$1N-34$

115667

$P.66$

NASA Contractor Report 4462

al Gas Flow Parameters
NASA Langley 22-Inch
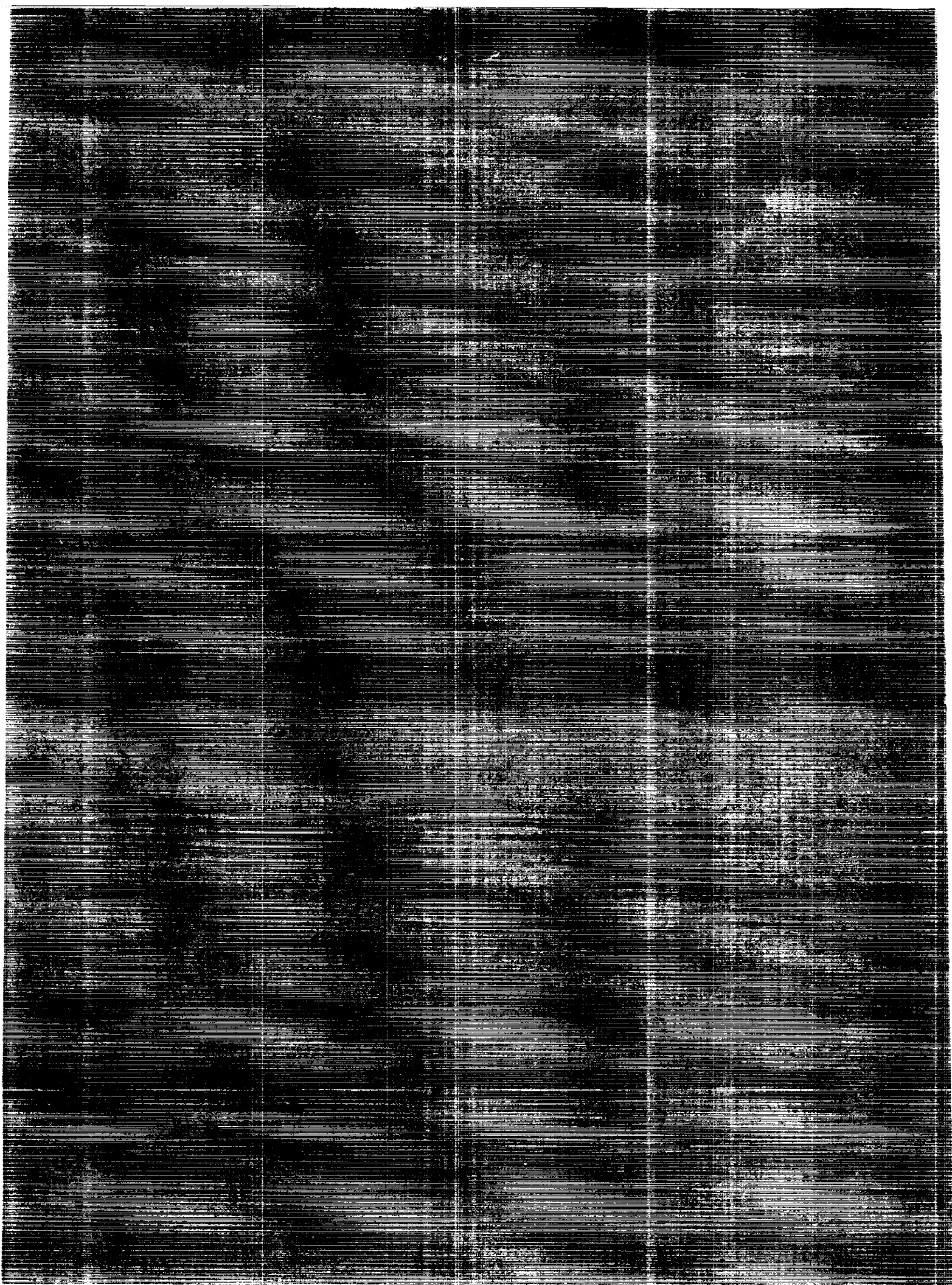20 Helium Tunnel

(NASA-CR-4462)   REAL GAS FLOW
PARAMETERS FOR NASA LANGLEY 22-INCH
MACH 20 HELIUM TUNNEL Report, Jun.
1991 - Jun. 1992 (North Carolina
State Univ.) 66 p

N92-30751

Unclas

H1/34   0115667

NASA Contractor Report 4462

Real Gas Flow Parameters for NASA Langley 22-Inch Mach 20 Helium Tunnel

Brian R. Hollis
North Carolina State University
Raleigh, North Carolina

NASA

National Aeronautics and
Space Administration

Office of Management

Scientific and Technical
Information Program

1992

TABLE OF CONTENTS

iii

ACKNOWLEDGEMENTS

SUMMARY

A computational procedure has been developed which can be used to determine the flow properties in hypersonic helium wind tunnels in which real gas behavior is significant. In this procedure, a three-coefficient virial equation of state and the assumption of isentropic nozzle flow are employed to determine the tunnel reservoir, nozzle, throat, freestream and post-normal shock conditions. This method has been applied to a range of conditions which encompasses the operational capabilities of the NASA Langley 22-Inch Mach 20 Helium Tunnel. Results are presented graphically in the form of real gas correction factors which can be applied to perfect gas calculations. Important thermodynamic properties of helium are also plotted versus pressure and temperature. The computational scheme used to determine the real-helium flow parameters has been incorporated into a FORTRAN code which is discussed in this paper.

INTRODUCTION

At high densities, intermolecular forces can have a significant effect on the pressure-density-temperature relationship and other thermodynamic properties of interest. For example, at typical reservoir conditions for the Langley 22-Inch Mach 20 Helium Tunnel of 137 atm. and 295° K, perfect gas theory yields a density of 22.65 kg/m^3 and a specific heat ratio of 1.667. However, the actual values of these properties at the specified conditions would be 21.26 kg/m^3 and 1.652, which amount to errors of +6.6% and +0.9% in the values obtained from perfect gas theory. For a measured test-section pitot pressure of 0.326 atm, the perfect gas Rayleigh pitot formula can be used to calculate a Mach number of 21.2, however the actual Mach number would be 21.6; thus, the perfect gas value is off by -1.8%. For a perfect gas the test section stagnation temperature would still be 295° K; however, the real gas temperature under these conditions would be 302° K. Thus, the perfect gas computation results in an error of +2.3%. This tunnel can be operated at much more extreme conditions than these, which would cause a much greater deviation from perfect gas behavior.

A method has been devised which accounts for real gas effects through the use of the virial form of the equation of state and real gas thermodynamic relationships for expanding flows. In this method, the standard assumption of isentropic flow is made. It is also assumed that the flow is remains free of condensation during the expansion process, that it remains at temperatures below that at which electronic excitation becomes a significant factor (T < 10000° K)[1], and that the helium density remains below the critical point (ρ_{CR} = 69.64 kg/m^3)[2]. These conditions are all satisfied in the normal operating range of Langley's Mach 20 helium wind tunnel.

Analysis of real-helium flow parameters has been carried out by Erickson[3,4] at moderate enthalpies and at high enthalpies by Miller and Wilder[5]. In Erickson's work, the Beattie-Bridgeman equation of state was used to obtain results up to 410 atm. and 590° K. This work was later extended to obtain approximate results up to 3400 atm. and 5800° K. Miller and Wilder were concerned with

1

the prediction of hypervelocity flow parameters for Hotshot tunnels and used a virial equation of state with coefficients determined by theoretical analysis of high-temperature gas behavior to obtain results for up to 3600 atm. and 15,000°K. In the present study, a virial equation of state is also used, but with virial coefficients derived from both analysis of low and moderate temperature experimental data and high-temperature theoretical results. This method provides accurate results both in the operational range of the 22-Inch Mach 20 Helium Tunnel (20 - 285 atm, 270 - 600° K)[6] and in the high pressure, high temperature range of computations carried out in reference (5). However, in this paper the emphasis is on the range of 1 - 400 atm. and 50 - 600° K with Mach numbers greater than 10.

SYMBOLS

a	speed of sound (m/s)
B	second virial coefficient (m^3/kg)
C	third virial coefficient $(m^3/kg)^2$
c_p	specific heat at constant pressure (J/kg-°K)
c_v	specific heat at constant volume (J/kg-°K)
D	fourth virial coefficient $(m^3/kg)^3$
e	internal energy (J/kg)
h	enthalpy (J/kg)
M	Mach number
p	pressure (N/m^2)
R	gas constant for helium (2077.1 J/kg-°K)
Re	Reynolds number
s	entropy (J/kg)
U	velocity (m/s)
W_{He}	molecular weight of helium (4.0026 g/g-mole)
Z	compressibility factor
γ	specific heat ratio
μ	coefficient of viscosity (kg/m-s)
ρ	density (kg/m^3)

Subscripts:

0	total
1	freestream conditions
2	post-shock static conditions
P	perfect gas value
r	reference value
t,1	reservoir stagnation conditions
t,2	post-shock stagnation conditions
x	conditions at arbitrary point

Superscripts:

*	throat conditions

VIRIAL EQUATION OF STATE

Flow properties are determined through the use of the virial equation of state, real gas relationships for entropy, enthalpy, specific heats and speed of sound, and the shock conservation relations for mass, momentum and energy.

The equation used to represent the pressure-density-temperature relationship of real helium gas is the virial equation of state

$$p = Z\rho RT = \rho RT[1 + \rho B(T) + \rho^2 C(T) + \rho^3 D(T) + ...] \qquad (1)$$

A relatively large amount of experimental data[7-11] on the second virial coefficient, $B(T)$, exists for low and moderate temperatures (up to about 1000° K). Experimental results for the third virial coefficient[7,10,11], $C(T)$, are few and show a great deal of scatter. No experimental data on the fourth and higher virial coefficients are available. At high temperatures (1000° K+), theoretical results for the second, third and fourth virial coefficients were derived from models of the intermolecular force potential of helium presented by Amdur and Mason[12].

Expressions were derived for $B(T)$ and $C(T)$ which match both the theoretical results and the experimental data, while that for $D(T)$ was based is a theoretical relationship taken from reference (12). At the conditions dealt with in this work, the contribution of $C(T)$ to the equation of state is relatively small and that of $D(T)$ essentially negligible, thus the virial equation of state is truncated at the third virial coefficient. If calculations are to be carried out at high temperatures and densities, the fourth virial coefficient should be included.

3

The expressions derived for the virial coefficients are

$$B(T) = b_0 + b_1(T^{-1/4}) + b_2(T^{-3/4}) + b_3(T^{-5/4}) + b_4(T^{-7/4}) \qquad (2)$$

$$\underline{T \leq 1300°K}$$
$$b_0 = -13.4067$$
$$b_1 = 165.4459$$
$$b_2 = -1357.92$$
$$b_3 = 5959.061$$
$$b_4 = -12340.8$$

$$\underline{T > 1300°K}$$
$$b_0 = 1.178236$$
$$b_1 = -7.57134$$
$$b_2 = 5225.701$$
$$b_3 = -188923.$$
$$b_4 = 2460461.$$

$$C(T) = c_0 + c_1(T^{-1/4}) + c_2(T^{-3/4}) + c_3(T^{-5/4})$$

$$c_0 = -13.7898$$
$$c_1 = 139.7339 \qquad\qquad (3)$$
$$c_2 = 8114.259$$
$$c_3 = -17456.9$$

where the units of T, B(T) and C(T) are °K, cm^3/mole and $(cm^3/mole)^2$ respectively. Expressions for B(T), C(T) and D(T) given by previous authors are:
Amdur and Mason[12]

$$B(T) = 1.3436 \times 10^{-2}(15.8922 - \ln T)^3 \qquad\qquad (4)$$

$$C(T) = 9.0263 \times 10^{-5}(15.8922 - \ln T)^6 \qquad\qquad (5)$$

$$D(T) = 7.0341 \times 10^{-7}(15.8922 - \ln T)^9 \qquad\qquad (6)$$

4

Harrison[13]

$$B(T) = 1.3436 \times 10^{-2}(15.8922 - \ln T)^3 - 4.39\exp[-2.4177 \times 10^{-3}T]$$

(7)

Miller and Wilder[5]

$$B(T) = 1.3436 \times 10^{-2}(15.8922 - \ln T)^3 - 8.04\exp[-3.7156 \times 10^{-3}T]$$

(8)

The values of B(T) and C(T) computed from equations (2) and (3) are plotted in figures (1) and (2). Also plotted are the experimental data and theoretical results on which these expressions are based and the values of B(T) and C(T) obtained using equations (4), (5), (7) and (8). Note that for ease of comparision with existing data, the virial coefficients are plotted in terms of $(cm^3/mole)$, where: $(cm^3/mole) = W_{He}/1000$ (m^3/kg).

At high temperatures, the various expressions for B(T) approach the same values because they are all based on Amdur and Mason's theoretical computations. However, in that work only the effect of the intermolecular repulsive force, which dominates at high temperature, is taken into account; the effect of the intermolecular attractive force, which dominates at low temperature, is ignored. Thus, the values of B(T) from equation (5) become increasingly inaccurate at lower temperatures. Amdur and Mason's expression for B(T) was later corrected empirically to reflect the effect of the attractive force through the addition of logarithmic terms as shown in equations (7) and (8).

Between 500 and 1000° K, the expression used herein for B(T) and those from the other references can be used to obtain approximately the same values. However, as shown in figure (1), these other expressions cannot be used to model the behavior of B(T) below 200°. Between 1000 and 2000° K, the results obtained from these expressions also differ significantly from the experimental data. The effects of the second (and higher) virial coefficients are small in the first range of temperatures stated for the densities generally associated with them in hypersonic flows, and thus their accuracy is not of great concern. However, for temperatures between 1000 and 2000° K the influence of B(T) is not negligible, and the use of these expressions may introduce a considerable error. The expression presented in this work, equation (2), can be used to accurately calculate the value of B(T) across the temperature range of interest.

As seen in figure (2), third virial coefficient data shows a considerable amount of scatter, but the accuracy of the expression for C(T) is not as great of a concern since its effects on the virial equation of state are small over the temperature range of interest. The expression used in the present study, equation (3), is fitted to the high temperature data of Amdur and Mason and also agrees with the trend of the experimental data at low and moderate temperatures.

The accuracy of these coefficients was checked by comparing the values of the resulting compressibility coefficient, Z, with

those given in the tabulation of helium properties prepared by the International Union of Pure and Applied Chemistry[2] (IUPAC). In figures (3a) and (3b) the data are compared along nine isobars ranging from 20 MPa to 0.001 MPa (197 atm to 0.00987 atm). The compressibility coefficient is also plotted versus pressure and temperature in figure (4). The only region where the computed values deviate significantly from the IUPAC values is in the very low temperature-very high density range, and this is generally not a condition dealt with in hypersonic research.

THERMODYNAMIC PROPERTIES

Real gas thermodynamic relationships based on the virial form of the equation of state were derived in references (1) and (3). The derivations are repeated here for completeness.

Enthalpy

Expressing enthalpy as a function of density and temperature gives

$$dh = \left(\frac{\delta h}{\delta \rho}\right)_T d\rho + \left(\frac{\delta h}{\delta T}\right)_\rho dT \tag{9}$$

And by definition

$$h = e + \frac{p}{\rho} \tag{10}$$

Differentiating this equation with respect to T at constant ρ

$$\left(\frac{\delta h}{\delta T}\right)_\rho = \left(\frac{\delta e}{\delta T}\right)_\rho + \frac{1}{\rho}\left(\frac{\delta p}{\delta T}\right)_\rho \tag{11}$$

And by definition

$$c_v = \left(\frac{\delta e}{\delta T}\right)_\rho \tag{12}$$

By the first law of thermodynamics

$$\delta q + \delta w = de \tag{13}$$

Or for isentropic compression or expansion

6

$$Tds + \frac{1}{\rho^2}pd\rho = de \qquad (14)$$

Differentiating equation (10) and substituting into equation (14) yields

$$dh = Tds + \frac{1}{\rho}dp \qquad (15)$$

At constant temperature, equations (9) and (15) give

$$\left(\frac{\delta h}{\delta \rho}\right)_T = T\left(\frac{\delta s}{\delta \rho}\right)_T + \frac{1}{\rho}\left(\frac{\delta p}{\delta \rho}\right)_T \qquad (16)$$

And by the Maxwell relation

$$\left(\frac{\delta s}{\delta \rho}\right)_T = \frac{-1}{\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho \qquad (17)$$

Then

$$\left(\frac{\delta h}{\delta \rho}\right)_T = \frac{-T}{\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho + \frac{1}{\rho}\left(\frac{\delta p}{\delta \rho}\right)_T \qquad (18)$$

By substitution of equations (11) and (18), equation (9) becomes

$$dh = \left[c_v + \frac{1}{\rho}\left(\frac{\delta p}{\delta T}\right)_\rho\right]dT + \left[\frac{-T}{\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho + \frac{1}{\rho}\left(\frac{\delta p}{\delta \rho}\right)_T\right]d\rho \qquad (19)$$

Where, from equation (1), the virial equation of state

$$\left(\frac{\delta p}{\delta T}\right)_\rho = \rho R\left[1 + \rho\left(B(T) + T\frac{dB(T)}{dT}\right) + \rho^2\left(C(T) + T\frac{dC(T)}{dT}\right)\right] \qquad (20)$$

and

$$\left(\frac{\delta p}{\delta \rho}\right)_T = RT[1+2\rho B(T) + 3\rho^2 C(T)] \qquad (21)$$

Since enthalpy is a potential function, the integral of dh is independent of the path of integration. Therefore, integrate equation (19) in two steps: the first step at constant density and the second at constant temperature.

$$\int_{T_r, \rho_r}^{T, \rho} dh = \int_{T_r, \rho_r}^{T, \rho_r} dh + \int_{T, \rho_r}^{T, \rho} dh \qquad (22)$$

For the first step, integrate to the desired temperature at an arbitrary, constant density which is low enough that perfect gas behavior can be assumed. Thus, for this step, equation (11) reduces to:

$$\left(\frac{\delta h}{\delta T}\right)_\rho = \left(\frac{\delta e}{\delta T}\right)_\rho + \frac{1}{\rho}\left(\frac{\delta p}{\delta T}\right)_\rho = c_v + R = c_p \qquad (11)$$

then

$$\int_{T_r, \rho_r}^{T, \rho_r} dh = \int_{T_r}^{T} c_p = c_p(T - T_r) = \frac{R\gamma_p}{\gamma_p - 1}(T - T_r) \qquad (23)$$

The temperature term evaluated at the reference temperature is eliminated by the value of enthalpy at the reference condition (h = 1548.2 kJ/kg at T = 298.15 °K)[2] since it is chosen to be a point of perfect gas behavior.

For the second step, integrate from the reference density to the desired density at constant temperature to get

$$\int_{T, \rho_r}^{T, \rho} dh = \int_{T, \rho_r}^{T, \rho}\left[\frac{-T}{\rho^2}(\frac{\delta p}{\delta T})_\rho + \frac{1}{P}(\frac{\delta P}{\delta T})_T\right]d\rho = RT\rho\left[B(T) - T\frac{dB(T)}{dT}\right]_{\rho_r}^{\rho}$$
$$+ RT\frac{\rho^2}{2}\left[2C(T) - T\frac{dC(T)}{dT}\right]_{\rho_r}^{\rho} \qquad (24)$$

At the reference density, perfect gas behavior is assumed. Thus the virial coefficients and their derivatives at this point can be neglected. Then, by substitution of equations (23) and (24) into (22), the expression for enthalpy becomes

$$h = RT\left[\frac{\gamma_p}{\gamma_p - 1} + \rho\left(B(T) - T\frac{dB(T)}{dT}\right) + \frac{\rho^2}{2}\left(2C(T) - T\frac{dC(T)}{dT}\right)\right] \qquad (25)$$

Entropy

An equation for entropy can also be derived by first

8

expressing s as a function of ρ and T

$$ds = \left(\frac{\delta s}{\delta T}\right)_\rho dT + \left(\frac{\delta s}{\delta \rho}\right)_T d\rho \qquad (26)$$

The second term has already been given by equation (17). Defining internal energy as a function of ρ and T gives

$$de = \left(\frac{\delta e}{\delta T}\right)_\rho dT + \left(\frac{\delta e}{\delta \rho}\right)_T d\rho \qquad (27)$$

Substituting equations (12) and (14) gives

$$ds = \frac{c_v}{T} dT + \frac{1}{T}\left[\left(\frac{\delta e}{\delta \rho}\right)_T - \frac{p}{\rho^2}\right] d\rho \qquad (28)$$

And since equations (26) and (28) are equivalent

$$\left(\frac{\delta s}{\delta T}\right)_\rho = \frac{c_v}{T} \qquad (29)$$

By substituting equations (17) and (29) into equation (26), the expression for entropy is obtained

$$ds = \frac{c_v}{T} dT - \frac{1}{\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho d\rho \qquad (30)$$

The expression for entropy can be integrated in the same manner as that for enthalpy to get

$$s = R\left[\frac{\ln(T)}{\gamma_p - 1} - \ln\rho - \rho\left(B(T) + T\frac{dB(T)}{dT}\right) - \frac{\rho^2}{2}\left(C(T) + T\frac{dC(T)}{dT}\right)\right] + s_{ref}$$
$$(31)$$

The value of the constant, s_{ref}, is determined by substituting the values for reference entropy, temperature and density (31.489 kJ/kg-°K at 298.15° K, 0.16361 kg/m³, respectively)[2] into equation (31) to get s_{ref} = 9.977 kJ/kg-°K.

Other Properties

The specific heat at constant volume can then be found using equations (29) and (31)

9

$$c_v = \frac{R}{\gamma_p - 1} - RT\left[\rho\left(2\frac{db(T)}{dT} + T\frac{d^2B(T)}{dT^2}\right) + \frac{\rho^2}{2}\left(2\frac{dC(T)}{dT} + T\frac{d^2C(T)}{dT^2}\right)\right]$$

(32)

The speed of sound and specific heat at constant pressure are found through the thermodynamic relationships

$$a^2 = \left(\frac{\delta p}{\delta \rho}\right)_s = \frac{c_p}{c_v}\left(\frac{\delta p}{\delta \rho}\right)_T$$

(33)

$$\frac{c_p}{c_v} = \frac{-T}{c_v\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho\left(\frac{\delta \rho}{\delta T}\right)_p + 1$$

(34)

$$\left(\frac{\delta p}{\delta \rho}\right)_T = -\left(\frac{\delta p}{\delta T}\right)_\rho\left(\frac{\delta T}{\delta \rho}\right)_p$$

(35)

Substitution of equations (34) and (35) into (33) leads to

$$a^2 = \left(\frac{\delta p}{\delta \rho}\right)_T + \frac{T}{c_v\rho^2}\left(\frac{\delta p}{\delta T}\right)_\rho^2$$

(36)

The speed of sound can then be computed using equations (20) and (21) with equation (36).
The specific heat at constant pressure is found by substituting equation (35) into (34) to obtain

$$c_p = c_v + \frac{\dfrac{T}{\rho^2}\left(\dfrac{\delta p}{\delta T}\right)_\rho^2}{\left(\dfrac{\delta p}{\delta \rho}\right)_T}$$

(37)

The final relations required are the one-dimensional shock conservation equations

mass:
$$\rho_2 u_2 = \rho_1 u_1$$
(38)

momentum:
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$
(39)

10

energy:

$$h_0 = h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad (40)$$

In figure (5), values of enthalpy and entropy calculated using the procedure described are plotted along lines of constant pressure and temperature. The speed of sound is plotted in figure (6) and the specific heats in figures (7) and (8).

COMPUTATIONAL PROCEDURE

The relationships from the previous section can be used to determine the reservoir, nozzle, throat, freestream and post-normal shock flow parameters in a helium wind tunnel from the measured reservoir pressure and temperature and the measured pitot pressure or freestream Mach number. The procedure described herein has been incorporated into a FORTRAN program called HEPROPS which is listed in the appendix.

