A}_1 \cos(y+\delta) + (\beta\lambda - 1) f((\lambda+\beta)Y) - (\beta\lambda + 1) f((\lambda-\beta)Y) \\ \hat{A}_2 \cos(y+\delta) - (\lambda+\beta) f((\lambda+\beta)Y) - (\lambda-\beta) f((\lambda-\beta)Y) \\ \hat{A}_3 \cos(y+\delta) + \sqrt{1+\lambda^2} U_0 \ (f((\lambda+\beta)Y) + f((\lambda-\beta)Y)) \end{pmatrix}
$$

where

$$
\begin{aligned}
\hat{A}_1 &= \tilde{U}_0 \sqrt{1+\lambda^2} \sin(\chi) A_1, \\
\hat{A}_2 &= \tilde{U}_0 \sqrt{1+\lambda^2} \cos(\chi) A_2, \\
\hat{A}_3 &= \tilde{U}_0 \sqrt{1+\lambda^2} \sin(\chi) A_3.
\end{aligned}
$$

The above system is easily reduced to

$$
\begin{pmatrix} 1 & 0 \\ 0 & -\alpha_3 N_0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} S(Y) \\ h_Y \end{pmatrix} \tag{15}
$$

$$
= \begin{pmatrix} (\hat{A}_1 - \frac{\alpha_1}{\alpha_3} \hat{A}_3) \cos(y+\delta) - \frac{\alpha_-}{\alpha 3} f((\lambda+\beta)Y) - \frac{\alpha_+}{\alpha 3} f((\lambda-\beta)Y) \\ \hat{A}_3 \cos(y+\delta) + \sqrt{1+\lambda^2} U_0 \ \{f((\lambda+\beta)Y) + f((\lambda-\beta)Y)\} \\ [\alpha_3 N_0 (\hat{A}_2 - \lambda \hat{A}_1) + (\lambda \alpha_1 N_0 - \alpha_2) \hat{A}_3] \cos(y+\delta) \\ + P_- f((\lambda+\beta)Y) + P_+ f((\lambda-\beta)Y) \end{pmatrix}
$$

where

$$
\begin{aligned}
\alpha_\pm &= \alpha_3 + \alpha_1 U_0 \sqrt{1+\lambda^2} \pm \beta\lambda\alpha_3, \\
P_\pm &= \sqrt{1+\lambda^2} \{U_0 (\lambda \alpha_1 N_0 - \alpha_2) \pm \beta \sqrt{1+\lambda^2} \alpha_3 N_0\}.
\end{aligned}
$$

The last equation provides the solvability condition which determines the unknown function $f(z)$. If the function $f(z)$ is expanded as

$$
f(z) = \sum_{i=0} P_i \cos(\nu_i z + \delta), \tag{16}
$$

it is found that

$$\nu_0 = \frac{\cos\theta + \lambda \sin\theta}{\lambda + \beta}, \tag{17a}$$

$$\nu_{i+1} = r\nu_i = r^{i+1}\nu_0, \tag{17b}$$

$$P_0 = -\frac{\alpha_3 N_0(\hat{A}_2 - \lambda \hat{A}_1) + (\lambda\alpha_1 N_0 - \alpha_2)\hat{A}_3}{P_-}, \tag{17c}$$

$$P_{i+1} = -\frac{P_+}{P_-}P_i = \left(-\frac{P_+}{P_-}\right)^{i+1} P_0, \tag{17d}$$

where

$$r = \frac{\lambda - \beta}{\lambda + \beta}.$$

Utilizing the above quantities, the displacement of the detonation and the vorticity behind the shock can be conveniently written as

$$\begin{aligned} h_Y = &-\frac{\hat{A}_3 + U_0\sqrt{1+\lambda^2}P_0}{\alpha_3 N_0}\cos((\lambda+\beta)\nu_0 Y + \delta) + \frac{(P_- - P_+)U_0\sqrt{1+\lambda^2}P_0}{(P_- + P_+)\alpha_3 N_0}\cos(\delta) \\ &+ \frac{(P_- - P_+)\sqrt{1+\lambda^2}U_0}{\alpha_3 N_0 P_-}P_0\cos((\lambda-\beta)\nu_0 Y + \delta) - \frac{(P_- - P_+)\sqrt{1+\lambda^2}U_0}{\alpha_3 N_0 P_-}P_0\cos(\delta) \\ &+ \frac{(P_- - P_+)\sqrt{1+\lambda^2}U_0}{\alpha_3 N_0 P_-}P_0\sum_{i=1}\left(-\frac{P_+}{P_-}\right)^i\left[\cos((\lambda-\beta)r^i\nu_0 Y + \delta) - \cos(\delta)\right], \end{aligned} \tag{18a}$$

$$\begin{aligned} S(Y) = &-\frac{(\alpha_3\hat{A}_1 - \alpha_1\hat{A}_3) - \alpha_- P_0}{\alpha_3}\cos((\lambda+\beta)\nu_0 Y + \delta) \\ &+ \frac{\alpha_- P_+ - \alpha_+ P_-}{\alpha_3 P_-}P_0\sum_{i=0}\left(-\frac{P_+}{P_-}\right)^i\cos((\lambda-\beta)r^i\nu_0 Y + \delta). \end{aligned} \tag{18b}$$

All quantities have been determined, and the results are presented in Section 4.

B. Oscillating Wedge Problem. For the problem where the wedge pivots about a point with a time periodic motion of small amplitudes, the disturbances are also time periodic. This implies that any transients due to the initiation of the motion have had sufficient time to propagate out of the region of interest. The solutions are sought in the form

$$U = U_0 + \epsilon e^{i\omega t} U', \tag{19a}$$

$$V = \epsilon e^{i\omega t} V', \tag{19b}$$

$$p = 1 + \epsilon e^{i\omega t} p', \tag{19c}$$

$$\rho = 1 + \epsilon e^{i\omega t} \rho', \tag{19d}$$

and the linearized Euler equations become

$$icU' + U_0 U'_X + \frac{1}{\gamma} p'_X = 0, \tag{20a}$$

$$i\omega V' + U_0 V'_X + \frac{1}{\gamma} p'_Y = 0, \tag{20b}$$

$$i\omega p' + U_0 p'_X + \frac{1}{\gamma}(U'_X + V'_Y) = 0, \tag{20c}$$

$$i\omega(p' - \gamma\rho') + U_0(p' - \gamma\rho')_X = 0. \tag{20d}$$

The first three govern the acoustic and vorticity disturbances, while the fourth governs the convection of the entropy disturbance. In addition, the detonation position is assumed to be a function of Y ($\eta = \sqrt{1+\lambda^2}Y$ along the detonation) and t of the form

$$h' = \epsilon e^{i\omega t} h_0(Y). \tag{21}$$

It is only necessary to utilize the equations for U', V' and p' to completely determine all quantities.

