# Modeling 3-D Objects with Planar Surfaces for Prediction of Electromagnetic Scattering 

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#### Abstract

Electromagnetic scattering analysis of objects at resonance is difficult because low frequency techniques are slow and computer intensive and high frequency techniques may not be reliable. In this paper, a new technique for predicting the electromagnetic backscatter from electrically conducting objects at resonance is investigated. This technique is based on modeling three-dimensional objects as a combination of flat plates where some of the plates are blocking the scattering from others. A cube is analyzed as a simple example. The preliminary results compare well with the Geometrical Theory of Diffraction and with measured data.


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## INTRODUCTION

Many techniques exist for the solution of electromagnetic scattering problems. Solution techniques are determined by the electrical region into which the scattering object falls. These regions are determined according to the size of the scattering object in relation to the wavelength of the incident radiation. Three electrical regions exist: the low frequency region, the resonant region, and the high frequency region. In the low frequency region the scattering object is smaller than the incident wavelength. In the resonant region the incident wavelength is on the same order of size as the scattering obstacle. Objects in the high frequency region are much larger than the incident wavelength.

High and low frequency techniques make trade-offs between accuracy and speed of calculation. Low frequency techniques, such as the Method of Moments (MoM) (ref.1,2) and the Finite-Difference Time-Domain (FD-TD) (ref. 3), are sometimes referred to as "exact" solutions since they solve Maxwell's Equations numerically. The only approximation involved is in the numerical implementation of the integral or differential equations. Low frequency techniques are very accurate but are computer intensive in memory and CPU-time. In contrast, high frequency techniques, such as Geometrical Optics (GO), Physical Optics (PO) and the Geometrical Theory of Diffraction (GTD), make simplifying assumptions or approximations in the formulation of the model (ref.2). These simplifying approximations are usually based on the largeness or smoothness of the scattering object, hence high frequency techniques are unreliable if applied to an object in the low frequency region. High frequency techniques are generally faster than low frequency techniques.

Objects in the resonance region are difficult to analyze because low frequency techniques are slow and computer intensive and high frequency techniques may not be reliable. The objective of our research was to investigate a different technique for predicting the electromagnetic backscatter from a perfectly electrically conducting object that would be more suitable for scatterers in the resonance region. The new technique we have investigated is based on an
intuitive approach we call "blocking." We have investigated scattering from an electrically conducting cube as a simple example.

Consider a simple electrically conducting cube in an electromagnetic field with sides of length "a" as shown in figure 1. The incident field is normal to one face and we are observing the backscattered field normal to that face. We know that all faces of the cube are connected and that all faces will affect the scattered field. The sides of the cube will not contribute to the scattered field directly as the front and back faces will, but the sides do affect the scattered field since they couple the front and back faces. If we were to solve this scattering problem via some low frequency method such as the Method of Moments, we would have to include all six sides of the cube in order to obtain an accurate solution. But if we could account for the coupling between the front and back faces without having to perform calculations over the side, top, and bottom faces we could significantly reduce the amount of computer calculation necessary to obtain a solution.


Figure 1 Blocking Model
In other methods, such as the Method of Moments, the cube is analyzed as a single scattering unit. The blocking technique analyzes the cube as a combination of flat plate units, where some of the flat plates are blocking the scattering from other plates in their shadow. If the incident field is normal to the front face then the sides do not contribute directly to the backscatter,
so they are not included in the blocking calculation for scattered fields, but are accounted for in the calculation over only the front and back faces.

## THE SOLUTION

The solution procedure is illustrated in figure 2 which shows the front and back plates in an edge profile. The blocking model is solved as the superposition of two problems--scattering from the back plate and scattering from the front plate. First, we calculate the fields scattered from the back plate over the plane of the front plate as if the front and sides were not there. Over the area occupied by the front plate, the fields are blocked and set equal to zero, and the remaining fields are transformed to the far field. Next, the far fields scattered from the front plate alone are calculated. Finally, the two solutions are superimposed to obtain the total scattered field.

