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APPLICATION OF NONLINEAR FEEDBACK CONTROL THEORY TO  
SUPERMANEUVERABLE AIRCRAFT

FINAL REPORT

SEPTEMBER 21, 1988 TO DECEMBER 21, 1991  
NASA GRANT NAG - 1 - 821

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(NASA-CR-190336) APPLICATION OF NONLINEAR  
FEEDBACK CONTROL THEORY TO SUPERMANEUVERABLE  
AIRCRAFT Final Report, 21 Sep. 1988 - 21  
Dec. 1991 (Minnesota Univ.) 10 p

N92-25646

Unclass

93/08 0091248

# APPLICATION OF NONLINEAR FEEDBACK CONTROL THEORY TO SUPERMANEUVERABLE AIRCRAFT

## INTRODUCTION

Controlled flight at extremely high angles of attack, far exceeding the stall angle, and/or at high angular rates is sometimes referred to as supermaneuvering flight. During supermaneuvers, transient angles of attack may reach 90 degrees. Studies have shown that fighter aircraft which have the capability of supermaneuverability may have tactical advantages over aircraft which are not capable of being controlled during high angle of attack maneuvers. The unaugmented flying qualities of aircraft at high angles of attack can be quite different from those at low angles of attack. The dutch-roll mode becomes less well damped and the spiral mode becomes less stable. The natural frequency of the phugoid increases with the low speeds which are typical of high angle of attack operations and the coupling between phugoid and short period longitudinal modes is more pronounced than at lower angles of attack. Effectiveness of the control surfaces is reduced making the aircraft less responsive to pilot inputs. The rudder, which is critical in controlling sideslip and providing yaw damping becomes almost totally ineffective at angles of attack near stall. Modern combat aircraft are usually designed to be statically unstable in the longitudinal mode in order to minimize trim drag. Longitudinal short period instability combined with lightly damped dutch-roll makes it virtually impossible for a pilot to maintain control of the aircraft without a closed loop flight control system to provide stability augmentation. The importance of turn coordination is also increased at high angles of attack since excessive asymmetry in the airflow over the wing can lead to spin. These effects make it difficult to maintain control of the unaugmented aircraft during supermaneuvers, and thus it is necessary to have a flight control system which can maintain predictable dynamic response characteristics throughout the extended flight envelope.

The objective of this study was to examine methods for design of control laws for aircraft performing supermaneuvers. Since the equations which govern the motion of aircraft during supermaneuvers are nonlinear, this study concentrated on nonlinear control law design procedures. The two nonlinear techniques which were considered were Nonlinear Quadratic Regulator (NLQR) theory and nonlinear dynamic inversion. A conventional gain scheduled proportional plus integral ( P + I ) controller was also developed to serve as a baseline design typical of current control laws used in aircraft.

At the time the research was initiated, no data base for an aircraft operating at high angles of attack was available to the investigators, so a mathematical model of a generic supermaneuverable aircraft similar to the X-31A was developed from data obtained from the literature. This aircraft had aileron, rudder, and canard aerodynamic control surfaces and longitudinal and lateral thrust vectoring control (TVC). The longitudinal dynamics of the aircraft were statically unstable. The mathematical model contained nonlinearities due to (1) the aerodynamics, (2) kinematics, and (3) nonlinear inertial coupling.

A detailed computer simulation of the aircraft model was developed. This simulation allowed us to fly proposed supermaneuvers and was used to (1) evaluate the performance of the control law designs and (2) generate linearized models of the aircraft at different flight conditions. The control laws were tested with numerous simulations.

Since the open loop aircraft was statically unstable, it was necessary to design a baseline controller in order to be able to fly various maneuvers using the simulation. This was the P + I

control law described above. The P + I control law was developed based on linearized models of the aircraft at various flight conditions ( velocity and angle of attack ). Conventional frequency response methods were used. The details of this design are given in Ref. 1. NLQR theory was applied to the design of longitudinal control law for the aircraft and compared with the P + I controller [2-3]. Similar performance was obtained with both control laws. Lateral control laws were designed with the NLQR technique but the results were no better than those obtainable with Linear Quadratic Regulator theory (LQR). Since the NLQR appeared to yield results which were no better than those obtainable with linear theories, this methodology was not pursued in any further. Nonlinear dynamic inversion was applied to design of both lateral and longitudinal control laws and yielded excellent results [4 - 10]. Most of the research effort concentrated on this methodology.

