

Old Dominion University Research Foundation

DEPARTMENT OF MECHANICAL ENGINEERING & MECHANICS
COLLEGE OF ENGINEERING & TECHNOLOGY
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23529

**AN ANALYTICAL APPROACH TO GRID SENSITIVITY ANALYSIS
FOR NACA FOUR-DIGIT WING SECTIONS**

By
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and
S.N. Tiwari, Principal Investigator

100-100000-100000
SA-CR-190251
S.N. Tiwari
P.69

Progress Report
For the period ended December 1991

Prepared for
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
Cooperative Agreement NCC1-68
Dr. Robert E. Smith Jr., Technical Monitor
ACD-Computer Applications Branch

(NASA-CR-190251) AN ANALYTICAL APPROACH TO
GRID SENSITIVITY ANALYSIS FOR NACA
FOUR-DIGIT WING SECTIONS Progress Report, 1
Jan. - 31 Dec. 1991 (Old Dominion Univ.)

N92-25175

70P

Unclas

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CSCL OIA G3/02 0089405

April 1992

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Submitted by the
Old Dominion University Research Foundation
P.O. Box 6369
Norfolk, Virginia 23508-0369

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FOREWORD

This is the progress report on the research project "Numerical Solutions of Three-Dimensional Navier-Stokes Equations for Closed-Bluff Bodies". Within the guidelines of the project, special attention was directed toward research activities in the area of "Grid Sensitivity in Airplane Design." The period of performance of this specific research was January 1, 1991 through December 31, 1991. This work was supported by the NASA Langley Research Center through Cooperate Agreement NCC1-68. The cooperate agreement was monitored by Dr. Robert E. Smith Jr. of Analysis and Computation Division (Computer Applications Branch), NASA Langley Research Center, Mail Stop 125.

ABSTRACT

**An Analytical Approach To Grid Sensitivity
Analysis For NACA Four-Digit Wing Sections**

Ideen Sadrehaghighi*
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Sensitivity analysis in Computational Fluid Dynamics with emphasis on grids and surface parameterization is described. An interactive algebraic grid-generation technique is employed to generate C-type grids around NACA four-digit wing sections. An analytical procedure is developed for calculating grid sensitivity with respect to design parameters of a wing section. A comparison of the sensitivity with that obtained using a finite-difference approach is made. Grid sensitivity with respect to grid parameters, such as grid-stretching coefficients, are also investigated. Using the resultant grid sensitivity, aerodynamic sensitivity is obtained using the compressible two-dimensional thin-layer Navier-Stokes equations.

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NOMENCLATURE

Q^*	= vector of field variables
X^*	= vector of physical coordinates
R	= steady-state residual
P	= vector of control parameters
X_D	= vector of design parameters
U	= horizontal interpolant
V	= normal interpolant
x, y, z	= physical coordinates
x_1, y_1	= airfoil surface coordinates
r	= surface grid distribution function
s	= far-field boundary grid distribution function
t	= grid stretching function
$R(r)$	= surface orthogonality function
$S(s)$	= far-field boundary orthogonality function
K_1, K_2	= orthogonality parameters
\bar{k}	= stretching parameter
T	= maximum thickness parameter
M	= maximum camber parameter
C	= camber location parameter
C_P	= pressure coefficient
C_D	= drag coefficient

C_L	= lift coefficient
C_f	= skin friction coefficient
C_X, C_Y	= force coefficients in x and y directions
\bar{x}, \bar{y}_c	= camber line coordinates
y_T	= section thickness distribution
P_i	= surface pressure
u, v, w	= velocity components in x , y and z directions
ρ	= density
e	= energy per unit volume
M_∞	= free-stream Mach number
ρ_∞	= free-stream density
U_∞	= free-stream velocity
Re_∞	= free-stream Reynolds number
ξ, η, ζ	= computational coordinates
$\alpha_I^n(\xi, \mathbf{P}_0^\xi)$	= horizontal blending function
$\beta_J^m(\eta, \mathbf{P}_0^\eta)$	= normal blending function
θ	= surface slope
α	= angle of attack
τ_i	= surface skin friction

1. INTRODUCTION

Over the past several years, Computational Fluid Dynamics (CFD) has rapidly evolved. This has been the result of the immense advances in computational algorithms [1] and the development of supercomputers. These innovations have had a major impact on obtaining numerical flow simulations about complex geometries. On current supercomputers, viscous-compressible flow simulations about aircraft configuration can require several hours per steady-state solution [2]. Such large amounts of computational time are acceptable for proof-of-concept studies and selective analysis, but they are not acceptable for optimization and design. With advent of the next generation of parallel supercomputers [3], airplane design and optimization using nonlinear CFD (Euler and Navier-Stokes equations) should become routine. For an individual component, such as a wing, it is now reasonable to consider design and optimization in conjunction with nonlinear CFD [4].

An essential element in design and optimization is acquiring the sensitivity of functions of CFD solutions with respect to control parameters. For aerodynamic surfaces, the control parameters specify the shapes of the surfaces. This affects the surface grid and the field grid which, in turn, affects the flow- field solution. There are two basic components in obtaining aerodynamic sensitivity. They are: (1) obtaining the sensitivity of the governing equations with respect to the state variables; and (2) obtaining the sensitivity of the grid with respect to the defining parameters. The sensitivity of the state variables with respect to the defining parameters are described by a linear-algebraic relation [5]. This study concentrates on the grid sensitivity and parameterization of aerodynamic surfaces.

The simplest method for calculating grid sensitivity is based upon finite-difference approximation. In this approach, a design parameter is perturbed from the nominal value, a new grid is obtained, and the difference between the new and the old grid is used to obtain the grid sensitivity derivatives. This direct, or brute force technique, has the disadvantages of being computationally intensive. It is the objective of the analytical approach to avoid the time consuming and costly numerical differentiation. In addition, the analytic derivatives are exact instead of approximate.

Taylor et al. [6] set the stage for the development of the technique used in this study. For a steady-state solution of the Euler or Navier-Stokes equations, the sensitivity of a function of the solution with respect to the control parameters is to be found. The problem includes the determination of sensitivity of the grid with respect to control parameters. For grid generation, algebraic transfinite interpolation [7] is ideally suited for the study of parameterization. Parameters are sub-grouped according to their purposes (grid spacing control and surface shape control). The objective is to cast the surface parameterization in terms of design parameters rather than geometric variables. Geometric parameterization has only local control and consequently requires large number of parameters to define a grid. A specialized application of transfinite interpolation [8] is cast in terms of design parameters for a class of wing sections and wings. The grid sensitivity of this system is discussed.

2. ALGEBRAIC GRID GENERATION

2.1 Basic Formulation

Structured algebraic grid generation techniques can be thought of as transformations from a rectangular computational domain to an arbitrarily-shaped physical domain [7]. The transformations are governed by the vector of control parameters \mathbf{P} . That is,

$$\mathbf{X}(\xi, \eta, \zeta, \mathbf{P}) = \left\{ x(\xi, \eta, \zeta, \mathbf{P}) \quad y(\xi, \eta, \zeta, \mathbf{P}) \quad z(\xi, \eta, \zeta, \mathbf{P}) \right\}^T \quad (2.1)$$

where

$$0 \leq \xi \leq 1, \quad 0 \leq \eta \leq 1, \quad \text{and} \quad 0 \leq \zeta \leq 1.$$

A discrete subset of the vector-valued function $\mathbf{X}(\xi_i, \eta_j, \zeta_k, \mathbf{P}) \equiv \mathbf{X} \{ x \ y \ z \}_{i,j,k}^T \equiv \mathbf{X}^*$ is a structured grid for $\xi_i = \frac{i-1}{L-1}$, $\eta_j = \frac{j-1}{M-1}$, $\zeta_k = \frac{k-1}{N-1}$, where $i = 1, 2, 3, \dots, L$, $j = 1, 2, 3, \dots, M$ and $k = 1, 2, 3, \dots, N$.

The dominant algebraic approach for grid generation is the *Transfinite Interpolation* scheme. The general methodology was first described by Gordon [9], and there have been numerous variations applied to particular problems. The methodology can be presented as recursive formulas composed of univariate interpolations [10] or as the Boolean sum of univariate interpolations [7]. Here, we follow the Boolean sum representation but; for brevity, we restrict the process to two dimensions and omit some of the details that can be found in Ref. [7]. Also, to be consistent with the example considered below, the parameterization is restricted to functions and first

derivatives at the boundaries ($\xi = 0, 1$) and ($\eta = 0, 1$) and control in the interpolation functions. The transformation is

$$\mathbf{X}(\xi, \eta, \mathbf{P}) = \mathbf{U} \oplus \mathbf{V} = \mathbf{U} + \mathbf{V} - \mathbf{UV} \quad (2.2)$$

where

$$\mathbf{U} = \sum_{I=1}^2 \sum_{n=0}^1 \alpha_I^n(\xi, \mathbf{P}_0^\xi) \frac{\partial^n \mathbf{X}(\xi_I, \eta, \mathbf{P}_I^\eta)}{\partial \xi^n} \quad (2.3)$$

and

$$\mathbf{V} = \sum_{J=1}^2 \sum_{m=0}^1 \beta_J^m(\eta, \mathbf{P}_0^\eta) \frac{\partial^m \mathbf{X}(\xi, \eta_J, \mathbf{P}_J^\xi)}{\partial \eta^m}. \quad (2.4)$$

The term \mathbf{UV} (not expanded here) is the tensor product of the two univariate interpolations. The boundary curves and their derivatives ($\frac{\partial^n \mathbf{X}(\xi_I, \eta, \mathbf{P}_I^\eta)}{\partial \xi^n}$ and $\frac{\partial^m \mathbf{X}(\xi, \eta_J, \mathbf{P}_J^\xi)}{\partial \eta^m}$) $I, J = 1, 2$ $m, n = 0, 1$) are blended into the interior of the physical domain by the interpolation functions $\alpha_I(\xi, \mathbf{P}_0^\xi)$ and $\beta_J(\eta, \mathbf{P}_0^\eta)$. The boundary grid, the derivatives at the boundary grid and the spacing between points are governed by the parameters $\{\mathbf{P}_J^\xi \mathbf{P}_I^\eta\}^T$. The interpolation functions are controlled with the parameters $\{\mathbf{P}_0^\xi \mathbf{P}_0^\eta\}^T$. The entire set of control parameters can be thought of as a vector $\{\mathbf{P}_0^\xi \mathbf{P}_0^\eta \mathbf{P}_J^\xi \mathbf{P}_I^\eta\}^T \equiv \mathbf{P}$.

2.2 Grid Algorithm

An interactive univariate version of Eq.(2.2) using only the normal interpolant \mathbf{V} is developed. This, known as *Hermite Cubic Interpolation*, matches both the function and its first derivative at two boundaries. An analytical approximation of the physical coordinates for a class of wing sections can be expressed as

$$\begin{aligned} x = & x_1(r, \mathbf{P}_1^\xi) \beta_1^0(t, \mathbf{P}_0^\eta) + R(r) \frac{\partial x_1(r, \mathbf{P}_1^\xi)}{\partial t} \beta_1^1(t, \mathbf{P}_0^\eta) \\ & + x_2(s, \mathbf{P}_2^\xi) \beta_2^0(t, \mathbf{P}_0^\eta) + S(s) \frac{\partial x_2(s, \mathbf{P}_2^\xi)}{\partial t} \beta_2^1(t, \mathbf{P}_0^\eta) \end{aligned} \quad (2.5)$$

$$\begin{aligned}
y &= y_1(r, \mathbf{P}_1^\xi) \beta_1^0(t, \mathbf{P}_0^\eta) + R(r) \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial t} \beta_1^1(t, \mathbf{P}_0^\eta) \\
&+ y_2(s, \mathbf{P}_2^\xi) \beta_2^0(t, \mathbf{P}_0^\eta) + S(s) \frac{\partial y_2(s, \mathbf{P}_2^\xi)}{\partial t} \beta_2^1(t, \mathbf{P}_0^\eta)
\end{aligned} \tag{2.6}$$

where

$$\beta_1^0(t, \mathbf{P}_0^\eta) = 2t^3 - 3t^2 + 1,$$

$$\beta_1^1(t, \mathbf{P}_0^\eta) = t^3 - 2t^2 + t,$$

$$\beta_2^0(t, \mathbf{P}_0^\eta) = -2t^3 + 3t^2,$$

$$\beta_2^1(t, \mathbf{P}_0^\eta) = t^3 - t$$

and

$$0 \leq t \leq 1.$$

Five functions $r = f_1(\xi)$, $s = f_2(\xi)$, $R(r) = K_1 f_3(\xi)$, $S(s) = K_2 f_4(\xi)$ and $t = f_5(\eta)$ and their implied defining parameters control the grid spacing on the boundaries and the interior grid. Functions r and s define the grid spacing for lower and upper boundaries respectively, while $R(r)$ and $S(s)$ specify the magnitude of orthogonality along those boundaries. The parameter t defines the grid distribution for the connecting curves between the two boundary. The quantities K_1 and K_2 are parameters that scale the magnitude of the orthogonality at the boundaries. Increasing K_1 and K_2 extends the orthogonality of the grid into the interior domain. Excessively large values of K_1 and K_2 can also cause the grid lines to intersect themselves, which is not a desirable phenomena.

A discrete uniform distribution of the computational coordinate, can be mapped into an arbitrary distribution of the physical coordinate, using the specified control function. The essence of mapping, is that the abscissa corresponds to the percentage of grid points and the ordinate corresponds to a particular control function which, in turn, relates to the geometric definition of the physical domain. The control function, can be either a specified analytical function, or for more general purposes,

a cubic- spline function. For example, the function

$$f_s(\eta) = \frac{e^{K\eta} - 1}{e^K - 1} \quad (2.7)$$

would concentrate grid points close to the bottom or the top boundary depending on the magnitude and the sign of the constant K . Figure 2.1 shows a unit square used as a control domain for grid spacing. Figure 2.2 presents the parametric representation of the boundaries and the cubic connecting function of Eqs.(2.5) and (2.6). Appendix A provides a complete listing of the FORTRAN source module for this type of grid generation algorithm.