Reservoir Conditions

Given the reservoir pressure and temperature, which are the properties normally measured during tunnel operation, and assuming zero flow velocity in the reservoir, the corresponding reservoir density can be obtained through Newtonian iteration on the virial equation of state, where

$$F = p_0{}^n - \rho_0 R T_0 \left[1 + \rho_0 B(T_0) + \rho_0^2 C(T_0) \right] \qquad (41)$$

$$F' = -R T_0 \left[1 + 2\rho_0 B(T_0) + 3\rho_0^2 C(T_0) \right] \qquad (42)$$

$$p_0^{n+1} = p_0{}^n - \frac{F}{F'} \qquad (43)$$

At this point, if the freestream Mach number is known, the computations can be carried out in a straightforward manner for the throat, freestream, and post-normal shock conditions. If instead, the pitot pressure is given, a Mach number must be assumed and all computations carried out for that Mach number. Newtonian iteration must then be carried out on the Mach number until the measured and computed pitot pressures are in agreement.

Conditions at An Arbitrary Point

Conditions at any point in the nozzle can be determined by specifying a Mach number and assuming that the expansion through the nozzle is an isentropic. This procedure is used to determine the conditions at the throat ($M^* = 1$) and in the freestream (M_1

11

either given or assumed during iteration). Additionally, the option exists in the HEPROPS program to compute the conditions at any arbitrary point (denoted "x") in the nozzle by specifying a Mach number for that point.

From the total enthalpy and entropy, h_0 and s_0, (which can be computed from the supply conditions) and a specified Mach number (for instance, at the throat, $M_x = 1$). Static density and temperature, ρ_x and T_x, can then be determined through Newtonian iteration on two-variables, where

$$G = \begin{bmatrix} \rho_x \\ T_x \end{bmatrix} \tag{44}$$

$$W = \begin{bmatrix} (h_0 - RT_x \left[\dfrac{\gamma_p}{\gamma_p - 1} + \rho_x \left(B(T_x) - T_x \dfrac{dB(T_x)}{dT_x} \right) + \rho_x^2 \left(2C(T_x) - T_x \dfrac{dC(T_x)}{dT_x} \right) \right] \\ \quad - \dfrac{u_x^2}{2}) \\ \\ s_0 - R \left[\dfrac{1}{\gamma_p - 1} - \ln\rho_x - \rho_x \left(B(T_x) + T \dfrac{dB(T_x)}{dT_x} \right) - \dfrac{\rho_x^2}{2} \left(C(T_x) T_x \dfrac{dC(T_x)}{dT_x} \right) \right] \end{bmatrix}$$

$$\tag{45}$$

$$W^{n+1} = W^n - \left[\frac{\delta W}{\delta G} \right]^{-1} G \tag{46}$$

Where the Jacobian of W is given by

$$\left[\frac{\delta W}{\delta G} \right] = \begin{bmatrix} \dfrac{\delta w_1}{\delta g_1} & \dfrac{\delta w_2}{\delta g_1} \\ \dfrac{\delta w_1}{\delta g_2} & \dfrac{\delta w_2}{\delta g_2} \end{bmatrix} \tag{47}$$

Post-Shock Static Conditions

Post-shock static conditions can be determined from the computed freestream conditions using the shock conservation equations. From these equations, implicit expressions for ρ_2 and T_2 can be derived and then solved iteratively, where

$$G = \begin{bmatrix} \rho_2 \\ T_2 \end{bmatrix} \tag{48}$$

12

$$W = \begin{bmatrix} \left(h_0 - RT_2\left[\dfrac{\gamma_p}{\gamma_p - 1} + \rho_2\left(B(T_2) - T_2\dfrac{dB(T_2)}{dT_2}\right) + \rho_2^2\left(2C(T_2) - T_2\dfrac{dC(T_2)}{dT_2}\right)\right]\right. \\ \left. - \dfrac{u_2^2}{2}\right) \\[2ex] p_\infty + \rho_\infty u_\infty^2 - \rho_2 RT_2\left[1 + \rho_2 B(T_2) + \rho_2^2 C(T_2)\right] - \dfrac{(\rho_\infty u\infty)^2}{\rho_2} \end{bmatrix} \tag{49}$$

Post-Shock Stagnation Conditions

By assuming isentropic compression from the post-shock static conditions, $\rho_{t,2}$ and $T_{t,2}$ can be computed using

$$W = \begin{bmatrix} \left(h_0 - RT_{t,2}\left[\dfrac{\gamma_p}{\gamma_p - 1} + \rho_{t,2}\left(B(T_{t,2}) - T_{t,2}\dfrac{dB(T_{t,2})}{dT_{t,2}}\right)\right]\right. \\ \left. - \rho_{t,2}^2\left(2C(T_{t,2}) - T_{t,2}\dfrac{dC(T_{t,2})}{dT_{t,2}}\right)\right) \\[2ex] \left(s_2 - R\left[\dfrac{1}{\gamma_p - 1} - \ln(\rho_{t,2}) - \rho_{t,2}\left(B(T_{t,2}) + T\dfrac{dB(T_{t,2})}{dT_{t,2}}\right)\right]\right. \\ \left. - \dfrac{\rho_{t,2}^2}{2}\left(C(T_{t,2}) + T_{t,2}\dfrac{dC(T_{t,2})}{dT_{t,2}}\right)\right) \end{bmatrix} \tag{50}$$

$$G = \begin{bmatrix} \rho_{t,2} \\ T_{t,2} \end{bmatrix} \tag{51}$$

At each point, the remaining thermodynamic properties can be evaluated using the equations from the previous section and by

$$M = \frac{u}{a} \tag{52}$$

$$Re = \frac{\rho \, u x}{\mu} \tag{53}$$

13

from reference (14)

$$T \leq 1.2°K$$
$$\mu = (2.1630 - 26.665\,T + 120.54\,T^2 - 187.41\,T^3$$
$$+ 126.82\,T^4 - 31.823\,T^5) \times 10^{-7}\,\frac{kg}{m-s}$$

$$1.2°K < T \leq 3.6°K$$
$$\mu = (5.02 - 3.2241\,T + 2.0308\,T^2 - 0.22351\,T^3) \times 10^{-7}\,\frac{kg}{m-s}$$

$$3.6°K < T \leq 8°K$$
$$\mu = (-1.5691 + 3.4167\,T - 0.10317\,T^2) \times 10^{-7}\,\frac{kg}{m-s}$$

$$T > 8°K$$
$$\mu = 5.023 \times 10^{-7}\,T^{0.647}\,\frac{kg}{m-s} \tag{54}$$

The expression used for viscosity[14] is plotted versus temperature in figures (9a) and (9b) and is compared with experimental and theoretical data[7,12,16-18].

At this point, the computation is complete if the Mach number was originally specified. However, if instead the pitot pressure was given, Newtonian iteration for the Mach number must be carried out where

$$F = p_{t,2_{calc}} - p_{t,2_{meas}} \tag{55}$$

F' is evaluated numerically

$$F' = \frac{\delta p_{t,2_{calc}}}{\delta M_\infty} = \frac{p_{t,2}(M_\infty + \Delta M_\infty) - p_{t,2}(M_\infty)}{\Delta M_\infty} \tag{56}$$

$$M_\infty^{n+1} = M_\infty^n - \frac{F}{F'} \tag{57}$$

It should be noted that this method provides a most accurate means of computing flow properties over a wide range of conditions since the assumption made in previous works of perfect gas behavior in the freestream (which is fairly accurate for the Langley 22-Inch

14

Mach 20 Helium Tunnel) is not made in these computations.

RESULTS AND DISCUSSION

   The HEPROPS program was used to determine helium flow properties for reservoir conditions of 1 to 400 atm. pressure and 50 to 600 °K temperature, which encompasses the operating range of the Langley 22-Inch Mach 20 Helium Tunnel (20 - 285 atm. and 290 - 600° K). Results are tabulated graphically in figures (10) through (13). In each figure are shown correction factors for ratios of thermodynamic properties at different points in the flow. These correction factors represent the actual ratio for the given thermodynamic property (as determined using HEPROPS) divided by the ratio calculated from perfect gas theory. The correction factors are given for Mach 10 and 20. The correction factors for Mach numbers greater than 20 are essentially constant and can be taken as their values at Mach 20. These figures should not be used to estimate correction factors for flows with Mach numbers much less than 10; however the operating conditions of low Mach number flows are generally such that real gas deviations are insignificant.

   As part of the computational procedure, the specific heats at constant pressure and temperature, the speed of sound, and the entropy and enthalpy were calculated across the entire range of flow conditions. The values of the properties can be estimated at any given pressure and temperature using figures (5) to (8). Also, the virial coefficients and the compressibility coefficient are given in figures (1) through (4). Viscosity is given in figures (9a) and (9b).

   Ideally, the HEPROPS program should be used to evaluate wind tunnel flow properties. However, if desired, flow properties can be estimated by applying these real gas correction factors to perfect gas calculations. As an example, consider a helium wind tunnel flow for which the freestream Mach number, M_∞, is equal to 20, and the measured supply pressure and temperature are $p_0 = 30.4$ MPa (300 atm) and $T_0 = 300$ K. The supply density is

$$\rho_0 = \frac{p_0}{Z_0 R T_0}$$

From figure (4)

$$\rho_0 = \frac{30.4\,MPa}{(1.14)\cdot(2079\,J/kg\text{-}K)\cdot(300\,K)} = 42.8\,kg/m^3$$

The freestream pressure is

$$p_1 = \left[\frac{\left(\dfrac{p_0}{p_1}\right)_p}{\left(\dfrac{p_0}{p_1}\right)}\right]\left(\dfrac{p_0}{\dfrac{p_0}{p_1}}\right)_p = \left[\frac{\left(\dfrac{p_0}{p_1}\right)_p}{\left(\dfrac{p_0}{p_1}\right)}\right]\frac{p_0}{\left(1 + \dfrac{\gamma_p - 1}{2}M_\infty^2\right)^{\frac{\gamma_p}{\gamma_p - 1}}}$$

15

From figure (11a)

$$p_1 = (1.14) \frac{30.4\,MPa}{\left(1 + \frac{5/3 - 1}{2}\,20^2\right)^{\frac{5/3}{5/3 - 1}}} = 166\,Pa$$

Note from figure (11e) that the correction factor for the freestream Mach number was found to be between 0.99 and 1 through most of the range of conditions studied, thus for rough estimates no correction to the Mach number is required.

The post-normal shock stagnation temperature is

$$T_{t,2} = \left[\frac{\left(\dfrac{T_0}{T_{t,2}}\right)_p}{\left(\dfrac{T_0}{T_{t,2}}\right)}\right] \frac{T_0}{\left(\dfrac{T_0}{T_{t,2}}\right)_p}$$

From figure (13b)

$$T_{t,2} = (1.06) \frac{300\,K}{1} = 318\,K$$

CONCLUDING REMARKS

A method for computing real gas flow parameters in helium wind tunnels has been developed which utilizes a three-coefficient virial equation of state. Comparisons of the compressibility factors obtained using this equation were made with those in published tabulations and were found to be in very close agreement. This method has been incorporated into a FORTRAN program designed to be used in the determination of wind tunnel flow properties in the reservoir, nozzle, throat, freestream and post-normal shock regions. This program can also be used to calculate thermodynamic properties at any arbitrary point in the nozzle which is specified by a Mach number. Correction factors have been tabulated for the range of conditions in which the Langley 22-Inch Mach 20 Helium Tunnel is operated. These correction factors can be applied to perfect gas theory computations to estimate real flow properties. Values for relevant thermodynamic properties of helium as functions of pressure and temperature are also presented.

REFERENCES

1. Olstad, W. B.; Kemper, J. T.; and Bengtson, R. D.: Equilibrium Normal-Shock and Stagnation-Point Properties of Helium for Incident-Shock Mach Numbers from 1 to 30. NASA TN D-4754, 1968.
2. International Union of Applied and Pure Chemistry.: International Thermodynamic Tables of the Fluid State of Helium-4. S. Angus and K. M. de Reuck eds., Pergamon Press, 1975.
3. Erickson, Wayne D.: Real-Gas Correction Factors for Hypersonic Flow Parameters in Helium. NASA TN D-462, 1960.
4. Erickson, Wayne D.: An Extension of Estimated Hypersonic Flow Parameters for Helium as a Real Gas. NASA TN D-1632, 1963.
5. Miller, Charles G.; and Wilder, Susan E.: Real-Helium Hypersonic Flow Parameters for Pressures and Temperatures to 3600 Atmospheres and 15,000° K. NASA TN D-4869, 1968.
6. Miller, C. G.: Langley Hypersonic/Aerothermodynamic Testing Capabilities - Present and Future. AIAA Paper 90-1376.
7. Keesom, W. H.: Helium. Elsevier, 1942.
8. Schneider, W.G.; and Duffie, J. A. H.: Compressibility of Gases at High Temperatures. II. The Second Virial Coefficient of Helium in the Temperature range of 0°C to 600°C. J. Chem. Phys., vol. 17, no.9, Sept. 1949, pp.751-754.
9. Yntema, J.L.; and Schneider, W. G.: Compressibility of Gases at High Temperatures. III. The Second Virial Coefficient of Helium in the Temperature Range of 600°C to 1200°C. J. Chem. Phys., vol. 18, no.5, May 1950, pp.641-646.
10. Beattie, James A.: Volumetric Behavior and Thermodynamic Properties of the Gas Phase. Argon, Helium, and the Rare Gases, Vol. I, Gerhard A. Cook, ed., Interscience Publ., 1961, pp.251-312.
11. Michels, A.; and Wouters, H.: Isotherms of Helium Between 0° and 150° C up to 200 Amagat. Physica, vol. VIII, no.8, Sept. 1941, pp.370-383.
12. Amdur, I.; and Mason, E. A.: Properties of Gases at Very High Temperatures. Phys. Fluids, vol. 1, no. 5, Sept.-Oct. 1958, pp. 370-383.
13. Harrison, Edwin F.: Intermolecular-Force Effects on the Thermodynamic Properties of Helium with Application. AIAA J. (Tech Notes), vol. 2, no.10, Oct. 1964, pp. 1854-1856.
14. Miller, C. G.; and Maddalon, D. V.: Errors in Free-Stream Reynolds Number of Helium Wind Tunnels. AIAA J., vol 8, no 5, May 1970, pp 968-970.
15. Touloukian, Y. S.; Saxon, S. C.; and Hestermans, P.: Viscosity. Thermophysical Properties of Matter, vol. 11, IFI/Plenum, 1975.
16. Keller. W. E.: Calculation of the Viscosity of Gaseous He^3 and He^4 at Low Temperatures. Phys. Rev., vol.105, no.1, Jan., 1957.
17. Handbook of Heat Transfer. W. E. Rohsenow and J. P. Hartnett eds., McGraw-Hill Book Company, 1973.
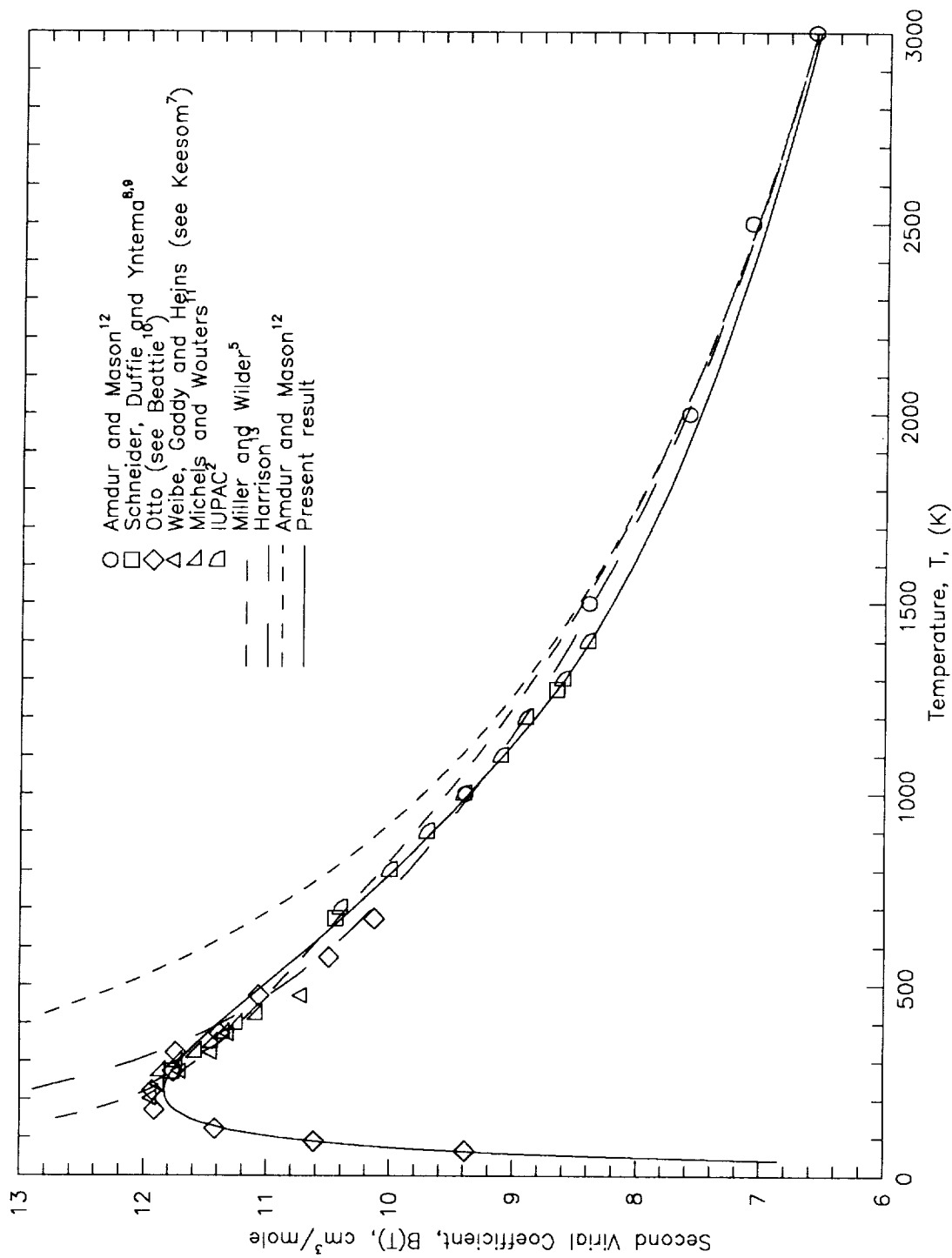
Figure 1. – Second Virial Coefficient as a Function of Temperature

