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## TEXAS A&M UNIVERSITY

### DETERMINATION OF AERODYNAMIC SENSITIVITY COEFFICIENTS FOR WINGS IN TRANSONIC FLOW

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# DETERMINATION OF AERODYNAMIC SENSITIVITY COEFFICIENTS FOR WINGS IN TRANSONIC FLOW

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## ABSTRACT

The quasianalytical approach is applied to the three-dimensional full potential equation to compute wing aerodynamic sensitivity coefficients in the transonic regime. Symbolic manipulation is used to reduce the effort associated with obtaining the sensitivity equations, and the large sensitivity system is solved using "state of the art" routines. The quasianalytical approach is believed to be reasonably accurate and computationally efficient for three-dimensional problems.

## INTRODUCTION

To design transonic vehicles using codes which utilize optimization techniques requires aerodynamic sensitivity coefficients, which are defined as the derivatives of the aerodynamic functions with respect to the design variables. In most cases, the main contributor to the optimization effort is the calculation of these derivatives; and, thus, it is desirable to have numerical methods which easily, efficiently, and accurately determine these coefficients for large complex problems. The primary purpose of the present study is to investigate the application of the quasianalytical method [1,2] to three-dimensional transonic flows using as the fundamental flow solver the three-dimensional transonic full potential fully conservative code, ZEBRA [3].

## PROBLEM STATEMENT

Application of the quasianalytical method to the full potential equation yields the sensitivity equation

$$\left[ \frac{\partial R_{i,j,k}}{\partial \phi_{ii,jj,kk}} \right] \left( \frac{\partial \phi_{ii,jj,kk}}{\partial XD} \right) = - \left( \frac{\partial R_{i,j,k}}{\partial XD} \right) \quad (1)$$

where  $XD$  is the vector of design variables and the residual expression,  $R_{i,j,k}$ , of the full potential equation in the computational plane,  $X, Y, Z$ , in terms of backward differences is

$$R_{i,j,k} = \bar{\delta}_X \left( \frac{\bar{\rho}U}{J} \right)_{i+1/2,j,k} + \bar{\delta}_Y \left( \frac{\bar{\rho}V}{J} \right)_{i,j+1/2,k} + \bar{\delta}_Z \left( \frac{\bar{\rho}W}{J} \right)_{i,j,k+1/2} \quad (2)$$

Here, the retarded density  $\bar{\rho}$  and the contravariant velocity components  $U, V$ , and  $W$ , are lengthy functions of the reduced potential function,  $\phi$ . The boundary conditions for Eq.(2) are the surface condition,  $W = U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y}$ , the Kutta condition along the wing semispan,  $\Gamma = \Delta \phi$ ,  $x_{TE} < x \leq \infty$ , and the farfield condition. Additional conditions are the downstream boundary potential  $\phi_x = 0$  and the wing symmetry condition,  $V = 0$ .

The discretized form of Eq.(2) contains lengthy expressions, and mathematical symbolic manipulation [4-6] was used to determine the functional dependencies of the residual, the analytical forms of the derivatives, and to generate the corresponding computer code. The basic approach used to differentiate the residual expression was to treat the main expression in terms of smaller subexpressions, each of which was examined in terms of its constituents. This process was extended until simple functional forms for the derivatives were obtained. This subdivision and chain rule differentiation by symbolic manipulation efficiently generated source code for the jacobian and vectors in Eq.(1). The resultant large sparse system, typically  $17500 * 17500$ , of algebraic equations is then efficiently solved for  $\frac{\partial \phi}{\partial XD}$  using either the iterative conjugate gradient method or the generalized minimum residual algorithm [7-8]. From these, the pressure and lift coefficient sensitivities to the design variables can be computed. Notice that the effort associated with this approach is essentially independent of the number of design variables considered on the right-hand-side of Eq.(1).

## EXAMPLE AND DISCUSSION

Consider the ONERA M6 wing planform with NACA 1406 airfoil sections at a supercritical condition of  $M_\infty = 0.84$  and  $\alpha = 3$  deg, which has subcritical lower surface flow and exhibits an upper surface shock wave located at 70 % chord at the root to 10 % chord at the tip that increases in strength from the root to a point near the wing tip. Basic design variables for the current problem include freestream design variables, Mach number  $M_\infty$  and angle of attack  $\alpha$ ; cross-section design variables of maximum thickness,  $T$ , maximum camber,  $C$ , and location of maximum camber,  $L$ ; variables that define wing twist,  $T1, T2, T3$ , and  $T4$ ; and planform tip coordinates,  $XLE_{tip}$ ,  $XTE_{tip}$ , and  $Y_{tip}$ . Knowing the sensitivities to these basic design parameters permits subsequent evaluation of the derivatives with respect to the nonbasic variables taper ratio, aspect ratio, wing area, and sweepback angles. Thus, the present method determines sensitivity coefficients for twelve design variables and five derived design variables.