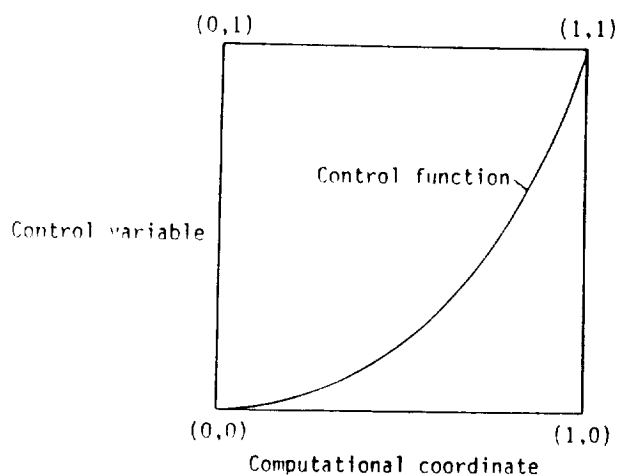


Fig. 2.1 Unit square control domain

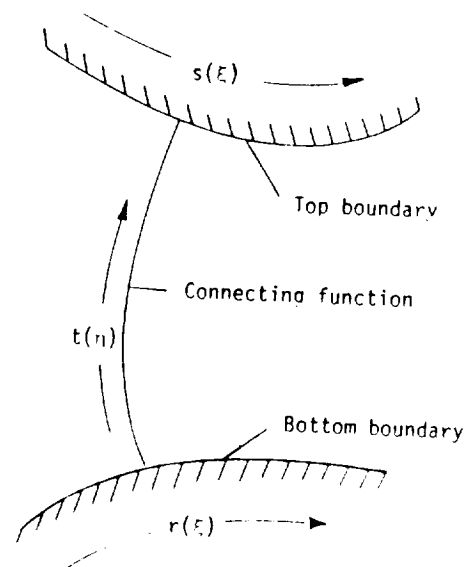


Fig. 2.2 Cubic connecting function

3. PROBLEM STATEMENT AND METHOD OF SOLUTION

3.1 Theoretical Formulation

Let \mathbf{Q} be a solution of the Euler or Navier-Stokes equations on the domain \mathbf{X} and \mathbf{Q}^* be a discrete solution on the grid \mathbf{X}^* where

$$\mathbf{Q}^* = \left\{ \begin{matrix} \rho & \rho u & \rho v & \rho w & \rho e \end{matrix} \right\}_{i,j,k}^T \quad \mathbf{X}^* = \left\{ \begin{matrix} x & y & z \end{matrix} \right\}_{i,j,k}^T \quad (3.1)$$

and

$$\frac{\partial \mathbf{Q}}{\partial t} = \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*) = 0. \quad (3.2)$$

Here, $\mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)$ is the residual of a steady-state solution as $t \rightarrow \infty$. Let \mathbf{P} be a vector of parameters that controls the grid \mathbf{X}^* such that $\mathbf{X}^* = \mathbf{X}(\xi_i, \eta_j, \zeta_k, \mathbf{P})$ where ξ, η and ζ are computational coordinates [7]. The numerical sensitivity of a function $F(\mathbf{Q}(\mathbf{X}))$ with respect to the control parameters is

$$F_p(\mathbf{Q}^*(\mathbf{X}^*)) = \left\{ \frac{\partial F(\mathbf{Q}^*(\mathbf{X}^*))}{\partial \mathbf{P}} \right\} = \left\{ \frac{\partial F(\mathbf{Q}^*(\mathbf{X}^*))}{\partial \mathbf{Q}^*} \right\} \left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}. \quad (3.3)$$

The fundamental sensitivity equation containing $\left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}$ and described by Taylor et al. [6] is

$$\left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{Q}^*(\mathbf{X}^*)} \right] \left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\} + \left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{X}^*} \right] \left\{ \frac{\partial \mathbf{X}^*}{\partial \mathbf{P}} \right\} = 0. \quad (3.4)$$

It is important to notice that Eq.(3.4) is a set of linear, algebraic equations, and the matrices $\left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{Q}^*(\mathbf{X}^*)} \right]$ and $\left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{X}^*} \right]$ are well understood [6]. The

quantity $\left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}$ is the solution to Eq.(3.4) given the sensitivity of the grid with respect to the parameters. Therefore, the grid sensitivity problem is described by

$$\left\{ \frac{\partial \mathbf{X}^*}{\partial \mathbf{P}} \equiv \text{GridSensitivity} \right\}.$$

3.2 Surface Parameterization

In Eq.(2.2), the parameterization is on the boundaries and in the interpolation functions. The most general parameterization of the boundaries would be to specify every grid point $\mathbf{X}_{I,J}^*$ (i.e., each boundary grid point is a parameter). This conceivably could be desirable for the boundaries corresponding to an airplane surface to allow a design procedure to have the greatest possible flexibility. This, however, is impractical from a computational point of view. A compromise is to specify the knots of a spline function or the control polygon for a Bézier function. Even with this compromise, it could require hundreds of parameters for a wing. Here we propose a quasi-analytical parameterization in terms of design variables. For instance, a class of wing sections is specified by two camber-line parameters and a thickness distribution parameter; a wing is specified by several wing sections; and the wing surface is interpolated from the sections. In this manner, an airplane component can be specified by tens of parameters instead of hundreds or thousands of parameters. The disadvantage is that a great deal of generality is not available, but the generality is a moot point if computational capacity cannot accommodate it. For design and optimization with CFD, at the present time, it is advocated here to use a small number of design parameters for boundary definition.

3.3 Grid Sensitivity

As it is stated in the introduction, the simplest way to obtain grid sensitivity with respect to the parameterization is to vary the parameters and finite difference the

results. This is computationally expensive compared to analytically differentiating Eq.(2.2). Therefore, we propose the latter. Grid sensitivity can be used for grid adaptation, or it can be used for boundary design. For adaptation, the grid sensitivity with respect to those parameters that control the grid spacing and the shape of the field grid away from fixed boundaries are desired. The sensitivity is used to improve some grid-quality function of the solution. For design and optimization the sensitivity of the grid with respect to the parameters that define the shape of a boundary is desired. The sensitivity is used to improve a design function of the solution.

4. WING-SECTION EXAMPLE

4.1 Wing-Section Parameterization

Much research has been devoted to the development and representation of wing sections. The NACA four-digit wing sections are examined for grid-generation parameterization. Families of wing sections are described by combining a mean line and a thickness distribution. The resultant expressions possess the necessary features that suit the problem, mainly the concise description of a wing section in terms of several design parameters. Reference 11 provides the general equations which define a mean line and a thickness distribution about the mean line. The design parameters are: $\mathbf{T} \equiv$ the maximum thickness, $\mathbf{M} \equiv$ the maximum ordinate of the mean line or camber, and $\mathbf{C} \equiv$ chordwise position of maximum ordinate. The numbering system for NACA four-digit wing-section is based on the geometry of the section. The first and second integers represent \mathbf{M} and \mathbf{C} respectively, while the third and fourth integers represent \mathbf{T} . Symmetrical sections are designated by zeros for the first two integers, as in the case of NACA 0012 wing-section. Figure 4.1 provides a schematic of the section definition. The ξ -coordinate is first mapped into the chord line $\bar{x} = \bar{x}(r) = \bar{x}(f_1(\xi))$ forward and then reversed to cover both the top and bottom of the section. The mean line equation is

$$\bar{y}_c(\bar{x}) = \frac{\mathbf{M}}{\mathbf{C}^2}(2\mathbf{C}\bar{x} - \bar{x}^2), \quad \bar{x} \leq \mathbf{C} \quad (4.1)$$

$$\bar{y}_c(\bar{x}) = \mathbf{M} \frac{(1 - 2\mathbf{C} + 2\mathbf{C}\bar{x} - \bar{x}^2)}{(1 - \mathbf{C})^2}, \quad \bar{x} \geq \mathbf{C}. \quad (4.2)$$

The section thickness is given by

$$y_{\mathbf{T}}(\bar{x}) = \frac{\mathbf{T}}{0.2}(0.2969\bar{x}^{\frac{1}{2}} - 0.126\bar{x} - 0.3516\bar{x}^2 + 0.2843\bar{x}^3 - 0.1015\bar{x}^4). \quad (4.3)$$

The section coordinates are

$$x_1(r, \mathbf{P}_1^\xi) = \bar{x} \quad y_1(r, \mathbf{P}_1^\xi) = \bar{y}_c(\bar{x}) \pm y_{\mathbf{T}}(\bar{x}). \quad (4.4)$$

Figures 4.2 and 4.3 show sample grids for NACA 0012 and NACA 8512 section using this procedure. The orthogonality at the far-field boundary is ignored. For solid boundary, the orthogonality is obtained using the components of unit normal vector at the surface

$$\frac{\partial x_1(r, \mathbf{P}_1^\xi)}{\partial t} = \mp \sin\theta \quad \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial t} = \pm \cos\theta \quad \theta = \tan^{-1} \left(\frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)} \right). \quad (4.5)$$

Figure 4.4 shows a wing-surface grid derived from three differently-specified sections in the spanwise direction. The surface grid results from the distribution function $f_1(\xi)$, and interpolation of the design parameters for the three wing sections in the spanwise direction. In addition to the three design parameters for each wing section, it is necessary to specify their relative chord lengths and positions. The additional design parameters can be: leading edge sweep, trailing edge sweep, dihedral angle, reference chord length, and section-span locations.

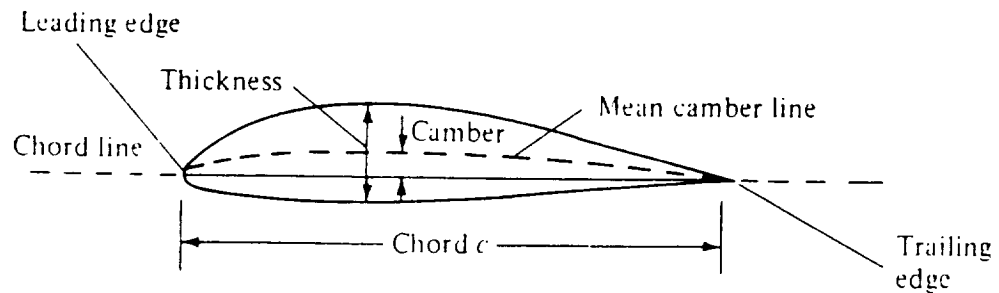


Fig. 4.1 Schematic of wing-section

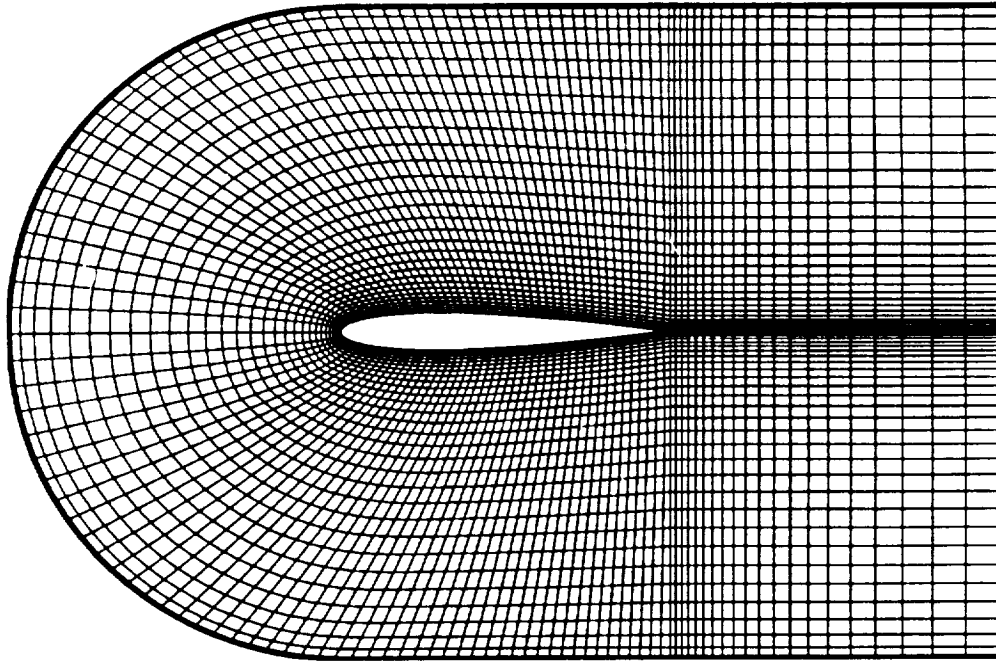


Fig. 4.2 Example grid for NACA 0012 wing-section

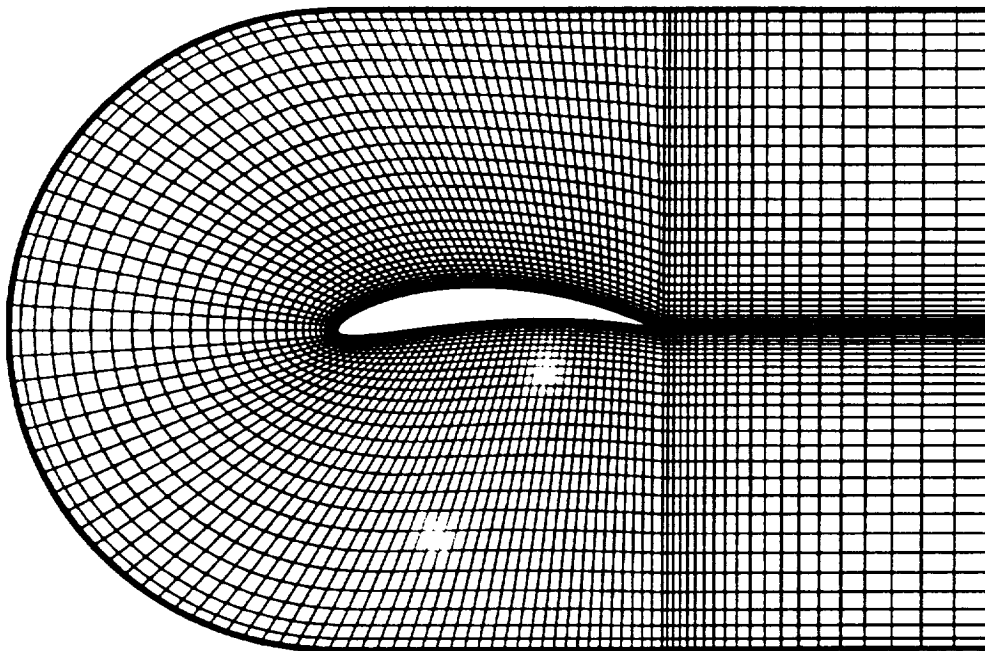


Fig. 4.3 Example grid for NACA 8512 wing-section

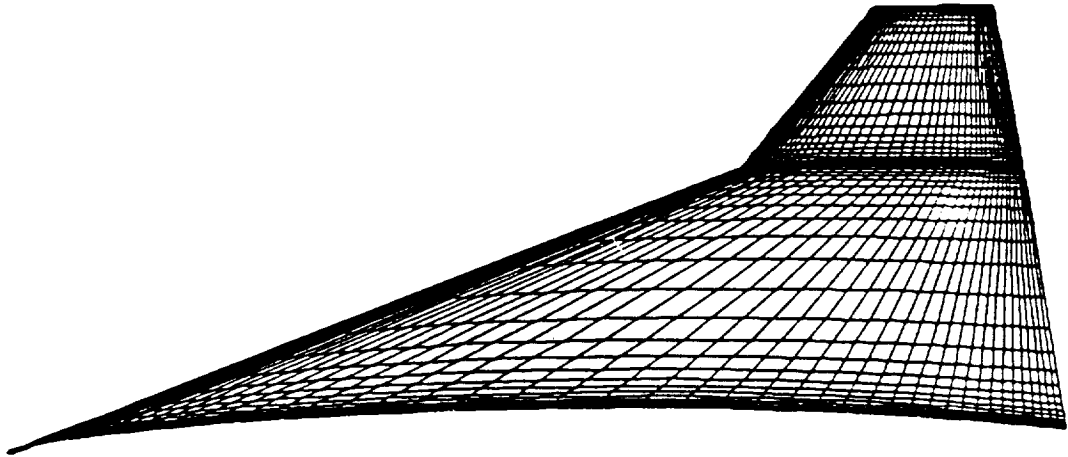


Fig. 4.4 Example wing surface grid (top view)

Appendix B provides the source module for generating the surface of NACA four-digit wing-sections.