18
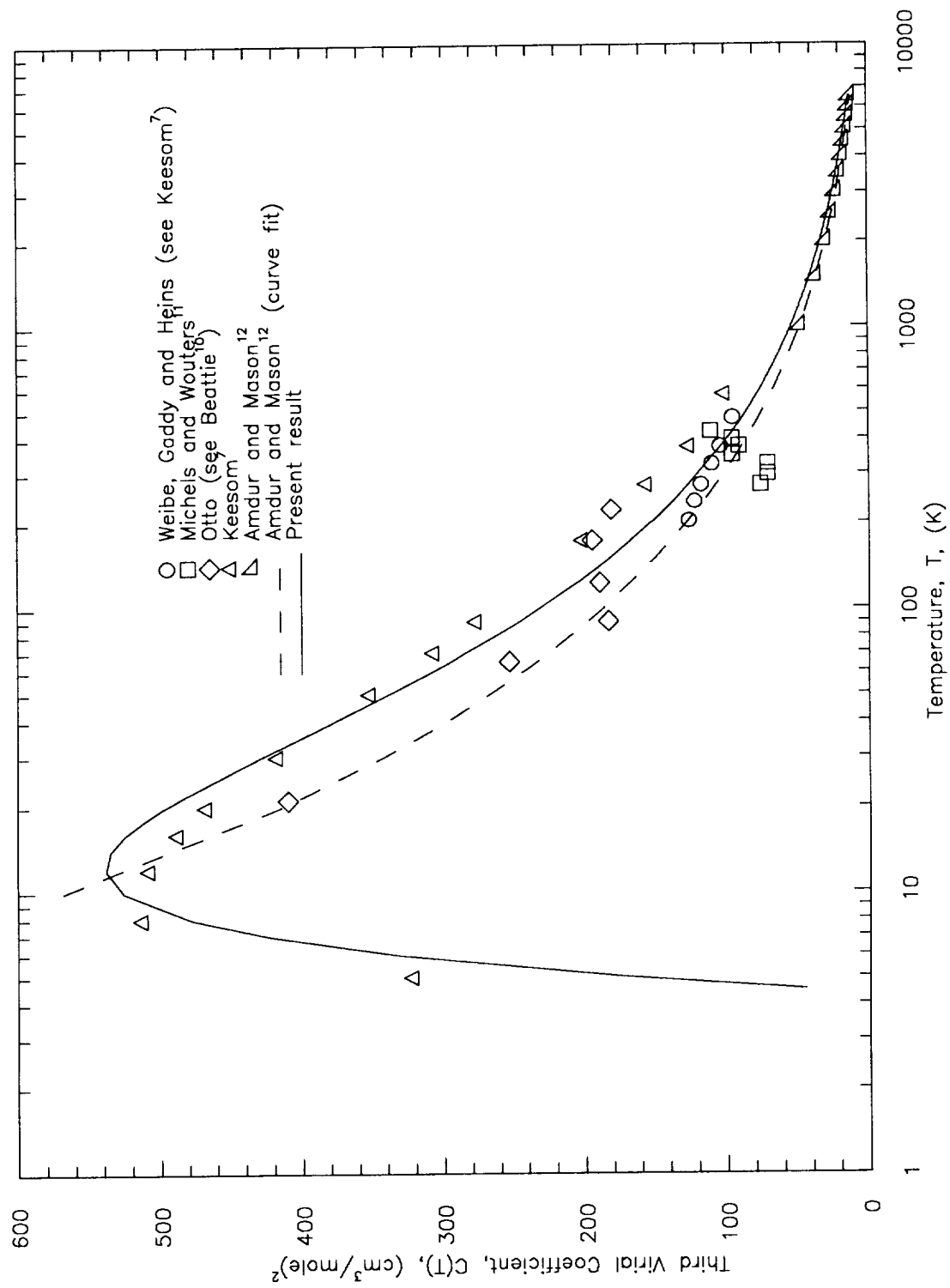
Figure 2. – Third Virial Coefficient as a Function of Temperature
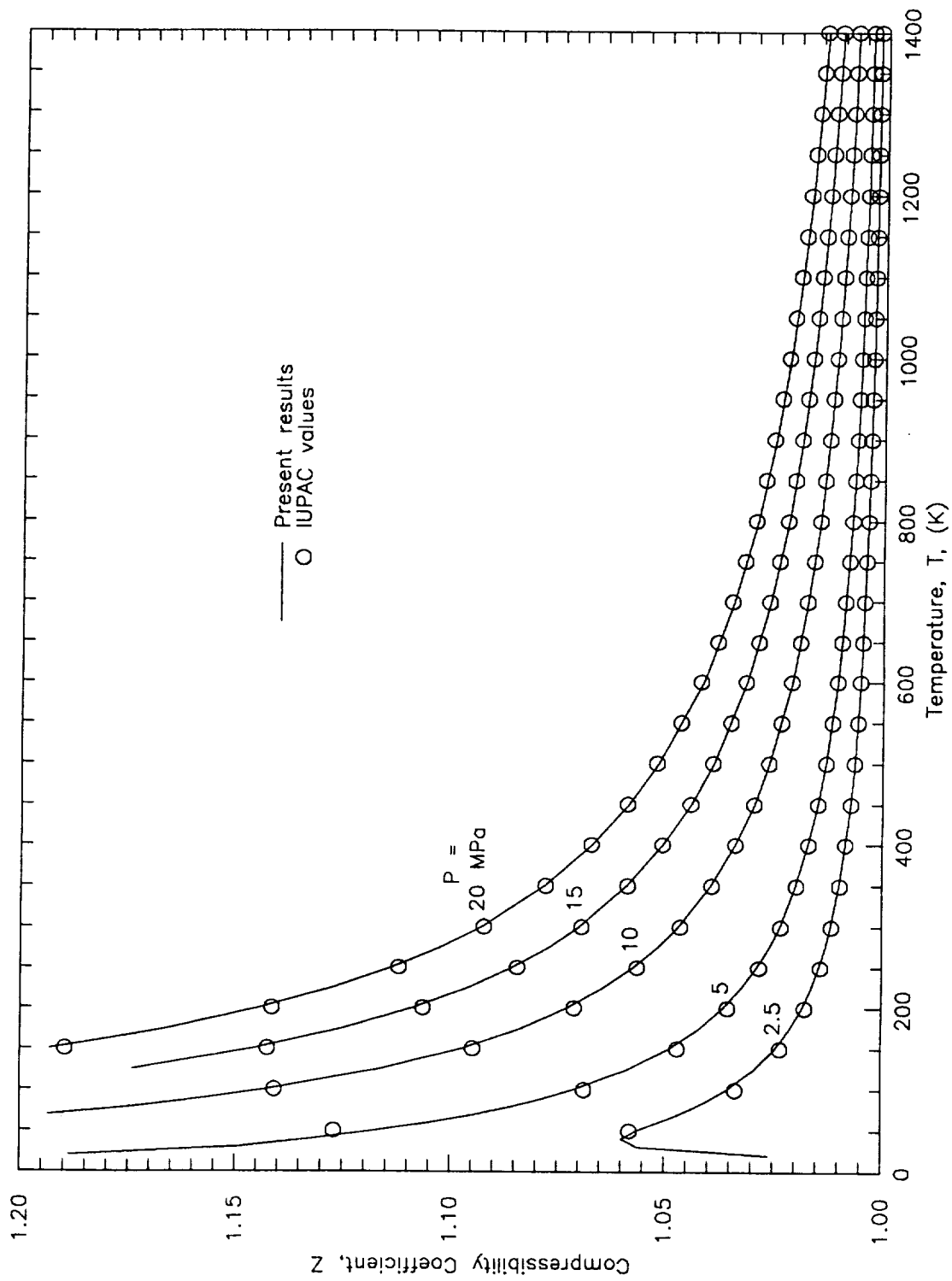
Figure 3. — Comparison of Compressibility Coefficient with IUPAC Data
(a) P ≥ 2.5 MPa
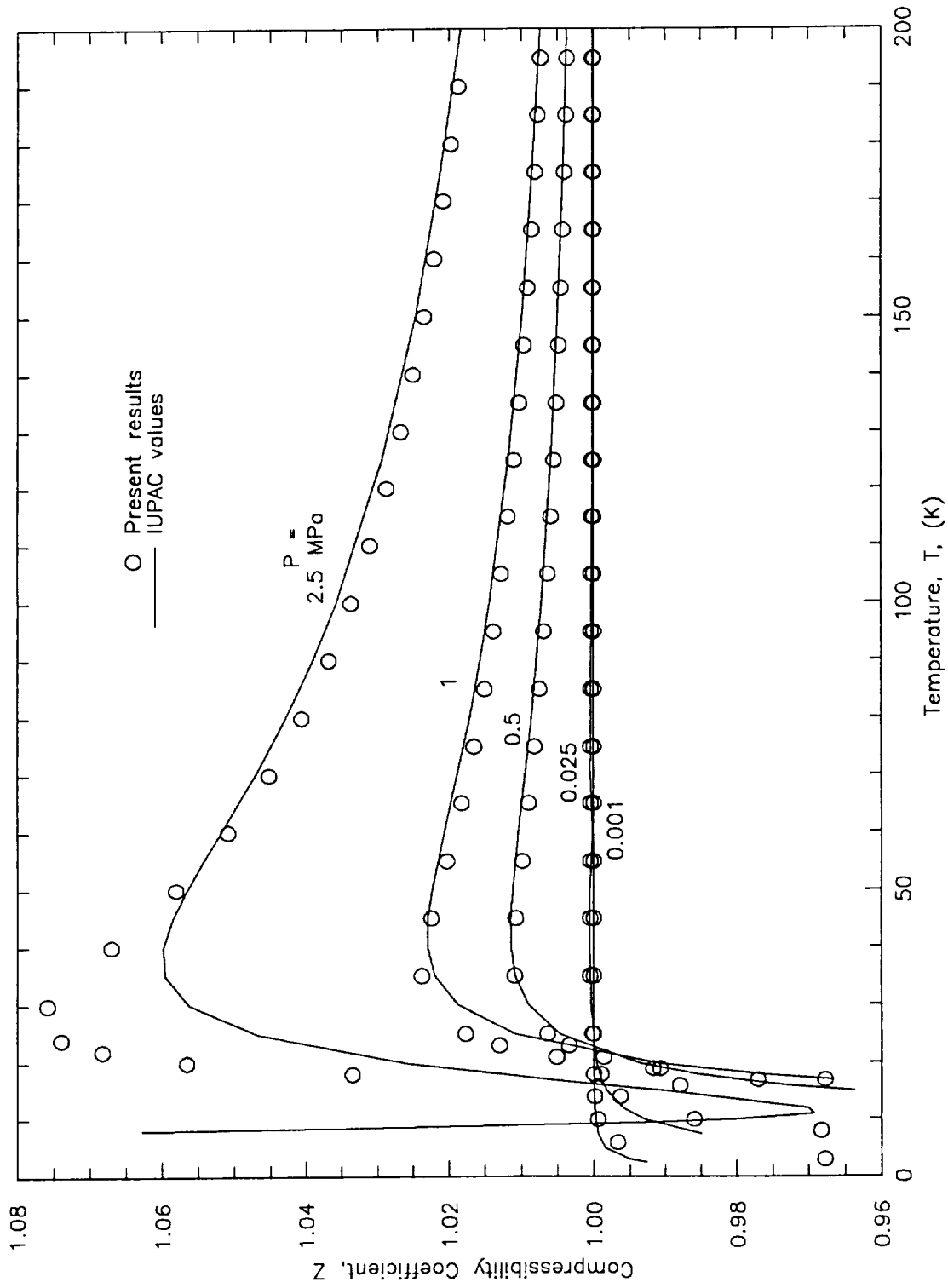
Figure 3. -- continued
(b) P ≤ 2.5 MPa
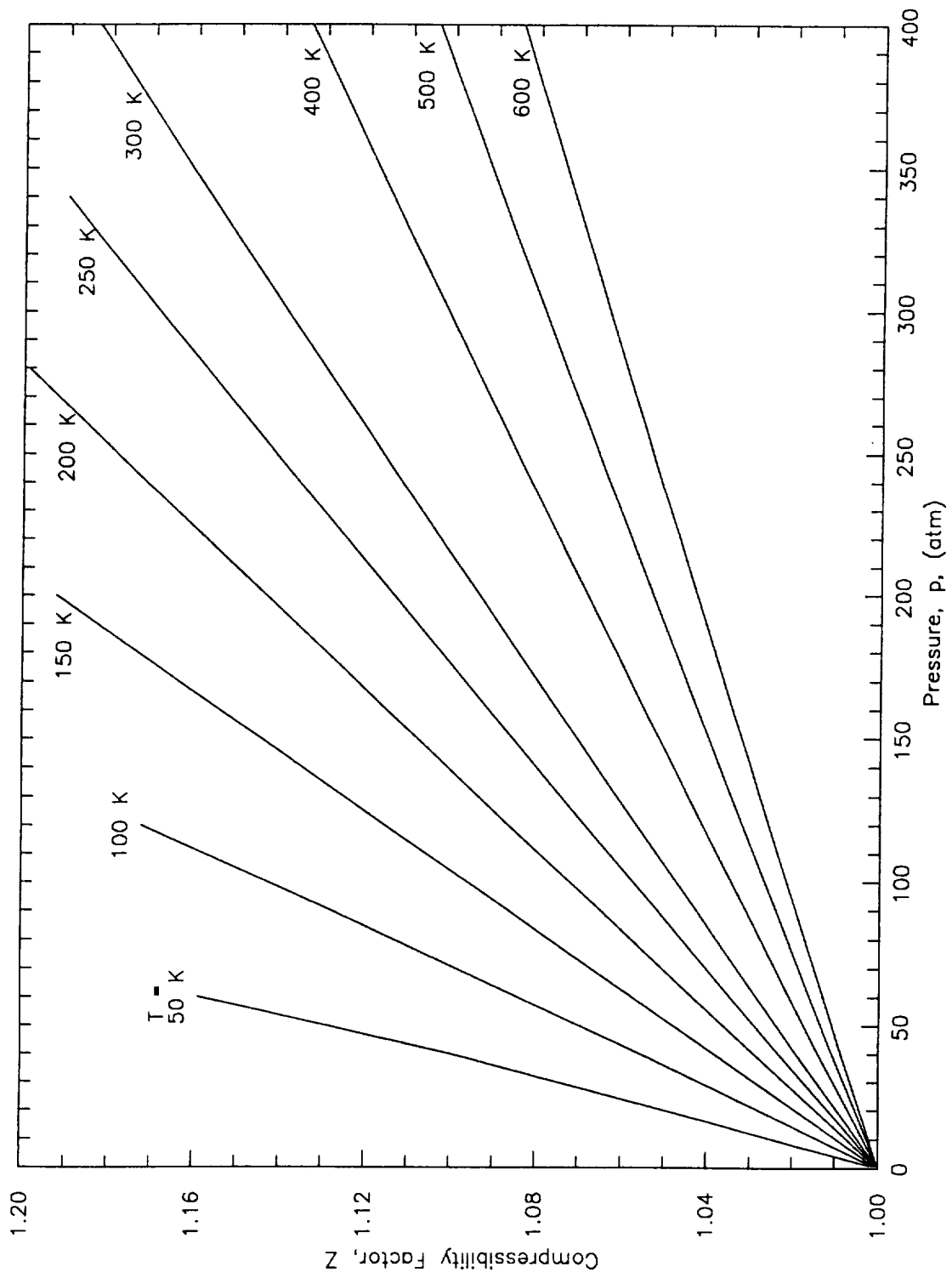
Figure 4. — Compressibility Coefficient vs Temperature and Pressure

22
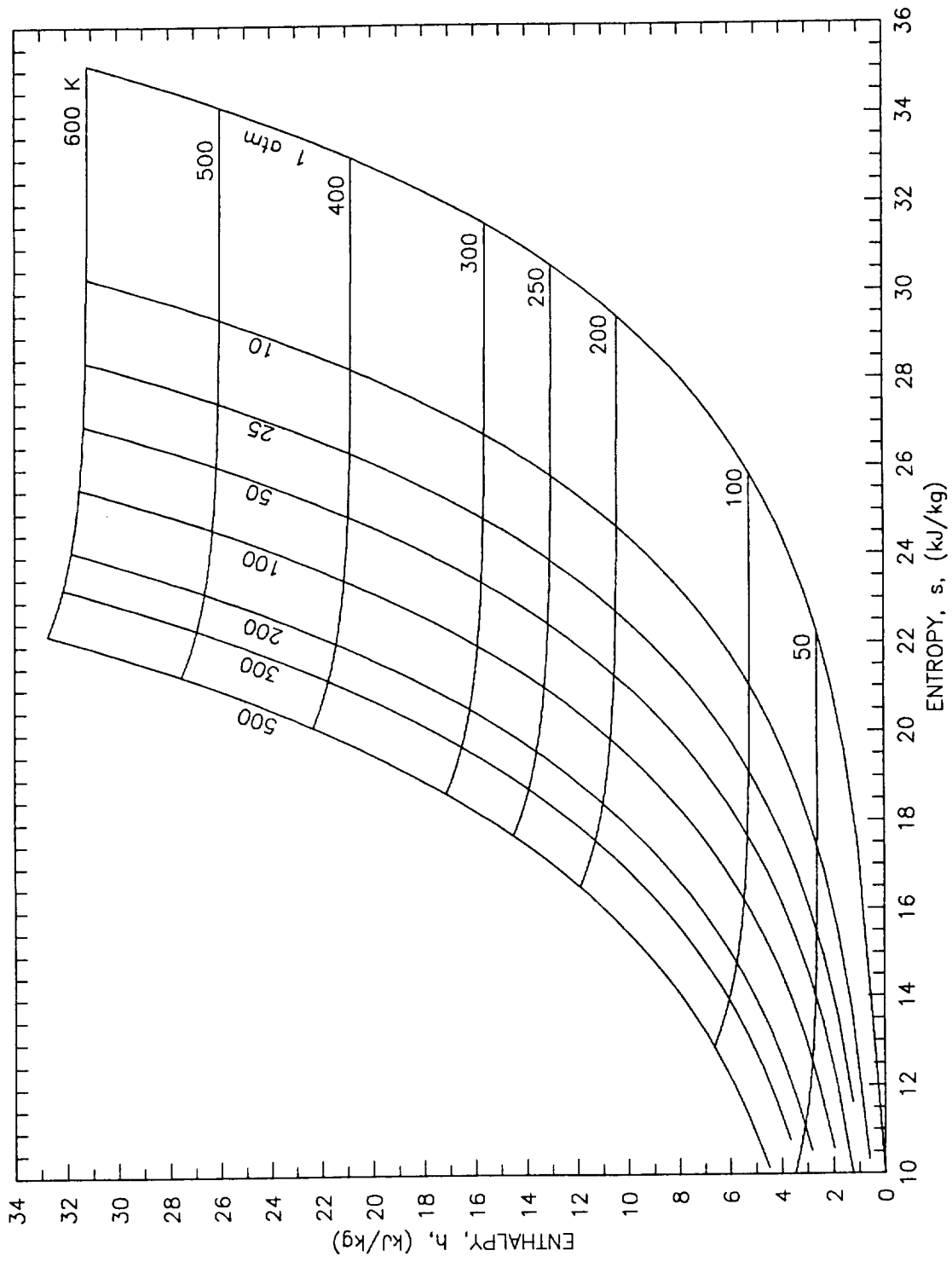
Figure 5. – Entropy vs Enthalpy Chart
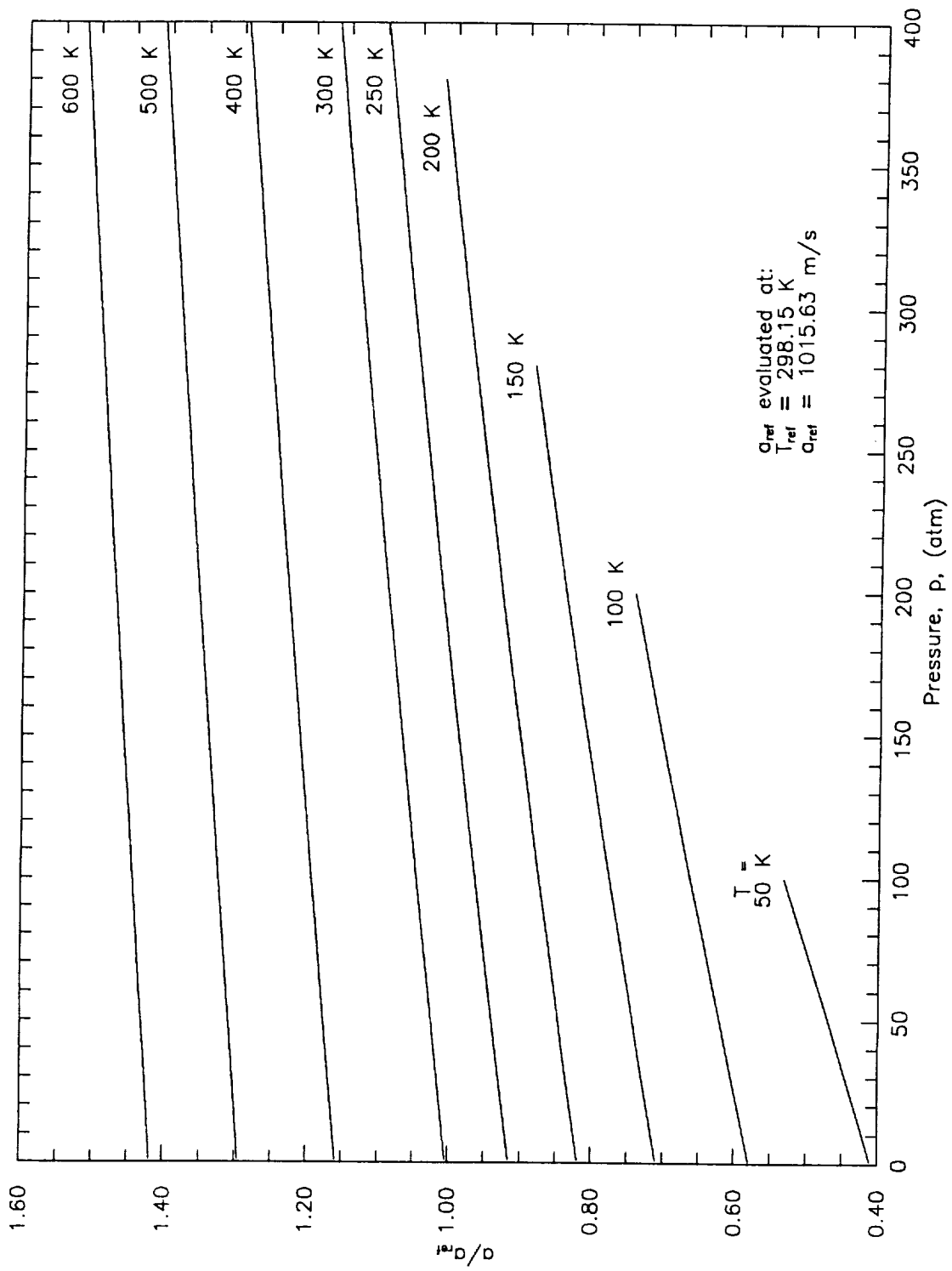
Figure 6. – Speed of Sound as a Function of Temperature and Pressure
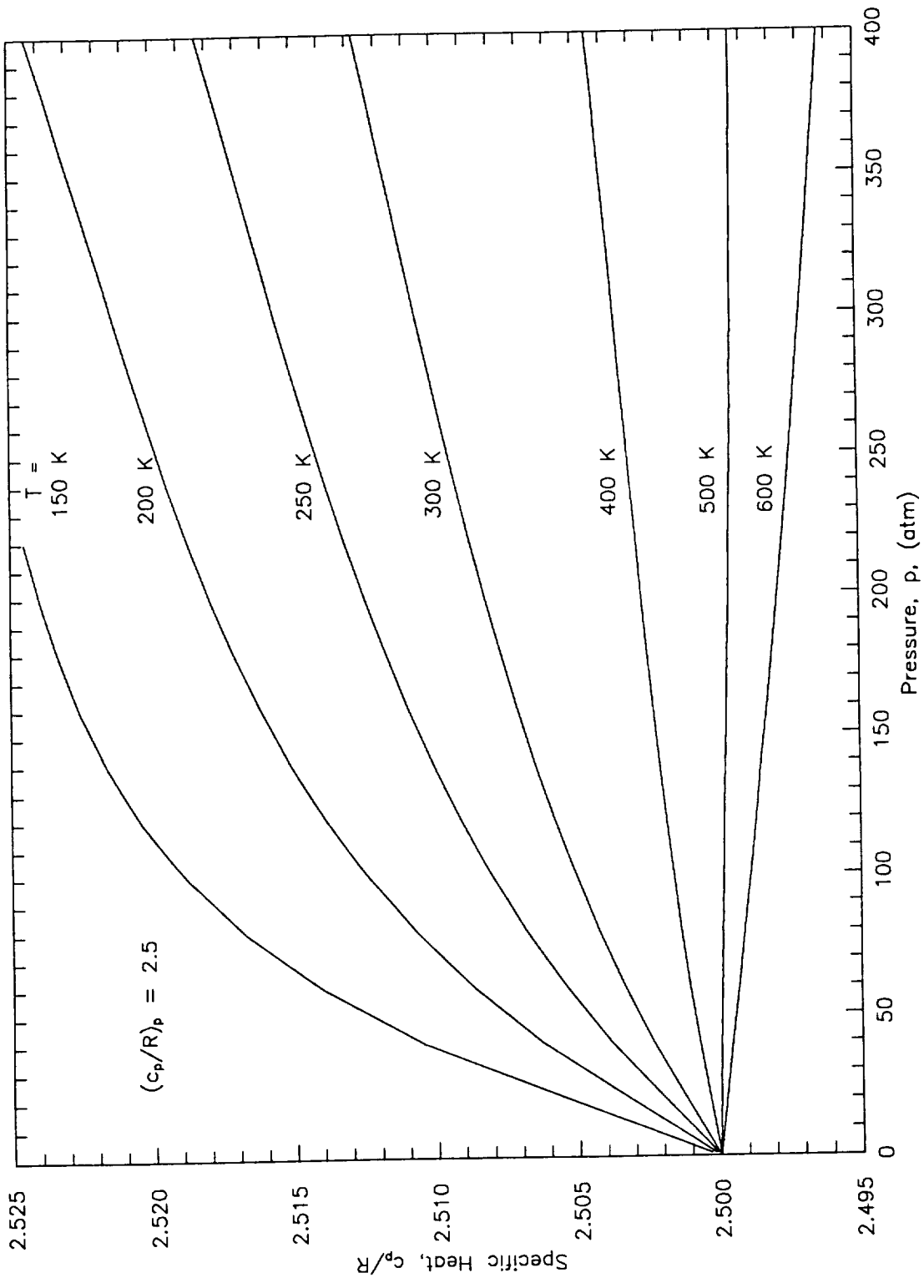
Figure 7. – Specific Heat at Constant Pressure
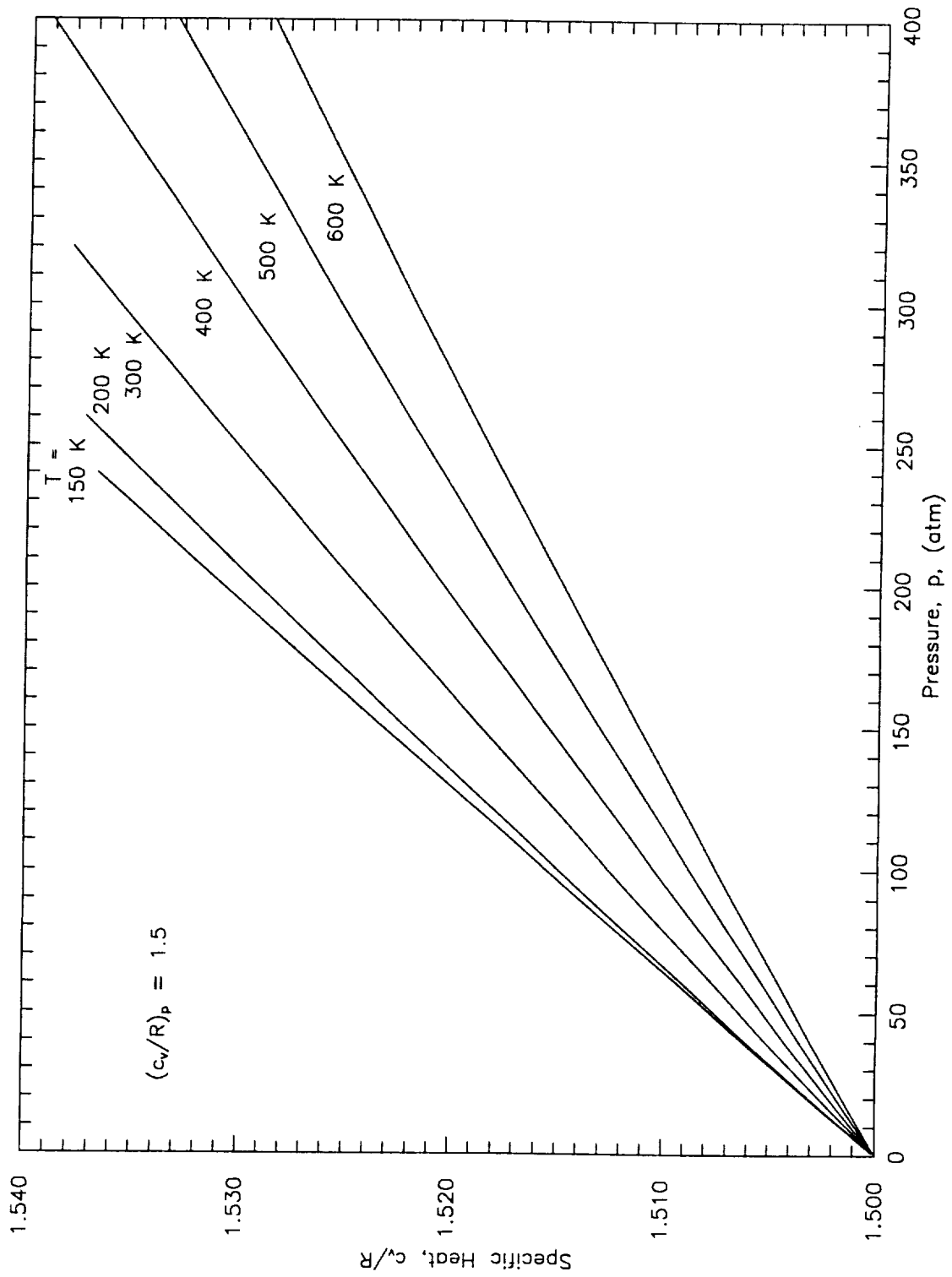
Figure 8. – Specific Heat at Constant Volume
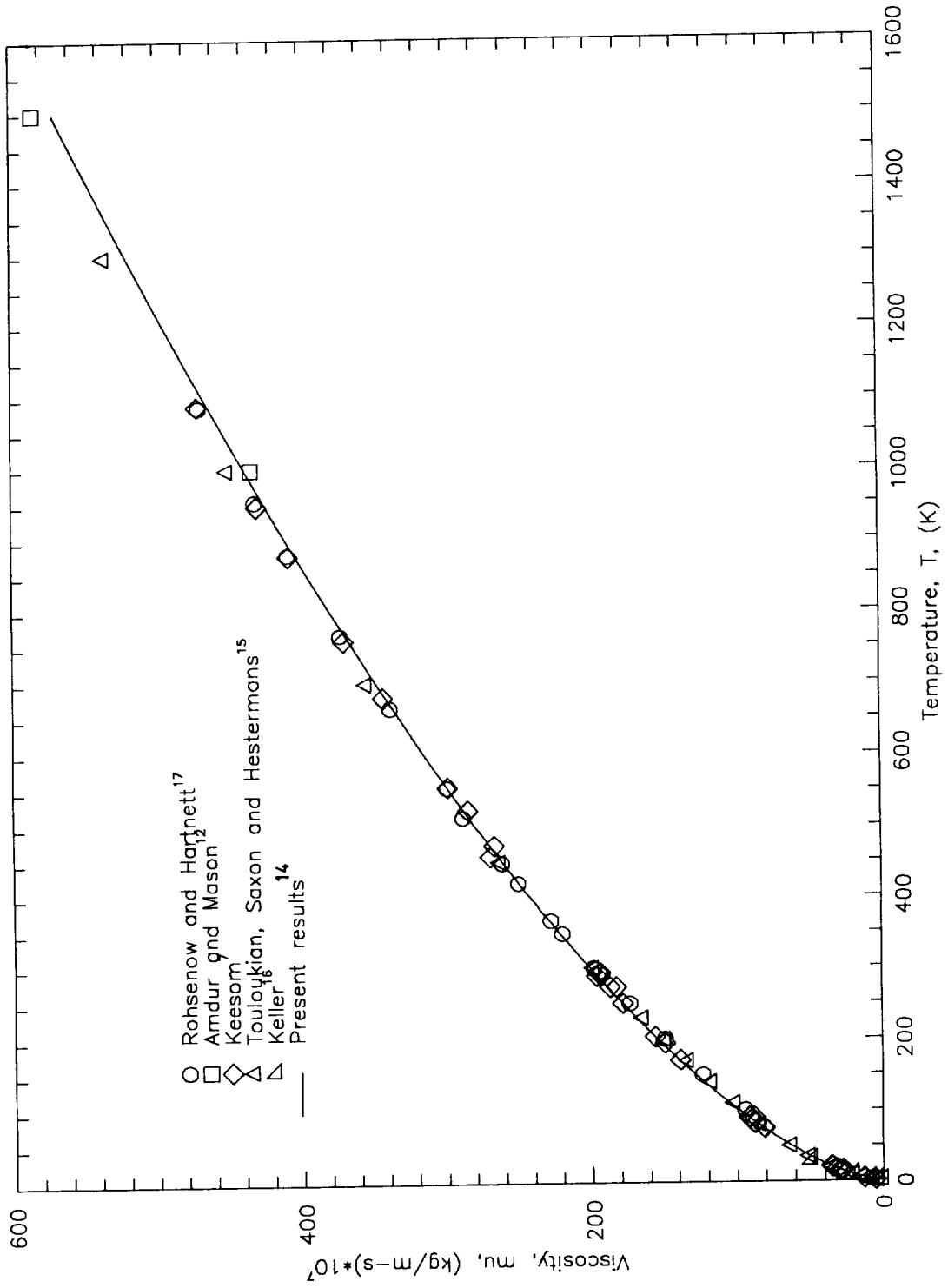
Figure 9. — Viscosity as a Function of Temperature
(a) High Temperature Range
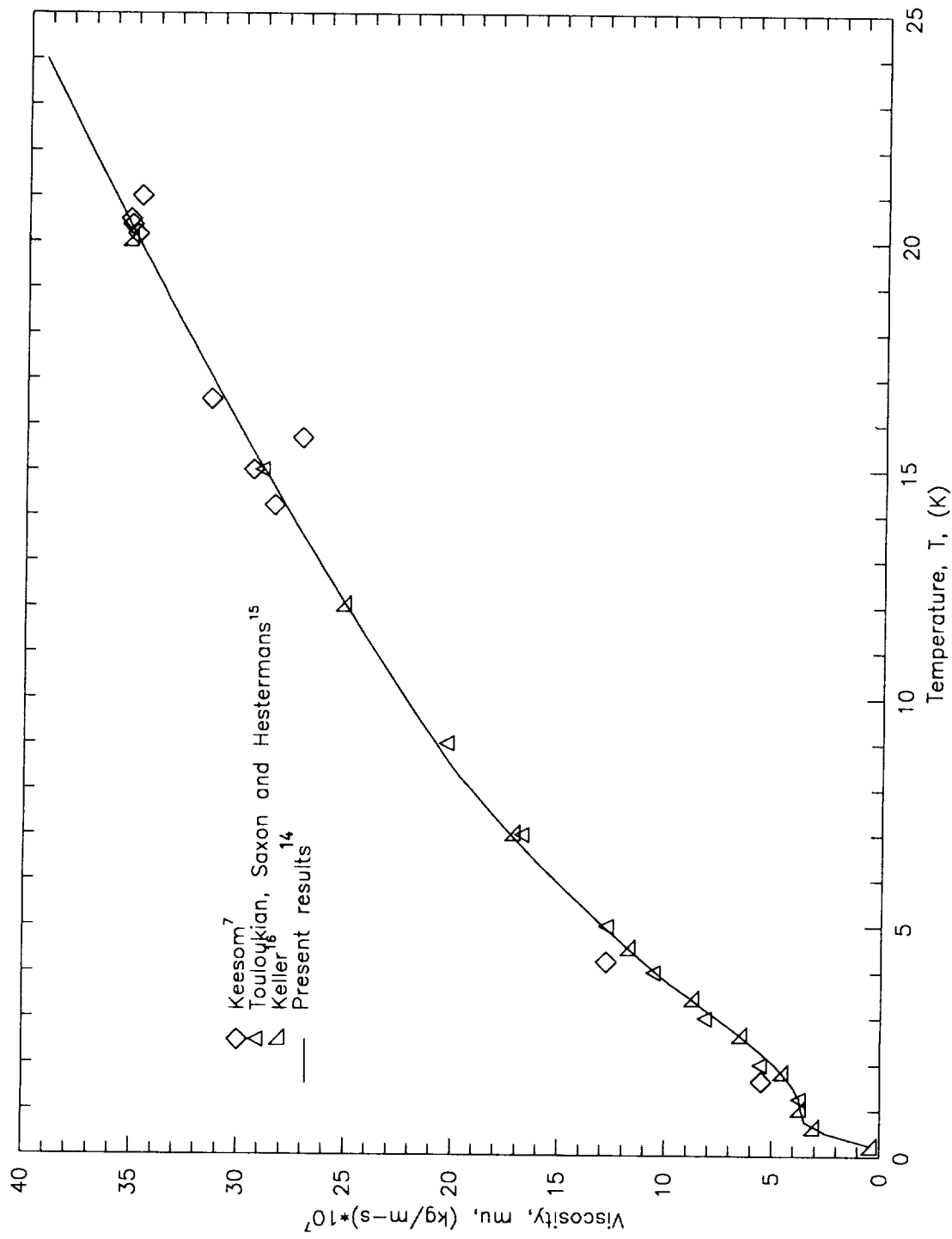
Figure 9. — Continued
(b) Low Temperature Range
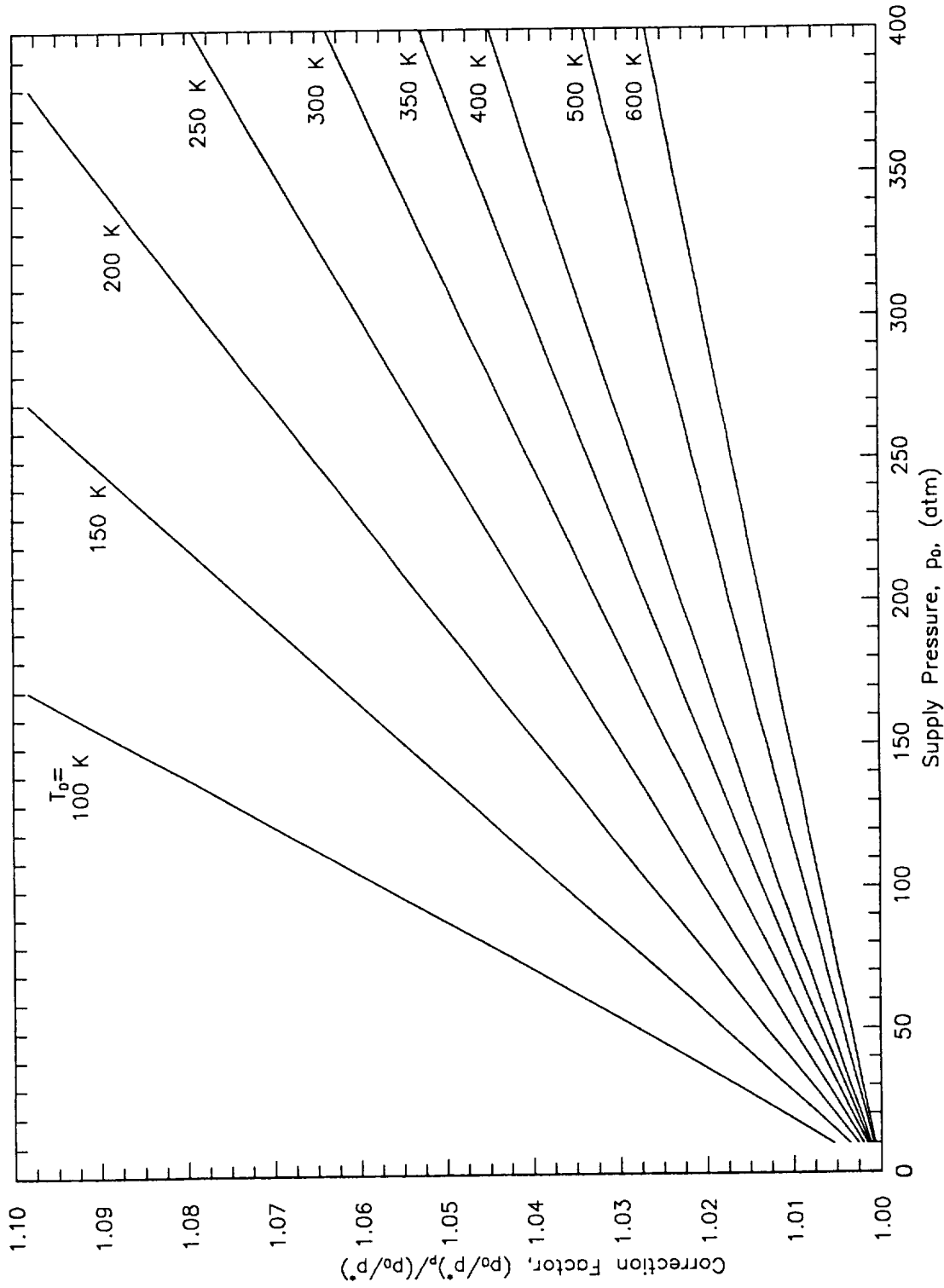
Figure 10. – Ratios of Real to Ideal Flow Parameters
(a) Pressure Ratio, $(p_o/p^\bullet)$