## MATHEMATICAL MODEL

The mathematical model used for this study was based on data collected from the literature. It is representative of a modern fighter type aircraft but does not represent any particular aircraft although the general configuration is that of the X-31A. The aircraft is modeled by 6 degree of freedom, nonlinear, rigid body dynamics. The model has 12 states. Six states give the position and velocity of the center of mass in space and six states specify the angular orientation and angular velocity. The wind axis angles of heading, flight path angle, and bank angle about the velocity vector were used instead of the body-axis Euler angles. The standard roll, pitch and yaw rates give the angular velocity, and the translational velocity of the center of mass is given by its magnitude, the angle of attack and the side slip angle. A flat non-rotating earth and uniform gravitational field are used. The standard Euler equations of motion are used to model the rigid body motions of the aircraft. The forces and moments acting on the aircraft are due to gravity (force only), aerodynamics, and the propulsive system. The aircraft aerodynamics were obtained from graphs and tables taken from various sources in the literature and are representative of an aircraft similar to the X-31A. Although in general the aerodynamic coefficients are nonlinear functions of the all of the system states as well as the control deflections, the literature from which the data used in this study were taken modeled the aerodynamic and control coefficients as functions of the angle of attack only. This simplifies the modeling considerably as it allows modeling of the aerodynamic coefficients as polynomial functions of the angle of attack only. The aerodynamic forces are affine in all other variables except velocity which appears quadratically in the dynamic pressure. The aircraft has an unstable longitudinal static margin of 4.65%.

Aerodynamic control moments are provided by ailerons, a single rudder, and canards. Moment analyses of high angle of attack maneuvers reported in the literature indicate that thrust vectoring control (TVC) is necessary for some high angle of attack maneuvers since aerodynamic control surfaces become ineffective at high angles of attack. Lateral TVC is provided to augment the rudder, since the rudder is ineffective for angles of attack greater than 40 degrees. Longitudinal TVC is provided to enhance the canards at high angles of attack. No facility is provided to allow TVC to generate rolling moments.

## CONTROL LAW DESIGNS

The control laws designed in this study consisted of a maneuver generator which was used to simulate pilot commands, outer loop control laws which were designed to control aircraft attitude angles, and inner loop control laws which were designed to control attitude rates.

## Maneuver Generator

The supermaneuvers which were simulated in this study were obtained from optimization studies reported in the literature. These studies were based on a three degree of freedom, point mass model of the aircraft. The output of this model is an optimal velocity vector which results in a minimum time maneuver. The outputs of the optimization studies are (1) the magnitude of the velocity vector, (2) the flight path angle and (3) the heading angle. Our six degree of freedom simulation required as input simulated pilot commands which consisted of thrust level, angle of attack, and bank angle about the velocity vector. A method for converting the optimal velocity vector to simulated pilot commands was required before control laws could be tested in simulated supermaneuvers. These pilot commands were generated by a maneuver generator which solved the inverse problem of computing the thrust, angle of attack and bank angle required to produce the rate of change of velocity vector necessary to achieve the optimal trajectory. The maneuver generator uses three control loops which compare the commanded velocity magnitude, heading angle, and flight path angle from the optimization studies with the actual values of these variables obtained from the simulation. These differences are considered to be proportional to the desired accelerations and are multiplied by feedback gains. The outputs of the gains are the desired accelerations which are fed into the inversion controller which uses the point mass equations of motion of the aircraft to calculate commanded thrust level, angle of attack and bank angle about the velocity vector. The maneuver generator does not form any part of the onboard, flight control system of the aircraft but is used simply to allow supermaneuvers to be "flown" on the computer. The onboard flight control systems are used to produce accurate tracking in the angle of attack, bank angle rate about the velocity vector, and side slip (in this study commanded side slip is zero).

## Gain-Scheduled P + I Controller

A baseline, gain scheduled P + I controller similar to those used on existing aircraft is implemented for comparison with the other control laws developed. This P + I controller is designed using linear, frequency response techniques. P + I elements are used to provide good tracking and desensitization to modeling errors. Scheduling gains with angle of attack and dynamic pressure is important because the wide range of operating conditions result in significant changes in the aerodynamic properties during supermaneuvers. Desensitization to modeling errors is beneficial because the aircraft cannot be modeled precisely. The design is somewhat unconventional in that gains are scheduled with angle of attack, a rapidly changing variable, as well as with the more conventional dynamic pressure, a slowly changing variable.

To track a desired trajectory accurately, the aircraft must respond precisely to the pilot commands for angle of attack, side slip angle and bank angle. Two outer loops were implemented to control angle of attack and bank angle. The loops had relatively low bandwidths of 1 rad/sec and 2 rad/sec. respectively. Because there was no tracking requirement on side slip (it was to be maintained at zero), it was regulated by one of the inner loops. In addition to the outer loops, there are three inner loops each having a cross over frequency of 10 rad/sec. These are used to augment the stability of the fast dynamics associated with the longitudinal short period, dutch roll and roll subsistence modes. The inner loops interface directly with the nonlinear aircraft dynamics and are used to control three regulated variables which characterize the aircraft motion. One of the inner loops controls the unstable longitudinal dynamics and the other two control the lateral-directional modes. The crossover frequency for the inner loops was selected as 10 rad/sec so that the loop gain was as high as possible in order to achieve good tracking and desensitization but with low enough gain at higher frequencies to allow the effects of unmodeled structural and actuator dynamics to be neglected.