Following Carrier[5] and Van Dyke[6], the functions ϕ and E are introduced such that

$$U' = \phi_X + E_Y, \tag{22a}$$

$$V' = \phi_Y - E_X, \tag{22b}$$

$$p' = -\gamma(i\omega\phi + U_0\phi_X), \tag{22c}$$

which are found to satisfy

$$\nabla^2\phi - U_0^2\phi_{XX} - 2i\omega U_0\phi_X + \omega^2\phi = 0, \tag{23a}$$

$$i\omega E + U_0 E_X = 0. \tag{23b}$$

The eigenfunctions for the first equation are

$$e^{-i\omega U_0 X/\beta^2}\cosh(\nu\Theta)J_\nu(\frac{\omega}{\beta^2}r), \quad e^{-i\omega U_0 X/\beta^2}\sinh(\nu\Theta)J_\nu(\frac{\omega}{\beta^2}r)$$

with $r^2 = X^2 - \beta^2 Y^2$ and $\tanh(\Theta) = \beta Y/X$. In order to satisfy the boundary conditions, the rotational component E and the detonation position h_0 are expanded in convenient series of Bessel Functions. The final form of the solution is

$$\phi = e^{-i\omega U_0 X/\beta^2}\sum_{\nu=1}\{a_\nu \cosh(\nu\Theta) + b_\nu \sinh(\nu\Theta)\}J_\nu(kr) \tag{24a}$$

$$E = e^{-i\omega(X/U_0+\lambda Y/(U_0\beta^2))}\sum_{\nu=1} c_\nu J_\nu(k\xi Y) \tag{24b}$$

$$h_0 = e^{-i\omega U_0\lambda Y/\beta^2}\sum_{\nu=0} d_\nu J_\nu(k\xi Y) \tag{24b}$$

where $k = \frac{\omega}{\beta^2}$ and $\xi = \sqrt{\lambda^2 - \beta^2}$. It is left to determine a_ν, b_ν, c_ν and d_ν using the boundary condition at the wedge surface and the appropriate conditions at the detonation. It should be noted that solutions exist when a_0 and c_0 are not zero, but these solutions represent modes which are excited by the initial start up. By assumption these modes are neglected, and the summations for ϕ and E start at $\nu = 1$. The assumption that the detonation is attached to the wedge apex determines d_0 and the condition at the surface (11) determines the b_ν. For the wedge which oscillates about the pivot point,

$$d_0 = -b\cos(\chi), \qquad b_\nu = i\beta\nu\left[\tau^\nu + (-\tau)^{-\nu}\right] + b\cos\theta\left[\tau^\nu - (-\tau)^{-\nu}\right] \tag{25}$$

where $\tau = iU_0 + i\sqrt{U_0^2 - 1}$. The conditions at the detonation (4) provide the recursive relations (see Appendix) for a_ν, c_ν and d_ν ($\nu \geq 1$) in terms of the known quantities b_ν and d_0. For the wedge undergoing lateral periodic motion, the frequency ω has been normalized to unity and the constants d_0 and b_ν are found to satisfy

$$d_0 = \sin(\chi), \qquad b_\nu = \sin\theta\left[(\tau)^\nu + (-\tau)^{-\nu}\right] \tag{26}$$

where $\tau = iU_0 + i\sqrt{U_0^2 - 1}$.

4. Results. It is possible to compute the velocity and pressure fields for the cases presented above. After determining the various coefficients, the summations (18) or (24) are computed. This paper focuses on the detonation position and the vorticity and pressure disturbances generated or transmitted at the detonation as a function of the length along the detonation. Furthermore, the variation of these quantities as a function of exothermicity and inflow Mach number is investigated.

A. Stationary Wedge Problem Figures 3(a-c) show the detonation position h_0, vorticity response and pressure response for inflow Mach number U_{in} equal to nine with q $= 1$, $\delta = 0$ and $\theta = 20$. The results of Jackson, Kapila, and Hussaini[7] are shown in dashed lines and represent the lead term in each of the equations (18a) and (18b) as well as the first generated pressure wave. The differences between the oblique detonation attached to the wedge and the unobstructed oblique detonation are seen. First, the pressure for the attached detonation has two discernible wavelengths, a result of a perfect reflection off the wedge surface with the reflected pressure wave being just as strong as the incoming wave. When the reflection returns to the detonation, it distorts the detonation further, as is seen by the two discernible wavelengths in the detonation position, and some of the energy is transferred to new entropy and vorticity waves. This second interaction is weak as seen from the small difference between the vorticity response of the attached oblique detonation and the detonation in the unobstructed flow. The second difference is in the detonation position. There appears to be an overall change of slope for the attached detonation. This change of slope can be determined from the sum of the second and fourth terms in equation (18a) where it is found that this apparent change of slope is proportional to $\cos\delta$. The summation in (18a) is approximately zero since the quantity $(\lambda - \beta)\nu_i Y$ is small for the scale chosen in Figure 3. The second term of (18a) represents the change in slope at the apex of the wedge also proportional to $\cos\delta$. For the unobstructed detonation, the solution demonstrates no overall dependence on the phase δ of the incoming disturbance owing to the invariance under a vertical translation; however, with a wedge present, the problem is no longer invariant under such a translation which results in the effects just mentioned.

The dependence of the responses on inflow Mach number and exothermicity is explored in Figures 4. Since for fixed wedge angle, a change in any of the other parameters automatically changes the angle at which the detonation meets the incoming flow, it is not possible to isolate individual effects in a single figure. Therefore, the Mach number of the velocity component normal to the detonation on the outflow side is chosen as the independent variable. This independent variable provides an easy measure of the degree of overdrive since the Chapman-Jouget point corresponds to the outflow normal Mach number equal to unity. The input parameters are chosen in terms of the incoming stream although the nondimensionalization has been given in terms of the outflow variables. The nondimensionalization was chosen for convenience of the algebraic manipulations even though it is conceptionally easier to think about specifying inflow conditions. Figure 4a is the amplitude of the first term in equation (18a) and Figure 4b is the second term of (18a) which gives the change of slope at the apex owing to the presence of the wedge. Figure 4c is the amplitudes of the first term of (18b) which is the transmission coefficient for the vorticity, and Figure 4d is the amplitude of the first term in the summation of (18b) which represents the strength of the vorticity generated by the first reflection. Figures 4e and 4f are the first and second generation coefficients for the pressure. The curves of Figure 4 are produced by fixing the inflow Mach number and the exothermicity parameter; then, the half angle of the wedge is varied from its minimum to maximum value. ¿From Figures 4, it is determined that for constant exothermicity or constant inflow Mach number, the response decreases as the other parameters are changed such that the C-J point is approached. Furthermore, the summations do not converge as the the sonic point is reached which corresponds to a wedge angle near the maximum allowable wedge angle and therefore the maximum degree of overdrive for a given inflow Mach number and reaction. At the sonic point, most of the assumptions inherent in performing the linear theory do not hold and a nonlinear theory is required. Four carefully chosen points are indicated in Figures 4. These correspond to $U_{in} = 5$, $\theta = 20^o$; $U_{in} = 9$, $\theta = 10^o$; $U_{in} = 9$, $\theta = 20^o$; and $U_{in} = 9$, $\theta = 26^o$. The first two compare in degree of overdrive. The first and third consider the trend used in many numerical calculations (*i.e.* the same reaction, the same wedge angle, same freestream conditions except for inflow Mach number). Finally, the first and fourth correspond to approximately the same angle between the detonation and the freestream. The relevant data is summarized in Table 1. From these points, it is determined that the transmission coefficient of the vorticity and the vorticity generated by the first reflection are dependent on the inflow Mach number. The transmission coefficient for Mach 5 is greater than the transmission coefficient for all three indicated points with Mach 9. The inflow Mach number dependence is not as great for the vorticity generated by the first reflection; this is consistent with the results for the Oscillating Wedge Problem discussed below. The first and second pressure coefficients are almost identical owing to the perfect reflection off the wedge surface. The second coefficient is only slightly lower than the first, since some of the energy of the reflected wave is transferred to vorticity, entropy and detonation curvature. The pressure coefficients show a dependence on the inflow Mach number owing to the inflow Mach number dependence of the vorticity response which generates the pressure disturbance.