4.2 Grid Sensitivity With Respect To Control Parameters

There are two types of control parameters involved in this analysis. First, there are the design parameters (\mathbf{T} , \mathbf{M} , and \mathbf{C}) which specify the shape of the primary boundary and secondly, there are the parameters that define the other boundaries and the the spacing between grid points. Here we express, in part, the sensitivity of the grid with respect to the design parameter vector \mathbf{X}_D , and with $\bar{\mathbf{k}}$ a stretching parameter in the interpolation functions related by $f_5(\eta, \bar{\mathbf{k}})$. The grid sensitivity with respect to design parameters at the outer boundary has been ignored. Also, due to zero orthogonality at the outer boundary, a direct differentiation of Eqs.(2.5) and (2.6) with respect to \mathbf{X}_D yields to

$$\frac{\partial x}{\partial \mathbf{X}_D} = \frac{\partial x}{\partial x_1} \frac{\partial x_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} + \frac{\partial x}{\partial x'_1} \frac{\partial x'_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} \quad (4.6)$$

$$\frac{\partial y}{\partial \mathbf{X}_D} = \frac{\partial y}{\partial y'_1} \frac{\partial y'_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} + \frac{\partial y}{\partial y_1} \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} \quad (1.7)$$

where

$$\mathbf{X}_D = (\mathbf{T}, \mathbf{M}, \mathbf{C}) \quad (1.8)$$

$$\frac{\partial x}{\partial x_1} = \frac{\partial y}{\partial y_1} = \beta_1^0(t, \mathbf{P}_0^\eta) \quad \frac{\partial x}{\partial x'_1} = \frac{\partial y}{\partial y'_1} = R(r)\beta_1^1(t, \mathbf{P}_0^\eta). \quad (1.9)$$

The prime indicates differentiation with respect to t and can be substituted from Eq.(4.5). Since $x_1(r, \mathbf{P}_1^\xi)$ is independent of design parameters \mathbf{X}_D , then

$$\frac{\partial x_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} = 0.0. \quad (1.10)$$

The x coordinate sensitivity, Eq.(4.6), can now be reduced to

$$\frac{\partial x}{\partial \mathbf{X}_D} = R(r)\beta_1^1(t, \mathbf{P}_0^\eta) \frac{\partial(\mp \sin\theta)}{\partial \mathbf{X}_D} \quad \theta = \tan^{-1} \left(\frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)} \right). \quad (1.11)$$

Using the relation

$$\frac{\partial}{\partial \mathbf{X}_D} \tan^{-1} u = \frac{1}{1+u^2} \frac{\partial u}{\partial \mathbf{X}_D} \quad (1.12)$$

the x coordinate sensitivity becomes

$$\frac{\partial x}{\partial \mathbf{X}_D} = \mp R(r)\beta_1^1(t, \mathbf{P}_0^\eta) \cos\theta \frac{1}{1 + \left(\frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)} \right)^2} \frac{\partial}{\partial \mathbf{X}_D} \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)}. \quad (1.13)$$

The term $\frac{\partial}{\partial \mathbf{X}_D} \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)}$ can be evaluated by direct differentiation of Eq.(4.4). The y coordinate sensitivity with respect to design parameters can be obtained using similar procedure. Equation (4.7) can be modified to

$$\frac{\partial y}{\partial \mathbf{X}_D} = \beta_1^0(t, \mathbf{P}_0^\eta) \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial \mathbf{X}_D} \mp R(r)\beta_1^1(t, \mathbf{P}_0^\eta) \sin\theta \frac{1}{1 + \left(\frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)} \right)^2} \frac{\partial}{\partial \mathbf{X}_D} \frac{\partial y_1(r, \mathbf{P}_1^\xi)}{\partial x_1(r, \mathbf{P}_1^\xi)}. \quad (1.14)$$

All terms at the right hand side of Eqs.(4.13) and (4.14) can be evaluated explicitly due to analytical parameterization of the surface for this particular example. The grid sensitivity with respect to the stretching parameter \bar{k} are

$$\frac{\partial x}{\partial \bar{k}} = x_1(r, \mathbf{P}_1^\xi) \frac{\partial \beta_1^0(t, \mathbf{P}_0^\eta)}{\partial t} \frac{\partial t}{\partial \bar{k}} + R(r) \frac{\partial x_1(r, \mathbf{P}_1^\xi)}{\partial \eta} \frac{\partial \beta_1^1(t, \mathbf{P}_0^\eta)}{\partial t} \frac{\partial t}{\partial \bar{k}}$$

$$+ x_2(r, \mathbf{P}_2^\xi) \frac{\partial \beta_2^0(t, \mathbf{P}_0^\eta)}{\partial t} \frac{\partial t}{\partial \bar{\mathbf{k}}} + S(s) \frac{\partial x_2(s, \mathbf{P}_2^\xi)}{\partial \eta} \frac{\partial \beta_2^1(t, \mathbf{P}_0^\eta)}{\partial t} \frac{\partial t}{\partial \bar{\mathbf{k}}} \quad (4.15)$$

where

$$\begin{aligned} \frac{\partial \beta_1^0(t, \mathbf{P}_0^\eta)}{\partial t} &= 6t^2 - 6t & \frac{\partial \beta_1^1(t, \mathbf{P}_0^\eta)}{\partial t} &= 3t^2 - 4t + 1 \\ \frac{\partial \beta_2^0(t, \mathbf{P}_0^\eta)}{\partial t} &= -6t^2 + 6t & \frac{\partial \beta_2^2(t, \mathbf{P}_0^\eta)}{\partial t} &= 3t^2 - 1. \end{aligned}$$

An example of the stretching function is

$$t = \frac{e^{\bar{\mathbf{k}}\eta} - 1}{e^{\bar{\mathbf{k}}} - 1} \quad (4.16)$$

$$\frac{\partial t}{\partial \bar{\mathbf{k}}} = \frac{(e^{\bar{\mathbf{k}}} - 1)\eta e^{\bar{\mathbf{k}}\eta} - (e^{\bar{\mathbf{k}}\eta} - 1)e^{\bar{\mathbf{k}}}}{(e^{\bar{\mathbf{k}}} - 1)^2}. \quad (4.17)$$

Similar developments can be extended to other grid control parameters such as the distribution of grid point around the wing section and magnitude of orthogonality at the boundaries. Appendix C provides the source module for grid sensitivity of NACA four-digit wing-sections with respect to design parameters.

4.3 Flow Sensitivity With Respect To Control Parameters

The flow sensitivity coefficient $\left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}$ can now be evaluated using the fundamental sensitivity equation, Eq.(3.4). The Jacobian matrices $\left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{Q}^*(\mathbf{X}^*)} \right]$ and $\left[\frac{\partial \mathbf{R}(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*)}{\partial \mathbf{X}^*} \right]$ are obtained using an implicit time integration of the 2-D thin-layer Navier-Stokes equations [10]. These equations are solved in their conservation form using an upwind cell-centered finite-volume formulation. A detailed description of the procedure is found in Refs. [12-17] and is not repeated here. A third-order accurate upwind biased inviscid flux balance is used in both streamwise and normal

directions. The finite-volume equivalent of second-order accurate central differences is used for viscous terms. For a typical design analysis of an airfoil, the flow sensitivity coefficient $\left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}$ provides far more information than needed. In most cases, the sensitivity of aerodynamic forces on the surface, such as lift and drag coefficients, are sought. For such analysis, only a small subset of the flow sensitivity coefficient $\left\{ \frac{\partial \mathbf{Q}^*(\mathbf{X}^*)}{\partial \mathbf{P}} \right\}$ (i.e. surface properties) is needed since the lift and drag coefficients can be expressed as

$$C_L = C_Y \cos \alpha - C_X \sin \alpha \quad (4.18)$$

$$C_D = C_Y \sin \alpha + C_X \cos \alpha \quad (4.19)$$

where α is the flow angle of attack. The quantities C_X and C_Y are the total force coefficients along x and y directions respectively and can be expressed as

$$C_X = \sum_{i=1}^{NE} C_{p_i} (y_{i+1} - y_i) + C_{f_i} (x_{i+1} - x_i) \quad (4.20)$$

$$C_Y = \sum_{i=1}^{NE} C_{p_i} (x_{i+1} - x_i) + C_{f_i} (y_{i+1} - y_i) \quad (4.21)$$

where C_{p_i} and C_{f_i} are pressure and skin friction coefficients respectively

$$C_{p_i} = \frac{P_i}{\frac{1}{2} \rho_\infty U_\infty^2} \quad C_{f_i} = \frac{\tau_i}{\frac{1}{2} \rho_\infty U_\infty^2} \quad (4.22)$$

and NE represents total number of boundary cells along airfoil surface. The terms P_i and τ_i are pressure and shear stress associated with boundary cell i and the quantity $\frac{1}{2} \rho_\infty U_\infty^2$ is known as dynamic pressure of the free stream. Finally, the drag and lift sensitivity coefficients with respect to \mathbf{X}_D are obtained by differentiating Eqs.(4.18) and (4.19) as

$$\frac{\partial C_L}{\partial \mathbf{X}_D} = \frac{\partial C_Y}{\partial \mathbf{X}_D} \cos \alpha - \frac{\partial C_X}{\partial \mathbf{X}_D} \sin \alpha \quad (4.23)$$

$$\frac{\partial C_D}{\partial \mathbf{X}_D} = \frac{\partial C_Y}{\partial \mathbf{X}_D} \sin \alpha + \frac{\partial C_X}{\partial \mathbf{X}_D} \cos \alpha. \quad (4.24)$$

5. RESULTS AND DISCUSSION

5.1 NACA 0012 Airfoil Test Case

5.1.1 Grid Sensitivity

The first test case considered is the NACA 0012 symmetrical airfoil. The previously obtained grid, as shown in Fig. 4.2, is considered for grid sensitivity analysis. The grid sensitivity with respect to the vector of design parameters \mathbf{X}_D , are obtained using Eqs. (4.13) and (4.14). The maximum thickness \mathbf{T} is the only design parameter for this case.

Figure 5.1 shows the contour levels of the y-coordinate sensitivity with respect to the thickness parameter, \mathbf{T} . The highest contour levels are, understandably, located in the vicinity of the chordwise location for the maximum thickness of the wing section. For a NACA four-digit wing section, this is positioned about 0.3 of the chord length from the leading edge [11]. The positive and negative contour levels corresponding to the upper and lower surfaces are the direct consequence of Eq.(4.4) and the second term on the right hand side of Eq.(4.7). The sensitivity levels decrease when approaching the far-field boundary due to diminishing effects of the interpolation function $\beta_1^0(t, \mathbf{P}_0^?)$. The first term on the right hand side of Eq.(4.7) is responsible for the sensitivity effects due to orthogonality on the surface, and it is directly proportional to the magnitude of the orthogonality vector K_1 . The wake region is not sufficiently affected by any of the design parameters, and no major sensitivity gradient should be expected there.

Figure 5.2 demonstrates the contour levels of the x-coordinate sensitivity with respect to thickness parameter, \mathbf{T} . An interesting observation can be made here regarding the contour levels adjacent to the surface. Unlike the y-coordinate sensitivity, the x-coordinate sensitivity has its lowest value on the surface. This can be traced back to Eq.(4.4), which indicates that x-coordinates on the surface are basically independent of the design parameters. The only remaining factor is the second term on the right hand side of Eq.(4.6) which is the effect of the surface orthogonality vector. There are some negative pockets of contour levels on the forward section and some corresponding positive pockets on the rear section. The dividing line between these pockets is located near the location of the maximum thickness (i.e., 0.3 of chord from leading edge). A simple conclusion from Fig. 5.2 is that by increasing the thickness parameter, \mathbf{T} points on the forward section will move to the left, while at the same time, points at rear section will move to the right.

For comparison purposes, the grid sensitivity for this case is obtained using the finite difference approach. The design parameter(i.e. \mathbf{T} for this case) are perturbed, one at a time, and a new grid is obtained using Eqs.(2.5) and (2.6). The sensitivity is then computed using a central difference approximation and the results are presented in Figs. 5.3 and 5.4 . A side by side comparison of both results indicates good agreement between the two approaches.

5.1.2 Flow Sensitivity

The second phase of the problem is obtaining the flow sensitivity coefficients using the previously obtained grid sensitivity coefficients. In order to achieve this, according to Eq. (3.4), a converged flow field solution about a fixed design point should be obtained. The computation is performed on a C-type grid composed of 141 points in the streamwise direction and 31 points in the normal direction. It is apparent that such a coarse grid is inadequate for capturing the full physics of the viscous

flow over an airfoil. Therefore, it should be understood that the main objective here is not to produce a highly accurate flow field solution rather than to demonstrate the feasibility of the approach.

A free stream Mach number of $M_\infty = 0.8$, Reynolds number $Re_\infty = 10^6$, and angle of attack $\alpha = 0^\circ$ is used. Figures 5.5 and 5.6 demonstrate the pressure and Mach number contours of the converged solution. Figure 5.7 shows the surface pressure coefficient C_p , where the lift and drag coefficient for this particular example are $C_L = 1.53 \times 10^{-8}$, $C_D = 4.82 \times 10^{-2}$. The sensitivities of the aerodynamic forces, such as drag and lift coefficients with respect to thickness parameter \mathbf{T} , are obtained utilizing Eqs.(4.18-4.24). The corresponding results are presented in Table 5.1. Again, for comparison purposes, a finite difference approximation has been implemented to validate the results. A nominal perturbation of 10^{-3} for design parameter \mathbf{T} has been chosen and the corresponding results are included in Table 5.1. The good agreement between the two sets of numbers verifies the accuracy of the approach.

Another important goal of using sensitivity analysis, apart from optimization, is the approximation analysis. An approximate version of Eq.(3.4) can be used to predict the steady-state solution changes which occur in response to geometric shape changes. Such a method is valid as long as the changes in geometric shape (i.e., design parameter) are small. Figure 5.8 shows the non-linear relation between drag coefficient and thickness parameter \mathbf{T} verifying the above argument.

5.2 NACA 8512 Airfoil Test Case

5.2.1 Grid Sensitivity

The second test case considered is the NACA 8512 cambered airfoil. Again, the previously obtained grid, as shown in Fig. 4.3, is considered for grid sensitivity analysis. Figures 5.9 and 5.10 show the y and x-coordinate sensitivity with respect to parameter \mathbf{T} respectively. Their characteristics are similar to the previous sym-

metrical airfoil case; hence, detailed description of their behavior is omitted here.

Figure 5.11 represents the y -coordinate sensitivity with respect to camber, M . It appears that the highest sensitivity contour levels are located at the chordwise location of camber, C (i.e., 0.5 of chord length). The contour levels decrease toward the far-field boundary, again as a consequence of interpolation function. However, unlike Fig. 5.9 here they possess positive values on both upper and lower surfaces. Consequently, an increase in camber, M , shifts all the points upward. Again, minimum activity can be detected in the wake region. Figure 5.12 shows the x -coordinate sensitivity contours with respect to camber, M . Here, as in Fig. 5.10, the sensitivities are minimum on the surface of the wing-section. There is a small gradient on the forward section, but by far, the strongest gradient is in the rearward section due to orthogonality effects.

Figure 5.13 illustrates the y -coordinate sensitivity with respect to camber location, C . A dividing line between positive and negative contour levels appears near the chordwise position of the camber. Like previous cases, there is no significant activity in the wake region. The result indicates that a positive change of C will cause the movement of points downward on the forward section, while at the same time, the points on the rear section will respond by moving upward. Figure 5.14 illustrates the x -coordinate sensitivity with respect to camber location C . The two major features are attributed to chordwise location of the camber and the orthogonality effects on the tail section. It is interesting to notice that the sensitivity level for camber location is considerably less than the other two design parameters.

Similar developments can be extended to other grid control parameters such as the distribution of grid point around the wing section and magnitude of orthogonality at the boundaries. For example, the grid sensitivity with respect to stretching parameter \bar{k} , using Eqs.(4.15-4.17), is obtained and the results are presented on Figs. 5.15 and 5.16.

5.2.2 Flow Sensitivity

Using free stream conditions of $M_\infty = 0.8$, $Re_\infty = 10^6$, and $\alpha = 0^\circ$, a converged flow field solution is obtained. As in previous case, a C-type grid of 141x31 is used. Figures 5.17 and 5.18 illustrate the pressure and Mach number contours. Figure 5.19 shows the surface pressure coefficient C_p , where lift and drag coefficients are $C_L = 0.106$, and $C_D = 0.0738$. The aerodynamic sensitivity coefficients with respect to vector of design parameters \mathbf{X}_D , are obtained and presented in Table 5.2. A comparison with finite difference validates the accuracy of the approach.