Scattering from the back


Scattering from the front


Superposition


Figure 2 Solution procedure
First, we determine the fields scattered from the back plate over the plane of the front plate using the physical optics method with a modifying coefficient. We include this coefficient to
account for the fact that the back plate is not in direct illumination, but rather indirect illumination from the diffracted fields off the front edges. To proceed, a plane wave with wavenumber $k$ and $\mathrm{e}^{\mathrm{j} \omega \mathrm{t}}$ time dependency, lying in the $\mathrm{y}-\mathrm{z}$ plane with the magnetic field vector perpendicular to the x axis $\left(\mathrm{TM}^{\mathrm{x}}\right)$ is assumed incident at an angle theta on the back plate as though the sides and front plate were not there. The electric and magnetic vector fields are represented as

$$
\vec{E}_{i}=E_{0} \hat{a}_{x} e^{-j k\left(y \sin \theta_{i}-z \cos \theta_{i}\right)}
$$

and

$$
\vec{H}_{i}=-\frac{E_{0}}{\eta}\left(\hat{a}_{y} \cos \theta_{i}+\hat{a}_{z} \sin \theta_{i}\right) e^{-j k\left(y \sin \theta_{i}-z \cos \theta_{i}\right)}
$$

where $\eta$ is the intrinsic impedance of free space, and are depicted in figure 3.


Figure 3 Calculating current on back plate

The physical optics approximation for the current induced on the back plate due to the incident wave is

$$
\overrightarrow{\mathrm{J}}_{\mathrm{s}}=2 \hat{\mathrm{n}} \times\left.\overrightarrow{\mathrm{H}}\right|_{\mathrm{z}}=0, \mathrm{y}=\mathrm{y}^{\prime}
$$

where the unprimed coordinates represent the observation point and the primed coordinates represent points on the back plate. We have modified this approximation for the current on the back plate, $\stackrel{\rightharpoonup}{J}_{1}$, to be

$$
\overrightarrow{\mathrm{J}}_{\mathrm{l}}=\left.\frac{-1}{\mathrm{ka}}(2 \hat{\mathrm{n}} \times \overrightarrow{\mathrm{H}})\right|_{\mathrm{z}=0, \mathrm{y}=\mathrm{y}^{\prime}}
$$

where " a " is the length of the edge of the plate parallel to the x -axis. This evaluates to

$$
\stackrel{J}{J}_{1}=\frac{-1}{\mathrm{ka}} 2 \frac{\mathrm{E}_{0}}{\eta} \hat{\mathrm{a}}_{\mathrm{x}} \cos \theta_{\mathrm{i}} \mathrm{e}^{-\mathrm{jky} y^{\prime} \sin \theta_{\mathrm{i}}}
$$

and substituting $\theta=0$ for our example of normal incidence gives

$$
\overrightarrow{\mathrm{J}}_{1}=\frac{-1}{\mathrm{ka}} 2 \frac{\mathrm{E}_{0}}{\eta} \hat{\mathrm{a}}_{\mathbf{x}}
$$

The coefficient, $\frac{-1}{\mathrm{ka}}$ is the edge diffraction coefficient borrowed from Geometrical Theory of Diffraction, which is basically a ray tracing technique with additional terms provided to account for edge diffraction. The $\frac{-1}{\mathrm{ka}}$ edge condition originates on the front plate and gets transferred to the back plate through the conducting sides. This coefficient numerically accounts for the effect of the sides on the scattered field and reflects the fact that the back face is not in direct illumination as the front face is (See Appendix).

Once the current on the back plate is determined, the scattered fields, $\overrightarrow{\mathrm{E}}_{\mathrm{S} 1}$, are determined by radiation integrals over the plane of the front plate located at $\mathrm{z}=\mathrm{a}$. The radiation integrals must be valid in the near field since the front plate is in the near field of the back plate. Only the field components tangential to the $\mathrm{z}=$ a plane are required in order to transform these fields to the far field. Also, since $\mathrm{J}_{\mathrm{y}}=0, \mathrm{E}_{\mathrm{y}}$ is negligible compared to $\mathrm{E}_{\mathrm{x}}$. The appropriate integral for $\mathrm{E}_{\mathrm{x}}$ over the plane $\mathrm{z}=\mathrm{a}$ becomes

$$
\left.\vec{E}_{s 1}=\vec{E}_{x}(x, y, a)=-j \frac{\eta}{4 \pi k} \iint_{S}\left[G_{1} J_{x}+\left(x-x^{\prime}\right)^{2} G_{2} J_{x}\right]\right]^{-j k R_{d x}} d y^{\prime} \hat{a}_{x}
$$

where

$$
\begin{gathered}
R=\sqrt{\left(x-x^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}+z_{0}^{2}} \\
G_{1}=\frac{-1-j k R+k^{2} R^{2}}{R^{3}} \\
G_{2}=\frac{3+j 3 k R-k^{2} R^{2}}{R^{5}}
\end{gathered}
$$

and

$$
J_{x}=\frac{-1}{k a} 2 \frac{E_{0}}{\eta}
$$

In step 2 of figure 2, these fields are made zero over the area occupied by the front plate, as if they are being blocked. In step 3, the remaining fields are transformed to the far field by the method of stationary phase (ref. 4).