In addition to the effects of Mach number on the flow, the effects of exothermicity is also explored in Figures 4. The crosses represent the response for $U_{in} = 9$ and $q = 0$ and the filled squares represent the responses for $U_{in} = 9$ and $q = 2$. The angles chosen for the calculations are the same as for the $U_{in} = 9$ and $q = 1$ case already discussed. It is seen that increasing exothermicity weakly increases the vorticity and pressure responses for the same wedge angle; however, for approximately the same degree of overdrive as measured by the normal outflow Mach number, exothermicity is seen to significantly increase the vorticity and pressure responses. The response of the detonation position is seen to be almost completely determined by the degree of overdrive.

B. Oscillating Wedge Problem. For the wedge undergoing a periodic pivoting motion, equation (25) is used in the recurrence relation and the summations (24) are computed. The pressure response p', the vorticity response $E_{YY}+E_{XX}$, and the detonation position h' are shown as functions of the distance from the apex along the detonation for three flow conditions in Figures 5. The flow condition for the first column are $U_{in} = 5$, $\theta = 20^o$, and $q = 1$; for the second column, $U_{in} = 9$, $\theta = 10$, and $q = 1$; and for the third column, $U_{in} = 9$, $\theta = 20^o$ and $q = 1$. Many oscillations are shown in Figures 5 although for any device of practical length only a few such oscillations are expected to be present. However, by considering the limit of large distances from the wedge apex, it is easier to quantify the results. Each of the three curves were fit to the functional form of $A\eta\cos(k\eta + \Delta)$ for η large (η being the distance along the detonation). The amplitude of the pressure disturbance is directly proportional to the frequency of the oscillation of the wedge, the amplitude of the vorticity is proportional to the square of the frequency, and the wave number of the response is proportional to the frequency. This can be determined by the computed solutions and also by examination of the summations; the pressure involves first derivatives of ϕ while the vorticity involves second derivatives of E. In Figure 6, the amplitudes determined by the fitting are presented as functions of the outflow Mach number normal to the detonation. It is clear that the pressure and detonation position amplitudes depend almost completely on the outflow Mach number with very little dependence on the tangential components and hence on the inflow Mach number. The responses decrease quickly as the C-J point is approached. The dependence on the tangential component (and therefore the inflow Mach number) is seen in the plots of the frequencies and the vorticity. The marked points are for the same conditions given for the Stationary Wedge Problem. The effects of holding the wedge angle and the exothermicity fixed while increasing the inflow Mach number are opposite the trend observed for the transmission of the vorticity wave. The responses greatly increase with increasing inflow Mach number. While the vorticity does show a strong dependence on the outflow normal Mach number, mostly owing to the strong dependence of the pressure which generates the vorticity, there is an equally strong dependence on the inflow Mach number. The points corresponding to $U_{in} = 5$, $\theta = 20^o$ (square) and $U_{in} = 9$, $\theta = 10^o$ (triangle) have almost the same degree of overdrive, but differ substantially in the vorticity generated although there is almost no difference in the pressure response.

The effects of exothermicity are investigated by considering $U_{in} = 5$ and $q = 0$ for three wedge angles: θ is equal to 5^o, 10^o, and 20^o. The responses are given by the filled symbols in Figures 6. For $\theta = 10^o$, the effects of exothermicity is to significantly lower the degree of overdrive, and therefore there is a significant decrease in the responses as the exothermicity is increased for constant wedge angle and constant inflow Mach number. However, for the two cases where the inflow Mach number is constant and the degree of overdrive is approximately the same (the two squares), the higher the exothermicity, the larger the vorticity response; for the same two conditions there is little variation in the pressure response.

The above analysis is a limiting case since the disturbances far away from the wedge apex are mostly the result of the relatively large disturbances imposed by the large movement of the wedge downstream of the pivot point. Hence, only one wavelength is present which represents the disturbance generated by the movement of the wedge surface which propagates along the characteristic intersecting the detonation; disturbances with other wavelengths which would represent the regeneration of acoustic waves by curvature of the detonation front and the subsequent reflection off of the wedge or the displacement of the detonation by movement of the apex are very small by comparison and are not discernible in the figures. The response of the detonation front decreases rapidly as the C-J point is approached since the characteristic along which the pressure disturbance travels from the wedge surface to the detonation and the detonation position itself become almost parallel as the C-J point is approached with the C-J point being the limiting case when the two are exactly parallel. The problem of lateral periodic motion discussed below provides a counterexample to the above problem in which the disturbance strength does not increase as the distance from the apex of the wedge increases.

The summations (24) are again computed after solving the recurrence relations utilizing (12) for the boundary condition rather than (11); therefore, equations (26) are used in the recurrence relation in place of (25). The amplitude of the imposed disturbance does not increase as a function of the distance from the apex as it does for a pivoting motion of the wedge. Although the results are not as easily quantifiable in this case as in the previous case, Figures 7 (which have the same flow conditions as Figures 5) provide insight into the underlying physical processes. First, the pressure responses and the responses of the detonation position show a single overall wavelength which is identical to the wavelength observed for the pivoting wedge with the same frequency of motion. This results from the same physical process, that is, the movement of the wedge surface generates a pressure disturbance which propagates to the detonation position where it interacts with the detonation by distorting the position and initiating the generation of entropy and vorticity disturbances. Disturbances of shorter wavelengths result from the movement of the apex of the wedge to which the detonation is assumed to remain attached. This disturbance decays as a function of the distance from the wedge faster than the disturbance owing to the movement of the wedge surface. The second disturbance becomes weaker as the angle

χ is decreased since the constant d_0 is proportional to the sine of this angle. This type of disturbance is also present for the pivoting wedge, but in the limit of large distance from the apex, it is negligible compared to the disturbance generated by the movement of the wedge surface.

4. Conclusion. The linear response of an oblique overdriven detonation to imposed free stream disturbances or to periodic movements of the wedge surface and apex is examined. It is found that the strength of the disturbance is best characterized by considering the normal Mach number of the flow behind the detonation. In all cases, the response, as measured by the difference of the detonation position from its undisturbed position, the pressure at the detonation, and the vorticity at the detonation, is weaker as the C-J point is approached. This is a result of the detonation and the only characteristic which crosses the detonation becoming parallel as the C-J point is approached. It is also found that the vorticity response is in general dependent on the inflow Mach number, while the pressure response and the deviation of the detonation from its undisturbed position are mostly functions of the normal outflow Mach number.