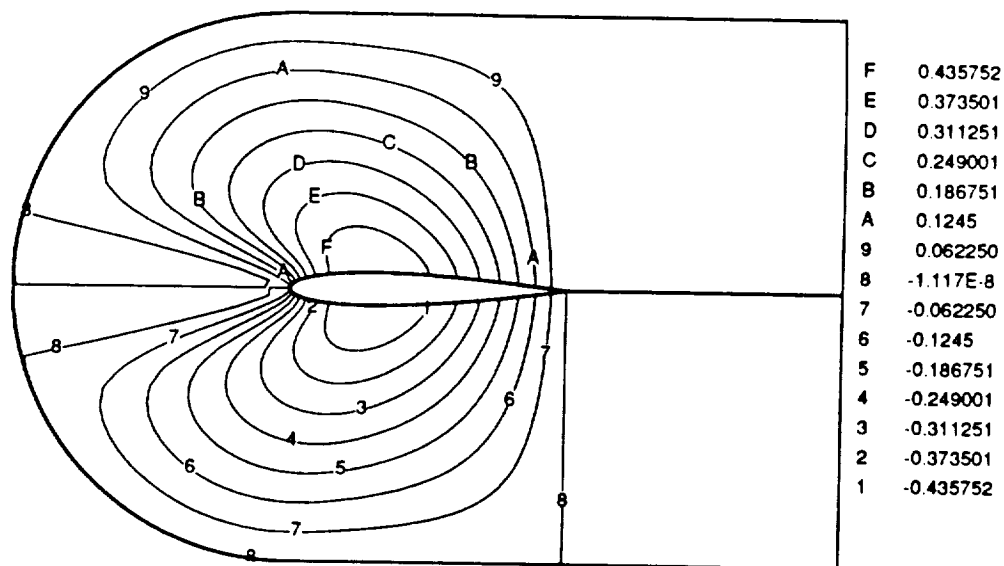


Fig. 5.1 Y-coordinate sensitivity with respect to thickness T (Analytical)

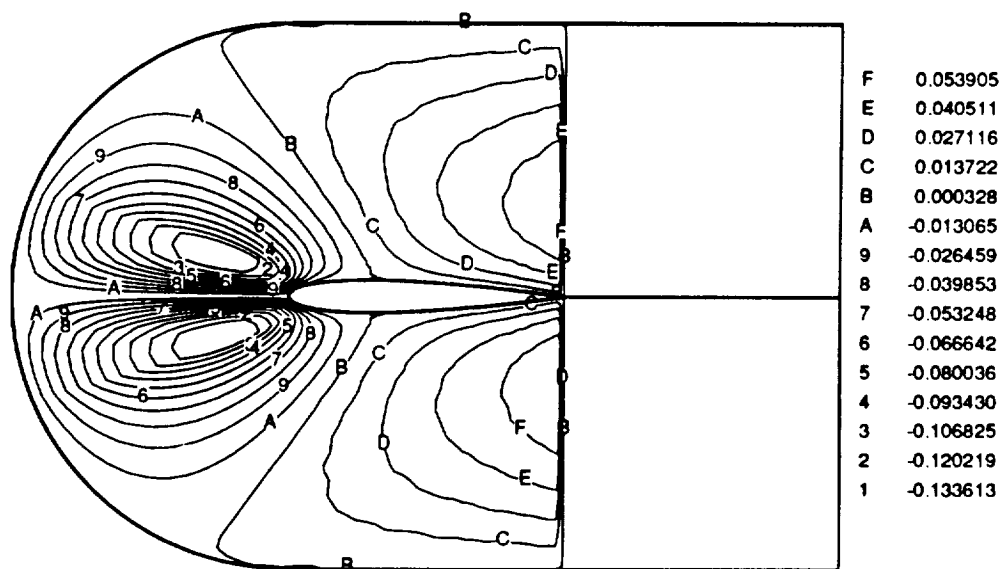


Fig. 5.2 X-coordinate sensitivity with respect to thickness T (Analytical)

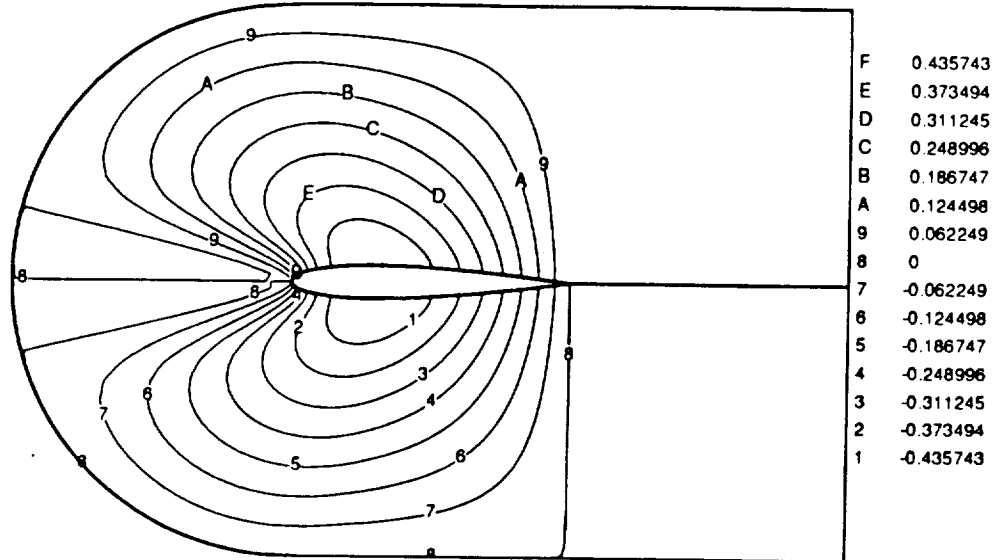


Fig. 5.3 Y-coordinate sensitivity with respect to thickness T (Finite-Difference)

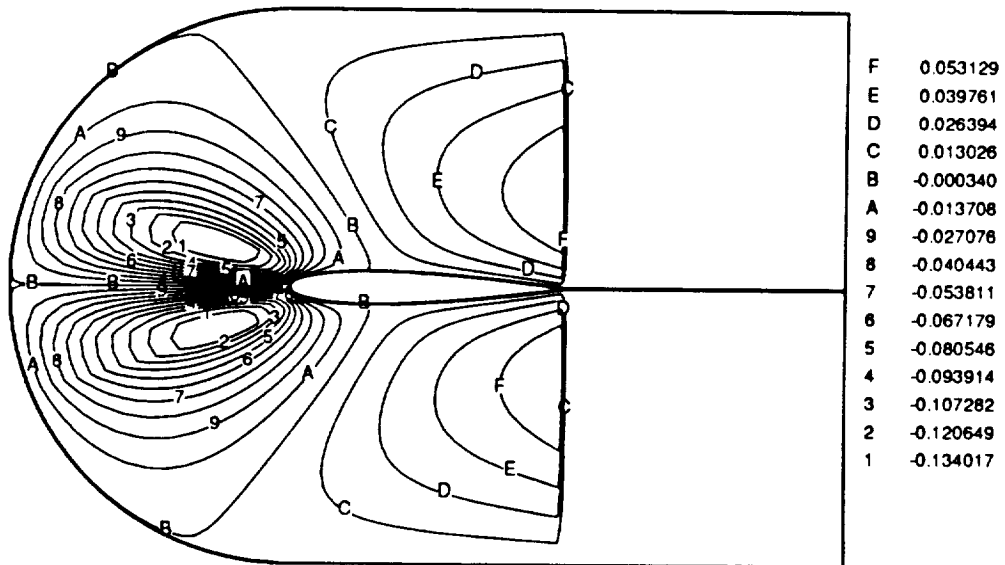


Fig. 5.4 X-coordinate sensitivity with respect to thickness T (Finite-Difference)

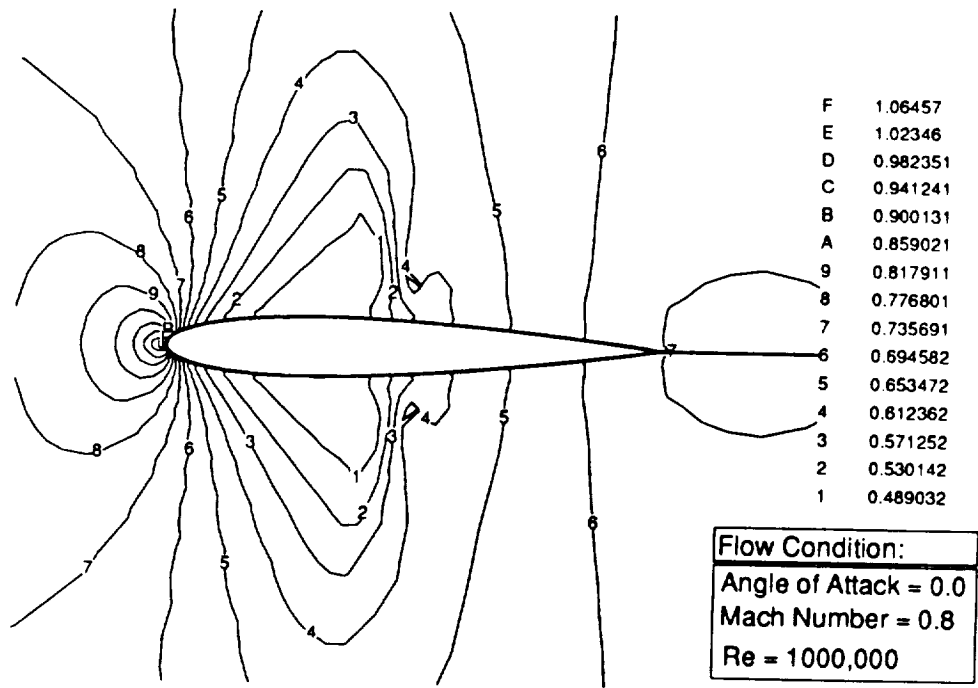


Fig. 5.5 Pressure contours for NACA 0012 wing-section

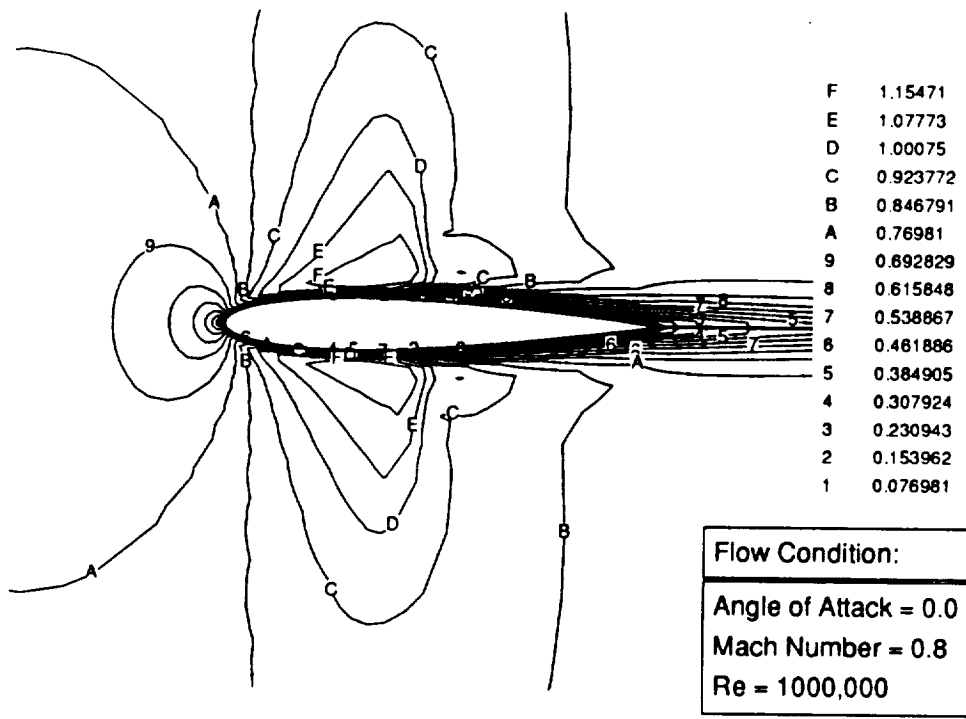


Fig. 5.6 Mach number contours for NACA 0012 wing-section

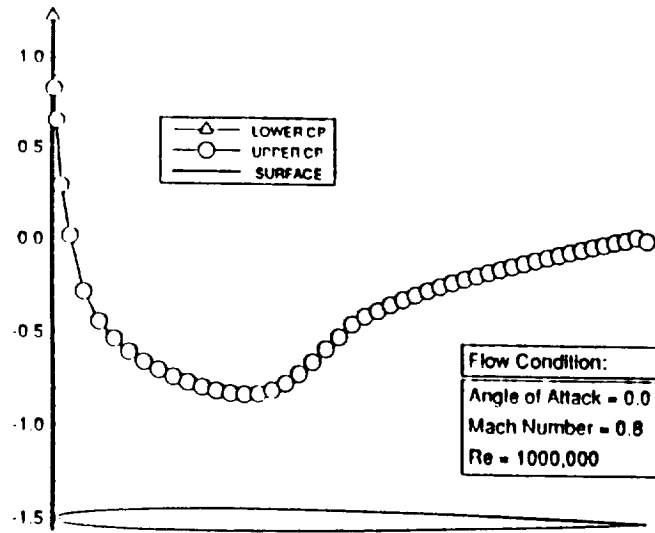


Fig. 5.7 Surface pressure coefficient for NACA 0012 wing-section

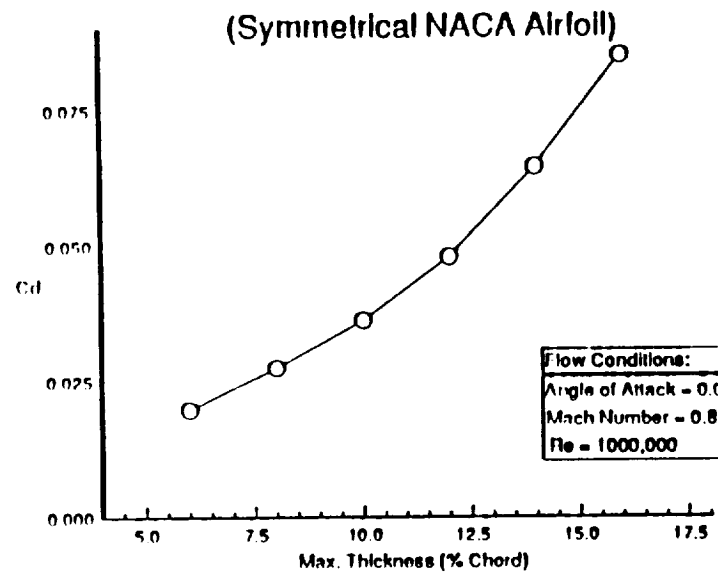


Fig. 5.8 Drag coefficient Vs Maximum Thickness

Table 5.1 Lift and Drag sensitivities with respect to design parameter T

NACA0012 Design Parameters	Direct Approach		Finite Difference	
	$\frac{\partial C_L}{\partial X_D}$	$\frac{\partial C_D}{\partial X_D}$	$\frac{\partial C_L}{\partial X_D}$	$\frac{\partial C_D}{\partial X_D}$
$X_D = \text{Thickness}$	-2.68×10^{-6}	0.709	-4.75×10^{-6}	0.707

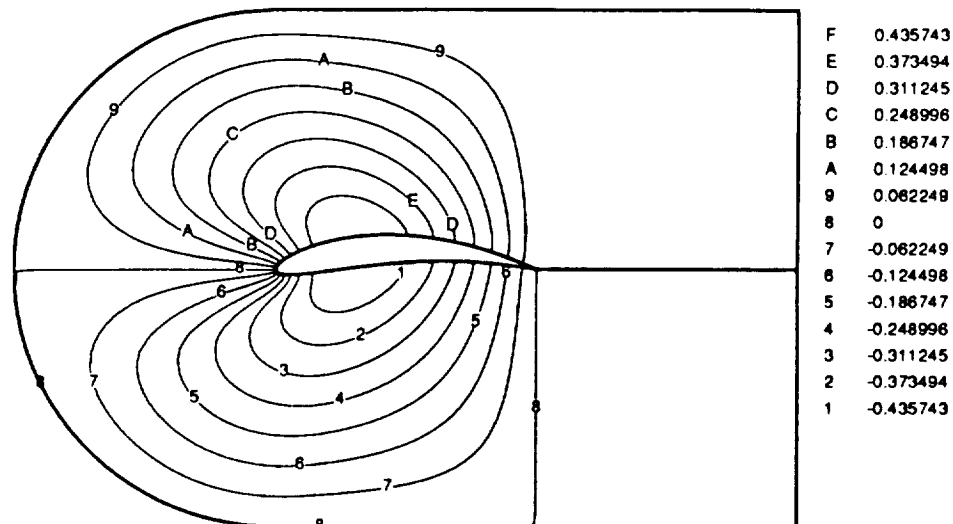


Fig. 5.9 Y-coordinate sensitivity with respect to thickness T (Analytical)

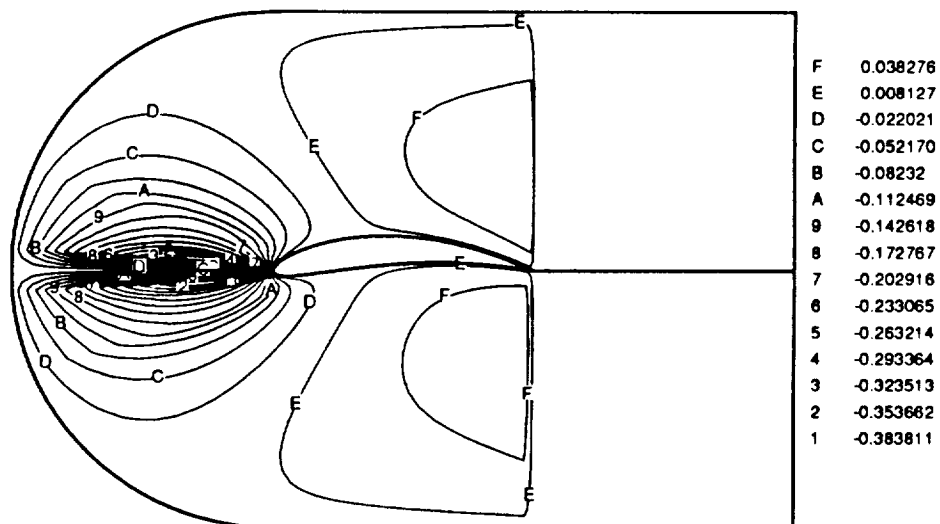


Fig. 5.10 X-coordinate sensitivity with respect to thickness T (Analytical)