x-axis: Supply Pressure, p_o, (atm)

y-axis: Correction Factor, $(p_o/p^\bullet)_p/(p_o/p^\bullet)$

Curve labels: $T_o = 100\ K$, $150\ K$, $200\ K$, $250\ K$, $300\ K$, $350\ K$, $400\ K$, $500\ K$, $600\ K$
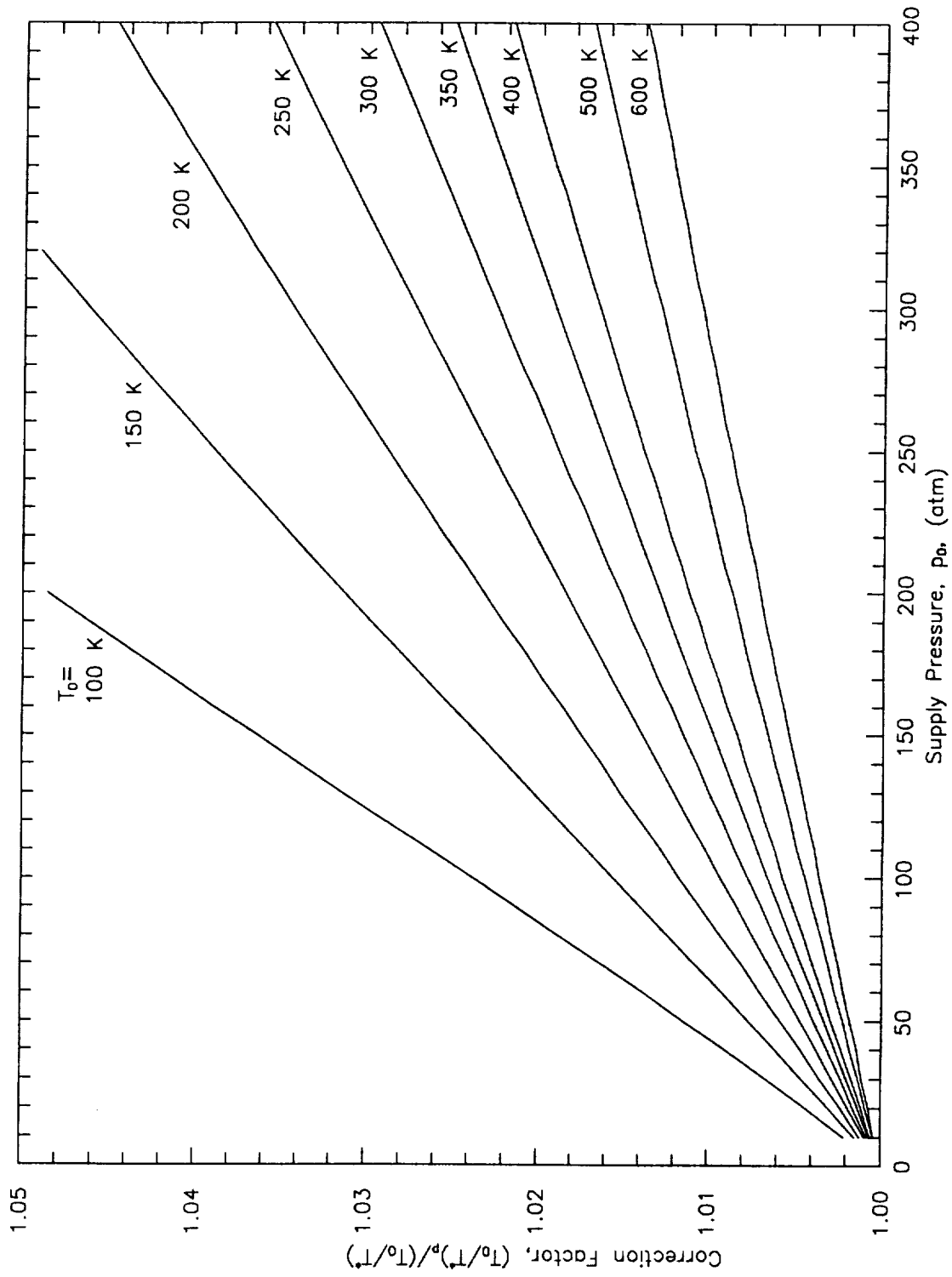
Figure 10. – Continued
(b) Temperature Ratio, (T_0/T^*)

30
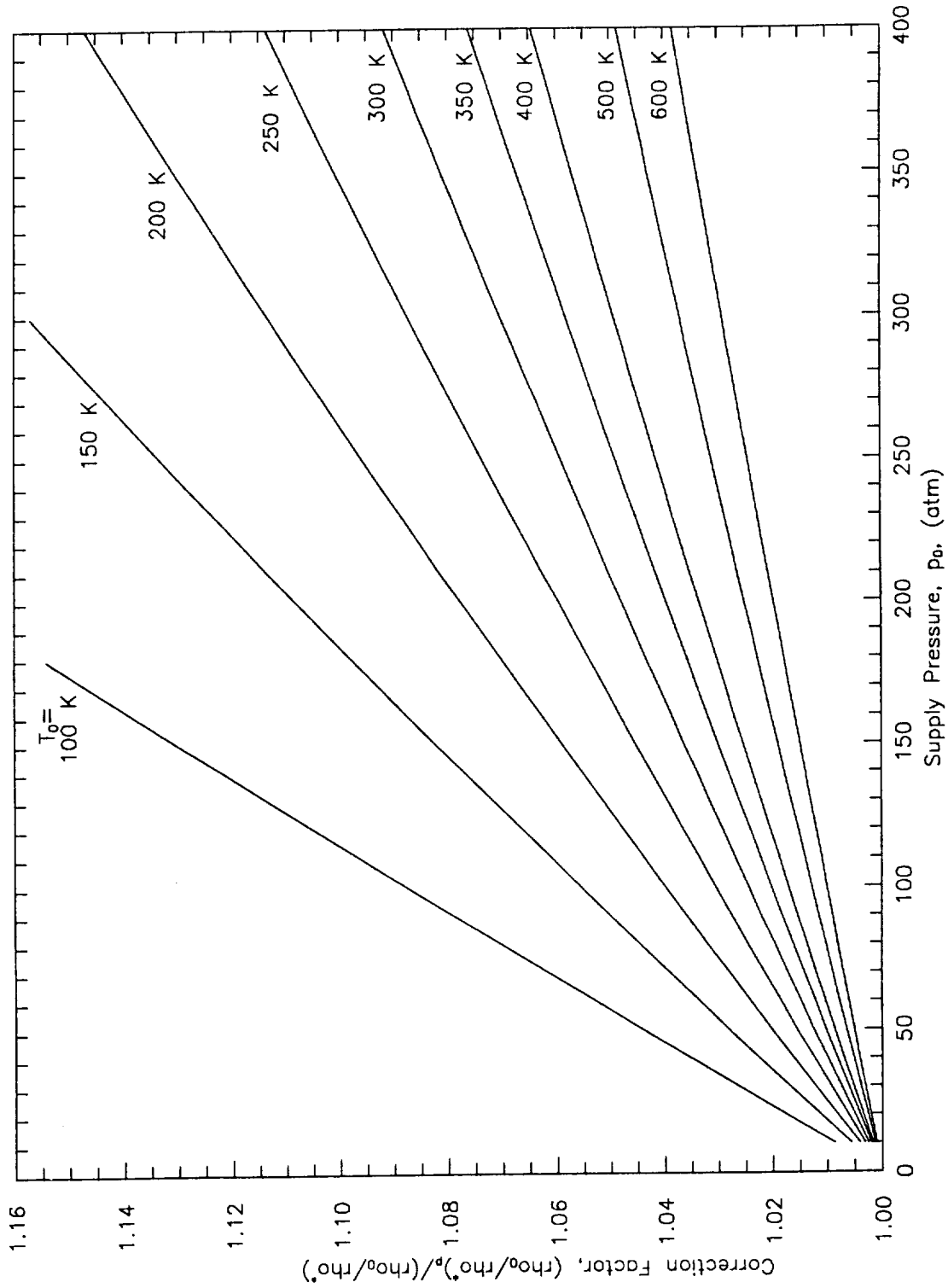
Figure 10. — Continued
(c) Density Ratio, (rho_0/rho^*)
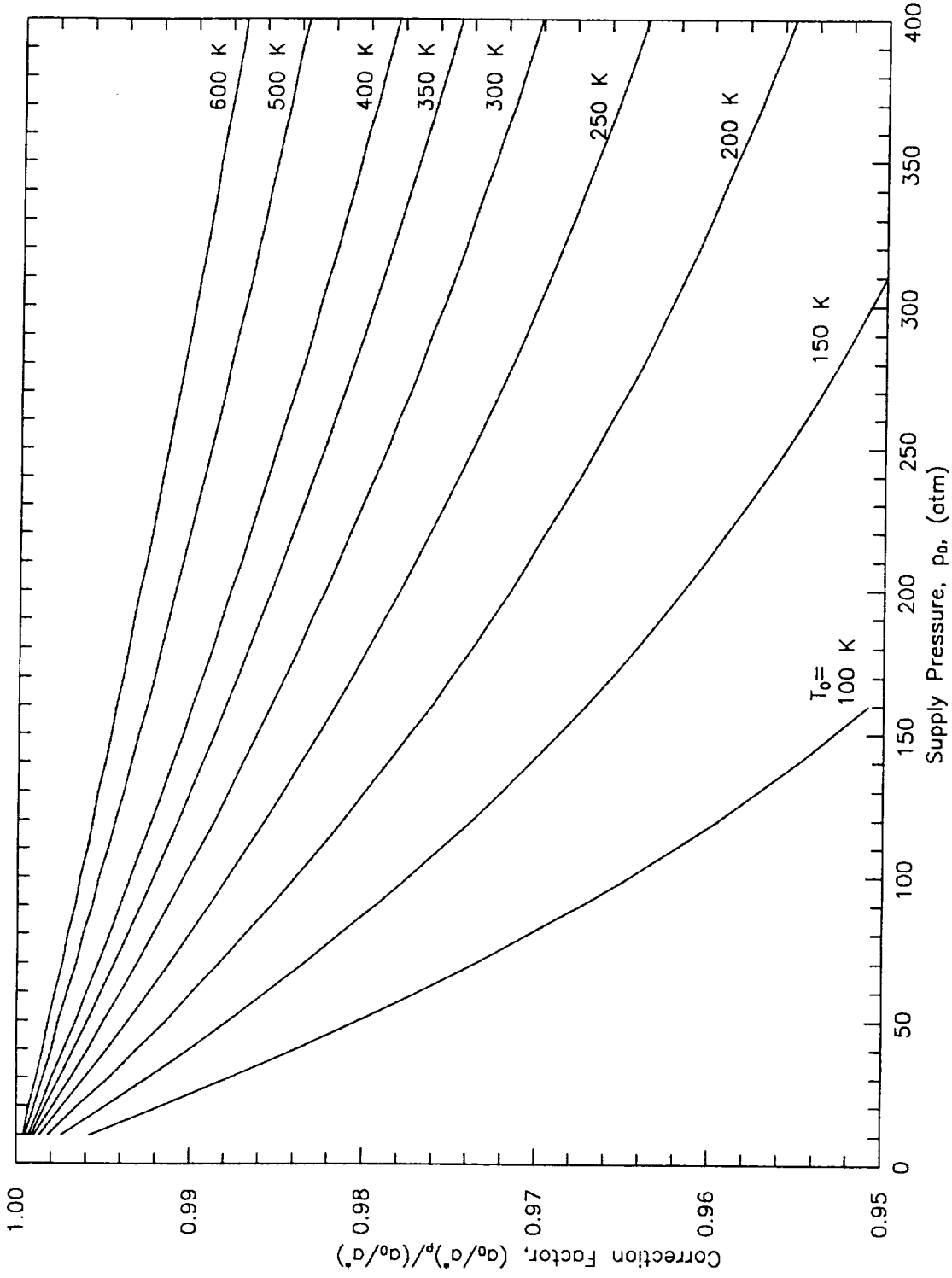
Figure 10. – Continued
(d) Speed of Sound Ratio (a_0/a^*)
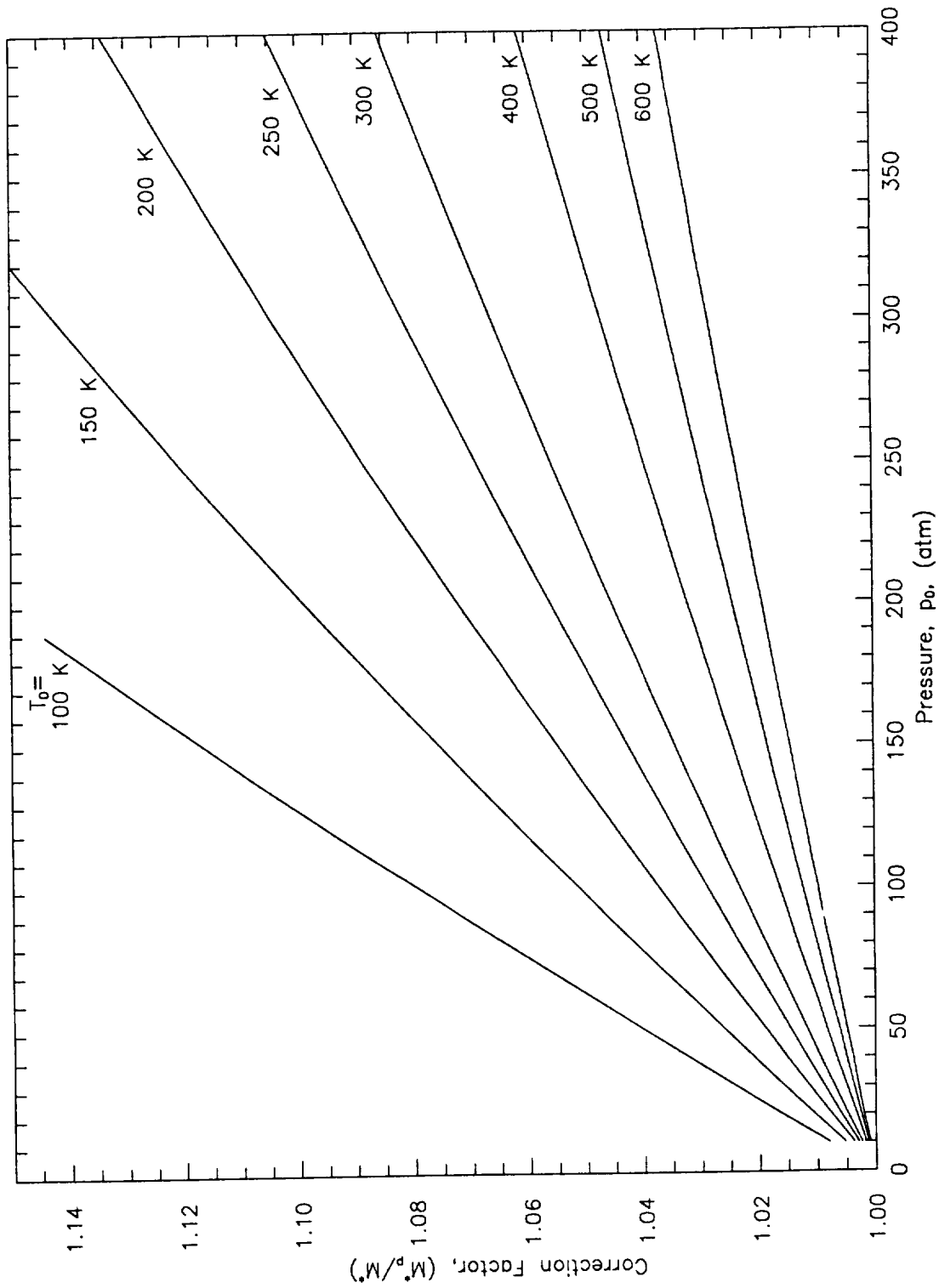
Figure 10. – Continued
(e) Mach Number Ratio, (M^*_p/M^*)