In step 4 of figure 2 , calculating the fields scattered from the front plate is a straightforward application of the physical optics approximation. The physical optics approximation for the current induced on the front plate is determined to be

$$
\overrightarrow{\mathrm{J}}_{2}=2 \hat{\mathrm{f}} \times\left.\overrightarrow{\mathrm{H}}\right|_{\mathrm{z}}=\mathrm{a}, \mathrm{y}=\mathrm{y}^{\prime}
$$

where the unprimed coordinates represent the observation point and the primed coordinates represent points on the front plate. The scattered fields are then found from radiation integrals which can be valid in the far field only. In the final step, the far fields from the back plate are superimposed on the far fields from the front plate to obtain the total scattered field.

## RESULTS

The fields scattered from an object are often given as the radar cross section (RCS) which is defined as

$$
\mathrm{RCS}=\lim _{\mathrm{r} \rightarrow \infty}\left[4 \pi \mathrm{r}^{2} \frac{\left|\mathrm{E}_{\mathrm{S}}\right|^{2}}{\left|\mathrm{E}_{\mathrm{i}}\right|^{2}}\right]
$$

Figure 4 shows a plot of the radar cross section of a 2 -inch cube both as a function of frequency and as a function of size in wavelength. Over the frequency range from 6 to 18 GHZ , the cube
varies in size from 1 to 3 wavelengths which is in the resonant region. The graph compares the blocking solution with the Geometrical Theory of Diffraction solution which is a high frequency technique, and with measured results. The measured data was taken in NASA Langley's Experimental Test Range, a compact range facility. The RCS data was taken on a 2 -inch cube test model made from high density foam and covered with silver conducting paint. Ideally, we would have liked to compare the blocking solution to a low frequency solution as well, such as the Method of Moments. We were unable to solve the 2 -inch cube problem at the higher frequencies ( $16-18 \mathrm{GHz}$ ) using the Method of Moments code on the Cray- 2 S supercomputer due to the enormous memory and processor time requirements. The blocking data agrees well with the GTD solution, and the measured data falls around the two solutions.

## CONCLUDING REMARKS

The preliminary results compare well to the Geometrical Theory of Diffraction. The fields scattered from each plate were obtained using the physical optics approximation which does not account for any edge effects, but when modified by the blocking coefficient, the blocking technique predicts the edge effects on the scattering very well. In order to include other angles of incidence and scattering besides normal to one face, other sides of the cube would have to be brought into the blocking technique. For example, if the field were incident at $\theta=45$ degrees and $\phi=0$, then the top and bottom plates would have to be included in addition to front and back plates. Over each face in direct illumination, blocking would have to be performed for each of the other two faces in the shadow. While the blocking solution is faster than low frequency techniques at normal incidence, at angles off normal, the calculations required for the blocking technique mount so quickly that it is uncertain whether or not blocking would be significantly faster than a low frequency technique.


## Appendix

## Source of Blocking Coefficient

In chapter 7 of Ruck (ref. 5), the first term of the backscatter cross section for a rectangular plate is given as

$$
\operatorname{RCS}=\frac{\mathrm{a}^{2}}{\pi}\left|\cos (\mathrm{ka} \sin \theta) \mp \mathrm{j} \frac{\sin (\mathrm{k} \sin \theta)}{\sin \theta}\right|^{2}
$$

which can be rewritten as

$$
\mathrm{RCS}=\frac{\mathrm{a}^{2}}{\pi}(\mathrm{ka})^{2}\left|\frac{\sin (\mathrm{k} a \sin \theta)}{\mathrm{ka} \sin \theta} \pm \mathrm{j} \frac{1}{\mathrm{ka}} \cos (\mathrm{ka} \sin \theta)\right|^{2}
$$

The first term of this equation is recognized as the physical optics term, and the second term is an edge diffraction term. When evaluated at $\theta=0$, the equation reduces to

$$
\operatorname{RCS}=\frac{\mathrm{a}^{2}}{\pi}(\mathrm{ka})^{2}\left|1 \pm \mathrm{j} \frac{1}{\mathrm{ka}}\right|^{2}
$$

where the edge diffraction coefficient is now $\frac{1}{\mathrm{ka}}$, the negative of which we assumed as our blocking coefficient.

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