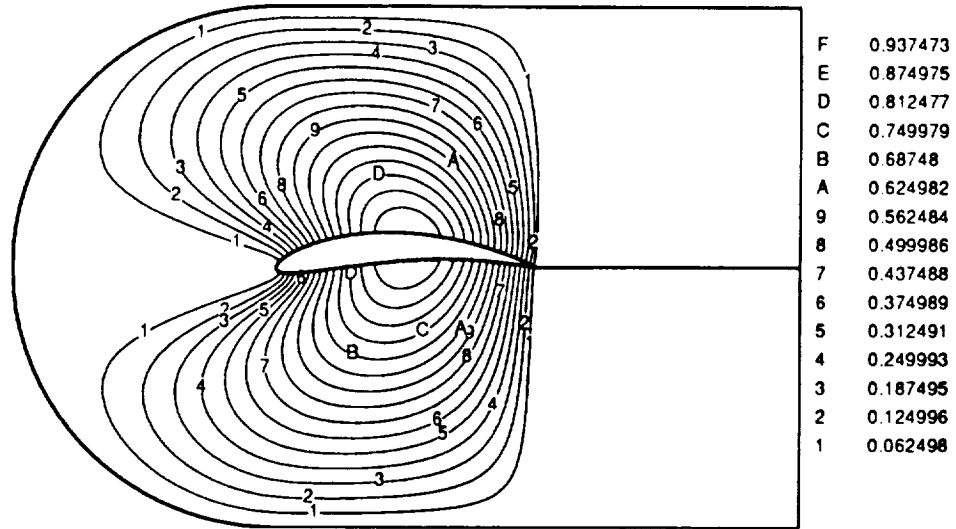


Fig. 5.11 Y-coordinate sensitivity with respect to camber M (Analytical)

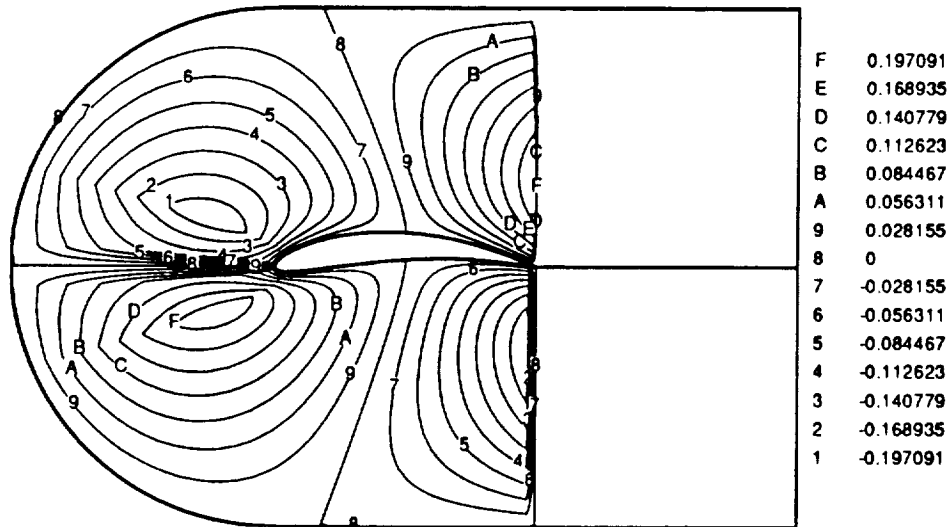


Fig. 5.12 X-coordinate sensitivity with respect to camber M (Analytical)

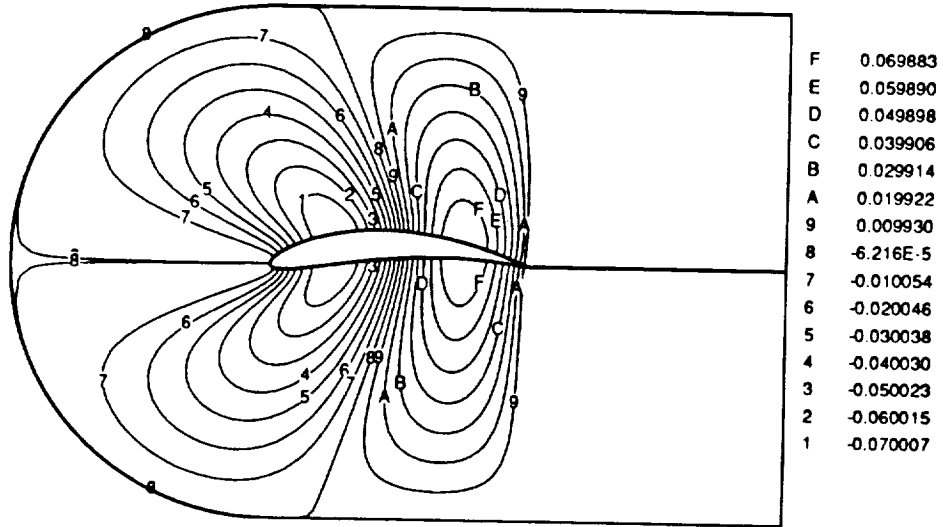


Fig. 5.13 Y-coordinate sensitivity with respect to camber location C (Analytical)

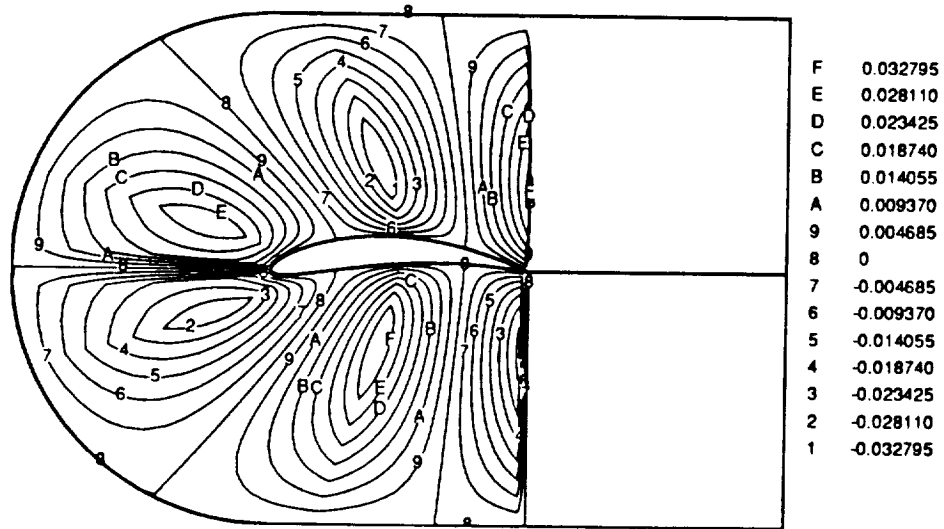


Fig. 5.14 X-coordinate sensitivity with respect to camber location C (Analytical)

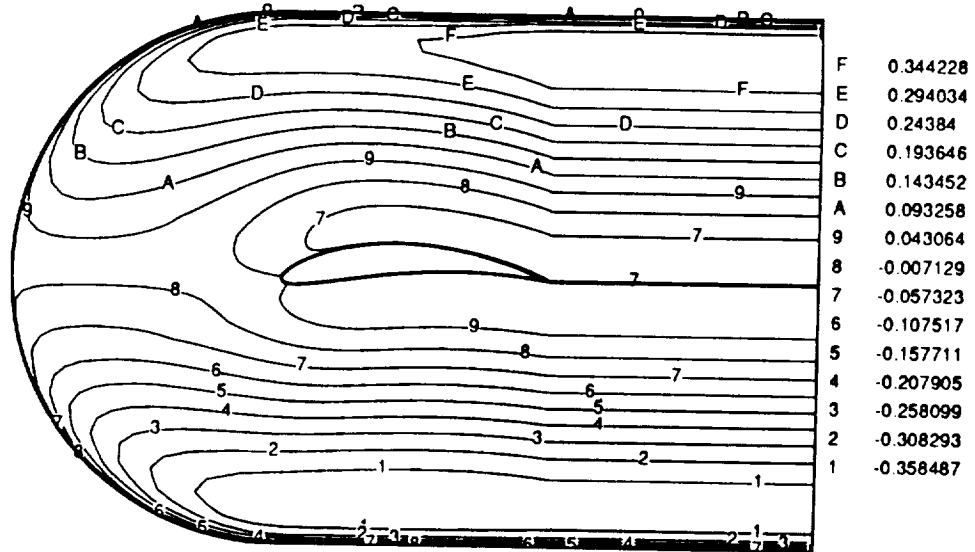


Fig. 5.15 Y-coordinate sensitivity with respect to stretching parameter \bar{k}

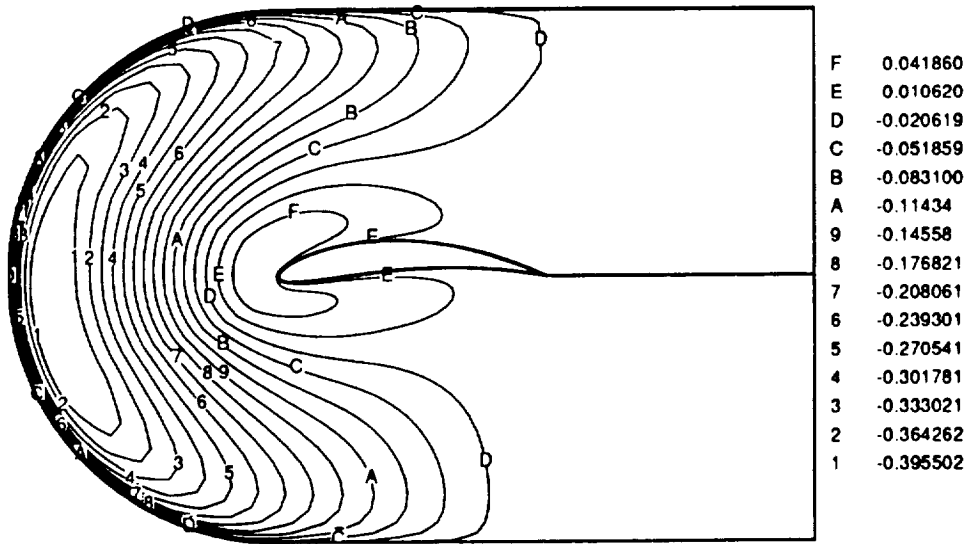


Fig. 5.16 X-coordinate sensitivity with respect to stretching parameter \bar{k}

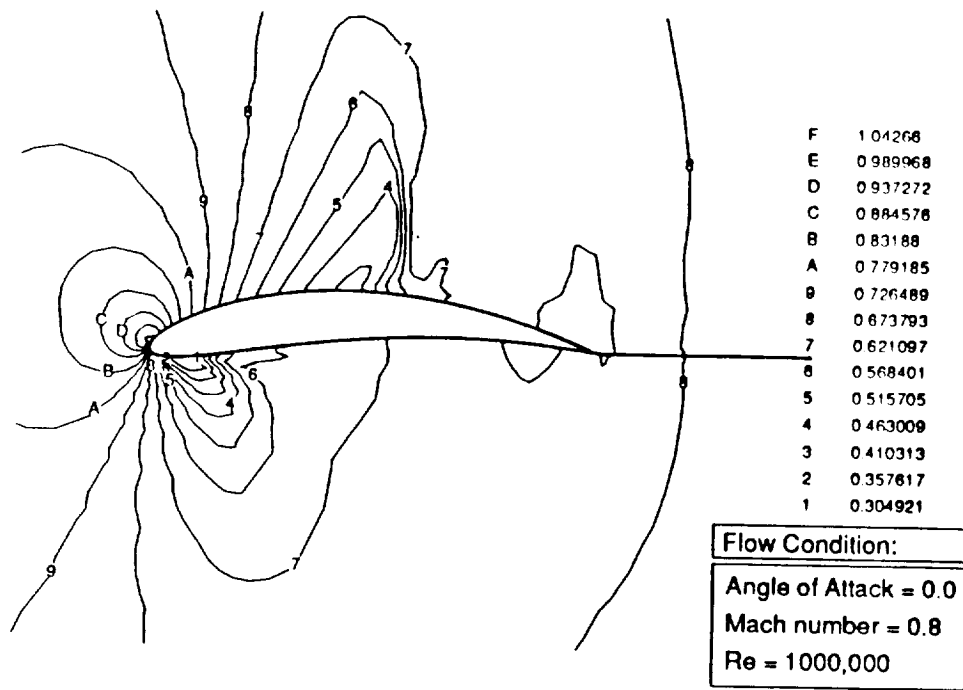


Fig. 5.17 Pressure contours for NACA 8512 wing-section

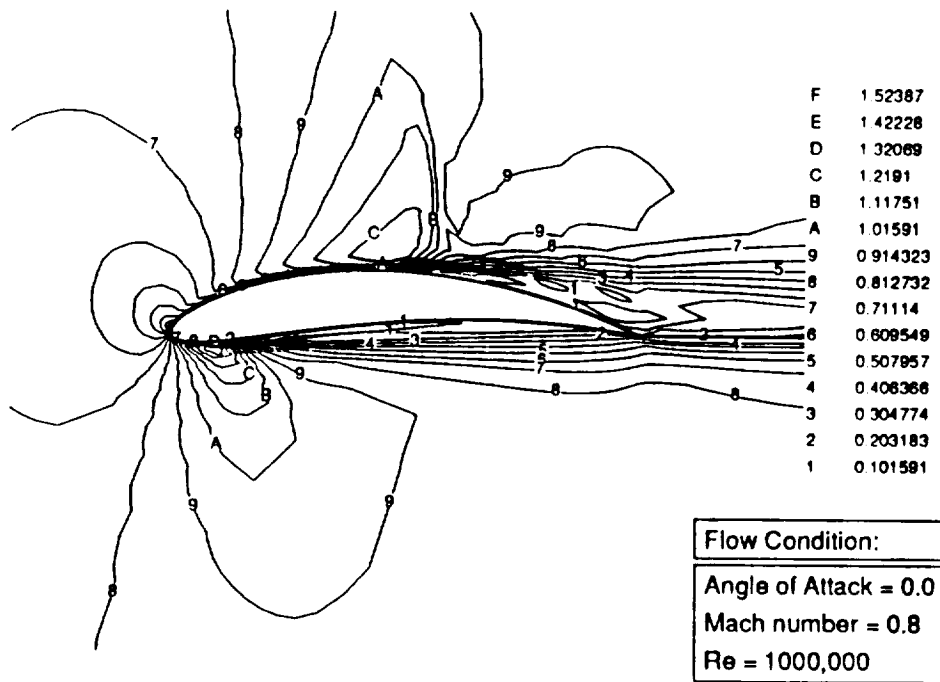


Fig. 5.18 Mach number contours for NACA 8512 wing-section

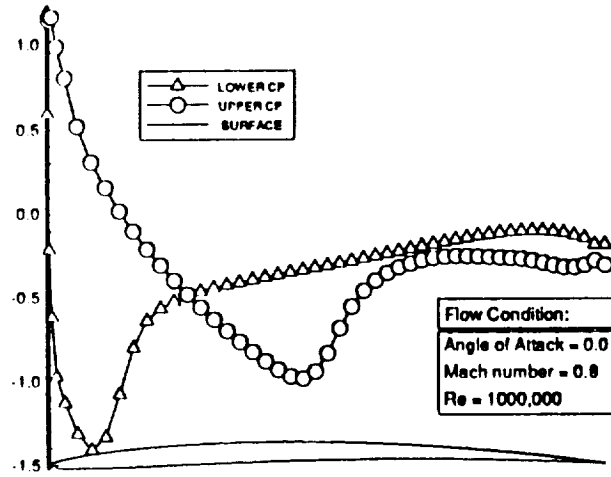


Fig. 5.19 Surface pressure coefficient for NACA 8512 wing-section

Table 5.2 Lift and Drag sensitivities with respect to vector of design parameters X_D

NACA 8512 Design Parameters	Direct Approach		Finite Difference	
	$\frac{\partial C_L}{\partial X_D}$	$\frac{\partial C_D}{\partial X_D}$	$\frac{\partial C_L}{\partial X_D}$	$\frac{\partial C_D}{\partial X_D}$
$X_D =$ Thickness	-1.058	0.410	-1.058	0.410
$X_D =$ Camber	7.840	0.322	7.838	0.322
$X_D =$ Camber location	-8.34×10^{-3}	-4.31×10^{-3}	-8.5×10^{-3}	-4.3×10^{-3}

6. CONCLUSION

The objective of this study has been to demonstrate an approach for obtaining grid sensitivity which can be used in aerodynamic design and optimization. It is shown that grid sensitivity is an essential ingredient in the calculation of aerodynamic sensitivity. The main supposition is that a grid is defined algebraically in terms of parameters and computational coordinates. Therefore, coordinates of the grid and derivatives of the coordinates with respect to the parameters (grid sensitivity) are computed directly as functions of the parameters and uniformly-discrete values of the computational coordinates. A subset of the parameters defines the shape of the grid boundaries which corresponds to the aerodynamic surfaces of interest. It is recommended that the aerodynamic surfaces be parameterized in terms of design parameters which have global control. As compared to a geometric parameterization, this drastically reduces the number of parameters. However, it limits the design flexibility. In addition to the aerodynamic surface parameters, the sensitivity with respect to parameters that define other boundaries, such as the far-field boundary or the spacing of grid points, is available for analysis or grid adaptation.