33
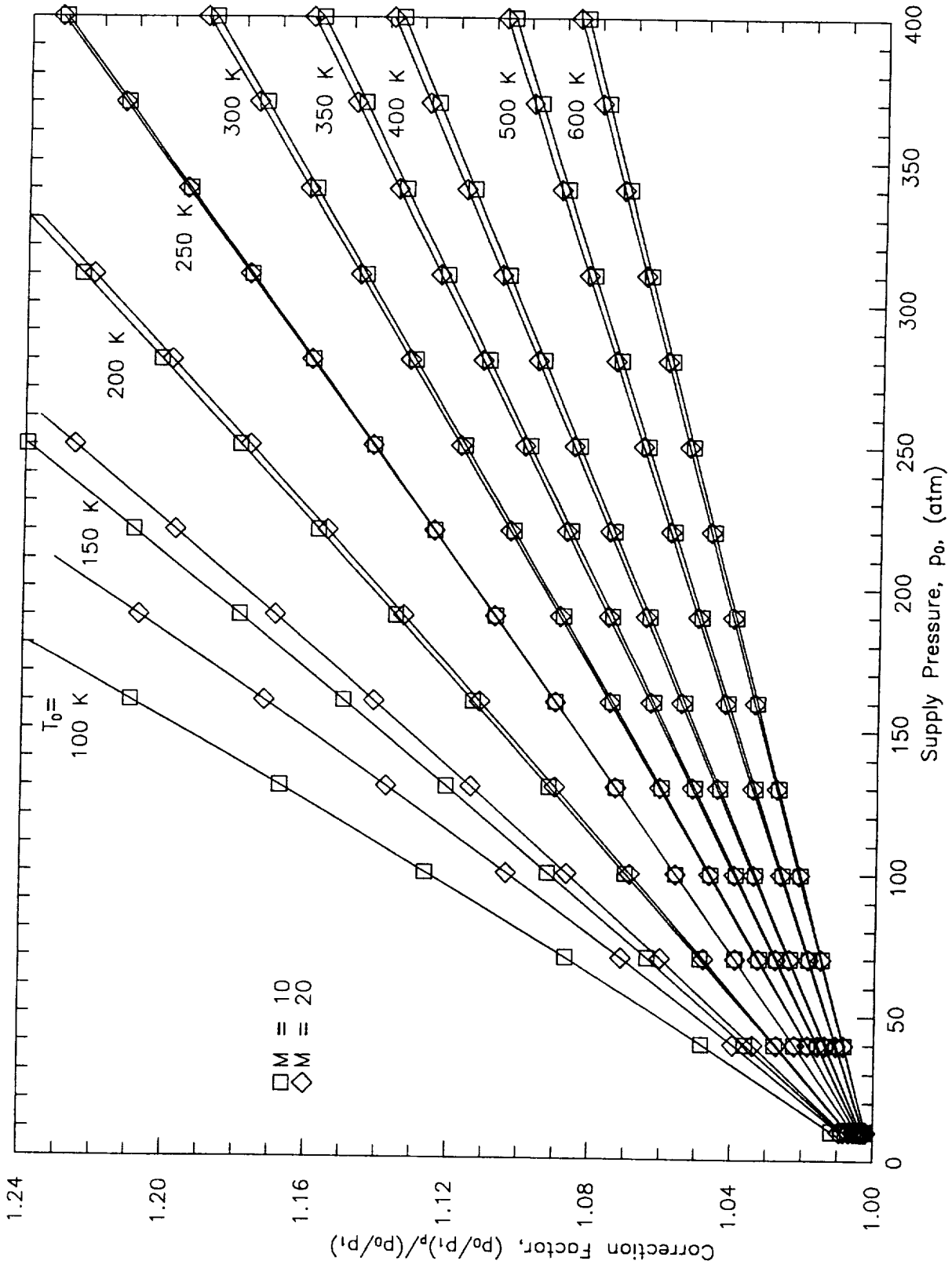
Figure 11. – Ratios of Real to Ideal Flow Parameters
(a) Pressure Ratio, (p_0/p_1)

34
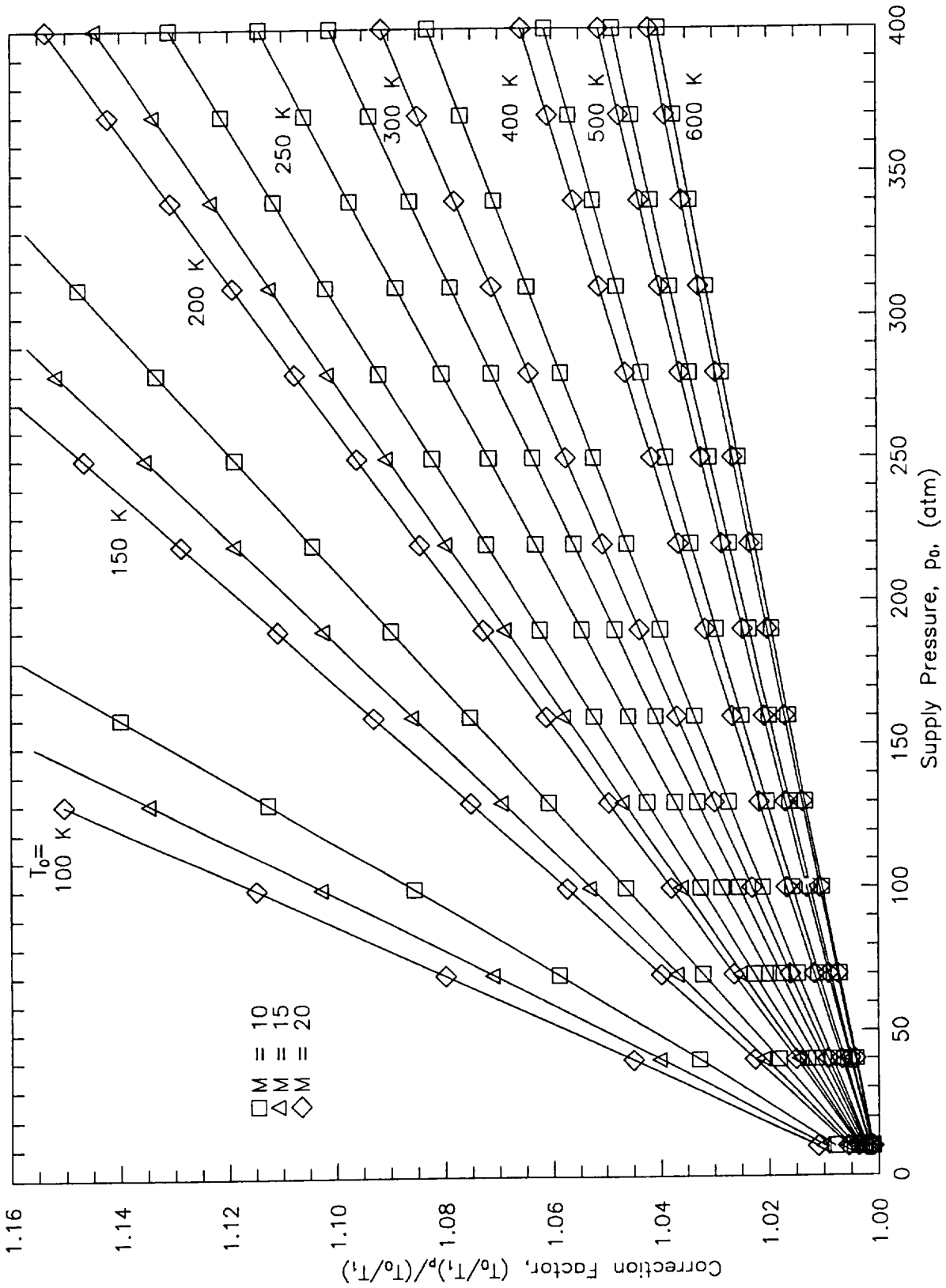
Figure 11. – Continued
(b) Temperature Ratio, (T_0/T_1)

35
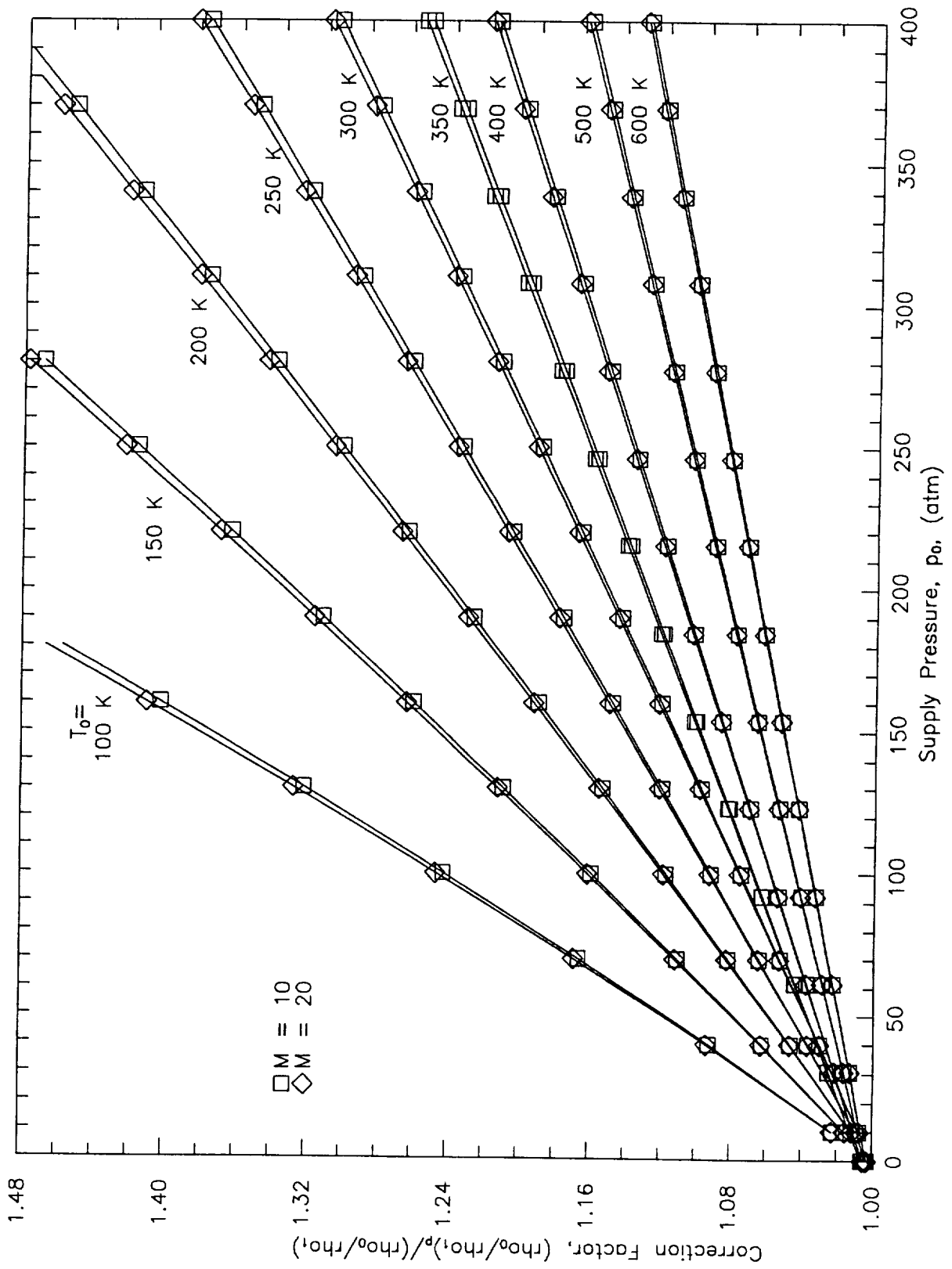
Figure 11. — Continued
(c) Density Ratio, (rho_o/rho_1)

Supply Pressure, p_o, (atm)

Correction Factor, $(rho_o/rho_1)_p/(rho_o/rho_1)$

$T_o = 100$ K

150 K

200 K

250 K

300 K

350 K

400 K

500 K

600 K

□ M = 10
◇ M = 20
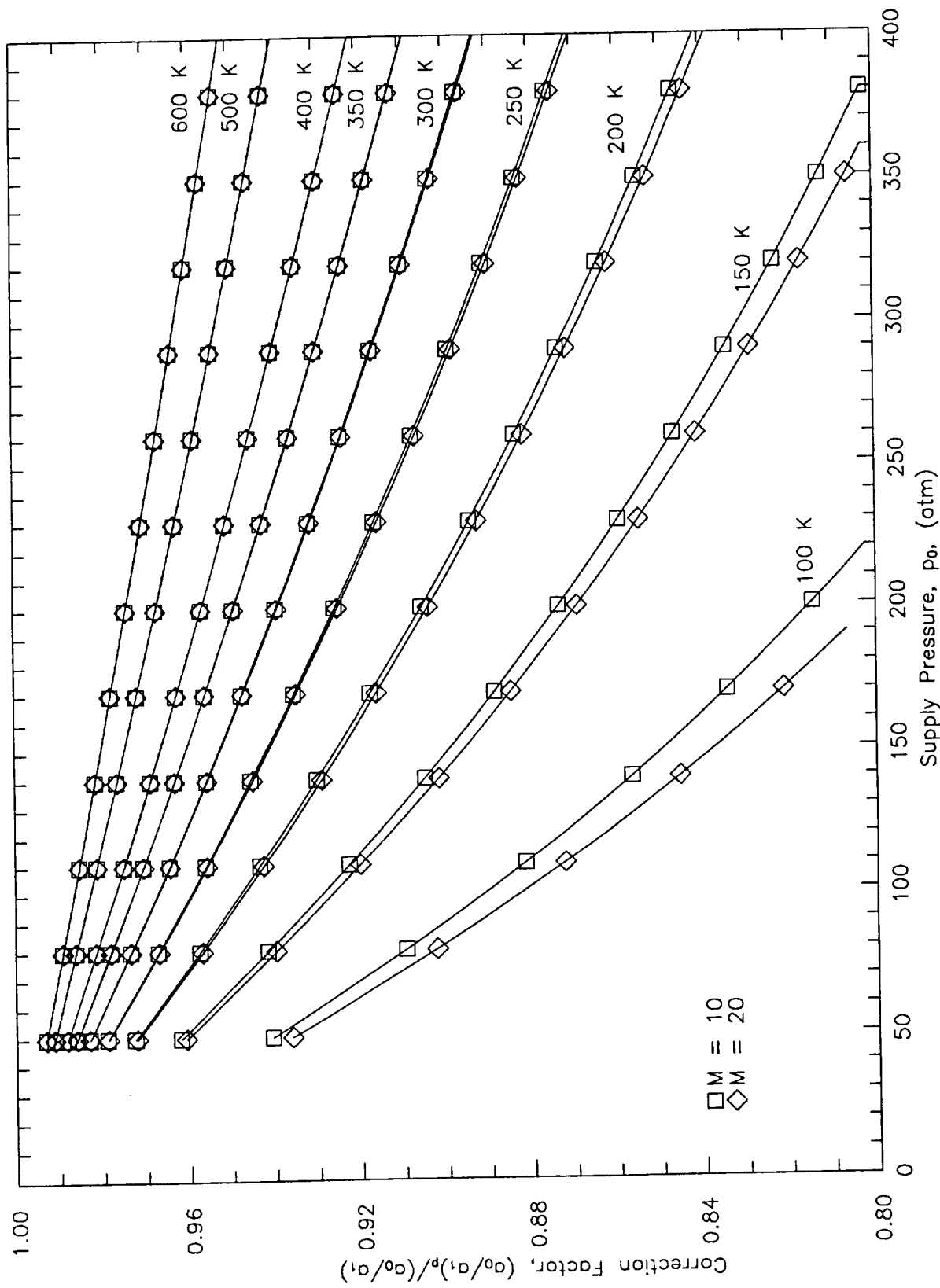
Figure 11. – Continued
(d) Speed of Sound Ratio (a_0/a_1)