To allow frequency response methods to be applied, a set of linearized models of the aircraft

were derived at various flight conditions. A linear P + I control law was designed for each flight condition. The controller gains for values of angle of attack between the design conditions were obtained by linear interpolation. This produced control law gain schedules which were piece-wise linear functions of angle of attack. The gain schedules also had an inverse dependence on dynamic pressure to account for the reduction of loop gain resulting from decreased control surface effectiveness at low speeds.

The flight control system used the canard to control the longitudinal motions. Thrust was controlled directly by the maneuver generator and was not used as part of the feedback system. The engine dynamics are slow and not suitable for use within control loops having bandwidths as high as 10 rad/sec. If necessary an outer loop could be added for thrust control. Since longitudinal TVC was not used, a single regulated variable,  $r_v$ , was defined. The control of  $r_v$  provided the required stability and response characteristics. The longitudinal  $r_v$  consisted of a blend of low-passed normal acceleration,  $n_z$ , and speed,  $V$ , combined with pitch rate,  $q$ .

$$r_v = K_q q + [3/(s + 3)][n_z + K_V(V_C - V)] \quad (1)$$

The regulated variable was designed to allow accurate control of  $n_z$  at steady state and provide feedback for gust rejection. It contains a term in  $q$  with a constant gain to provide for pitch rate damping. A small angle of attack dependent gain,  $K_V$ , multiplying the velocity error is used to stabilize the phugoid mode.

Normal acceleration is used in the regulated variable because it is usually one of the primary outputs that the pilot wishes to control. It is directly proportional to the structural loads on the aircraft and can be used to limit these loads to safe levels. Furthermore, normal acceleration gives an immediate measure of aerodynamic loads due to wind gusts, which makes it ideal to use in the gust rejection loop. Finally normal acceleration can be used to estimate angle of attack and can be measured by accelerometers which have a proven record for accuracy and reliability. This may not be true for direct angle of attack sensors. Pure  $n_z$  feedback is not desirable because (1) high frequency elastic modes may be excited, (2) a non-minimum phase transfer functions may result, and (3) accelerometers are inherently noisy at high frequencies. These problems can be alleviated by low passing the output of the accelerometer and combining it with a pitch rate signal as shown in Eq. 1. The regulated variable also includes a low passed term in speed error, which is required to stabilize the phugoid mode. The gain on the speed error,  $K_V$ , is critical and must be scheduled with angle of attack in order to stabilize the phugoid mode. The P + I compensator is implemented with the following transfer function

$$k(s) = [\omega_C/h_{rV}][s + 3]/s \quad (2)$$

The zero in the P + I controller cancels the pole in the low pass filter in Eq. 1,  $\omega_C$  is the desired cross over frequency, and  $h_{rV}$  is proportional to dynamic pressure. Selection of the various gains and parameters in the control law defined by Eqs. 1 and 2 must be verified for each design flight condition. Thus the design process can be time consuming.

Lateral-directional controllers are designed in much the same manner as the longitudinal control laws; however, there are three controls, aileron, rudder, and lateral thrust vectoring, and two outputs, roll rate and lateral acceleration. Thus the lateral-directional controller design problem is multiple input multiple output (MIMO), and is more difficult than the longitudinal controller design. A MIMO, P + I control law is developed which requires considerable gain scheduling.

The control laws described above are for the inner loops. Outer loop control laws with lower

bandwidths are then formulated for a tracking of angle of attack, bank angle, and side slip commands ( a zero side slip command is assumed ). Outer loop designs are performed using classical frequency response methods.

### NLQR Controller

NLQR theory is an extension of LQR theory to nonlinear systems. Nonlinear systems equations are used and control laws are obtained from the approximate solution of the Hamilton-Jacobi-Bellman partial differential equation of optimal control theory. The resulting control laws are nonlinear functions of the system state. As the nonlinearities approach zero, the NLQR control laws approach the standard LQR control laws and have the same desirable robustness properties. The NLQR control laws are nonlinear functions of the angle of attack, which is equivalent to scheduling gains with angle of attack.

The NLQR technique was applied to the design of an inner loop longitudinal control law for the mathematical model of the aircraft described above. The control law was designed to track normal acceleration commands by minimizing a quadratic performance index which consisted of the integral of the square of the normal acceleration and the control deflection. The control laws obtained from application of the NLQR methodology were very similar in structure to the gain scheduled P + I control laws and most of the gains resulting from the two methods were close to one another. The NLQR procedure was also applied to the design of lateral-directional control laws.