The algebraic grid-generation scheme and NACA wing sections presented here are intended to demonstrate the elements involved in obtaining grid sensitivity from an algebraic grid generation process. It is evident that each grid generation formulation would require considerable analytical differentiation. This implies that a symbolic manipulator, which directly produces computer code for derivative evaluation, should be considered. Also, there are trade-offs between analytical differen-

tiation and finite-difference differentiation. It may be feasible to obtain some of the derivatives by finite differences.

It is implied that airplane surfaces should be parameterized in terms of design variables. This is not simple or feasible for geometrically-complex airplanes in advanced stages of design. However, design parameterization is feasible during conceptual and preliminary design. The parameterization, which is the development of analytical formulas for part or all of a surface is critical for satisfactory results. It is always possible to create a geometric parameterization of a surface (collection of points or derivatives that define surface patches), but geometric parameterizations have very local sensitivities and a complete aerodynamic surface can require a large number of parameters for its definition.

As a compromise between totally analytic parameterization of surfaces and geometric parameterization, a hybrid approach is advocated. In a hybrid approach, certain sections or skeletal parts of a surface are specified analytically and interpolation formulas are used for the remainder of the surface. This is employed for the wing example described herein.

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APPENDIX A

FORTRAN LISTING FOR GRID GENERATION
ALGORITHM : HERMITE

```

SUBROUTINE HERMITE1 (XS, YS, NI, NJ, ALFA, XGRD, YGRD, PROPID, DOBATCH)
PARAMETER (NGRID=3)
$INCLUDE tbggl.inc
$INCLUDE tbgg5.inc
  DIMENSION XS (NGDIM, *), YS (NGDIM, *), T (NGDIM), XGRD (NGDIM, NGDIM)
1      , YGRD (NGDIM, NGDIM), ALFA (NGDIM), S1 (NGDIM, NGDIM)
2      , XLEFT (NGDIM), YLEFT (NGDIM), XRIGHT (NGDIM), YRIGHT (NGDIM)
3      , RI (NGDIM), RO (NGDIM), R (NGDIM), S (NGDIM), SK (NGDIM)
4      , X (NGDIM), Y (NGDIM), XVIEW (NGDIM, NGDIM), YVIEW (NGDIM, NGDIM)
COMMON/PQ/PS (MGDIM), QS (MGDIM), tS (MGDIM, MGDIM)
COMMON/DXYDETA/DXBDETA (NGDIM), DYBDETA (NGDIM)
COMMON/JAYS/J1, J2, J3
common/design/cm, p, th, rr, wlength, chord
CHARACTER PROPID*(*), PR(4)*80
SAVE RO, XLEFT, YLEFT, XRIGHT, YRIGHT
LOGICAL REDRAW, DONE, REDIST, DOBATCH, FIRST

C
C
C
C-----
C Does basic grid calculations, draws grid, and provides for user
C modifications in interactive loop.
C-----
C
C
C
C REDIST = .FALSE.
C REDIST = .TRUE.
C DONE = .FALSE.
C DONE = .TRUE.
C
C ..... Magnitude Of Orthogonality Vector
C
C
C WRITE(*,*)
C WRITE(*,*)' Magnitude of Normal Derivatives ?'
C READ(*,*) SKB
C
C DO 299 I = 1, NI
C SK(I) = SKB
299 CONTINUE
C
C 99 CONTINUE
C
C Distribution For Stretching Variable T
C Uses Linear Distribution For Initial Trial and Arc-length
C Distribution For Final Trial
C
C
C IF (REDIST.AND.(.NOT.DONE)) THEN
C
C DONE = .TRUE.
C
C IF (DOBATCH) THEN
C CALL BATCH (NJ, RO, 5)
C ELSE
C CALL DISTXI (NJ, RO, 5)
C ENDIF
C
C DO 100 I = 1, NI

```

```

C
      DO 199 J = 1 , NGDIM
      X(J) = XGRD(I,J)
      Y(J) = YGRD(I,J)
199  CONTINUE
C
      CALL ARC(X,Y,R,NGDIM,RMAX)
C
      DO 101 J = 1 , NGDIM
C
      RI(J) = R(J) / RMAX
      IF (RI(NGDIM) .NE. 1.0) RI(NGDIM)=1.0
C
101  CONTINUE
C
      CALL INTERPO(RI,T,NGDIM,RO,S,NJ)
C
      DO 102 J = 1 , NJ
C
      S1(I,J) = S(J)
      tS(I,J) = S1(I,J)
C
102  CONTINUE
C
100  CONTINUE
C
      ELSEIF((.NOT.REDIST).AND.(.NOT.DONE)) THEN
C
      REDIST = .TRUE.
C
C
      DO 201 I = 1 , NI
C
      DO 201 J = 1 , NGDIM
C
      T(J) = FLOAT(J-1)/FLOAT(NGDIM-1)
      S1(I,J) = T(J)
      tS(I,J) = S1(I,J)
C
201  CONTINUE
C
C ..... Base Line Distribution
C
      ELSEIF(REDIST.AND.DONE) THEN
C
      IF(DOBATCH) THEN
      CALL BATCH(NJ,RO,5)
      ELSE
      CALL DISTXI(NJ,RO,5)
      ENDIF
C
      DO 107 I = 1 , NI
      DO 107 J = 1 , NJ
      S1(I,J) = RO(J)
      tS(I,J) = S1(I,J)
107  CONTINUE
      ENDIF
C
      IF(MGDIM.NE.NGDIM) THEN
      WRITE(*,*)' >>>> tbgg1.inc & tbgg5.inc Should Have the

```

```

1                               Same Dimension'
STOP
ENDIF

C
C
C      -----
C      Hermite Interpolation Function (Eq.3 of manual)
C      -----
C
C      SKT = 0.0
C
C      DO 240 I=1,NI
C
C          XB = XS(I,1)
C          YB = YS(I,1)
C          XT = XS(I,2)
C          YT = YS(I,2)
C
C      ..... Orthogonality For Foil
C
C          IF(I.GE.1.AND.I.LE.J2) THEN
C
C              DXDETA = - SIN(ALFA(I))
C              DYDETA = + COS(ALFA(I))
C
C          ELSEIF(I.GT.J2.AND.I.LE.NI) THEN
C
C              DXDETA = + SIN(ALFA(I))
C              DYDETA = - COS(ALFA(I))
C
C          ENDIF
C
C
C          DXBDETA(I)=DXDETA
C          DYBDETA(I)=DYDETA
C
C
C          IF(I.EQ.J2-4) SK(I) = SKB * 0.85
C          IF(I.EQ.J2-3) SK(I) = SKB * 0.75
C          IF(I.EQ.J2-2) SK(I) = SKB * 0.65
C          IF(I.EQ.J2-1) SK(I) = SKB * 0.55
C          IF(I.EQ.J2)   SK(I) = SKB * 0.45
C          IF(I.EQ.J2+1) SK(I) = SKB * 0.55
C          IF(I.EQ.J2+2) SK(I) = SKB * 0.65
C          IF(I.EQ.J2+3) SK(I) = SKB * 0.75
C          IF(I.EQ.J2+4) SK(I) = SKB * 0.85
C
C          PS(I) = SK(I)
C          QS(I) = SKT
C
C      IF (.NOT.DONE) THEN
C
C          N=NGDIM
C
C          ELSE
C
C          N=NJ
C
C      ENDIF
C
C      DO 240 J = 1 , N

```


C

CALL BLEND (S1(I,J),F1,F2,F3,F4)

C

XGRD(I,J) = XB*F1 + XT*F2 + PS(I) * DXDETA * F3
YGRD(I,J) = YB*F1 + YT*F2 + PS(I) * DYDETA * F3

C

240 CONTINUE

C

IF (.NOT.DONE) GOTO 99

C

C

RETURN

END

APPENDIX B

FORTRAN LISTING FOR NACA FOUR-DIGIT SURFACE
GENERATION : NACA


```

P=P*TENTH
WRITE(*,*)' MAXIMUM THICKNESS OF AIRFOIL ( % CHORD) ?'
READ(*,*) TH
TH=TH*PERCENT

C
C-----
C
C AIRFOIL NOMENCLATURE:
C
C CM ..... CAMBER
C P ..... CAMBER LOCATION ALONG CHORD
C TH ..... MAXIMUM THICKNESS
C A ..... THICKNESS DISTRIBUTION COEFFICIENTS
C XCI , YC ..... MEAN LINE COORDINATES
C CHORD ..... CHORD LENGTH
C WLENGTH ..... WAKE LENGTH
C
C-----
C
C----- THICKNESS DISTRIBUTION
C
C      A1=0.29648
C      A2=-0.12642
C      A3=-0.35202
C      A4=0.28388
C      A5=-0.10192
C
C
C .....INITIAL DISTRIBUTION IN XI-DIRECTION FOR LOWER BOUNDARY
C
C      DO 5 I=1,NGDIM
C        C = FLOAT(I-1)/FLOAT(NGDIM-1)
C        X(I) = (EXP(CK*C)-1.)/(EXP(CK)-1.)
C        XCI(I) = X(I) * CHORD
C      5 CONTINUE
C
C .....GET INITIAL FOIL (UPPER & LOWER)
C
C      CALL FOIL(XCI, XUS, YUS, ALFAU, DYUDX, NGDIM, 1, CHEAT, 999)
C      CALL FOIL(XCI, XLS, YLS, ALFAL, DYLDX, NGDIM, 2, CHEAT, 999)
C
C----- OUTER BOUNDARY
C
C      XOUTER = CHORD+R
C
C
C ..... INITIAL DISTRIBUTION IN XI-DIRECTION FOR OUTER BOUNDARY
C
C      DO 35 I=1,NI
C
C        C = FLOAT(I-1)/FLOAT(NI-1)
C        XX(I) = (EXP(C*CK)-1.)/(EXP(CK)-1.)
C
C      35 CONTINUE
C
C      DO 36 I=1,NI
C        XOL(I) = XX(I) * XOUTER
C
C
C        XOU(I)=XOL(I)

```

```

                IF (XOL(I) .LE. R) THEN
                    YOU(I) = SQRT (R*R - (XOL(I) - R) * (XOL(I) - R))
                    YOL(I) = -YOU(I)
                ELSE
                    YOU(I) = YLENGTH
                    YOL(I) = -YLENGTH
                END IF
36      CONTINUE
C
C ----- OUTPUT TO HERMITE
C
C      NLI(J) = REPRESENTS NUMBER OF POINTS IN EACH BOUNDARY
C      J=1 ..... SOLID BOUNDARY
C      J=2 ..... OUTER BOUNDARY
C      J=3 ..... RIGHT BOUNDARY
C      J=4 ..... LEFT BOUNDARY
C
C
C      NLI(1) = NI*2
C      NLI(2) = NI*2
C      NLI(3) = NJ
C      NLI(4) = NJ
C
C ..... BOTTOM BOUNDARY DISTRIBUTION (ARC-LENGTH)
C
C      J=1
C
C      REDIST(J) = .TRUE.
C      REDIST(J) = .FALSE.
C
C      IF (REDIST(J)) THEN
C ..... INTERPOLATE UPPER PORTION
C
C      CALL ARC (XUS, YUS, RI, NGDIM, RMAX)
C      IF (DOBATCH) THEN
C          CALL BATCH (NI, ROBAR, 1)
C      ELSE
C          CALL DISTXI (NI, ROBAR, IS)
C      ENDIF
C      DO 100 I=1, NI
C          RO(I) = ROBAR(I) * RMAX
100     CONTINUE
C      CALL INTERPO (RI, XUS, NGDIM, RO, XDIST, NI)
C
C ..... GET FINAL FOIL (UPPER)
C
C      CALL FOIL (XDIST, XU, YU, ALFAU, DYUDX, NI, 1, CHEAT, 999)
C
C ..... INTERPOLATE LOWER PORTION
C
C      CALL ARC (XLS, YLS, RI, NGDIM, RMAX)
C      IF (DOBATCH) THEN
C          CALL BATCH (NI, ROBAR, 2)
C      ELSE
C          CALL DISTXI (NI, ROBAR, IS)
C      ENDIF
C      DO 101 I=1, NI
C          RO(I) = ROBAR(I) * RMAX
101     CONTINUE

```

```

CALL INTERPO (RI, XLS, NGDIM, RO, XDIST, NI)
C
C ..... GET FINAL FOIL (LOWER)
C
CALL FOIL (XDIST, XL, YL, ALFAL, DYLDX, NI, 2, CHEAT, 999)
C
ELSE
C
C ..... Base Line Distribution
C
IF (DOBATCH) THEN
CALL BATCH (NI, ROBAR, 1)
ELSE
CALL DISTXI (NI, ROBAR, IS)
ENDIF
CALL FOIL (ROBAR, XU, YU, ALFAU, DYUDX, NI, 1, CHEAT, 999)
IF (DOBATCH) THEN
CALL BATCH (NI, ROBAR, 2)
ELSE
CALL DISTXI (NI, ROBAR, IS)
ENDIF
CALL FOIL (ROBAR, XL, YL, ALFAL, DYLDX, NI, 2, CHEAT, 999)
ENDIF
C
C ..... SHIFT X-COORDINATES TO THE RIGHT FOR C-TYPE GRID
C
DO 30 I=1, NI
XU(I) = XU(I) + R
XL(I) = XL(I) + R
30 CONTINUE
C
C ..... ASSEMBLE COUNTER-CLOCKWISE
C
DO 40 I=1, NLI(J)
IF (I.LE.NI) THEN
XS(I, J)=XU(NI-I+1)
YS(I, J)=YU(NI-I+1)
ALFA(I)=ALFAU(NI-I+1)
ALFA(NI)=PI/2.
ELSE
XS(I, J)=XL(I-NI)
YS(I, J)=YL(I-NI)
ALFA(I)=ALFAL(I-NI)
END IF
40 CONTINUE
C
C ..... CHECK DOUBLE POINTS FOR BOTTOM BOUNDARY
C
DO 46 I=1, NLI(J)-1
IF (XS(I, J).EQ.XS(I+1, J).AND.YS(I, J).EQ.YS(I+1, J)) THEN
K=I+1
XS(K, J)=XS(K+1, J)
YS(K, J)=YS(K+1, J)
ALFA(K)=ALFA(K+1)
IF (I.EQ.NLI(J)-1) NLI(J)=NLI(J)-1
END IF
46 CONTINUE
C
C ----- OUTER BOUNDARY
C

```

```

      J=2
C
      REDIST(J) = .TRUE.
      DO 45 I=1,NLI(J)
        IF(I.LE.NI) THEN
          XI(I)=XOU(NI-I+1)
          YI(I)=YOU(NI-I+1)
        ELSE
          XI(I)=XOL(I-NI)
          YI(I)=YOL(I-NI)
        END IF
45    CONTINUE
C
C ----- CHECK FOR DOUBLE POINTS (SYMMETRY LINE)
C
      DO 48 I=1,NLI(J)-1
        IF(XI(I).EQ.XI(I+1).AND.YI(I).EQ.YI(I+1)) THEN
          K=I+1
          XI(K)=XI(K+1)
          YI(K)=YI(K+1)
          IF(I.EQ.NLI(J)-1) NLI(J)=NLI(J)-1
        END IF
48    CONTINUE
C
      IF(REDIST(J)) THEN
        CALL ARC(XI,YI,RI,NLI(J),RMAX)
        IF(DOBATCH) THEN
          CALL BATCH(NLI(J),ROBAR,6)
        ELSE
          CALL DISTXI(NLI(J),ROBAR,IS)
        ENDIF
        DO 103 I=1,NLI(J)
          RO(I) = ROBAR(I) * RMAX
103    CONTINUE
        CALL INTERPO(RI,XI,NLI(J),RO,XO,NLI(J))
        CALL INTERPO(RI,YI,NLI(J),RO,YO,NLI(J))
      ENDIF
C
      DO 201 I=1,NLI(J)
        IF(REDIST(J)) THEN
          XS(I,J) = XO(I)
          YS(I,J) = YO(I)
        ELSEIF(.NOT.REDIST(J)) THEN
          XS(I,J) = XI(I)
          YS(I,J) = YI(I)
        ENDIF
201    CONTINUE
C
C ..... ETA DISTRIBUTION
C
      DO 51 J = 1 , NJ
        ETA(J) = FLOAT(J-1)/FLOAT(NJ-1)
51    CONTINUE
C
C ----- RIGHT BOUNDARY
C
      J=3
      DO 49 I=1,NJ
        XS(I,J)=CHORD+R
        YS(I,J)=ETA(I)*R