37
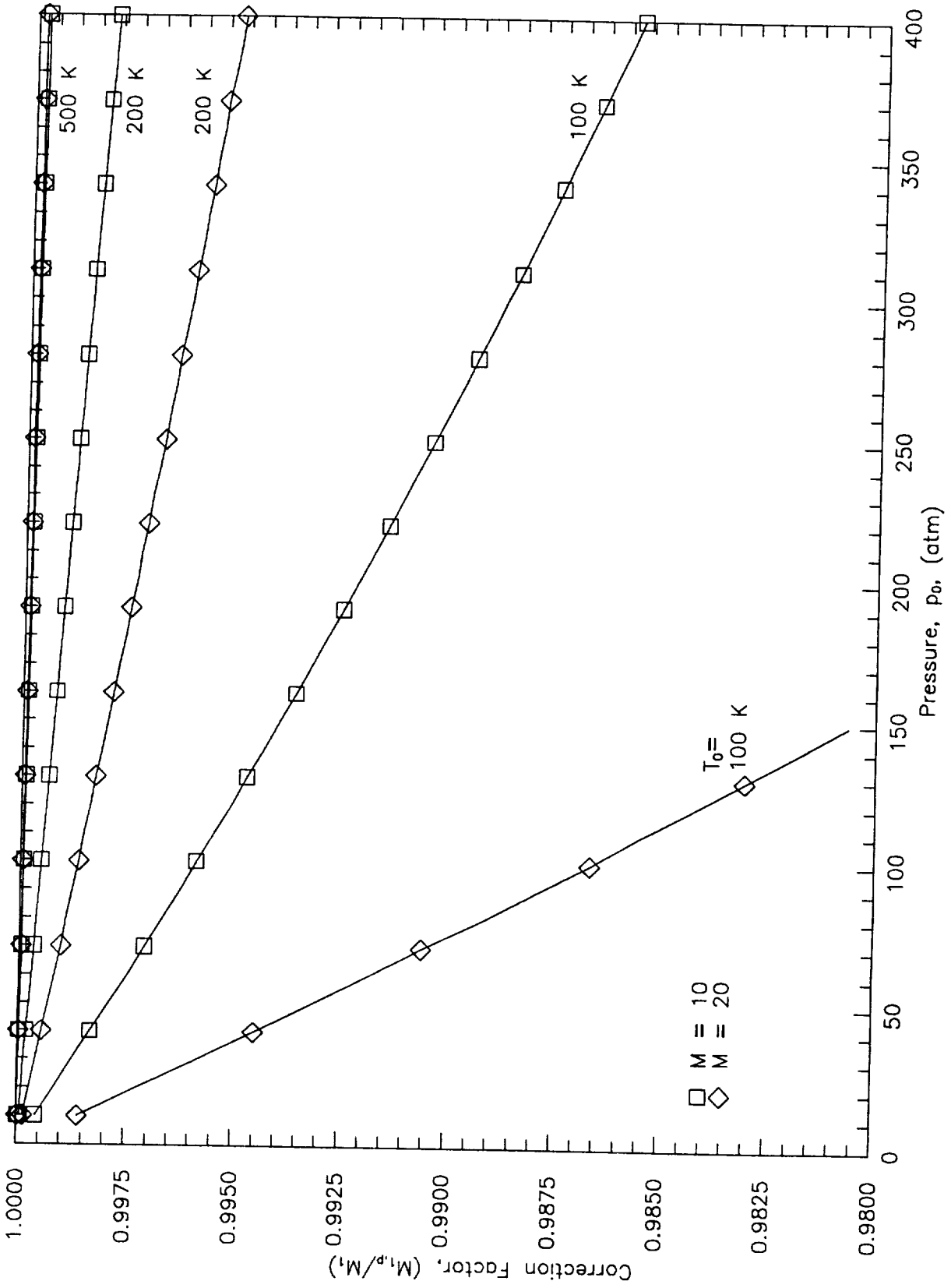
Figure 11. — Continued
(e) Mach Number Ratio, $(M_{1,p}/M_1)$
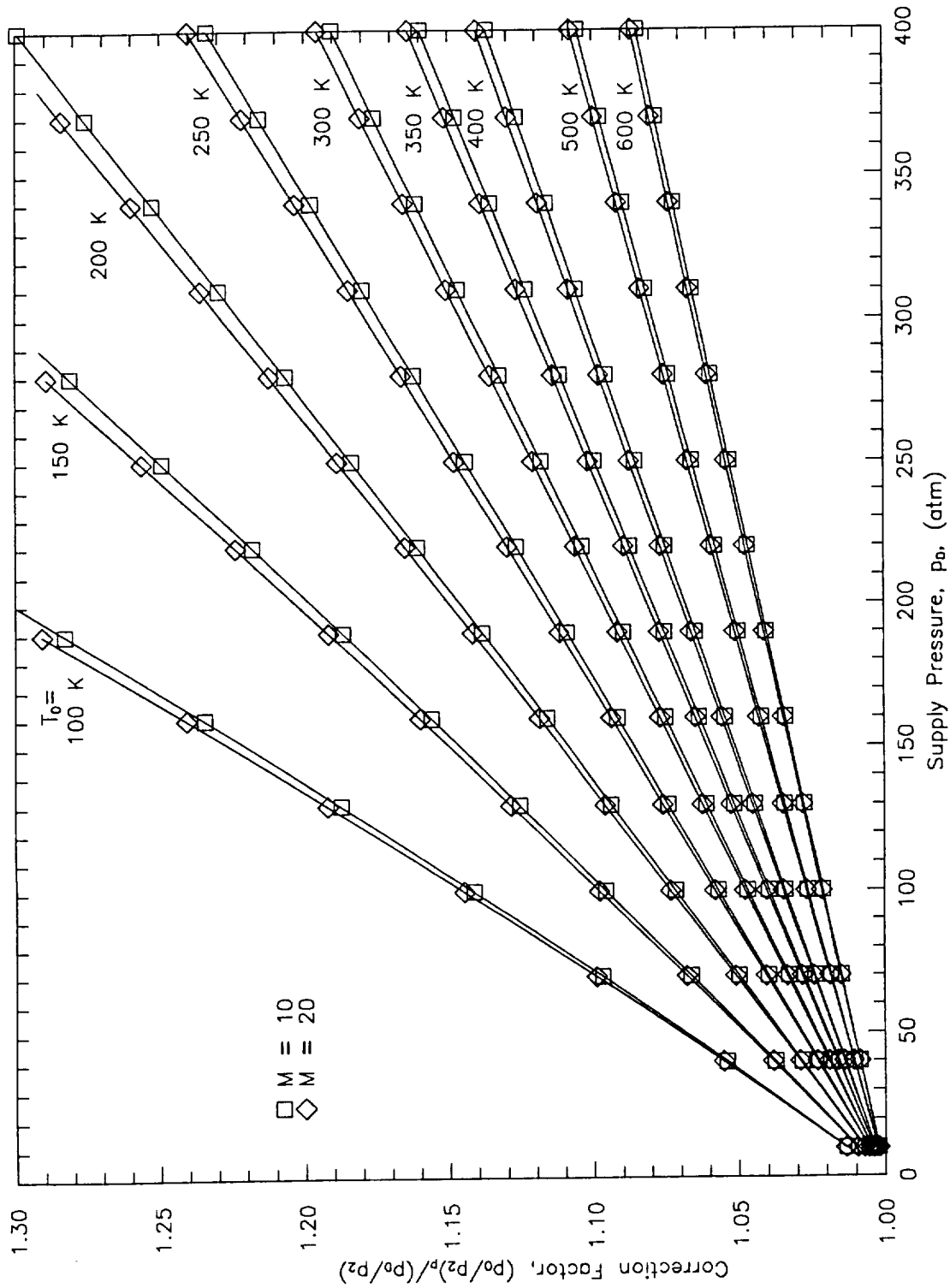
Figure 12. – Ratios of Real to Ideal Flow Parameters
(a) Pressure Ratio, (p_0/p_2)
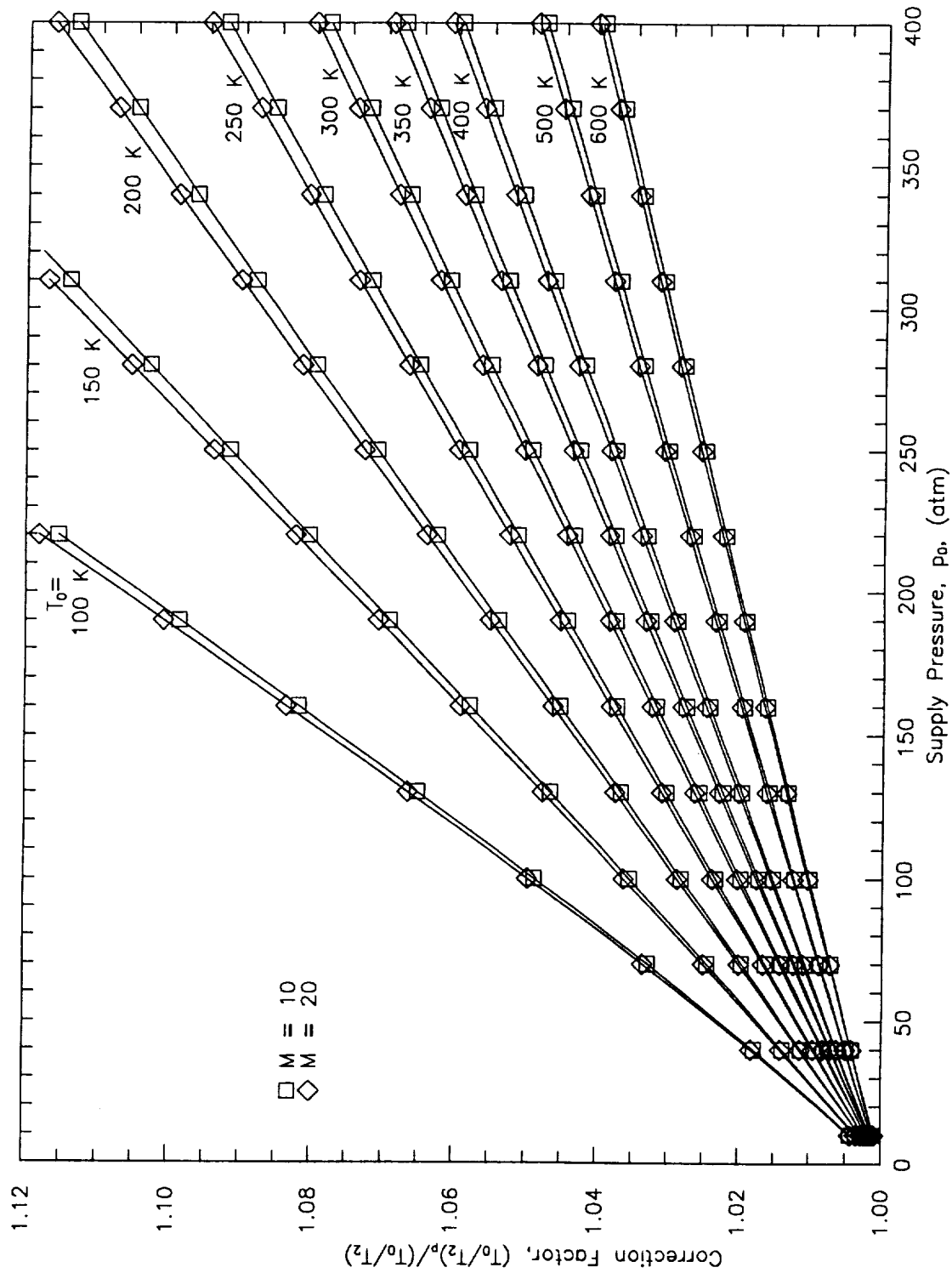
Figure 12. - Continued
(b) Temperature Ratio, (T_0/T_2)
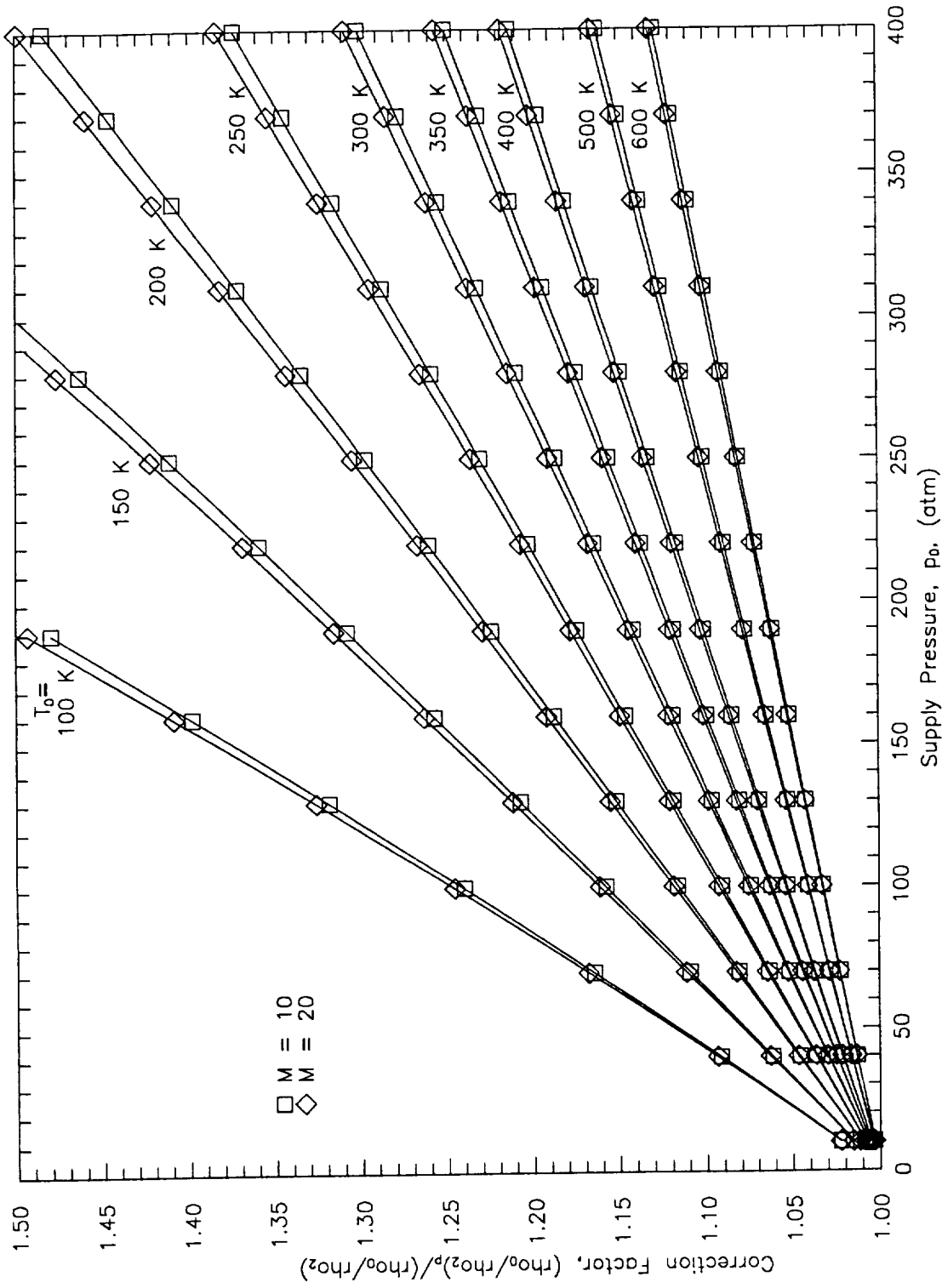
Figure 12. – Continued
(c) Density Ratio, (rho_o/rho_2)
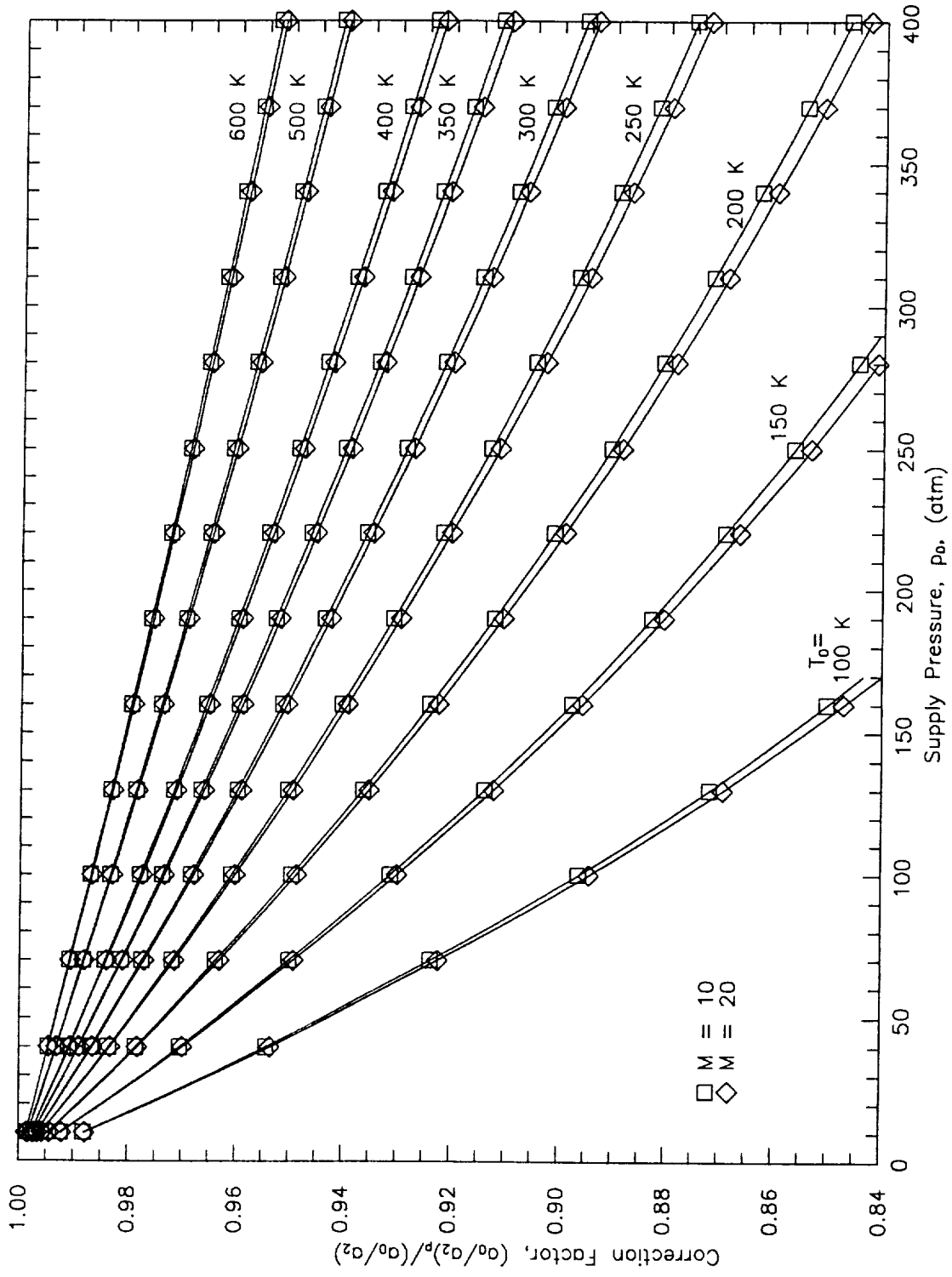
Figure 12. – Continued
(d) Speed of Sound Ratio, (a_0/a_2)
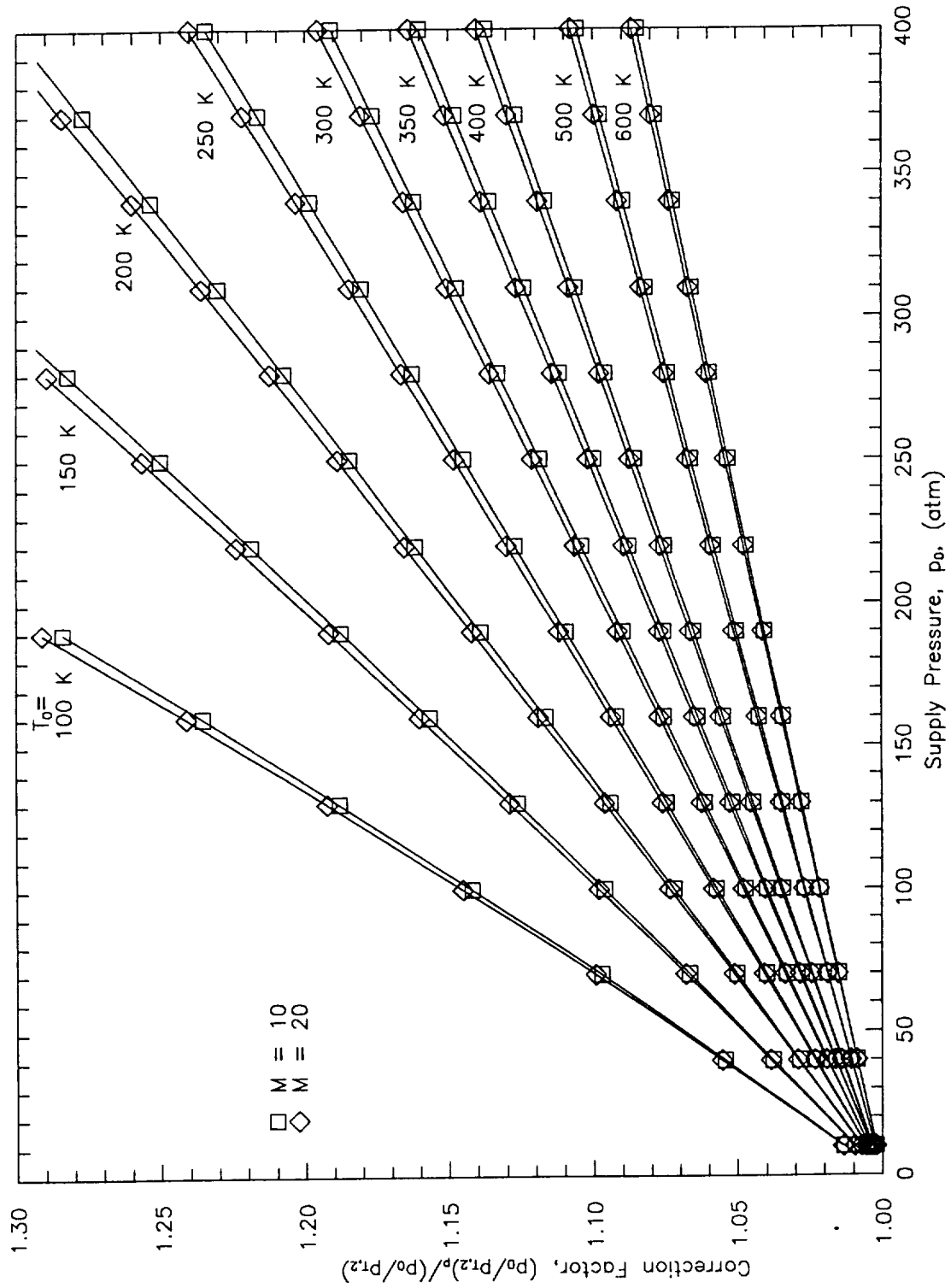
Figure 13. – Ratios of Real to Ideal Flow Parameters
(a) Pressure Ratio, $(p_0/p_{t,2})$
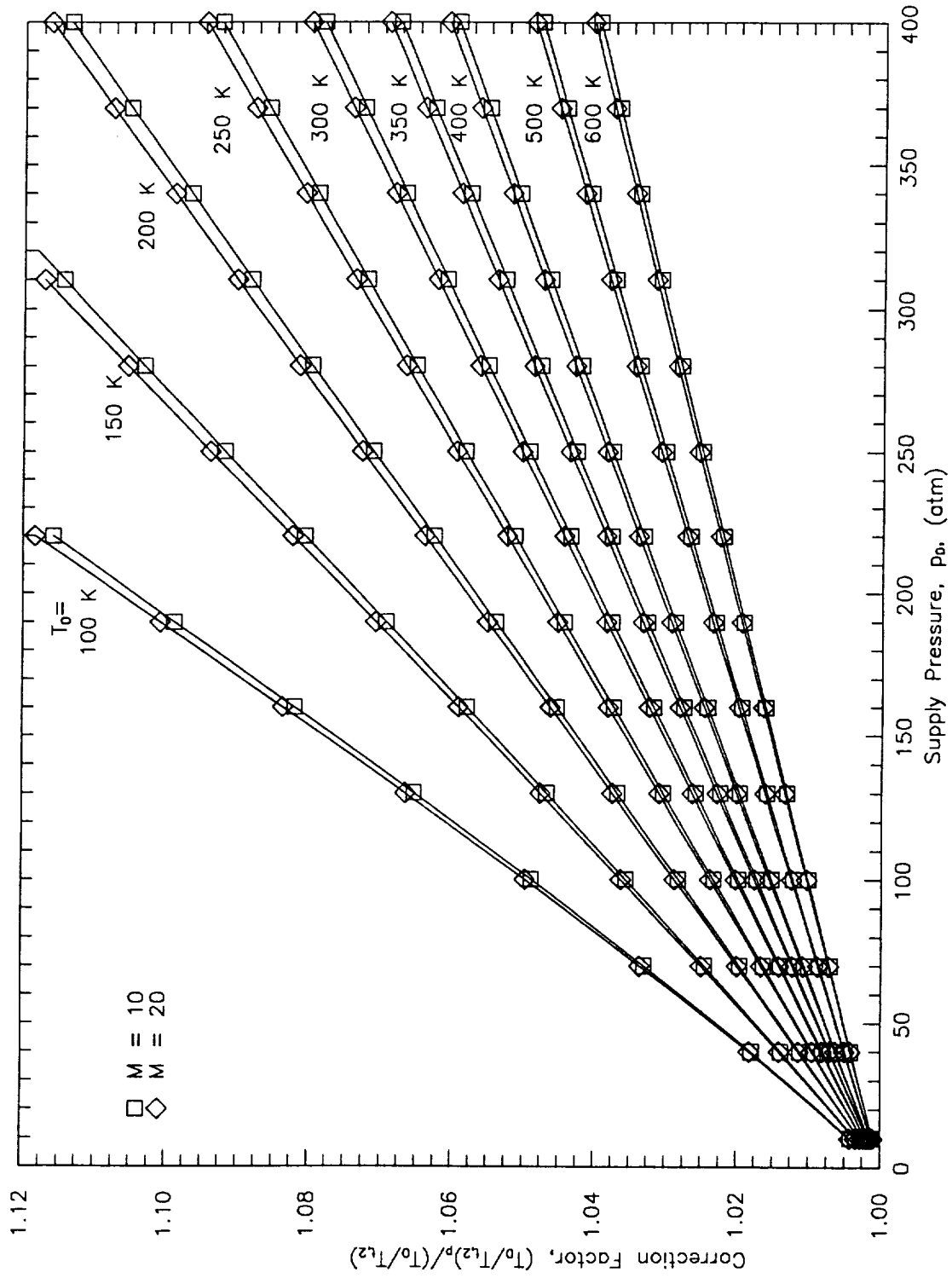
Figure 13. – Continued
(b) Temperature Ratio, $(T_0/T_{t,2})$