From this analysis, a fundamental difference in the behaviour of the responses as functions of the input parameters dependent upon whether the detonation is responding to a disturbance coming from the free stream or whether the disturbance is propagating from the wedge surface to the detonation is shown. In the latter case, the pressure and detonation position show almost no dependence on the inflow Mach number assuming that the degree of overdrive is the same, and the vorticity responses show a significant influence of the inflow Mach number. This same behaviour is seen in the response of the detonation to the first reflected pressure wave arriving from the wedge surface. The change in slope at the apex of the wedge is again independent of the inflow Mach number assuming the same degree of overdrive. Also, the vorticity generated by this reflected wave shows an increase for increasing inflow Mach number (wedge angle and exothermicity constant) whereas the immediate response to the incoming vorticity wave shows the opposite dependence on the inflow Mach number. The effects of increasing exothermicity while holding inflow Mach number and wedge angle constant show the same opposite behaviour depending on whether the detonation is responding to a free stream disturbance or to a disturbance emanating from the wedge surface.

In determining if an oblique detonation is stablized by increasing the Mach number of the incoming flow, this study shows that the computational domain will need to be sufficiently large in order to capture the response of the system to reflected acoustic waves. The response to these reflected waves increase with increasing Mach number, whereas the response to disturbances coming from the freestream decrease with increasing Mach number. It could be imagined that increasing inflow Mach number is stabilizing to freestream disturbances while at the same time is destabilizing to disturbances reflected from the wedge surface or generated by movements of the wedge itself.

Acknowledgements

The authors would like to thank T.L. Jackson for his helpful comments.

This work was supported by the National Aeronautics and Space Administration under Nasa Contract No. 18605 while the authors were in residence at the Institute of Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, Virginia 23665.

Appendix. The recurrence relation is the same as given in Van Dyke[6] with the matrix elements modified to allow for heat release across the discontinuity. Further algebraic simplifications could be made; however, the formulae given here can easily be compared to a direct application of the linearized conditions across the detonation.

$$\begin{pmatrix} \cosh(\nu\Theta_0) & \xi & -\xi(\lambda\alpha_2+\alpha_1 N_0)/(1+\lambda^2) \\ \beta\sinh(\nu\Theta_0) & 0 & \xi(\lambda\alpha_1 N_0-\alpha_2)/(1+\lambda^2) \\ -U_0\cosh(\nu\Theta_0) & 0 & -\xi\alpha_3 N_0/\sqrt{1+\lambda^2} \end{pmatrix} \begin{pmatrix} a_{\nu+1} \\ c_{\nu+1} \\ d_{\nu+1} \end{pmatrix} +$$

$$\begin{pmatrix} -2iU_0\cosh(\nu\Theta_0) & -2i\lambda/U_0 & \begin{matrix} 2i\lambda U_0(\lambda\alpha_2+\alpha_1 N_0)/(1+\lambda^2) \\ -2i\beta^2\alpha_1/\sqrt{1+\lambda^2} \end{matrix} \\ 0 & i\beta^2/U_0 & \begin{matrix} 2i\lambda U_0(\alpha_2-\lambda\alpha_1 N_0)/(1+\lambda^2) \\ +2i\beta^2\alpha_1\lambda/\sqrt{1+\lambda^2} \end{matrix} \\ 2i\cosh(\nu\Theta_0) & 0 & 2i\lambda U_0\alpha_3 N_0/\sqrt{1+\lambda^2}-2i\beta^2\alpha_3 \end{pmatrix} \begin{pmatrix} a_\nu \\ c_\nu \\ d_\nu \end{pmatrix}$$

$$
+\begin{pmatrix} -\cosh(\nu\Theta_0) & -\xi & \xi(\lambda\alpha_2+\alpha_1 N_0)/(1+\lambda^2) \\ \beta\sinh(\nu\Theta_0) & 0 & -\xi(\lambda\alpha_1 N_0-\alpha_2)/(1+\lambda^2) \\ U_0\cosh(\nu\Theta_0) & 0 & \xi\alpha_3 N_0/\sqrt{1+\lambda^2} \end{pmatrix}\begin{pmatrix} a_{\nu-1} \\ c_{\nu-1} \\ d_{\nu-1} \end{pmatrix}
$$

$$
+\begin{pmatrix} \sinh(\nu\Theta_0) & -2iU_0\sinh(\nu\Theta_0) & -\sinh(\nu\Theta_0) \\ \beta\cosh(\nu\Theta_0) & 0 & \beta\cosh(\nu\Theta_0) \\ -U_0\sinh(\nu\Theta_0) & 2i\sinh(\nu\Theta_0) & U_0\sinh(\nu\Theta_0) \end{pmatrix}\begin{pmatrix} b_{\nu+1} \\ b_{\nu} \\ b_{\nu-1} \end{pmatrix}
$$

References

[1]T. Fujiwara, A. Matsuo, and H. Nomoto, "A Two-Dimensional Detonation Supported by a Blunt Body or Wedge," AIAA Paper No. AIAA-88-0098, 1988.

[2]A. Kumar, D. M. Bushnell, and M.Y. Hussaini, "Mixing Augmentation Technique for Hypervelocity Scramjets," Journal of Propulsion and Power **5** 514 (1989).

[3]R. A. Gross, "Oblique Detonation Waves," AIAA J. **1** 1225 (1963).

[4]D. T. Pratt, J.W. Humphrey, and D.E. Glenn, "Morphology of a Standing Oblique Detonation Wave," AIAA Paper No. AIAA-87-1785, 1987.

[5]G. F. Carrier, "On the Stability of the Supersonics Flows Past a Wedge," Quart. Appl. Math. **6**, 367 (1948).

[6]Van Dyke, M. D., "On Supersonic Flow Past an Oscillating Wedge," Quart. Appl. Math. **11**, 360 (1953).

[7]M.Y. Hussaini, F. Collier, and D. M. Bushnell, "Turbulence Alteration due to Shock Motion", IUTAM Symposium, Turbulent Shear-Layer/Shock-Wave Interactions, edited by J. Delery, Springer-Verlag, New York, 1985, pp. 371-381.

[8]T.L. Jackson, A.K. Kapila, and M.Y. Hussaini, "Convection of a Pattern of Vorticity Through a Reacting Shock", Phys. Fluids A **2** 1260 (1990)

[9]D.G. Lasseigne , T.L. Jackson, and M.Y. Hussaini, "Non-Linear Interaction of a Detonation/Vorticity Wave", Phys. Fluids A. **3**, 1972 (1991)

U_{in}	θ	q	M_{in}/M_{CJ}	χ	Symbol (Figure)	
5	20	1	1.178	37.41	□	(4,6)
9	10	1	1.157	18.64	○	(4,6)
9	20	1	1.663	28.46	△	(4,6)
9	26	1	2.033	35.62	◇	(4)
9	10	0	2.314	14.90	+	(4)
9	20	0	3.967	26.15	+	(4)
9	26	0	4.977	33.57	+	(4)
9	10	2	1.019	22.61	■	(4)
9	20	2	1.360	30.86	■	(4)
9	26	2	1.625	37.78	■	(4)
5	5	0	1.300	15.07	●	(6)
5	10	0	1.659	19.37	■	(6)
5	20	0	2.485	29.80	▼	(6)

Table 1. Degree of overdrive M_{in}/M_{CJ} and detonation angle χ for the input parameters inflow Mach number U_{in}, wedge half-angle θ, and heat release parameter q. Also indicated are the corresponding symbol and figure number for each inlow condition.