```

```

49  CONTINUE
C
C ----- LEFT BOUNDARY
C
      J=4
      DO 50 I=1,NJ
        XS(I,J)=CHORD+R
        YS(I,J)=-ETA(I)*R
50  CONTINUE
C
C
      RETURN
      END
C
C*****
C
      SUBROUTINE FOIL(XCI,X,Y,ANGLE,DYDX,N,II,CHEAT,IGRID)
$INCLUDE tbggl.inc
      DIMENSION XCI(NGDIM),X(NGDIM),Y(NGDIM),YC(NGDIM),YT(NGDIM),XD(3)
1      ,DYCDXCI(NGDIM),DYTDXCI(NGDIM),ANGLE(NGDIM),DYDX(NGDIM)
      COMMON /A/A1,A2,A3,A4,A5
      COMMON /DESIGN/CM,P,TH,R,WLENGTH,CHORD
      COMMON /DELTA/D/TH1,CM1,P1,TH3,CM3,P3
      LOGICAL CHEAT
      DATA PI,CK/3.14159,3./
C
C
C
      XD(1) = TH
      XD(2) = CM
      XD(3) = P
C
      IF (CHEAT) THEN
        IF (IGRID.EQ.1) THEN
          XD(1) = TH1
          XD(2) = CM1
          XD(3) = P1
        ELSEIF (IGRID.EQ.3) THEN
          XD(1) = TH3
          XD(2) = CM3
          XD(3) = P3
        ENDIF
      ENDIF
C
      DO 10 I=1,N
C
C
C
        IF (XD(3).NE.0.0.AND.XCI(I).LE.XD(3)) THEN
C
          YC(I)=(XD(2)/(XD(3)*XD(3)))*(2.*XD(3)*XCI(I)-XCI(I)*XCI(I))
          DYCDXCI(I)=(XD(2)/(XD(3)*XD(3)))*(2.*XD(3)-2.*XCI(I))
C
          ELSEIF (XD(3).NE.0.0.AND.XCI(I).GT.XD(3)) THEN
C
          YC(I)=(XD(2)/((1.-XD(3))*(1.-XD(3))))*
1          (1.-2.*XD(3)+2.*XD(3)*XCI(I)-XCI(I)*XCI(I))
          DYCDXCI(I)=(XD(2)/((1.-XD(3))*(1.-XD(3))))*(2.*XD(3)-2.*XCI(I))
C
          ELSEIF (XD(3).EQ.0.0) THEN
C
            YC(I)=0.0

```



```

          DYCDXCI(I) = 0.0
C
          END IF
10      CONTINUE
C
C
          DO 15 I=1,N
C
          IF (XCI(I) .LE. CHORD) THEN
C
          YT(I) = (XD(1)/0.2) *
1          (A1*SQRT(XCI(I)) + A2*XCI(I) + A3*XCI(I)*XCI(I)
2          + A4*XCI(I)*XCI(I)*XCI(I) + A5*XCI(I)*XCI(I)*XCI(I))
C
          DYTDXCI(I) = (XD(1)/0.2) * (A1*(0.5/SQRT(XCI(I))) + A2 + 2.*A3*XCI(I)
1          + 3.*A4*XCI(I)*XCI(I) + 4.*A5*XCI(I)*XCI(I)*XCI(I))
C
          ELSE
C
          YT(I) = 0.0
          DYTDXCI(I) = 0.0
C
          END IF
15      CONTINUE
C
C
C ----- SURFACE COORDINATES (EQS. 5, 6)
C
          DO 25 I = 1, N
C
          X(I) = XCI(I)
          IF (II.EQ.1) THEN
          Y(I) = YC(I) + YT(I)
          DYDX(I) = DYCDXCI(I) + DYTDXCI(I)
          ANGLE(I) = ATAN(DYDX(I))
          ANGLE(1) = PI/2
          ELSEIF (II.EQ.2) THEN
          Y(I) = YC(I) - YT(I)
          DYDX(I) = DYCDXCI(I) - DYTDXCI(I)
          ANGLE(I) = ATAN(DYDX(I))
          ANGLE(1) = PI/2
          ELSE
          WRITE(*,*) ' Trouble in FOIL'
          STOP
          ENDIF
C
25      CONTINUE
C
          RETURN
          END

```

APPENDIX C

FORTRAN LISTING OF NACA FOUR-DIGIT GRID
SENSITIVITY : SENSIT

```

SUBROUTINE DXYDXYD (XX, YY, NI, NJ, ICUR, FLAG, ISENS, SRAN)
PARAMETER (ICHOICE=8)
$INCLUDE tbgg1.inc
$INCLUDE tbgg5.inc
DIMENSION XX (NGDIM, NGDIM), YY (NGDIM, NGDIM)
COMMON /XYDXYD/ DXDTH (NGDIM, NGDIM), DYDTH (NGDIM, NGDIM),
1          DXDCM (NGDIM, NGDIM), DYDCM (NGDIM, NGDIM),
2          DXDP (NGDIM, NGDIM), DYDP (NGDIM, NGDIM)
COMMON /DXYBDES/DXBDCM (NGDIM), DYBDCM (NGDIM), DXBDTH (NGDIM),
1          DYBDTH (NGDIM), DXBDP (NGDIM), DYBDP (NGDIM), DXBEDTH (NGDIM),
2          DYBEDTH (NGDIM), DXBEDCM (NGDIM), DYBEDCM (NGDIM), DXBEDP (NGDIM),
3          DYBEDP (NGDIM)
COMMON /PQ/P (MGDIM), Q (MGDIM), T (MGDIM, MGDIM)
COMMON /JAYS/J1, J2, J3
LOGICAL FLAG (ICHOICE)

C
C
C
C
C
C -----
C          GRID SENSITIVITY WITH RESPECT TO DESIGN VARIABLES
C
C          WRITTEN BY : IDEEN SADREHAGHIGHI
C
C -----
C ..... Get Bottom Boundary Sensitivity
C
C          CALL DXYBDD (XX, YY, NI, NJ, ICUR, SRAN, CHEAT)
C
C ..... Sensitivity Without Arc-length Distribution For Normal Direction
C
C          DO 10 J = 1, NJ
C
C              DO 10 I = 1, NI
C
C                  CALL BLENDP (T (I, J), ALFA1, ALFA2, ALFA3, ALFA4)
C
C
C          DXDTH (I, J) = ALFA1 * DXBDTH (I) + P (I) * ALFA3 * DXBEDTH (I)
C          DYDTH (I, J) = ALFA1 * DYBDTH (I) + P (I) * ALFA3 * DYBEDTH (I)
C          DXDCM (I, J) = ALFA1 * DXBDCM (I) + P (I) * ALFA3 * DXBEDCM (I)
C          DYDCM (I, J) = ALFA1 * DYBDCM (I) + P (I) * ALFA3 * DYBEDCM (I)
C          DXDP (I, J) = ALFA1 * DXBDP (I) + P (I) * ALFA3 * DXBEDP (I)
C          DYDP (I, J) = ALFA1 * DYBDP (I) + P (I) * ALFA3 * DYBEDP (I)
C
C
C 10 CONTINUE
C
C          FLAG (ISENS) = .TRUE.
C
C          RETURN
C          END
C
C *****
C
C          SUBROUTINE DXYBDD (XX, YY, NI, NJ, ICUR, SRAN, CHEAT)
C          PARAMETER (ICHOICE=8)
C          $INCLUDE tbgg1.inc
C          DIMENSION XX (NGDIM, NGDIM), YY (NGDIM, NGDIM),
C          1          DXBUDTH (NGDIM), DXBUDCM (NGDIM), DXBUDP (NGDIM),
C          2          DYBUDTH (NGDIM), DYBUDCM (NGDIM), DYBUDP (NGDIM),

```

```

3      DXBLDTH (NGDIM) , DXBLDCM (NGDIM) , DXBLDP (NGDIM) ,
4      DYBLDTH (NGDIM) , DYBLDCM (NGDIM) , DYBLDP (NGDIM) ,
5      DXUEDTH (NGDIM) , DXUEDCM (NGDIM) , DXUEDP (NGDIM) ,
6      DYUEDTH (NGDIM) , DYUEDCM (NGDIM) , DYUEDP (NGDIM) ,
7      DXLEDTH (NGDIM) , DXLEDCM (NGDIM) , DXLEDP (NGDIM) ,
8      DYLEDTH (NGDIM) , DYLEDCM (NGDIM) , DYLEDP (NGDIM) ,
9      XU (NGDIM) , XL (NGDIM) , YU (NGDIM) , YL (NGDIM)
COMMON /DXYBDES/DXBDCM (NGDIM) , DYBDCM (NGDIM) , DXBDTH (NGDIM) ,
1      DYBDTH (NGDIM) , DXBDP (NGDIM) , DYBDP (NGDIM) , DXBEDTH (NGDIM) ,
2      DYBEDTH (NGDIM) , DXBEDCM (NGDIM) , DYBEDCM (NGDIM) , DXBEDP (NGDIM) ,
3      DYBEDP (NGDIM)
COMMON/JAYS/J1, J2, J3
COMMON/TANGENT/THETAU (NGDIM) , THETA (NGDIM) , DYUDX (NGDIM) , DYLDX (NGDIM) , NFOIL
COMMON /A/A1, A2, A3, A4, A5
COMMON /DESIGN/CM, P, TH, R, WLENGTH, CHORD
COMMON /TEMP1/XNEW (NGDIM) , YNEW (NGDIM) , RNEW (NGDIM)
LOGICAL CHEAT

```

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

COMPUTES ANALITICALLY THE DERIVATIVE OF BOUNDARY COORDINATES
WITH RESPECT TO DESIGN VARIABLES DXB/DXD

<<< DESIGN VARIABLES >>>

- CM CAMBER
- P LOCATION OF CAMBER
- TH MAX. THICKNESS

```

IUPPER = J2
ILOWER = NI - J2 + 1
IF (IUPPER.NE.ILOWER) THEN
  WRITE (*, *) ' ERROR FROM DXYBDD '
  STOP
ENDIF

```

```

DO 1 I = 1 , IUPPER
  XU (I) = XX (J2-I+1, 1) - R
  YU (I) = YY (J2-I+1, 1)
1  CONTINUE

```

```

DO 2 I = 1 , ILOWER
  XL (I) = XX (I+J2-1, 1) - R
  YL (I) = YY (I+J2-1, 1)
2  CONTINUE

```

C
C
C
C
C
C
C

BOUNDARY DESIGN SENSITIVITY DERIVATIVES OF AIRFOIL

```

C
  IDUMMY = J2 - J1
  DO 16 I = 1, IUPPER
C
  IF(I.LE.IDUMMY) THEN
C
C ----- CAMBERED AIRFOIL -----
C
C ..... FORWARD OF MAX. CAMBER
C
  IF(P.NE.0.0.AND.XU(I).LE.P) THEN
C
  YC = (CM/(P*P))*(2.*P*XU(I)-XU(I)*XU(I))
  DYCDCM = (1./(P*P))*(2.*P*XU(I)-XU(I)*XU(I))
  DYCDP = (2.*CM/(P*P))*(-XU(I)+XU(I)*XU(I)/P)
  D2YCDCM= (1./(P*P))*(2.*P-2.*XU(I))
  D2YCDP = (2.*CM/(P*P))*(2.*XU(I)/P-1.)
C
  ELSEIF(P.NE.0.0.AND.XU(I).GT.P) THEN
C
C ..... AFT OF MAX. CAMBER
C
  YC = (CM/((1.-P)*(1.-P)))*(1.-2.*P+2.*P*XU(I)-
      XU(I)*XU(I))
  DYCDCM = (1./((1.-P)*(1.-P)))*(1.-2.*P+2.*P*XU(I)
      -XU(I)*XU(I))
  FACTOR1 = CM/((1.-P)*(1.-P))
  FACTOR2 = 2./(1.-P)
  FACTOR3 = 1.-2.*P+2.*P*XU(I)-XU(I)*XU(I)
  DYCDP = FACTOR1 * (FACTOR2 * FACTOR3 - 2. + 2.*XU(I))
  D2YCDCM= (1./((1.-P)*(1.-P)))*(2.*P-2.*XU(I))
  D2YCDP = (2.*CM/((1.-P)*(1.-P)))*(2.*((P-XU(I))/(1.-P))+1.)
C
  ELSEIF(CM.EQ.0.0.AND.P.EQ.0.0) THEN
C
C ----- SYMMETRICAL AIRFOIL -----
C
      YC      = 0.0
      DYCDCM = 0.0
      DYCDP   = 0.0
      D2YCDCM= 0.0
      D2YCDP  = 0.0
C
  ENDIF
C
C
C
  D2YTDTH = (1./0.2)*(0.5*A1/SQRT(XU(I))+A2
      +2.*A3*XU(I)+3.*A4*XU(I)*XU(I)+4.*A5*XU(I)*XU(I)*XU(I))
C
  FACTOR = A1*SQRT(XU(I))+A2*XU(I)+A3*XU(I)*XU(I)
      +A4*XU(I)*XU(I)*XU(I)+A5*XU(I)*XU(I)*XU(I)*XU(I)
C
  YT = (TH/0.2)*FACTOR
C
  DYDTH = YT/TH
  DYBUDTH(I) = DYDTH
  DYBUDCM(I) = DYCDCM
  DYBUDP(I) = DYCDP
C
C ..... EVALUATING DXETA/DTH , DYETA/DTH

```

```

C
C
      TIMESU      = COS (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DXUEDTH (I) = + D2YTDTH * - TIMESU
C
      TIMESU      = SIN (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DYUEDTH (I) = + D2YTDTH * - TIMESU
C
C ..... EVALUATING DXETA/DCM , DYETA/DCM
C
      TIMESU      = COS (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DXUEDCM (I) = + D2YCDCM * - TIMESU
C
      TIMESU      = SIN (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DYUEDCM (I) = + D2YCDCM * - TIMESU
C
C ..... EVALUATING DXETA/DP , DYETA/DP
C
      TIMESU      = COS (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DXUEDP (I)  = + D2YCDP * - TIMESU
C
      TIMESU      = SIN (THETAU (I)) / (1.+(DYUDX (I) *DYUDX (I)))
      DYUEDP (I)  = + D2YCDP * - TIMESU
C
C ..... Singularity At Nose .... Slope dy/dx = Infinite
C
      IF (I.EQ.1) THEN
        DXUEDTH (I) = 0.0
        DYUEDTH (I) = 0.0
        DXUEDCM (I) = 0.0
        DYUEDCM (I) = 0.0
        DXUEDP (I)  = 0.0
        DYUEDP (I)  = 0.0
      ENDIF
C
      ELSE
C ..... SET BOUNDARY SENSITIVITY DERIVATIVES TO ZERO IN WAKE REGION
C
        DXBUDCM (I) = 0.0
        DYBUDCM (I) = 0.0
        DXBUDP (I)  = 0.0
        DYBUDP (I)  = 0.0
        DXBUDTH (I) = 0.0
        DYBUDTH (I) = 0.0
        DXUEDTH (I) = 0.0
        DYUEDTH (I) = 0.0
        DXUEDCM (I) = 0.0
        DYUEDCM (I) = 0.0
        DXUEDP (I)  = 0.0
        DYUEDP (I)  = 0.0
C
      ENDIF
C
16 CONTINUE
C
      IL = J2 - J1 + 1
C
C ..... GET X-BOUNDARY COORDINATE SENSITIVITY FOR UPPER WAKE REGION