44
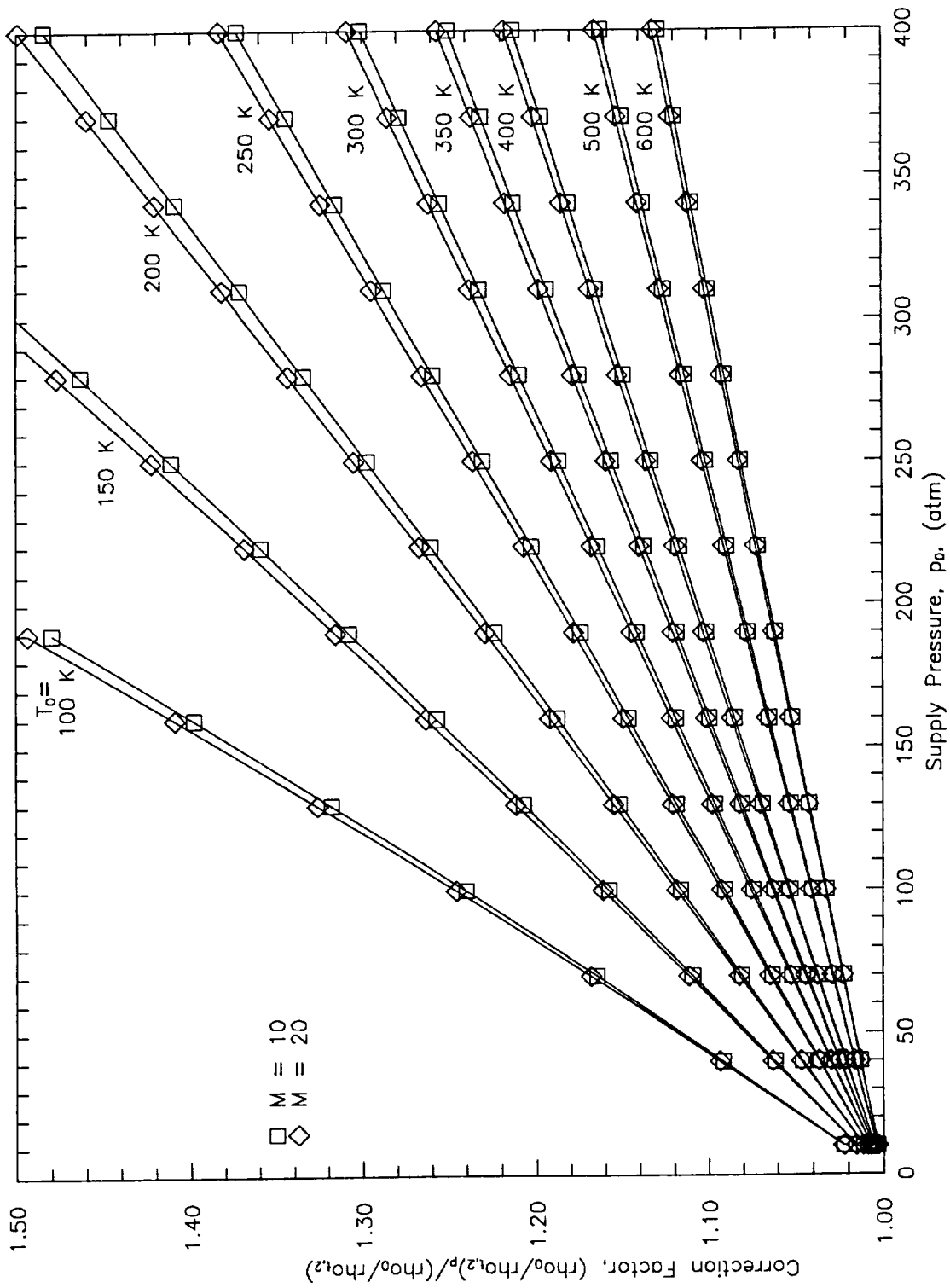
Figure 13. — Continued
(c) Density Ratio, $(rho_o/rho_{t,2})$

APPENDIX

```
        PROGRAM HEPROPS
C       BRIAN R. HOLLIS
C       MARCH 1992
C       ===========================================================
C       - PROGRAM HEPROPS IS DESIGNED TO EVALUATE THE WIND        =
C       = TUNNEL FLOW PROPERTIES OF HELIUM GAS.  IT IS            =
C       = ASSUMED THAT THE EXPANSION OF THE TEST GAS THROUGH THE  =
C       = TUNNEL IS ISENTROPIC AND THAT TOTAL ENTHALPY IS CONSERVED. =
C       - THE HELIUM IS TREATED AS A REAL GAS THROUGHOUT THE      =
C       = EXPANSION. THIS PROGRAM CAN ALSO BE USED TO EVALUATE THE =
C       - PROPERTIES OF REAL HELIUM AT A GIVEN TEMPERATURE AND    =
C       - PRESSURE OR TO COMPUTE FLOW PROPERTIES AT AN            =
C       = ARBITRARY MACH NUMBER IN THE TUNNEL EXPANSION SECTION.  =
C       ===========================================================
C
C       ===========================================================
C       =                 - PROGRAM VARIABLES AND CONSTANTS -     =
C       =                                                         =
C       -        A - SPEED OF SOUND                               =
C       =       CP - SPECIFIC HEAT AT CONSTANT PRESSURE           =
C       =       CV - SPECIFIC HEAT AT CONSTANT VOLUME             =
C       =     FLAG = SETS OPERATIONS TO BE CARRIED OUT IN PROGRAM =
C       =     HREF - REFERENCE ENTHALPY AT 298.15 K, 1atm         =
C       =        K - SPECIFIC HEAT RATIO                          =
C       -     MACH - MACH NUMBER                                  =
C       -      MHE - MOLAR MASS OF HELIUM                         =
C       =       MU - VISCOSITY                                    =
C       =        P - PRESSURE                                     =
C       - PITOT - TEST SECTION PITOT PRESSURE                     =
C       -        R - UNIVERSAL GAS CONSTANT                       =
C       -      RHO - DENSITY                                      =
C       =     SREF - REFERENCE ENTROPY AT 298.15 K, 1 atm         =
C       =       VB - SECOND VIRIAL COEFFICIENT                    =
C       =      VBP - FIRST DERIVATIVE OF VB WITH RESPECT TO TEMPERATURE  =
C       =     VBPP - SECOND DERIVATIVE OF VB WITH RESPECT TO TEMPERATURE =
C       =     VB3P = THIRD DERIVATIVE OF VB WITH RESPECT TO TEMPERATURE  =
C       =       VC - THIRD VIRIAL COEFFICIENT                     =
C       -      VCP - FIRST DERIVATIVE OF VC WITH RESPECT TO TEMPERATURE  =
C       =     VCPP = SECOND DERIVATIVE OF VC WITH RESPECT TO TEMPERATURE =
C       =     VC3P = THIRD DERIVATIVE OF VC WITH RESPECT TO TEMPERATURE  =
C       =        Z = COMPRESSIBILTY FACTOR                        =
C       =          THE VARIOUS ARRAYS APPEARING IN THE PROGRAM ARE FOR =
C       =        STORAGE OF DATA DURING ITERATIONS.               =
C       =                                                         =
C       -                  - VARIABLE SUFFIXES -                  =
C       =                                                         =
C       -        0 - SUPPLY RESERVOIR                             =
C       =        1 = FREESTREAM                                   =
C       =        2 - POST NORMAL SHOCK                            =
C       -        S - SONIC POINT                                  =
C       =        X = ARBITRARY POINT IN NOZZLE SPECIFIED BY MACH NUMBER =
C       =        T - STAGNATION CONDITION                         =
C       =        P - EQUIVALENT PERFECT GAS CONDITION             =
```

```
C       =           THUS, P2 IS STATIC PRESSURE BEHIND THE NORMAL SHOCK =
C       =       WHILE PT2P IS THE EQUIVALENT PERFECT GAS PITOT PRESSURE.=
C       =       CP, CV, AND K WITHOUT SUBSCRIPTS ARE THE PERFECT GAS    =
C       =       VALUES FOR HELIUM.                                      =
C       =                                                               =
C       =                       - UNITS -                               =
C       =                                                               =
C       =       ALL COMPUTATIONS IN THE PROGRAM ARE CARRIED OUT IN      =
C       =       S.I. UNITS.  INPUT DATA UNITS ARE NOTED BELOW.  THE     =
C       =       UNITS OF THE OUTPUT ARE UP TO THE USER TO FORMAT.       =
C       =                                                               =
C       ================================================================

        IMPLICIT REAL (A-Z)
        INTEGER I,J,L,N
        DIMENSION W(2),G(2),DWDG(4,4),OLDW(2),INV(2,2),OLDG(2),FP(2)
        COMMON K,MHE,R,SREF

C       ***** CONSTANTS *****
        HREF = 0
        MHE = 4.0026
        R = 8314
        SREF =  9.977E3

C       ***** IDEAL GAS VALUES *****
        K = 5/3.
        CP = K*(R/MHE)/(K-1)
        CV = (R/MHE)/(K-1)

C       ***************************************************
C       *                  - DATA INPUT -                *
C       *                                                *
C       *  FLAG = 0 COMPUTE PROPERTIES USING MACH1       *
C       *       = 1 COMPUTE PROPERTIES USING PITOT       *
C       *       = 2 JUST COMPUTE FLOW PROPERTIES AT      *
C       *           AN ARBITRARY, MACH NUMBER SPECIFIED  *
C       *           POINT IN THE NOZZLE.                 *
C       *       = 3 OUTPUT ALL CORRECTION FACTORS        *
C       *           FOR A GIVEN MACH NUMBER.             *
C       * MACH1 = TEST SECTION MACH NUMBER               *
C       *   P01 = SUPPLY PRESSURE (atm)                  *
C       * PITOT = TEST SECTION PITOT PRESSURE (atm)      *
C       *   T01 = SUPPLY TEMPERATURE (R)                 *
C       *                                                *
C       * ALL QUANTITIES ARE CONVERTED TO S.I. UNITS     *
C       ***************************************************


C       ***********************************************
C       * USERS CAN SUPPLY A DATA INPUT ROUTINE       *
C       * APPLICABLE TO THEIR SYSTEM AT THIS POINT,   *
C       * THE ROUTINE MUST READ IN THE QUANTITIES     *
C       * NOTED ABOVE AND CONVERT TO THEM S.I. UNITS. *
C       ***********************************************

C       ***** SAMPLE INPUT FOR TESTING PROGRAM *****
```

47

```
      PO = 300*1.01325E5
      TO = 300
      FLAG = 3
      MACH1 = 20
      PITOT = 1.01325E5*6.04042/14.697

C     ==================================================================
C     =                                                                =
C     =                     - SUPPLY CONDITIONS -                      =
C     =                                                                =
C     =      SUPPLY RESERVOIR HELIUM BEHAVES AS A REAL GAS             =
C     =      SUPPLY RESERVOIR PRESSURE AND TEMPERATURE                 =
C     =      ARE MEASURED.                                             =
C     =                                                                =
C     ==================================================================


C     *************************************************
C     * SOLVE FOR DENSITY FROM VIRIAL GAS EQUATION    *
C     * OF STATE.   USE IDEAL GAS EQUATION OF STATE   *
C     * FOR INITIAL ESTIMATE.                         *
C     *************************************************

      RHOO = PO/((R/MHE)*TO)
      CALL VIRIAL (TO,VBO,VBPO,VBPPO,VB3PO,VCO,VCPO,VCPPO,VC3PO)
100   F = PO - RHOO*(R/MHE)*TO*(1 + VBO*RHOO + VCO*RHOO**2)
      DFDRHO = -(R/MHE)*TO*(1 + 2*VBO*RHOO + 3*VCO*RHOO**2)
      OLDRHO = RHOO
      RHOO = RHOO - F/DFDRHO
      ERR = ABS((RHOO-OLDRHO)/OLDRHO)
      IF (ERR.GT.0.1E-5) GOTO 100

C     ***** COMPUTE RESERVOIR THERMODYNAMIC PROPERTIES *****

      ZO = 1 + VBO*RHOO + VCO*RHOO**2
      HO = (R/MHE)*TO*(K/(K-1) + RHOO*(VBO - TO*VBPO) + RHOO**2
C          /2*(2*VCO - TO*VCPO)) + HREF
      SO = (R/MHE)*(1/(K-1)*LOG(TO) - LOG(RHOO) - RHOO*(VBO + TO*
C          VBPO) - RHOO**2/2*(VCO + TO*VCPO)) + SREF
      CVO = (R/MHE)/(K-1) - (R/MHE)*TO*(RHOO*(2*VBPO + TO*VBPPO) +
C          RHOO**2/2*(2*VCPO + TO*VCPPO))
      DPDT = RHOO*(R/MHE)*(1 + RHOO*(VBO + TO*VBPO) + RHOO**2*(VCO +
C          TO*VCPO))
      DPDRHO = (R/MHE)*TO*(1 + 2*RHOO*VBO + 3*RHOO**2*VCO)
      CPO = CVO + (TO/RHOO**2)*DPDT**2/DPDRHO
      KO = CPO/CVO
      AO = (CPO*(DPDRHO)/CVO)**.5
      IF (FLAG.EQ.2) GOTO 750


C     *************************************************
C     * USERS'S CHOICE OF INPUTTING FREESTREAM MACH  *
C     * NUMBER OR PITOT PRESSURE.   GIVEN PITOT       *
C     * PRESSURE, MUST ITERATE FOR MACH NUMBER,       *
C     * GIVEN MACH NUMBER, NO ITERATION REQUIRED.     *
C     *************************************************
```

48

```
500       CONTINUE
          L = 1
          DELMACH = MACH1/1E5
          IF (FLAG.EQ.1) THEN
              MACH1 = MACH1 + DELMACH
          END IF
510       CONTINUE

C         ===================================================================
C         =                                                                 =
C         =                    - FREESTREAM CONDITIONS -                     =
C         =                                                                 =
C         =          ASSUME FREESTREAM BEHAVIOR IS REAL, TOTAL              =
C         =          ENTHALPY AND ENTROPY ARE CONSERVED.                     =
C         =                                                                 =
C         ===================================================================

          S1 = SO
          HT1 = HO


C         ***** COMPUTE REAL GAS PROPERTIES *****


C         ***** INITIAL GUESS AT (RHO1, T1) FROM PERFECT GAS EQUATIONS *****
          T1 = TO/(1 + (K-1)/2*MACH1**2)
          RHO1 = RHOO/(1 + (K-1)/2*MACH1**2)**(1/(K-1))
          CALL REALEXP (P1,T1,RHO1,Z1,MACH1,U1,A1,H1,HT1,S1,RE1,MU1,Q1)



C         ===================================================================
C         =                    - POST-SHOCK STATIC CONDITIONS -             =
C         =                                                                 =
C         =          ASSUME POST-SHOCK STATIC GAS BEHAVIOR                   =
C         =          IS REAL AND TOTAL ENTHALPY IS CONSERVED.               =
C         =                                                                 =
C         ===================================================================

          HT2 = HO

C         ***********************************************************
C         * ITERATE FOR RHO2, T2 TO SATISFY CONSERVATION OF       *
C         * MOMENTUM ACROSS SHOCK AND TOTAL ENTHALPY USING TWO-   *
C         * VARIABLE (RHO2, T2) SHOOTING TECHNIQUE.   INTIAL      *
C         * ESTIMATE FROM PERFECT GAS RELATIONS.                  *
C         ***********************************************************

          RHO2 = RHO1*(K+1)*MACH1**2/((K-1)*MACH1**2 + 2)
          T2 = T1*(2*K*MACH1**2 - (K-1))*((K-1)*MACH1**2 + 2)/((K+1)
     C         **2*MACH1**2)

          G(1) = RHO2
          G(2) = T2
150       CONTINUE
          CALL VIRIAL (T2,VB2,VBP2,VBPP2,VB3P2,VC2,VCP2,VCPP2,VC3P2)
          W(1) = P1 + RHO1*U1**2 - RHO2*(R/MHE)*T2*(1 + RHO2*VB2 +
     C          RHO2**2*VC2) - (RHO1*U1)**2/RHO2
```

49

```
        W(2) = HT2 - (R/MHE)*T2*(K/(K-1) + RHO2*(VB2 - T2*VBP2) +
   C           RHO2**2/2*(2*VC2 - T2*VCP2)) - (RHO1*U1/RHO2)**2/2


   C     ***** COMPUTE JACOBIAN OF MATRIX [W] *****
         DWDG(1,1) = -(R/MHE)*T2*(1 + 2*RHO2*VB2 + 3*RHO2**2*VC2) +
   C           (RHO1*U1/RHO2)**2
         DWDG(1,2) = -(R/MHE)*(RHO2 + RHO2**2*(VB2 + T2*VBP2) + RHO2**3
   C           *(VC2 + T2*VCP2))
         DWDG(2,1) = -(R/MHE)*T2*((VB2 - T2*VBP2) + RHO2*(2*VC2 - T2*
   C           VCP2)) + (RHO1*U1)**2/RHO2**3
         DWDG(2,2) =  -(R/MHE)*(K/(K-1) + RHO2*(VB2 - T2*VBP2 + T2**2
   C           *VBPP2) + RHO2**2/2*(VC2 - T2**2*VCPP2))


   C     ***** INVERT [DWDG] *****
         INV(2,1) = 1/(DWDG(1,2) - DWDG(1,1)*DWDG(2,2)/DWDG(2,1))
         INV(1,1) = -INV(2,1)*DWDG(2,2)/DWDG(2,1)
         INV(2,2) = 1/(DWDG(2,2) - DWDG(1,2)*DWDG(2,1)/DWDG(1,1))
         INV(1,2) = -INV(2,2)*DWDG(1,2)/DWDG(1,1)


   C     ***** COMPUTE NEW RHO2, T2 *****
         DO 200 I = 1,2
            OLDG(I) = G(I)
            DO 210 J = 1,2
               G(I) = G(I) - INV(I,J)*W(J)
   210      CONTINUE
   200   CONTINUE
         RHO2 = G(1)
         T2 = G(2)
         ERR = ((G(1) - OLDG(1))**2 + (G(2) - OLDG(2))**2)**.5
         IF (ERR.GT.1E-5) GOTO 150


   C     ***** COMPUTE REMAINING THERMODYNAMIC PROPERTIES *****
         Z2 = 1 + RHO2*VB2 + RHO2**2*VC2
         P2 = Z2*RHO2*(R/MHE)*T2
         U2 = ((P1 + RHO1*U1**2 - P2)/RHO2)**.5
         CV2 = (R/MHE)/(K-1) - (R/MHE)*T2*(RHO2*(2*VBP2 + T2*VBPP2) +
   C         RHO2**2/2*(2*VCP2 + T2*VCPP2))
         DPDT = RHO2*(R/MHE)*(1 + RHO2*(VB2 + T2*VBP2) + RHO2**2*(VC2 +
   C           T2*VCP2))
         DPDRHO = (R/MHE)*T2*(1 + 2*RHO2*VB2 + 3*RHO2**2*VC2)
         CP2 = CV2 + (T2/RHO2**2)*DPDT**2/DPDRHO
         K2 = CP2/CV2
         A2 = (CP2*(DPDRHO)/CV2)**.5
         MACH2 = U2/A2
         H2  = HT2 - U2**2/2
         CALL VISCOS(T2,MU2)
         RE2 = RHO2*U2/MU2
         Q2 = RHO2*U2**2/2
         S2 = (R/MHE)*(1/(K-1)*LOG(T2) - LOG(RHO2) - RHO2*(VB2 + T2*
   C           VBP2) - RHO2**2/2*(VC2 + T2*VCP2)) + SREF


   C     ============================================================
   C     =                                                          =
   C     =            - POST-SHOCK STAGNATION CONDITIONS -          =
   C     =                                                          =
```

```
C        =         ASSUME POST-SHOCK STAGNATION POINT GAS          =
C        =         BEHAVIOR IS REAL, ENTROPY AND TOTAL             =
C        =         ENTHALPY ARE CONSERVED FROM STATIC VALUES.      =
C        =                                                         =
C        ==========================================================


C        ************************************************
C        * ITERATE FOR RHOT2, TT2 TO SATISFY CONSERVATION    *
C        * OF ENTROPY AND TOTAL ENTHALPY USING TWO-VARIABLE   *
C        * (RHOT2, TT2) SHOOTING TECHNIQUE.   INITIAL         *
C        * ESTIMATE FROM PERFECT GAS RELATIONS.               *
C        ************************************************

         TT2 = TO
         RHOT2 = RHO2*(1 + (K-1)/2*MACH2**2)**(1/(K-1))*2


         G(1) = RHOT2
         G(2) = TT2


350      CONTINUE
         CALL VIRIAL (TT2,VBT2,VBPT2,VBPPT2,VB3PT2,VCT2,VCPT2,VCPPT2,
   C                  VC3PT2)
         W(1) = HT2 - (R/MHE)*TT2*(K/(K-1) + RHOT2*(VBT2 - TT2*VBPT2) +
   C         RHOT2**2/2*(2*VCT2 - TT2*VCPT2))
         W(2) = S2 - (R/MHE)*(1/(K-1)*LOG(TT2) - LOG(RHOT2) - RHOT2*
   C           (VBT2 + TT2*VBPT2) - RHOT2**2/2*(VCT2 + TT2*VCPT2))-SREF


C        ***** COMPUTE JACOBIAN OF MATRIX [W] *****
         DWDG(1,1) = -(R/MHE)*TT2*((VBT2 - TT2*VBPT2) + RHOT2*(2*VCT2 -
   C              TT2*VCPT2))
         DWDG(1,2) = -(R/MHE)*(K/(K-1) + RHOT2*(VBT2 - VBPT2*TT2 - TT2
   C              **2*VBPPT2) + RHOT2**2/2*(2*VCT2-TT2**2*VCPPT2))
         DWDG(2,1) = -(R/MHE)*(-1/RHOT2 - (VBT2 + TT2*VBPT2) - RHOT2*
   C              (VCT2 + TT2*VCPT2))
         DWDG(2,2) = -(R/MHE)*(1/((K-1)*TT2) - RHOT2*(2*VBPT2
   C              + TT2*VBPPT2) - RHOT2**2/2*(2*VCPT2 + TT2*VCPPT2))


C        *** INVERT [DWDG] ***
         INV(2,1) = 1/(DWDG(1,2) - DWDG(1,1)*DWDG(2,2)/DWDG(2,1))
         INV(1,1) = -INV(2,1)*DWDG(2,2)/DWDG(2,1)
         INV(2,2) = 1/(DWDG(2,2) - DWDG(1,2)*DWDG(2,1)/DWDG(1,1))
         INV(1,2) = -INV(2,2)*DWDG(1,2)/DWDG(1,1)


C        ***** COMPUTE NEW RHOT2, TT2 *****
         DO 300 I = 1,2
            OLDG(I) = G(I)
            DO 310 J = 1,2
               G(I) = G(I) - INV(I,J)*W(J)
310         CONTINUE
300      CONTINUE
         RHOT2 = G(1)
         TT2 = G(2)
         ERR = ((G(1) - OLDG(1))**2 + (G(2) - OLDG(2))**2)**.5
         IF (ERR.GT.1E-5) GOTO 350
```

51

```
          ZT2 = 1 + RHOT2*VBT2 + RHOT2**2*VCT2
          PT2 = ZT2*RHOT2*(R/MHE)*TT2

C         ***** NEW ESTIMATE FOR MACH1 *****
          FP(L) = PITOT - PT2
          IF (FLAG.EQ.1 .AND. L.LT.2) THEN
              L = L + 1
              MACH1 = MACH1 - DELMACH
              GOTO 510
          END IF
          IF (FLAG.EQ.1) THEN
              OLDMACH = MACH1
              DELFP = (FP(1)-FP(2))/DELMACH
              MACH1 = MACH1 - FP(2)/DELFP
              ERR = ABS((MACH1-OLDMACH)/OLDMACH)
              IF (ERR.GT.1E-5) GOTO 500
          END IF

C         ***** COMPUTE REMAINING THERMODYNAMIC PROPERTIES *****
          CVT2 = (R/MHE)/(K-1) - (R/MHE)*TT2*(RHOT2*(2*VBPT2 + TT2*VBPPT2)
C                + RHOT2**2/2*(2*VCPT2 + TT2*VCPPT2))
          DPDT = RHOT2*(R/MHE)*(1 + RHOT2*(VBT2+TT2*VBPT2)+RHOT2**2*(VCT2
C                + TT2*VCPT2))
          DPDRHO = (R/MHE)*TT2*(1 + 2*RHOT2*VBT2 + 3*RHOT2**2*VCT2)
          CPT2 = CVT2 + (TT2/RHOT2**2)*DPDT**2/DPDRHO
          KT2 = CPT2/CVT2
          AT2 = (CPT2*(DPDRHO)/CVT2)**.5
          CALL VISCOS(TT2,MUT2)


C         ================================================================
C         =                                                              =
C         =                    - SONIC POINT PROPERTIES -                =
C         =                                                              =
C         =        ASSUME SONIC POINT GAS BEHAVIOR IS REAL,             =
C         =        ENTROPY AND TOTAL ENTHALPY ARE CONSERVED             =
C         =        FROM THEIR SUPPLY RESERVOIR VALUES.                  =
C         =                                                              =
C         ================================================================

          MACHS = 1
          HTS = H0
          SS = S0

C         ***** INITIAL GUESS AT RHOS, TS FROM PERFECT GAS RELATIONS *****

          RHOS = RHO0/(1 + (K-1)/2*MACHS**2)**(1/(K-1))
          TS = T0/(1 + (K-1)/2*MACHS**2)
          CALL REALEXP (PS,TS,RHOS,ZS,MACHS,US,AS,HS,HTS,SS,RES,MUS,QS)
          GOTO 800

750       CONTINUE
C         ================================================================
C         =                                                              =
C         =                    - PROPERTIES AT ARBITRARY POINT -         =
```