### Nonlinear Dynamic Inversion Control Laws

Exact dynamic inversion is based on an intuitively simple idea which can easily be demonstrated by a scalar example. Consider the first order system

$$\dot{x} = f(x) + g(x)u \quad (3)$$

This system can be given any desired dynamics by suitable choice of the control u. For example the stable first order dynamics given by Eq. 4 might be chosen.

$$\dot{x} = \dot{x}_d = \omega_c (x_c - x) \quad (4)$$

Here  $\omega_c$  is the desired bandwidth. The required control can then be computed by inverting Eq. 3 to give

$$u = g(x)^{-1}(\dot{x}_d - f(x)) \quad (5)$$

Substitution of Eq. 5 into Eq. 3 clearly yields the desired dynamics of Eq. 4. The method of dynamic inversion can be extended to higher order systems provided  $g(x)$  is invertible. In aircraft control problems,  $g(x)$  may be invertible if there are sufficient control effectors; however, there will often be conditions where  $g(x)$  is nearly singular. This would result in excessively large commands and saturation of control effectors. The near singularity of  $g(x)$  is due to the fact that the control moment effectors produce very small forces and thus provide very little direct control of attitude angles. Thus it is difficult to use dynamic inversion directly for aircraft with more or less standard control effectors. If direct lift and side force effectors are available, dynamic inversion may be directly applicable; however, such effectors are not included in the mathematical model used in this study.

The problems associated with the invertibility of  $g(x)$  was overcome by separating the dynamics into fast and slow subsystems each having only three states. The fast subsystem

corresponds to the body axis angular rates and the slow subsystem corresponds to the angle of attack, side slip angle, and bank angle about the velocity vector. An exact inner loop inversion was carried out by using the five control effectors, canards, ailerons, rudder, lateral TVC, and longitudinal TVC. Since there were more controls than states it was possible to select the controls in such a way as to minimize the norm of the control vector. The desired inner loop dynamics were  $P + I$  with a cross over frequency of 10 rad/sec.

Once the inner loop control law was formulated, an outer loop controller was designed using an approximate inversion. It was assumed that the dynamics of the fast states, the body axis angular rates, were so much faster than the slow states that the fast states reached their steady state values essentially instantaneously and could be used as control inputs for the slow states. The approximation used in the outer loop inversion neglected the effects of forces produced by control surface deflections. The desired dynamics for the outer loops were also  $P + I$  but with cross over frequencies of 2 rad/sec.

In fact the full order rigid body dynamics are decomposed into four subsystems. These are denoted as fast, slow, very slow and extremely slow. The the extremely slow states are the components of the position vector of the center of mass of the aircraft relative to the earth. These states are controlled by the translational velocity vector. The very slow states are the magnitude and direction of the translational velocity vector. These states are controlled by the pilot or maneuver generator which produces cockpit commands for angle of attack, thrust level, side slip angle and bank angle rate. The purpose of the onboard flight control system is to cause the aircraft to respond to cockpit commands in a desirable manner. This is accomplished by control of the slow states, the attitude angles, and the fast states, the body angular rates, using dynamic inversion as described above.

## RESULTS FOR SIMULATED SUPERMANEUVERS

A number of maneuvers were flown using a digital simulation which included all of the dynamic, kinematic, and aerodynamic nonlinearities. These maneuvers consisted of minimum time reversals in direction with various final conditions imposed. During these maneuvers all of the state variables underwent very large changes. For example transient angles of attack of over 80 degrees were observed. The  $P + I$  gain scheduled controller performed well in maneuvers in which the angular rates were small, however, the performance deteriorated when the aircraft was subjected to high angular rates at high angles of attack. Specifically, sideslip and lateral acceleration were significantly higher than expected. Gyroscopic coupling was found to account for part of this degradation. Significant coupling between roll rate and lateral acceleration was observed at large values of angle of attack.

In a vertical turn in which only longitudinal dynamics were simulated, the NLQR control law was shown to perform about the same as the  $P + I$  gain scheduled controller. Since there was no clear performance gain in using the NLQR control law over more conventional designs, this methodology was not considered further in the study.

It was observed that in all of the maneuvers, the response of the gain scheduled controller was more oscillatory in the angular rates than was the dynamic inversion system. This is due to the fact that the inner loops of the dynamic inversion system control the angular rates directly so that the responses of these states accurately track the inputs. Accurate control of side slip is very important in post-stall maneuvering. The dynamic inversion control is more successful in providing precise control of both bank angle rate and side slip angle than the gain scheduled control law, particularly at high angles of attack. This suggests that, in the gain scheduled control law, the directional control afforded by the inner loop controlling the lateral acceleration needs to be improved.