As part of the solution  $\partial \phi / \partial XD$  values are obtained for every grid point in the flowfield. Also, the method automatically computes  $\partial C_p / \partial XD$  at twenty-five chordwise locations at each of the twenty semi-span stations on the wing as well as  $\partial C_l / \partial XD$  at each of the span stations. Typical results for the example case are shown in Fig.1 for a midspan station. As expected, the sensitivity derivative profiles for the lower surface are typical of subcritical flow [2]; and the upper surface results exhibit large variations in the vicinity of the shock wave. The latter reflect the influence on the aerodynamic coefficients of the sensitivity of the upper surface shock wave location to the various design parameters. Currently, efforts are in progress to validate the present method by comparison with the finite difference

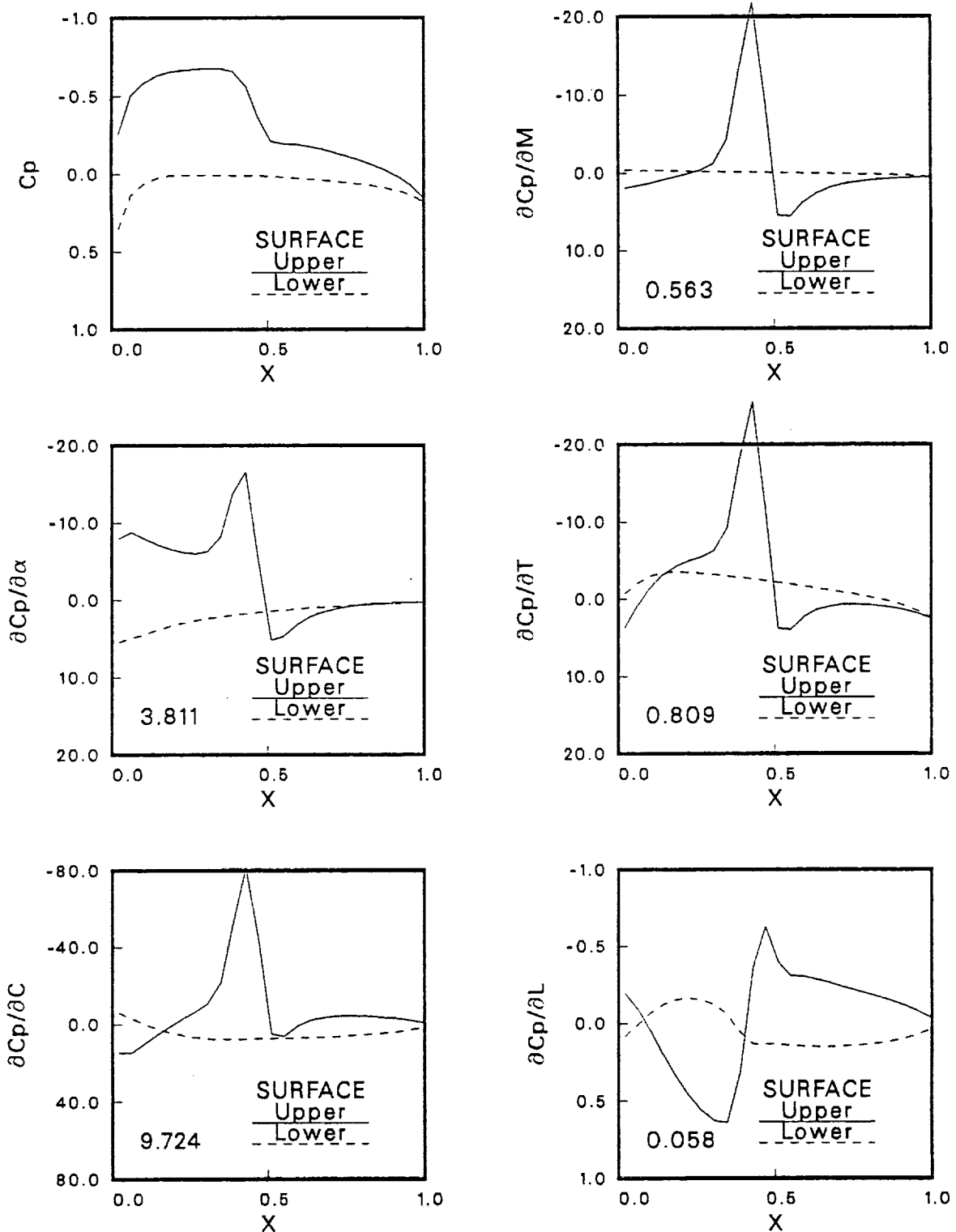


Fig.1 Pressure Coefficient and Pressure Coefficient Sensitivity Derivatives at 56 % Span

approach, which calculates sensitivities by perturbing a design variable from its previous value, obtaining a new solution, using the differences between the new and old solutions to obtain the sensitivity coefficients. While this direct technique is computer intensive and inefficient, it should serve as a check on the present method.

Based upon the present results, it is concluded that the quasianalytical method is a viable and efficient concept for the determination of three-dimensional transonic aerodynamic sensitivity coefficients. In addition, use of symbolic manipulation to evaluate the elements of the sensitivity equation is believed to be an efficient approach to the development of such methods. Finally, further studies are needed to determine the accuracy and range of applicability of the quasianalytical approach.

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