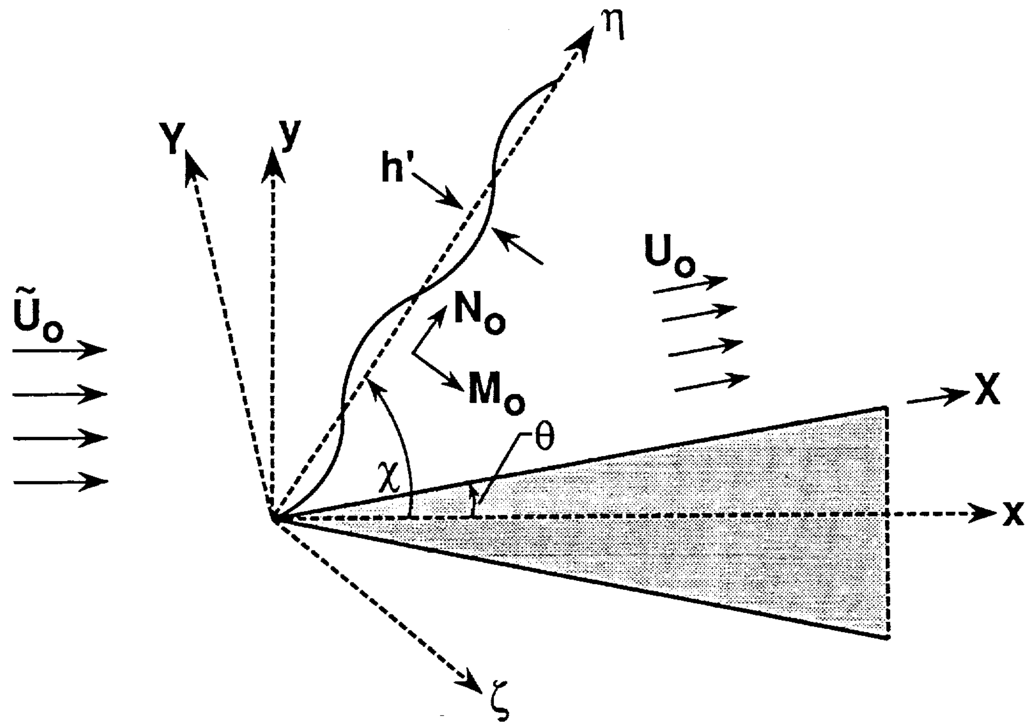

Figure 1. Schematic of the model problem.

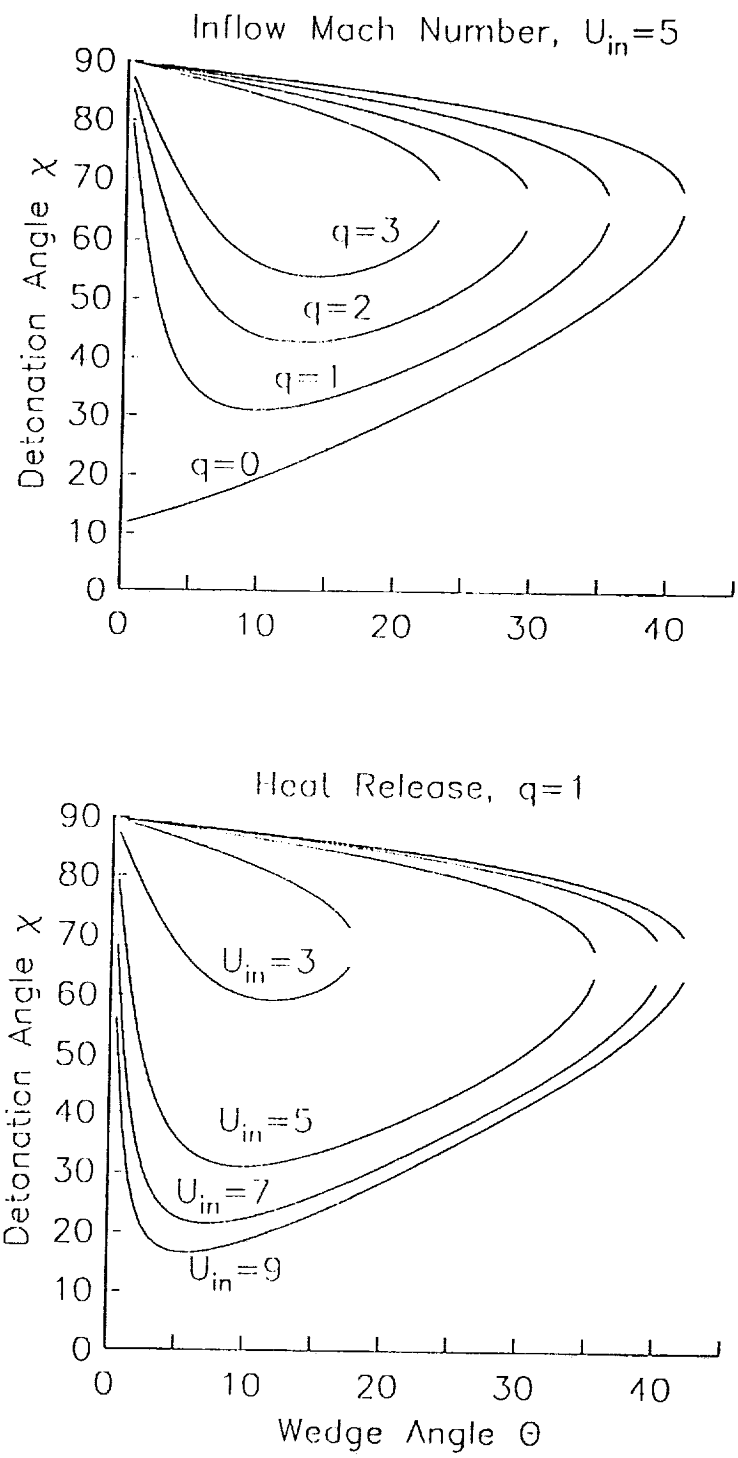

Figure 2. Detonation angle χ *vs.* wedge angle θ for various heat release parameters q and inflow Mach numbers U_{in}.