```

```

C          ..... DXB/DXD = DXB/DR * DR/DXD .....
C
C          CALL DXBDXD (1, IL, XU, YU, DXBUDCM, DXBUDTH, DXBUDP, CHEAT)
C
C          IDUMMY = J3 - J2
C          DO 17 I = 1, ILOWER
C
C          IF (I.LE.IDUMMY) THEN
C
C          ----- CAMBERED AIRFOIL -----
C
C          ..... FORWARD OF MAX. CAMBER
C
C          IF (P.NE.0.0.AND.XL(I).LE.P) THEN
C              YC = (CM/(P*P))*(2.*P*XL(I)-XL(I)*XL(I))
C              DYCDCM = (1./(P*P))*(2.*P*XL(I)-XL(I)*XL(I))
C              DYCDP = (2.*CM/(P*P))*(-XL(I)+XL(I)*XL(I)/P)
C              D2YCDCM= (1./(P*P))*(2.*P-2.*XL(I))
C              D2YCDP = (2.*CM/(P*P))*(2.*XL(I)/P-1.)
C
C          ELSEIF (P.NE.0.0.AND.XL(I).GT.P) THEN
C
C          ..... AFT OF MAX. CAMBER
C
C              YC = (CM/((1.-P)*(1.-P)))*(1.-2.*P+2.*P*XL(I)-
C                  XL(I)*XL(I))
C              DYCDCM = (1./((1.-P)*(1.-P)))*(1.-2.*P+2.*P*XL(I)
C                  -XL(I)*XL(I))
C              FACTOR1 = CM/((1.-P)*(1.-P))
C              FACTOR2 = 2./(1.-P)
C              FACTOR3 = 1.-2.*P+2.*P*XL(I)-XL(I)*XL(I)
C              DYCDP = FACTOR1 * (FACTOR2 * FACTOR3 - 2. + 2.*XL(I))
C              D2YCDCM= (1./((1.-P)*(1.-P)))*(2.*P-2.*XL(I))
C              D2YCDP = (2.*CM/((1.-P)*(1.-P)))*(2.*((P-XL(I))/(1.-P))+1.)
C
C          ELSEIF (CM.EQ.0.0.AND.P.EQ.0.0) THEN
C
C          ----- SYMMETRICAL AIRFOIL -----
C
C              YC          = 0.0
C              DYCDCM     = 0.0
C              DYCDP      = 0.0
C              D2YCDCM    = 0.0
C              D2YCDP     = 0.0
C
C          ENDIF
C
C          D2YTDTH = (1./0.2)*(0.5*A1/SQRT(XL(I))+A2
C              +2.*A3*XL(I)+3.*A4*XL(I)*XL(I)+4.*A5*XL(I)*XL(I)*XL(I))
C
C          FACTOR = A1*SQRT(XL(I))+A2*XL(I)+A3*XL(I)*XL(I)
C              +A4*XL(I)*XL(I)*XL(I)+A5*XL(I)*XL(I)*XL(I)*XL(I)
C
C          YT = (TH/0.2)*FACTOR
C
C          DYDTH = YT/TH
C          DYBLDTH(I) = - DYDTH
C          DYBLDCM(I) = DYCDCM
C          DYBLDP(I) = DYCDP

```

```

C
C ..... EVALUATING DXETA/DTH , DYETA/DTH
C
C
C      TIMESL      = COS (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DXLEDTH(I) = - D2YTDTH * + TIMESL
C
C      TIMESL      = SIN (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DYLEDTH(I) = - D2YTDTH * + TIMESL
C
C ..... EVALUATING DXETA/DCM , DYETA/DCM
C
C      TIMESL      = COS (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DXLEDCM(I) = + D2YCDCM * + TIMESL
C
C      TIMESL      = SIN (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DYLEDCM(I) = + D2YCDCM * + TIMESL
C
C ..... EVALUATING DXETA/DP , DYETA/DP
C
C      TIMESL      = COS (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DXLEDP (I)  = + D2YCDP * + TIMESL
C
C      TIMESL      = SIN (THETAL (I)) / (1.+(DYLDX(I)*DYLDX(I)))
      DYLEDP (I)  = + D2YCDP * + TIMESL
C
C ..... Singularity At Nose .... Slope dy/dx = Infinite
C
C      IF (I.EQ.1) THEN
C          DXLEDTH(I) = 0.0
C          DYLEDTH(I) = 0.0
C          DXLEDCM(I) = 0.0
C          DYLEDCM(I) = 0.0
C          DXLEDP (I) = 0.0
C          DYLEDP (I) = 0.0
C      ENDIF
C
C      ELSE
C
C ..... SET SENSITIVITY DERIVATIVES TO ZERO IN WAKE REGION
C
C          DXBLDCM(I) = 0.0
C          DYBLDCM(I) = 0.0
C          DXBLDP (I) = 0.0
C          DYBLDP (I) = 0.0
C          DXBLDTH(I) = 0.0
C          DYBLDTH(I) = 0.0
C          DXLEDTH(I) = 0.0
C          DYLEDTH(I) = 0.0
C          DXLEDCM(I) = 0.0
C          DYLEDCM(I) = 0.0
C          DXLEDP (I) = 0.0
C          DYLEDP (I) = 0.0
C
C      ENDIF
C
C 17 CONTINUE
C
C ..... GET X-BOUNDARY COORDINATE SENSITIVITY FOR LOWER WAKE REGION
C ..... DXB/DXD = DXB/DR * DR/DXD .....

```



```

C
CALL DXBDXD (2, IL, XL, YL, DXBLDCM, DXBLDTH, DXBLDP, CHEAT)
C
DO 50 I = 1, IUPPER
C
    DXBDTH (IUPPER - I + 1) = DXBUDTH (I)
    DYBDTH (IUPPER - I + 1) = DYBUDTH (I)
    DXBDCM (IUPPER - I + 1) = DXBUDCM (I)
    DYBDCM (IUPPER - I + 1) = DYBUDCM (I)
    DXBDP (IUPPER - I + 1) = DXBUDP (I)
    DYBDP (IUPPER - I + 1) = DYBUDP (I)
    DXBEDTH (IUPPER - I + 1) = DXUEDTH (I)
    DYBEDTH (IUPPER - I + 1) = DYUEDTH (I)
    DXBEDCM (IUPPER - I + 1) = DXUEDCM (I)
    DYBEDCM (IUPPER - I + 1) = DYUEDCM (I)
    DXBEDP (IUPPER - I + 1) = DXUEDP (I)
    DYBEDP (IUPPER - I + 1) = DYUEDP (I)
C
50 CONTINUE
C
DO 60 I = 1, ILOWER
C
    DXBDTH (IUPPER - 1 + I) = DXBLDTH (I)
    DYBDTH (IUPPER - 1 + I) = DYBLDTH (I)
    DXBDCM (IUPPER - 1 + I) = DXBLDCM (I)
    DYBDCM (IUPPER - 1 + I) = DYBLDCM (I)
    DXBDP (IUPPER - 1 + I) = DXBLDP (I)
    DYBDP (IUPPER - 1 + I) = DYBLDP (I)
    DXBEDTH (IUPPER - 1 + I) = DXLEDTH (I)
    DYBEDTH (IUPPER - 1 + I) = DYLEDTH (I)
    DXBEDCM (IUPPER - 1 + I) = DXLEDCM (I)
    DYBEDCM (IUPPER - 1 + I) = DYLEDCM (I)
    DXBEDP (IUPPER - 1 + I) = DXLEDP (I)
    DYBEDP (IUPPER - 1 + I) = DYLEDP (I)
C
60 CONTINUE
C
RETURN
END
C
C*****
C
SUBROUTINE DXBDXD (II, IL, XNEW, YNEW, DXBDCM, DXBDTH, DXBDP, CHEAT)
$INCLUDE tbgg1.inc
DIMENSION XNEW (NGDIM), YNEW (NGDIM), DXBDCM (NGDIM), DXBDTH (NGDIM)
1      , DXBDP (NGDIM), ROLD (NGDIM), RNEW (NGDIM), XOLD (NGDIM)
2      , YOLD (NGDIM), DYDTH (NGDIM), DYDCM (NGDIM), DYDP (NGDIM)
3      , DRDCM (NGDIM), DRDTH (NGDIM), DRDP (NGDIM), DRDCMN (NGDIM)
4      , DRDTHN (NGDIM), DRDPN (NGDIM)
COMMON/OLD/XUS (NGDIM), YUS (NGDIM), XLS (NGDIM), YLS (NGDIM)
COMMON/TANGENT/THETAU (NGDIM), THETA (NGDIM), DYUDX (NGDIM), DYLDX (NGDIM), NFOIL
COMMON/A/A1, A2, A3, A4, A5
COMMON /DESIGN/CM, P, TH, R, WLENGTH, CHORD
COMMON /DELTA/D/TH1, CM1, P1, TH3, CM3, P3
COMMON/DUMMY/X (NGDIM), Y (NGDIM), X1 (NGDIM), X3 (NGDIM), RO (NGDIM), RI (NGDIM),
1      XDIST (NGDIM), DUM1 (NGDIM), DUM2 (NGDIM), XCI (NGDIM), ROBAR (NGDIM)
LOGICAL CHEAT
DATA CK, DELTA/4.0, 0.0001/

```

C
C

```

C
  goto 999
  DO 10 I = 1 , NGDIM
C
  IF (II.EQ.1) THEN
C
    XOLD(I)=XUS(I)
    YOLD(I)=YUS(I)
C
  ELSE
C
    XOLD(I)=XLS(I)
    YOLD(I)=YLS(I)
C
  ENDIF
C
10  CONTINUE
C
C
C
  DO 15 I = 1,NGDIM
C
  IF (XOLD(I) .LE. CHORD) THEN
C
C ----- CAMBERED AIRFOIL -----
C
C ..... FORWARD OF MAX. CAMBER
C
  IF (P.NE.0.0.AND.XOLD(I) .LE. P) THEN
    YC = (CM/(P*P)) * (2.*P*XOLD(I) - XOLD(I) * XOLD(I))
    DYCDCM = (1./(P*P)) * (2.*P*XOLD(I) - XOLD(I) * XOLD(I))
    DYCDP = (2.*CM/(P*P)) * (-XOLD(I) + XOLD(I) * XOLD(I) / P)
C
  ELSEIF (P.NE.0.0.AND.XOLD(I) .GT. P) THEN
C
C ..... AFT OF MAX. CAMBER
C
    YC = (CM/((1.-P)*(1.-P))) * (1.-2.*P+2.*P*XOLD(I) -
      XOLD(I) * XOLD(I))
    DYCDCM = (1./((1.-P)*(1.-P))) * (1.-2.*P+2.*P*XOLD(I)
      - XOLD(I) * XOLD(I))
    FACTOR1 = CM/((1.-P)*(1.-P))
    FACTOR2 = 2./(1.-P)
    FACTOR3 = 1.-2.*P+2.*P*XOLD(I) - XOLD(I) * XOLD(I)
    DYCDP = FACTOR1 * (FACTOR2 * FACTOR3 - 2. + 2.*XOLD(I))
C
  ELSEIF (CM.EQ.0.0.AND.P.EQ.0.0) THEN
C
C ----- SYMMETRICAL AIRFOIL -----
C
    YC = 0.0
    DYCDCM = 0.0
    DYCDP = 0.0
C
  ENDIF
C
  ELSE
C
    WRITE(*,*) 'ERROR FROM DXBDX18..... XOLD IS OUT OF DOMAIN'
    STOP

```

```

C
      ENDIF
C
C
      FACTOR = A1*SQRT(XOLD(I))+A2*XOLD(I)+A3*XOLD(I)*XOLD(I)
      +A4*XOLD(I)*XOLD(I)*XOLD(I)+A5*XOLD(I)*XOLD(I)*XOLD(I)*XOLD(I)
C
      YT = (TH/0.2)*FACTOR
C
      IF(II.EQ.2)YT=-YT
C
      DYDTH(I) = YT/TH
      DYDCM(I) = DYDCM
      DYDP(I) = DYCDP
C
15  CONTINUE
C
C
      CALL ARC(XOLD,YOLD,ROLD,NGDIM,RMAX)
      CALL ARC(XNEW,YNEW,RNEW,IL,RMAX)
C
C
      DRDCM(1)=0.0
      DRDTH(1)=0.0
      DRDP(1)=0.0
C
      DO 20 I = 2 , NGDIM
C
      FACTOR = (YOLD(I) - YOLD(I-1))/(ROLD(I)-ROLD(I-1))
C
      DRDCM(I) = FACTOR * (DYDCM(I)-DYDCM(I-1)) + DRDCM(I-1)
      DRDTH(I) = FACTOR * (DYDTH(I)-DYDTH(I-1)) + DRDTH(I-1)
      DRDP(I) = FACTOR * (DYDP(I)-DYDP(I-1)) + DRDP(I-1)
C
20  CONTINUE
C
C
      CALL INTERPO(ROLD,DRDCM,NGDIM,RNEW,DRDCMN,IL)
      CALL INTERPO(ROLD,DRDTH,NGDIM,RNEW,DRDTHN,IL)
      CALL INTERPO(ROLD,DRDP,NGDIM,RNEW,DRDPN,IL)
C
C
      DO 25 I = 1 , IL
C
      IF(I.EQ.1) THEN
          DXBDR = (XNEW(I+1)-XNEW(I))/(RNEW(I+1)-RNEW(I))
      ELSEIF(I.EQ.IL) THEN
          DXBDR = (XNEW(I)-XNEW(I-1))/(RNEW(I)-RNEW(I-1))
      ELSE
C
          DXBDR = (XNEW(I+1)-XNEW(I-1))/(RNEW(I+1)-RNEW(I-1))
C
      ENDIF
C
      IF(II.EQ.1)DYDX=DYUDX(I)
      IF(II.EQ.2)DYDX=DYLDX(I)
      FACTOR=SQRT(1. + DYDX*DYDX)
      DXBDR=1./FACTOR
C
      DXBDCM(I) = DXBDR * DRDCMN(I)

```

```

      DXBDTH(I) = DXBDR * DRDTHN(I)
      DXBDP (I) = DXBDR * DRDPN (I)
C
  25  CONTINUE
C
  999  CONTINUE
      goto 1099
      IF(CHEAT) THEN
C
          TH1 = TH + DELTA
          TH3 = TH - DELTA
          IF(CM.NE.0.0) THEN
              CM1 = CM + DELTA
              CM3 = CM - DELTA
          ELSE
              CM1=0.0
              CM3=0.0
          ENDIF
          IF(P.NE.0.0) THEN
              P1 = P + DELTA
              P3 = P - DELTA
          ELSE
              P1=0.0
              P3=0.0
          ENDIF
C
          IGRID = 1
  1000 CONTINUE
C
          DO 35 I=1,NGDIM
C
              C = FLOAT(I-1)/FLOAT(NGDIM-1)
              XCI(I) = ((EXP(CK*C)-1.)/(EXP(CK)-1.))*CHORD
C
  35  CONTINUE
C
          CALL FOIL(XCI,X,Y,DUM1,DUM2,NGDIM,II,CHEAT,IGRID)
          CALL ARC(X,Y,RI,NGDIM,RMAX)
          CALL BATCH(IL,ROBAR,1)
C
          DO 45 I=1,IL
              RO(I)=ROBAR(I)*RMAX
  45  CONTINUE
C
          CALL INTERPO(RI,X,NGDIM,RO,XDIST,IL)
          CALL FOIL(XDIST,X,Y,DUM1,DUM2,IL,II,CHEAT,IGRID)
C
          IF(IGRID.EQ.1) THEN
              DO 65 I=1,IL
                  X1(I) = X(I)
  65  CONTINUE
                  IGRID = 3
                  GOTO 1000
          ELSEIF(IGRID.EQ.3) THEN
              DO 85 I=1,IL
                  X3(I) = X(I)
  85  CONTINUE
          ENDIF
C
C

```

C Finite Differencing

C

DELTATH = TH1 - TH3
DELTACM = CM1 - CM3
DELTAP = P1 - P3

C

```
DO 75 I=1,IL
  DXBDTH(I) = (X1(I) - X3(I))/(TH1 - TH3)
  IF (DELTACM.NE.0.0) THEN
    DXBDCM(I) = (X1(I) -X3(I))/(CM1 - CM3)
  ELSE
    DXBDCM(I) = 0.0
  ENDIF
  IF (DELTAP.NE.0.0) THEN
    DXBDP(I) = (X1(I) - X3(I))/(P1 - P3)
  ELSE
    DXBDP(I) = 0.0
  ENDIF
```

75 CONTINUE

C

ENDIF

C

```
1099 continue
  do 2000 I=1,IL
    DXBDTH(I) = 0.0
    DXBDCM(I) = 0.0
    DXBDP (I) = 0.0
```

2000 CONTINUE

RETURN

END