52

```
C        -                                                             =
C        -          ASSUME ARBITRARY POINT BEHAVIOR IS REAL,           =
C        -          ENTROPY AND TOTAL ENTHALPY ARE CONSERVED           =
C        -          FROM THEIR SUPPLY RESERVOIR VALUES. INPUT          =
C        -          MACH NUMBER TO SPECIFY POINT                       =
C        =                                                             =
C        ===============================================================

         WRITE (*,755)
755      FORMAT (1X,'ENTER MACH NUMBER AT ARBITRARY POINT IN NOZZLE ')
         READ (*,*) MACHX
         HTX = HO
         SX = SO


C        ***** INITIAL GUESS AT RHOX, TX FROM PERFECT GAS RELATIONS *****
         RHOX = RHOO/(1 + (K-1)/2*MACHX**2)**(1/(K-1))
         TX = TO/(1 + (K-1)/2*MACHX**2)

         CALL REALEXP (PX,TX,RHOX,ZX,MACHX,UX,AX,HX,HTX,SX,REX,MUX,QX)


800      CONTINUE
         IF (FLAG.NE.3) GOTO 900
C        ===============================================================
C        =                - PERFECT GAS CORRECTION FACTORS -           =
C        =                                                             =
C        =          COMPUTE CORRECTION FACTORS TO BE APPLIED TO        =
C        =          PERFECT GAS CALCULATIONS.                          =
C        =                                                             =
C        ===============================================================


C        *** SUPPLY ***
         TOP = HO/CP
         RHOOP = EXP(CV*LOG(TOP)/(R/MHE) - (SO - SREF)/(R/MHE))
         POP = RHOOP*(R/MHE)*TOP
         AOP = (K*(R/MHE)*TOP)**.5


C        *** SONIC POINT ***
         TSP = HS/CP
         ASP = (K*(R/MHE)*TSP)**.5
         MACHSP = ((TOP/TSP - 1)*2/(K-1))**.5
         USP = MACHSP*ASP
         PSP = POP/(1 + (K-1)/2*MACHSP**2)**(K/(K-1))
         RHOSP = PSP/((R/MHE)*TSP)


C        *** POST-SHOCK STAGNATION ***
         TT2P = HT2/CP
         RHOT2P = EXP(CV*LOG(TT2P)/(R/MHE) - (S2 - SREF)/(R/MHE))
         PT2P = RHOT2P*(R/MHE)*TT2P
         AT2P = (K*(R/MHE)*TT2P)**.5


C        *** FREESTREAM ***
         MACH1P = MACH1
400      DELMACH = MACH1P/10000
         CALL ITER(PT2P,POP,MACH1P,F1)
         CALL ITER(PT2P,POP,MACH1P+DELMACH,F2)
```

```
                F = (F2-F1)/DELMACH
                OLDMACH = MACH1P
                MACH1P = MACH1P - F1/F
                ERROR =  ABS((MACH1P-OLDMACH)/OLDMACH)
                IF (ERROR.GT.1E-5) GOTO 400
                P1P = POP/(1 + (K-1)/2*MACH1P**2)**(K/(K-1))
                T1P = TOP/(1 + (K-1)/2*MACH1P**2)
                RHO1P = P1P/((R/MHE)*T1P)
                A1P = (K*(R/MHE)*T1P)**.5
                U1P = MACH1P*A1P
                H1P = CP*T1P
                CALL VISCOS(T1P,MU1P)
                RE1P = RHO1P*U1P/MU1P
                Q1P = RHO1P*U1P**2/2

C               *** POST-SHOCK STATIC ***
                MACH2P = (((K-1)*MACH1P**2 + 2)/(2*K*MACH1P**2 - (K-1)))**.5
                T2P = TT2P/(1 + (K-1)/2*MACH2P**2)
                P2P = PT2P/(1 + (K-1)/2*MACH2P**2)**(K/(K-1))
                RHO2P = P2P/((R/MHE)*T2P)
                H2P = CP*T2P
                A2P = (K*(R/MHE)*T2P)**.5
                U2P = MACH2P*A2P
                CALL VISCOS(T2P,MU2P)
                RE2P = RHO2P*U2P/MU2P
                Q2P = RHO2P*U2P**2/2

900             CONTINUE

C       ***********************************************************************
C       * USERS CAN SUPPLY AN OUTPUT ROUTINE                                 *
C       * APPLICABLE TO THEIR REQUIREMENTS                                   *
C       * AT THIS POINT.                                                     *
C       *                                                                    *
C       *    OUTPUT ROUTINE MUST DO THE FOLLOWING:                           *
C       * IF FLAG = 0 OR 1 OUTPUT CONDITIONS AT ALL POINTS IN TUNNEL         *
C       * IF FLAG = 2 OUTPUT CONDITIONS AT ARBITRARY POINT                   *
C       * IF FLAG = 3 OUTPUT EQUIVALENT PERFECT GAS CORRECTION FACTORS       *
C       ***********************************************************************

C       *** SAMPLE OUTPUT FOR TESTING PROGRAM ***
C       ***** FILE OUTPUT *****
                OPEN (UNIT = 21, FILE = 'RUNDATA.DAT')
                WRITE (21,600)
600             FORMAT (1X,/,'SUPPLY CONDITIONS')
                WRITE (21,605)
605             FORMAT (2X,'PO (psi)      TO (R)      RHO0(slug/ft^3)      ZO')
                WRITE (21,700) P0*1.4504E-4,TO*1.8,RHO0*6.8521E-2/3.2808**3,ZO

                IF (FLAG.EQ.2) THEN
                  WRITE (21,645) MACHX
645               FORMAT (/,1X,'CONDITIONS AT M = ',F5.2,' POINT IN NOZZLE')
                  WRITE (21,650)
650               FORMAT (2X,'PX (psi)      TX (R)      RHOX(slug/ft^3)      ZX')
                  WRITE (21,700) PX*1.4504E-4,TX*1.8,RHOX*6.8521E-2/3.2808**3,ZX
```

```
         WRITE (21,655)
655      FORMAT (1X,' UX (ft/s)        MUX (slug/ft-s)        REX(1/ft) ')
         WRITE (21,710) UX*3.2808,MUX*6.8521E-2/3.2808,REX/3.2808
         GOTO 999
       END IF


       WRITE (21,610)
610      FORMAT (/,1X,'FREESTREAM CONDITIONS')
       WRITE (21,615) MACH1
615      FORMAT (1X,' Minf = ',F5.2)
       WRITE (21,620)
620      FORMAT (2X,'P1 (psi)        T1 (R)        RHO1(slug/ft^3)        Z1')
         WRITE (21,700) P1*1.4504E-4,T1*1.8,RHO1*6.8521E-2/3.2808**3,Z1
       WRITE (21,625)
625      FORMAT (1X,' U1 (ft/s)        MU1 (slug/ft-s)        RE1(1/ft)')
         WRITE (21,710) U1*3.2808,MU1*6.8521E-2/3.2808,RE1/3.2808


       WRITE (21,630)
630      FORMAT (/,1X,'SONIC POINT CONDITIONS')
       WRITE (21,635)
635      FORMAT (2X,'P* (psi)        T* (R)        RHO*(slug/ft^3)        Z*')
         WRITE (21,700) PS*1.4504E-4,TS*1.8,RHOS*6.8521E-2/3.2808**3,ZS
       WRITE (21,640)
640      FORMAT (1X,' U* (ft/s)        MU* (slug/ft-s)        RE*(1/ft) ')
         WRITE (21,710) US*3.2808,MUS*6.8521E-2/3.2808,RES/3.2808



       WRITE (21,660)
660      FORMAT (/,1X,'POST-SHOCK CONDITIONS')
       WRITE (21,665)
665      FORMAT (2X,'P2 (psi)        T2 (R)        RHO2(slug/ft^3)        Z2')
         WRITE (21,700) P2*1.4504E-4,T2*1.8,RHO2*6.8521E-2/3.2808**3,Z2
       WRITE (21,667)
667      FORMAT (1X,' U2 (ft/s)        MU2 (slug/ft-s)        RE2(1/ft) ')
         WRITE (21,710) U2*3.2808,MU2*6.8521E-2/3.2808,RE2/3.2808
       WRITE (21,670)
670      FORMAT (2X,'PT2 (psi)        TT2 (R)        RHOT2(slug/ft^3)        ZT2')
         WRITE (21,700)PT2*1.4504E-4,TT2*1.8,RHOT2*6.8521E-2/3.2808**3,
     C                 ZT2


       WRITE (21,675)
675      FORMAT (1X,'SHOCK RATIOS',/,3X,'PT2/PT1        P2/P1',
     C          '        T2/T1')
       WRITE (21,680) PT2/P0,P2/P1,T2/T1
680      FORMAT (3X,E10.4,3X,E10.4,3X,E10.4)

C        *** PERFECT GAS CORRECTION FACTORS ***
       IF (FLAG.EQ.3) THEN
         WRITE (21,689)
689        FORMAT (/,1X,'PERFECT GAS CORRECTION FACTORS')
         WRITE (21,690)
690        FORMAT (1X,' (P0/P1)p/(P0/P1) (T0/T1)p/(T0/T1) (RHO0'
     C          ,'/RHO1)p/(RHO0/RHO1) (A0/A1)p/(A0/A1)')
         WRITE (21,720) P0P*P1/(P1P*P0),T0P*T1/(T1P*T0),
     C          RHO0P*RHO1/(RHO0*RHO1P),A0P*A1/(A1P*A0)

                              55
```

```
            WRITE (21,691)
691         FORMAT (1X,' (P0/PS)p/(P0/PS) (T0/TS)p/(T0/TS) (RHOO'
     C             ,'/RHOS)p/(RHOO/RHOS) (A0/AS)p/(A0/AS)')
            WRITE (21,720) POP*PS/(PSP*PO),TOP*TS/(TSP*T0),
     C             (RHOOP*RHOS)/(RHOSP*RHOO),AOP*AS/(ASP*AO)
            WRITE (21,692)
692         FORMAT (1X,' (P0/P2)p/(P0/P2) (T0/T2)p/(T0/T2) (RHOO'
     C             ,'/RHO2)p/(RHOO/RHO2) (A0/A2)p/(A0/A2)')
            WRITE (21,720) POP*P2/(P2P*PO),TOP*T2/(T2P*T0),
     C             RHOOP*RHO2/(RHO2P*RHOO),AOP*A2/(A2P*AO)
            WRITE (21,693)
693         FORMAT (1X,' (P0/PT2)p/(P0/PT2) (T0/TT2)p/(T0/TT2) (RHOO'
     C             ,'/RHOT2)p/(RHOO/RHOT2) ')
            WRITE (21,721) POP*PT2/(PT2P*PO),TOP*TT2/(TT2P*T0),
     C             (RHOOP*RHOT2)/(RHOT2P*RHOO)
            END IF

700         FORMAT (1X,F10.5,5X,F8.4,7X,E9.4,6X,F7.5)
710         FORMAT (1X,3X,F7.2,9X,E9.4,11X,E9.4)
720         FORMAT (8X,F6.4,11X,F6.4,15X,F6.4,15X,F6.4)
721         FORMAT (8X,F6.4,12X,F6.4,16X,F6.4)

999         END


C####################################################################################

            SUBROUTINE VIRIAL(T,VB,VBP,VBPP,VB3P,VC,VCP,VCPP,VC3P)
     C      *************************************************
     C      * COMPUTE 2nd, AND 3rd VIRIAL COEFFICIENTS OF   *
     C      * HELIUM BASED ON THE CURVE FITS OF EMPIRICAL   *
     C      * DATA AND THEIR FIRST AND SECOND DERIVATIVES   *
     C      * WITH RESPECT TO TEMPERATRE.                   *
     C      *************************************************

            IMPLICIT REAL (A-Z)
            COMMON K,MHE,R,SREF

     C      ***** 2nd VIRIAL COEFFICIENT *****
            IF (T.LE.1300) THEN
                L0 = -13.4067
                L1 = 165.4459
                L2 = -1357.92
                L3 = 5959.061
                L4 = -12340.8
            ELSE
                L0 = 1.178236
                L1 = -7.57134
                L2 = 5225.701
                L3 = -188923
                L4 = 2460461
            END IF

            VB = (L0 + L1*T**(-1/4.) + L2*T**(-3/4.) +
     C           L3*T**(-5/4.) + L4*T**(-7/4.))/(1000*MHE)
```

```
      VBP = (-L1/4*T**(-5/4.) - L2*3/4*T**(-7/4.) -
C            L3*5/4*T**(-9/4.) - L4*7/4*T**(-11/4.))/
C          (1000*MHE)
     VBPP = (L1*5/16*T**(-9/4.) + L2*21/16*T**(-11/4.) +
C            L3*45/16*T**(-13/4.) + L4*77/16*T**(-15/4.))/
C          (1000*MHE)
     VB3P = (-L1*(45/64)*T**(-13/4.) - L2*231/64*T**(-15/4.) -
C            L3*585/64*T**(-17/4.) - L4*11155/64*T**(-19/4.))/
C          (1000*MHE)

C       ***** 3rd VIRIAL COEFFICIENT *****
        J0 = -13.7898
        J1 = 139.7339
        J2 = 8114.259
        J3 = -17456.9


        VC =  (J0 + J1*T**(-1/4.) + J2*T**(-3/4.)
C           +J3*T**(-5/4.))/(1000*MHE)**2
      VCP = (-J1/4*T**(-5/4.) - J2*3/4*T**(-7/4.)
C            - J3*5/4*T**(-9/4.))/(1000*MHE)**2
      VCPP = (+J1*5/16*T**(-9/4.) + J2*21/16*T**(-11/4.)
C            + J3*45/16*T**(-13/4.))/(1000*MHE)**2
      VC3P = (-J1*(45/64)*T**(-13/4.) - J2*231/64*T**(-15/4.) -
C            J3*585/64*T**(-17/4.))/(1000*MHE)**2


        RETURN
      END


C---------------------------------------------------------------------
      SUBROUTINE REALEXP(PX,TX,RHOX,ZX,MACHX,UX,AX,HX,HTX,SX,REX,
C                        MUX,QX)
      IMPLICIT REAL (A-Z)
      INTEGER I,J,L
      DIMENSION W(2),G(2),DWDG(4,4),OLDW(2),INV(2,2),OLDG(2),FP(2)
C              ,HITER(2),SITER(2)
      COMMON K,MHE,R,SREF
C    ================================================================
C    =                                                              =
C    =            - CONDITIONS AT AN ARBITRARY POINT -              =
C    =                                                              =
C    =        THE FLOW PROPERTIES OF REAL HELIUM AT AN              =
C    =        ARBITRARY POINT ARE DETERMINED.  THE POINT            =
C    =        IS IDENTIFIED BY PICKING A MACH NUMBER                =
C    =        (USUALLY M = 1, THE SONIC POINT, OR M = Minf) AND     =
C    =        DETERMINING FLOW PROPERTIES FROM CONSERVATION         =
C    =        OF ENTROPY AND TOTAL ENTHALPY.  BECAUSE OF            =
C    =        THE EXTREME SENSITIVITIES OF SOME OF THE              =
C    =        DERIVATIVES TO DIFFERENT MACH NUMBERS, THE            =
C    =        DERIVATIVES REQUIRED IN THE NEWTON ITERATION          =
C    =        ARE EVELUTED NUMERICALLY INSTEAD OF ANALYTICALLY      =
C    =        TO PREVENT PROGRAM CRASHING AND TO INSURE             =
C    =        RAPID CONVERGENCE.                                    =
C    =                                                              =
C    ================================================================
```

57

```
          G(1) = RHOX
          G(2) = TX

 110      CONTINUE
          CALL VIRIAL(TX,VBX,VBPX,VBPPX,VB3PX,VCX,VCPX,VCPPX,VC3PX)
          CVX = (R/MHE)/(K-1) - (R/MHE)*TX*(RHOX*(2*VBPX + TX*VBPPX) +
     C        RHOX**2/2*(2*VCPX + TX*VCPPX))
          DCVXDT = -(R/MHE)*(RHOX*(2*VBPX + 4*TX*VBPPX + TX**2*VB3PX) +
     C        RHOX**2/2*(2*VCPX + 4*TX*VCPPX + TX**2*VC3PX))
          DCVXDR = -(R/MHE)*TX*((2*VBPX + TX*VBPPX) + RHOX*(2*VCPX +
     C           TX*VCPPX))


          DPDT = RHOX*(R/MHE)*(1 + RHOX*(VBX + TX*VBPX) + RHOX**2*(VCX +
     C         TX*VCPX))
          DPDTDT = RHOX*(R/MHE)*(RHOX*(2*VBPX + TX*VBPPX) + RHOX**2*
     C         (2*VCPX + TX*VCPPX))
          DPDTDR = (R/MHE)*(1 + 2*RHOX*(VBX + TX*VBPX) + 3*RHOX**2*
     C         (VCX + TX*VCPX))



          DPDRHO = (R/MHE)*TX*(1 + 2*RHOX*VBX + 3*RHOX**2*VCX)
          DPDRDT = (R/MHE)*(1 + 2*RHOX*(VBX + TX*VBPX) + 3*RHOX**2*
     C         (VCX + TX*VCPX))
          DPDRDR = (R/MHE)*TX*(2*VBX + 6*RHOX*VCX)


          AX = (DPDRHO + TX*DPDT**2/(CVX*RHOX**2))**.5
          DAXDT = (DPDRDT + TX*DPDT**2/(CVX*RHOX**2)*(1/TX + 2*DPDTDT/
     C         DPDT - DCVXDT/CVX))/(AX*2)
          DAXDR = (DPDRDR + TX*DPDT**2/(CVX*RHOX**2)*(2*DPDTDR/DPDT -
     C         DCVXDR/CVX - 2/RHOX))/(AX*2)



          W(1) = HTX - (R/MHE)*TX*(K/(K-1) + RHOX*(VBX - TX*VBPX) +
     C        RHOX**2/2*(2*VCX - TX*VCPX)) - (MACHX*AX)**2/2
          W(2) = SX - (R/MHE)*(1/(K-1)*LOG(TX) - LOG(RHOX) - RHOX*
     C         (VBX + TX*VBPX) - RHOX**2/2*(VCX + TX*VCPX))-SREF

 C        ***** COMPUTE JACOBIAN OF MATRIX [W] *****
          DWDG(1,1) = -(R/MHE)*TX*((VBX - TX*VBPX) + RHOX*(2*VCX -
     C           TX*VCPX)) - MACHX**2*AX*DAXDR
          DWDG(1,2) = -(R/MHE)*(K/(K-1) + RHOX*(VBX - VBPX*TX - TX
     C           **2*VBPPX) + RHOX**2/2*(2*VCX-TX**2*VCPPX)) -
     C           MACHX**2*AX*DAXDT
          DWDG(2,1) = -(R/MHE)*(-1/RHOX - (VBX + TX*VBPX) - RHOX*
     C           (VCX + TX*VCPX))
          DWDG(2,2) = -(R/MHE)*(1/((K-1)*TX) - RHOX*(2*VBPX
     C           + TX*VBPPX) - RHOX**2/2*(2*VCPX + TX*VCPPX))
 C        ***** INVERT [DWDG] *****
          INV(2,1) = 1/(DWDG(1,2) - DWDG(1,1)*DWDG(2,2)/DWDG(2,1))
          INV(1,1) = -INV(2,1)*DWDG(2,2)/DWDG(2,1)
          INV(2,2) = 1/(DWDG(2,2) - DWDG(1,2)*DWDG(2,1)/DWDG(1,1))
          INV(1,2) = -INV(2,2)*DWDG(1,2)/DWDG(1,1)

 C        ***** COMPUTE NEW RHOX, TX *****
          DO 120 I = 1,2
```

```fortran
            OLDG(I) = G(I)
            DO 130 J = 1,2
               G(I) = G(I) - INV(I,J)*W(J)
130         CONTINUE
120      CONTINUE
         RHOX = G(1)
         TX = G(2)
         ERR = ((G(1) - OLDG(1))**2 + (G(2) - OLDG(2))**2)**.5
         IF (ERR.GT.1E-4) GOTO 110

C        ***** EVALUATE PROPERTIES AT RHOX,TX *****

         CALL VIRIAL (TX,VBX,VBPX,VBPPX,VB3PX,VCX,VCPX,VCPPX,VC3PX)
         DPDT = RHOX*(R/MHE)*(1 + RHOX*(VBX + TX*VBPX) + RHOX**2*
C              (VCX + TX*VCPX))
         DPDRHO = (R/MHE)*TX*(1 + 2*RHOX*VBX + 3*RHOX**2*VCX)
         CVX = (R/MHE)/(K-1) - (R/MHE)*TX*(RHOX*(2*VBPX + TX*VBPPX) +
C              RHOX**2/2*(2*VCPX + TX*VCPPX))
         CPX = CVX + TX*DPDT**2/(RHOX**2*DPDRHO)
         KX = CPX/CVX
         AX = (DPDRHO + TX*DPDT**2/(CVX*RHOX**2))**.5
         UX = AX*MACHX
         HX = HTX - UX**2/2
         ZX = 1 + RHOX*VBX + RHOX**2*VCX
         PX = ZX*RHOX*(R/MHE)*TX
         CALL VISCOS (TX,MUX)
         REX = RHOX*UX/MUX
         QX = RHOX*UX**2/2

         RETURN
         END


C-----------------------------------------------------------------------


         SUBROUTINE VISCOS(T,MU)
C        ******************************
C        * COMPUTE VISCOSITY FROM CURVE *
C        * FITS OF EXPERIMENTAL DATA.   *
C        ******************************
         REAL MU

         IF (T.LE.1.2) THEN
            MU = (2.1630-2.6665E1*T + 1.2054E2*T**2 - 1.8741E2*
C               T**3 + 1.2682E2*T**4 - 3.1823E1*T**5)*1E-7
         END IF

         IF (T.GT.1.2 .AND. T.LE.3.6) THEN
            MU = (5.02 - 3.2241*T + 2.0308*T**2 - 2.2351E-1
C               *T**3)*1E-7
         END IF

         IF (T.GT.3.6 .AND. T.LE.8) THEN
            MU = (-1.5691 + 3.4167*T - 1.0317E-1*T**2)*1E-7
         END IF
```

```fortran
      IF (T.GT.8) THEN
          MU = 5.023E-7*T**.647
      END IF

      RETURN
      END


      SUBROUTINE ITER(PITOT,P01,M1,F)
C     ***************************
C     * USE NEWTON'S METHOD TO  *
C     * ITERATE FOR MACH NUMBER. *
C     ***************************
      IMPLICIT REAL (A-Z)
      COMMON K,MHE,R,SREF

      PT2P01 = (((K+1)*M1**2)/((K-1)*M1**2 + 2))**(K/(K-1))*
C              ((K+1)/(2*K*M1**2 - (K-1)))**(1/(K-1))

      F = PITOT/P01 - PT2P01

      RETURN
      END
```

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE August 1992	3. REPORT TYPE AND DATES COVERED Contractor Report 6/91 – 6/92

4. TITLE AND SUBTITLE

Real Gas Flow Parameters for NASA Langley 22-Inch
Mach 20 Helium Tunnel

6. AUTHOR(S)

Brian R. Hollis

5. FUNDING NUMBERS

G NAGW-1072

WU 506-40-41-02

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

North Carolina State University
Dept. of Mechanical and Aerospace Engineering
Raleigh, NC 27695

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665-5225

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA CR-4462

11. SUPPLEMENTARY NOTES

Langley Technical Monitor: Charles G. Miller

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified – Unlimited

Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A computational procedure has been developed which can be used to determine the flow properties in hypersonic helium wind tunnels in which real gas behavior is significant. In this procedure, a three-coefficient virial equation of state and the assumption of isentropic nozzle flow are employed to determine the tunnel reservoir, nozzle, throat, freestream and post-normal shock conditions. This method has been applied to a range of conditions which encompasses the operational capabilities of the NASA Langley 22-Inch Mach 20 Helium Tunnel. Results are presented graphically in the form of real gas correction factors which can be applied to perfect gas calculations. Important thermodynamic properties of helium are also plotted versus pressure and temperature. The computational scheme used to determine the real-helium flow parameters has been incorporated into a FORTRAN code which is discussed in this paper.

14. SUBJECT TERMS Helium; Real Gas; Wind Tunnels; Hypersonics			15. NUMBER OF PAGES 68
			16. PRICE CODE A04

17. SECURITY CLASSIFICATION OF REPORT unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT

National Aeronautics and
Space Administration
JTT
Washington, D.C.
20546-0001

Official Business
Penalty for Private Use. $300

NASA

POSTMASTER: If Undeliverable (Section 158
Postal Manual) Do Not Return