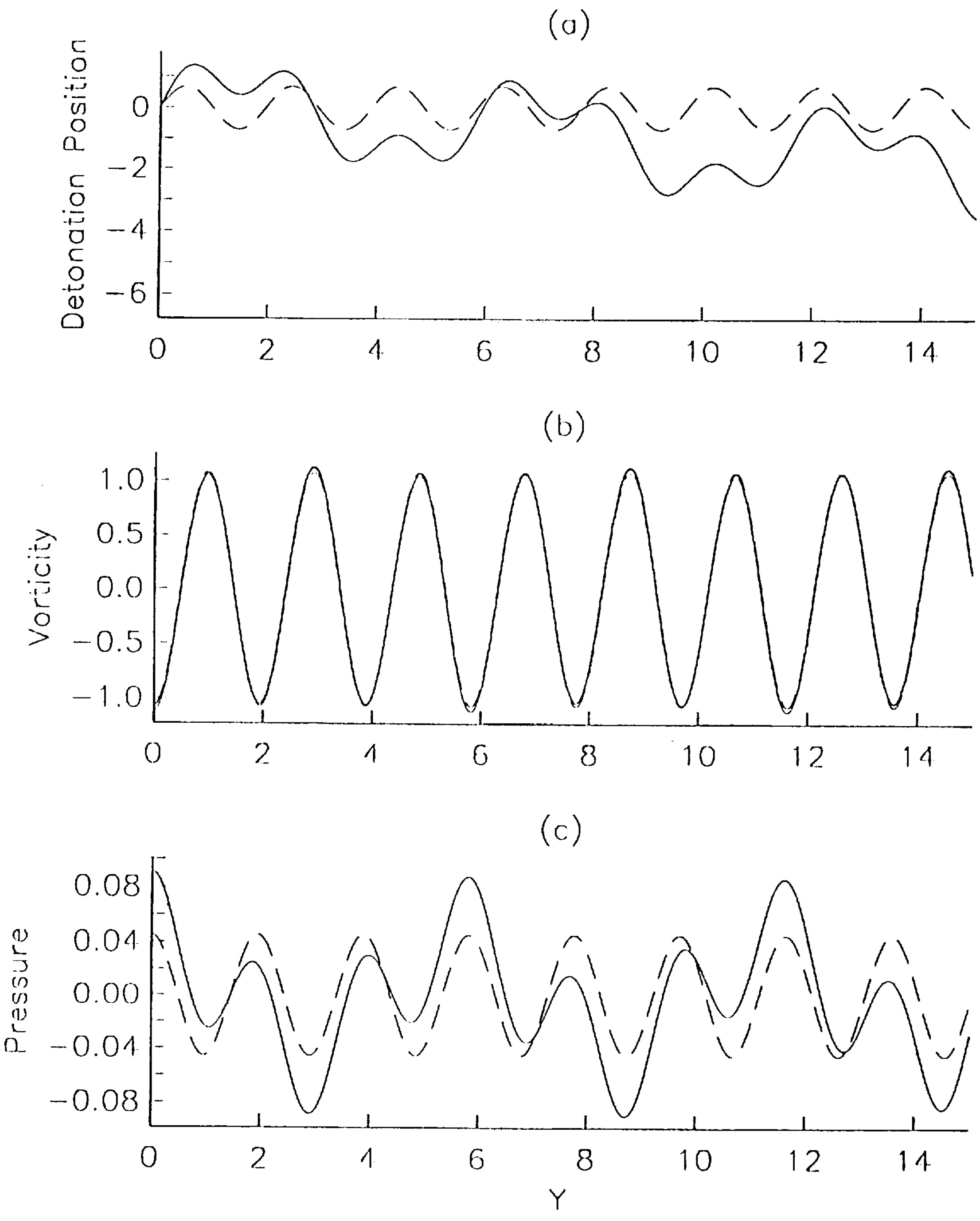

Figure 3. Plot of (a) Detonation position h', (b) Vorticity function $S/\tilde{U}_0$, and (c) Pressure function $p'/\tilde{U}_0$ along the detonation. Dashed lines represent the solution without the presence of the wedge.

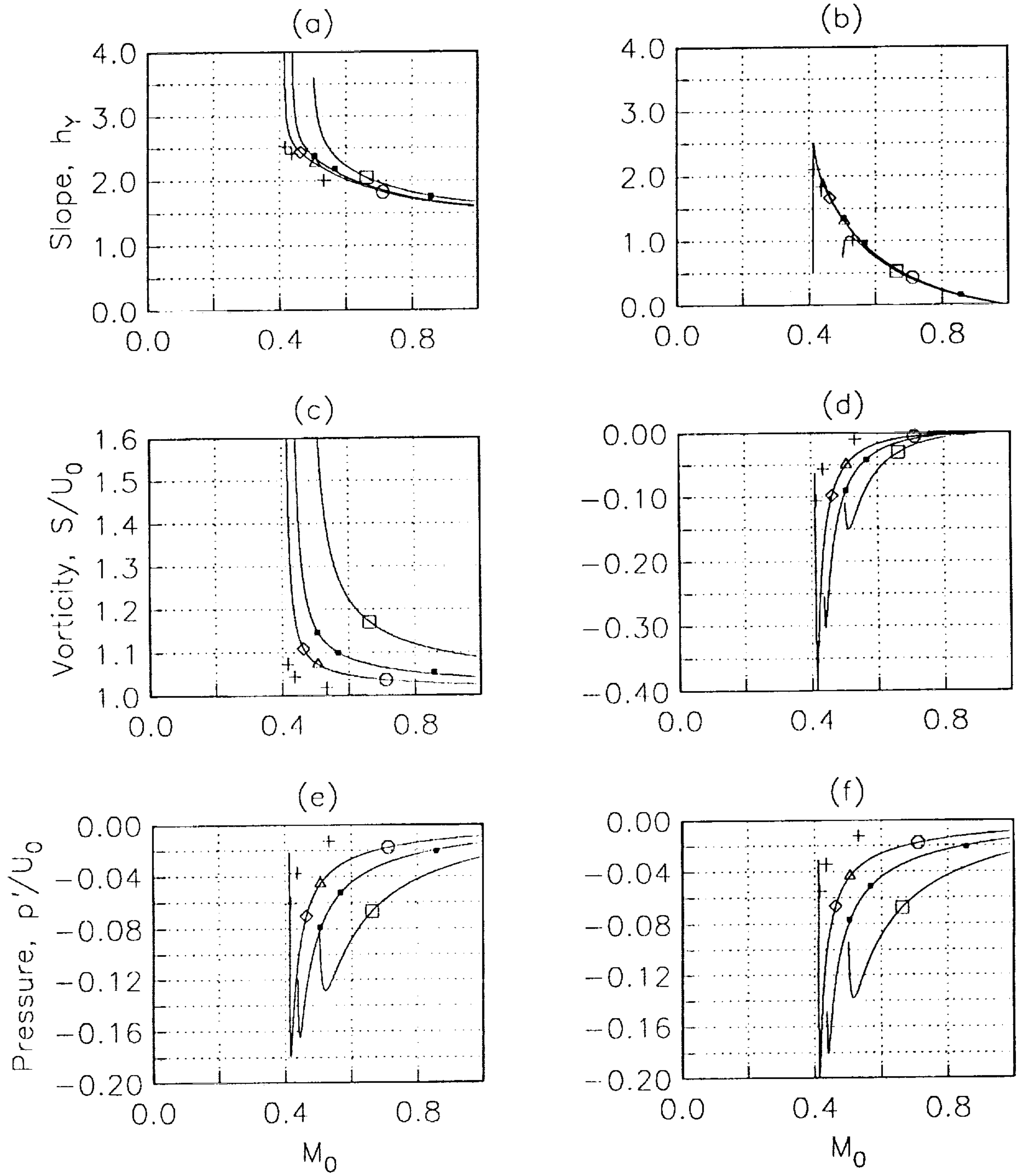

Figure 4. Plot of **(a)** First coefficient for the slope of the Detonation position h'; **(b)** The correction to the slope at the apex of the wedge; **(c)** First coefficient for the Vorticity function $S/\tilde{U}_0$; **(d)** Second coefficient for the Vorticity function; **(e)** First coefficient for the Pressure function $p'/\tilde{U}_0$; and **(f)** Second coefficient for the Pressure function *vs.* the downstream normal Mach number $M+0$. See Table 1 and text for meaning of the symbols.

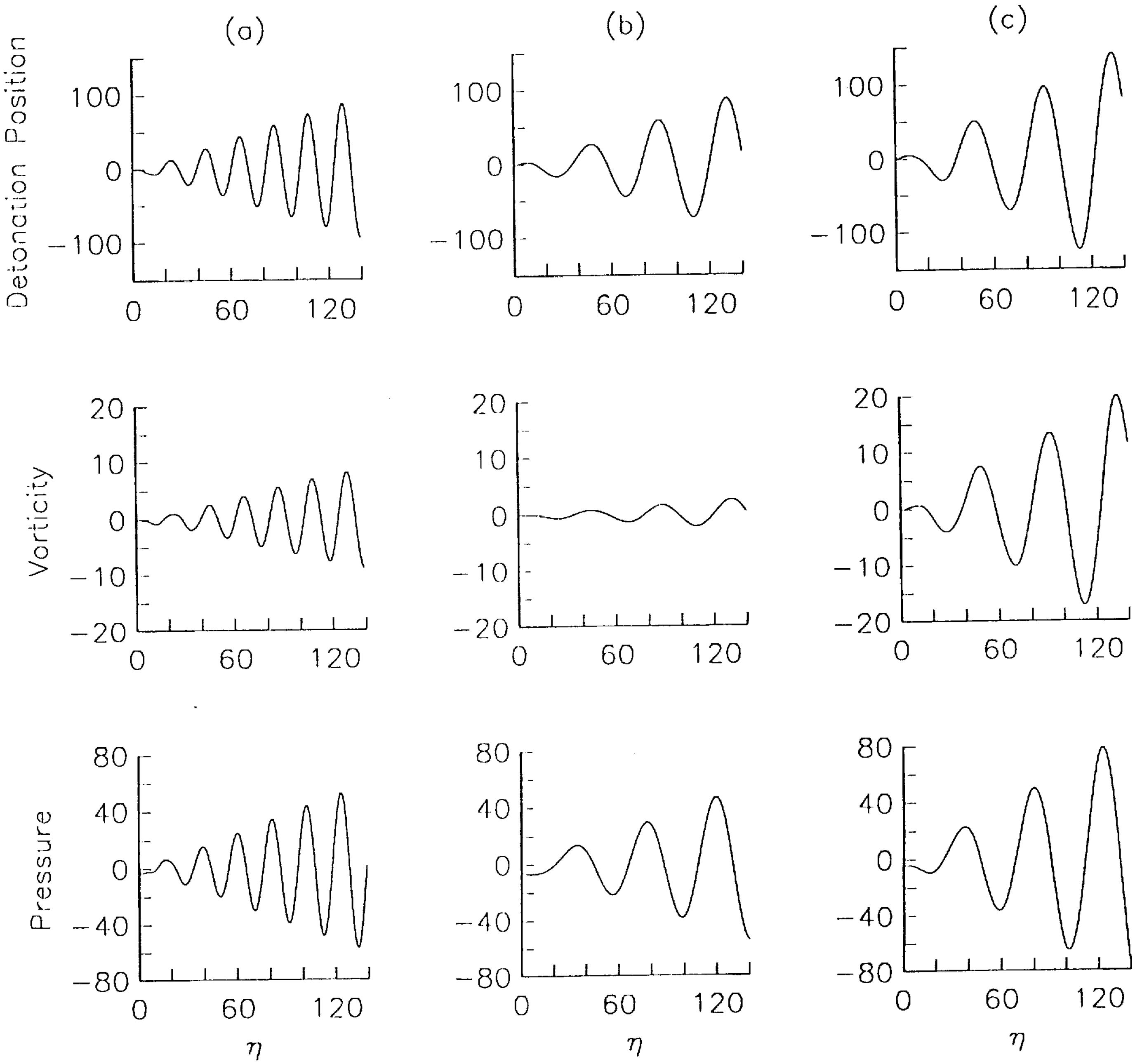

Figure 5. Detonation position h', vorticity $e^{i\omega t}(E_{XX} + E_{YY})$, and pressure $-\gamma e^{i\omega t}(i\omega\phi + U_0\phi_X)$ along the detonation for pivoting oscillations of the wedge at $t = \pi/4$. Flow conditions in each column are: **(a)** $U_{in} = 5$, $\theta = 20^o$, $q = 1$; **(b)** $U_{in} = 9$, $\theta = 10^o$, $q = 1$; and **(c)** $U_{in} = 9$, $\theta = 20^o$, $q = 1$.

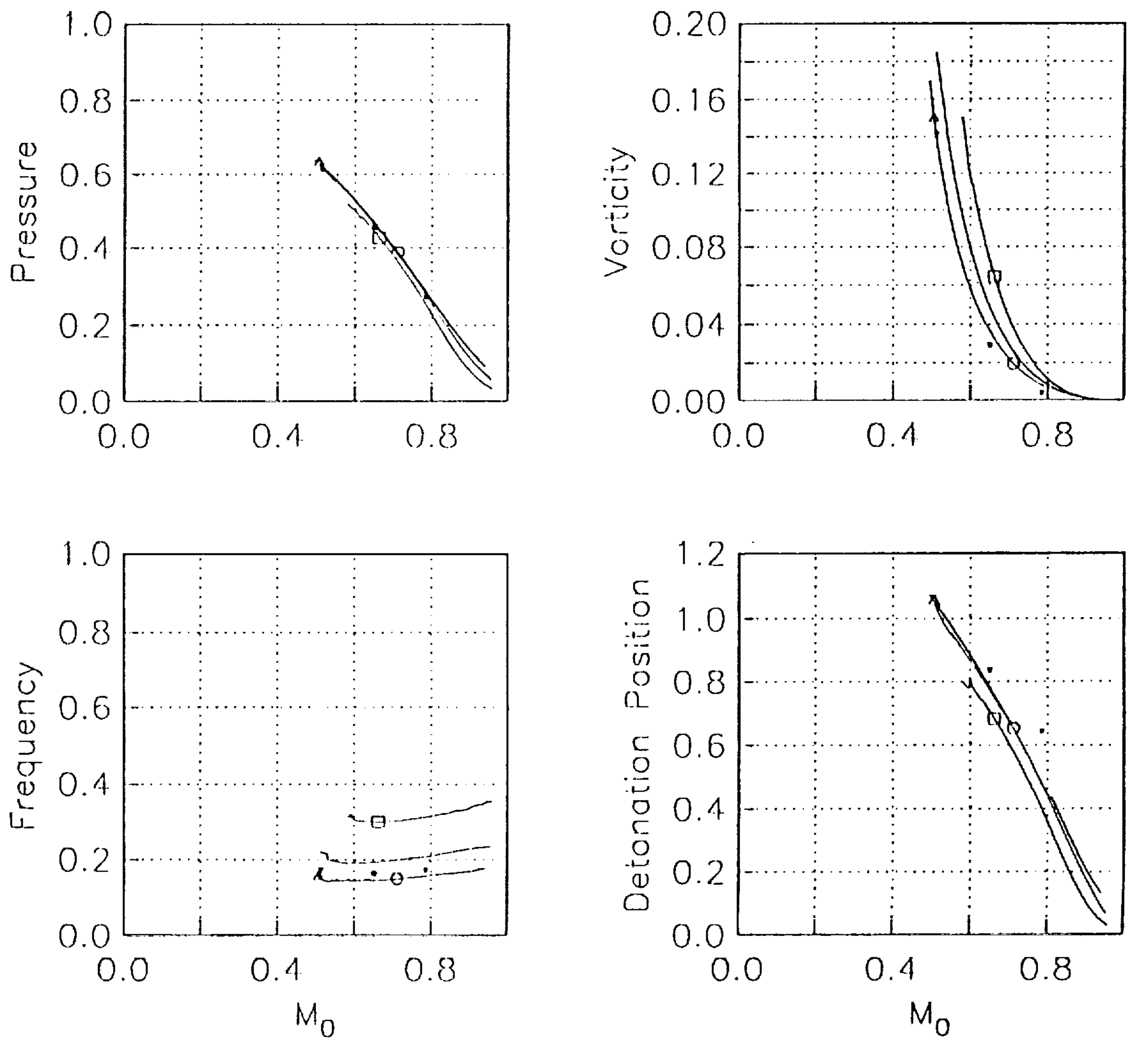

Figure 6. Frequency and magnitude of the pressure response, vorticity response, and detonation postiion for the pivoting motion of the wedge. Lines represent the response for $q = 1$ with inflow Mach numbers $U_{in} = 5, 7, \& \ 9$. See Table 1 for flow values associated with the marked points.

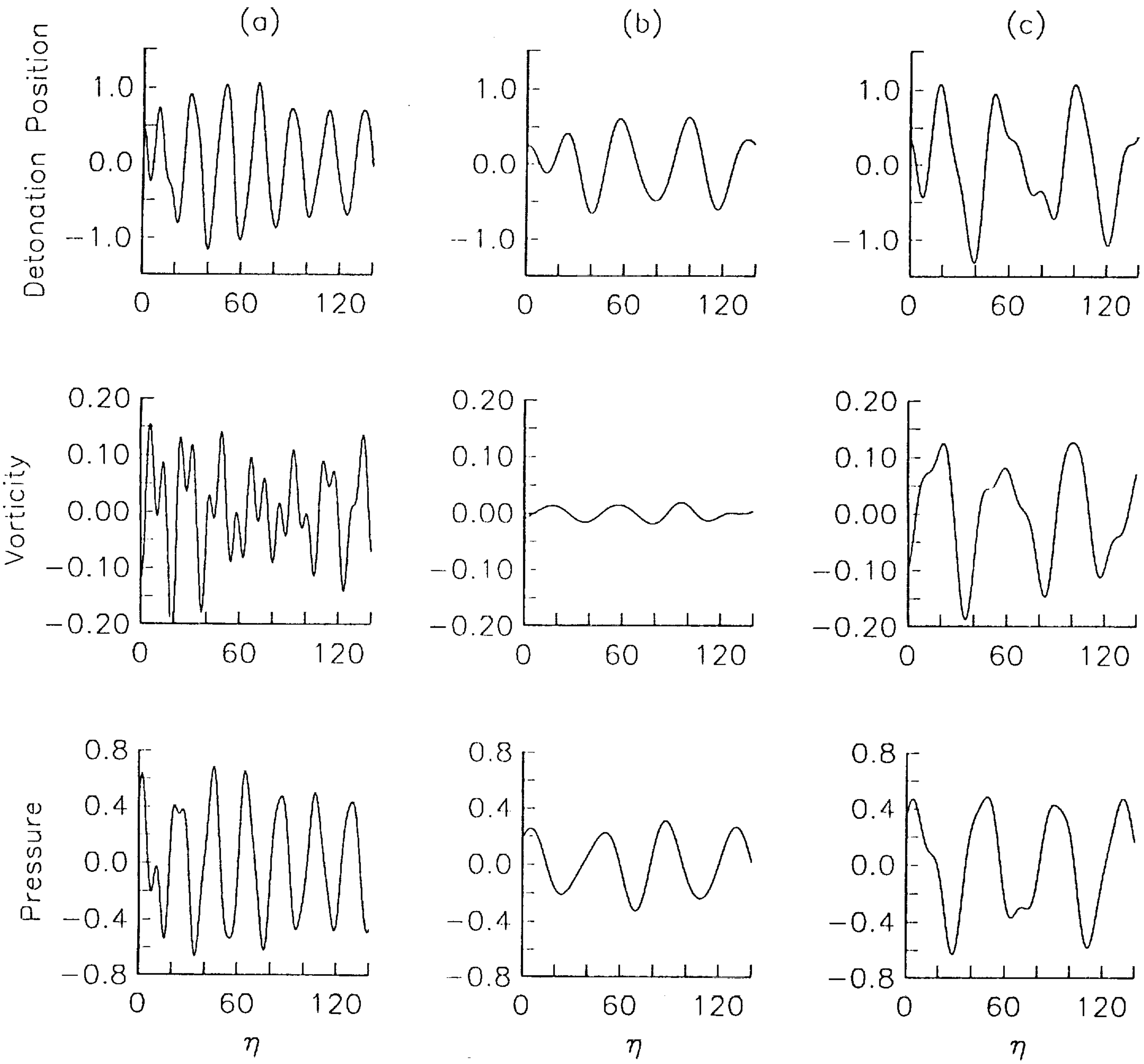

Figure 7. Plot of detonation position h', vorticity $e^{i\omega t}(E_{XX} + E_{YY})$, and pressure $-\gamma e^{i\omega t}(i\omega\phi + U_0\phi_X)$ along the detonation for lateral oscillations of the wedge. Same flow conditions as in Figure 5.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1992	Contractor Report

4. TITLE AND SUBTITLE
INTERACTION OF DISTURBANCES WITH AN OBLIQUE DETONATION WAVE ATTACHED TO A WEDGE

5. FUNDING NUMBERS
C NAS1-18605
WU 505-90-52-01

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Institute for Computer Applications in Science and Engineering
Mail Stop 132C, NASA Langley Research Center
Hampton, VA 23665-5225

8. PERFORMING ORGANIZATION REPORT NUMBER
ICASE Report No. 92-11

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665-5225

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-189624
ICASE Report No. 92-11

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Michael F. Card
Final Report
Submitted to Physics of Fluids A

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited

Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The linear response of an oblique overdriven detonation to imposed free stream disturbances or to periodic movements of the wedge is examined. The freestream disturbances are assumed to be steady vorticity waves and the wedge motions are considered to be time periodic oscillations either about a fixed pivot point or along the plane of symmetry of the wedge aligned with the incoming stream. The detonation is considered to be a region of infinitesimal thickness in which a finite amount of heat is released. The response to the imposed disturbances is a function of the Mach number of the incoming flow, the wedge angle, and the exothermocity of the reaction within the detonation. It is shown that as the degree of overdrive increases, the amplitude of the response increases significantly; furthermore, a fundamental difference in the dependence of the response on the parameters of the problem is found between the response to a free stream disturbance and to a disturbance emanating from the wedge surface.

14. SUBJECT TERMS
detonation

15. NUMBER OF PAGES
31

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified		