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## STEADY STATE RISETIMES OF SHOCK WAVES IN THE ATMOSPHERE

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## SUMMARY

A square wave shape is used in the Pestorius algorithm to calculate the risetime of a step shock in the atmosphere. These results agree closely with steady shock calculations. The healing distance of perturbed shocks due to finite wave effects is then investigated for quasi-steady shocks. Perturbed 100 Pa shocks require on the order of 1.0 km travel distance to return to within 10% of their steady shock risetime. For 30 Pa shocks the minimum recovery distance increases to 3.0 km. It is unlikely that finite wave effects can remove the longer risetimes and irregular features introduced into the sonic boom by turbulent scattering in the planetary boundary layer.

## INTRODUCTION

In a previous paper<sup>1</sup> we compared the risetimes calculated by the enhanced Pestorius<sup>2,3,4</sup> algorithm with risetimes calculated using the augmented Burger's<sup>5,6,7</sup> equation under the assumption of a steady step shock. Good agreement was obtained if the N-wave used in the enhanced Pestorius algorithm had a duration on the order of 100 times the risetime.

Sparrow<sup>8</sup> has applied his numerical method for general finite amplitude wave propagation to the propagation of square pulses as displayed in Fig. 1. These pulse shapes are particularly useful for performing the comparison described in Ref. 1 since the shock front is a better approximation of a steady

shock than a long duration N-wave. In addition, if the calculation is performed without geometric spreading, the shock overpressure will remain constant until the central linear position of the wave is convected to the shock front (see Fig. 1).  $t_a$  may be calculated using weak shock consideration as

$$t_a = \frac{\rho c^2 T_0}{4\beta P_0} \tag{1}$$

where  $P_0$  is the overpressure,  $T_0$  is the duration,  $c$  is the speed of sound,  $\rho$  is the density of air, and  $\beta = 1.2$ .

We have also used this waveform to investigate the distance a perturbed shock must propagate before its risetime approaches its steady state value. Plotkin and George<sup>9</sup> suggest that finite wave effects might significantly shorten the risetimes after scattering has initially increased them.

### CALCULATIONS

The numerical algorithm which includes vibrational relaxation absorption and dispersion<sup>3</sup> was applied to the square wave form displayed in Fig. 1. The initial rise portion of the wave was modeled by a hyperbolic tangent as illustrated in Fig. 2 for the initial wave shape. This rise shape is the solution to the Burger's equation for a steady shock in a non-relaxing media.

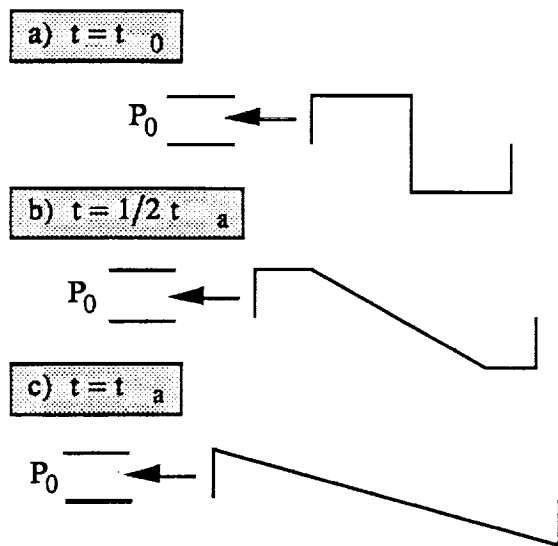


Figure 1. Square shock propagation without geometric spreading.

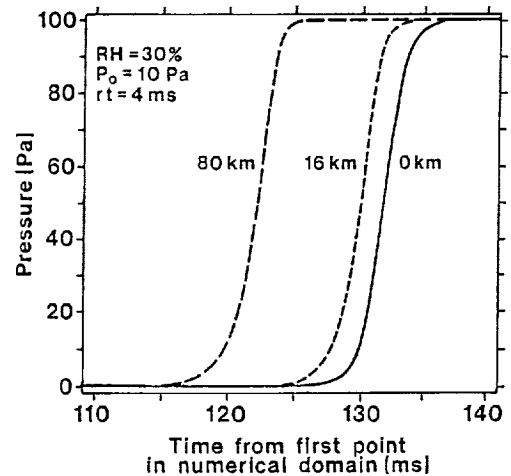


Figure 2. Development of the shock front with distance for a square wave with a 10 Pa overpressure at 295°K and a relative humidity of 30%.

For all calculations the 250 ms duration waveform is embedded in 125 ms of zeroes in front of and behind the pulse to allow for non-linear duration changes and to minimize sampling errors. 8192 points are used to assure that the rise portion of the wave contains sufficient points to model the physical properties accurately. We note that 8192 points were only required for 100 Pa overpressure waves but this number of points was used in all calculations.

## RISETIME OF STEADY SHOCKS

The numerical algorithm was started with a hyperbolic tangent rise portion with a best guess risetime for three steady state overpressures (100, 30 and 10 Pa), two relative humidities (30% and 10%) and a temperature of 295°K. These conditions were chosen for comparison with Kang and Pierce's calculations<sup>6,7</sup> and with our earlier work with non-steady shocks<sup>2</sup> in the atmosphere. The shocks were propagated until a steady risetime was achieved.

In some cases the wave had to be propagated extreme distances to achieve an equilibrium risetime since the rise shape was quite different from the initial hyperbolic tangent. We will discuss this in more detail in the next section.

Table I contains the risetimes computed from the enhanced Pestorius algorithms with results read from Fig. 5.12 of Kang's dissertation. These values are read from a log-log scale and are only accurate to two significant digits.

Table I. Comparison of rise times computed using a square wave in the enhanced Pestorius algorithm with Kang's steady state calculations.

	Pressure	Risetime (ms)	
		square wave	steady state
10% Relative Humidity			
	100 Pa	1.00	1.10
	30 Pa	3.90	4.70
	10 Pa	11.80	13.60
30% Relative Humidity			
	100 Pa	0.45	0.34
	30 Pa	1.40	1.60
	10 Pa	4.10	4.90

The two results agree within about 20%, confirming the validity of both methods as means of estimating the risetimes of steady shocks. This relatively good agreement is in contrast to the comparison of the steady state risetimes with the risetime of spherically decaying relatively short duration explosion waves calculated using the enhanced Pestorius algorithm. The risetimes of the spherically decaying short duration waves are significantly shorter than the square wave or steady state risetimes.

### HEALING DISTANCE OF PERTURBED WAVES

Square pulses were started with a range of risetimes about the equilibrium risetimes listed in Table I. All waves had a hyperbolic tangent rise function. Figure 3 displays the results for 30% relative humidity and 30 Pa overpressure. The pulse started with a 1.5 ms risetime, dips to a lower risetime, then approaches a 1.4 ms risetime. The pulse started at 1.0 ms has a risetime within 0.1 ms of the 1.5 ms risetime pulse after propagating about 2.5 km, while the pulse started at 2.0 ms is within 0.1 ms after 4.0 km.

The healing distance is longer for atmospheric conditions with larger attenuation. The 30 Pa pulses with a 10% relative humidity started with 4.0 ms and 3.5 ms risetimes which agree within 0.1 ms by 10 km; pulses started at 4.5 ms and 4.0 ms are within 0.1 ms at 12 km. The behavior of the risetime versus distance for different starting risetimes is displayed in Fig. 4.

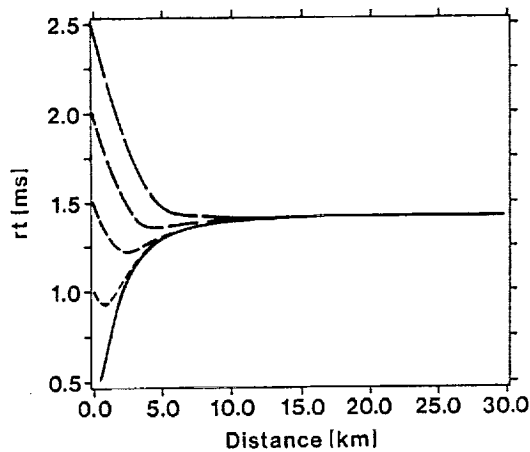


Figure 3. Development of risetime for a 30 Pa overpressure shock wave for  $T=295^{\circ}\text{K}$  and a relative humidity of 30%.

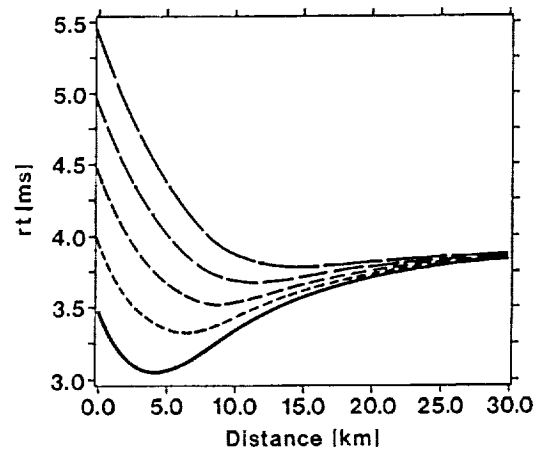


Figure 4. Development of risetime for a 30 Pa overpressure shock wave for  $T=295^{\circ}\text{K}$  and a relative humidity of 10%.

The dip in risetime displayed in Figs. 3 and 4 occurs because the steady state rise function does not resemble a hyperbolic tangent function. Figure 5 displays the behavior of the risetime for the case which displayed the largest relative decrease in risetime. The risetime for the 10% relative humidity case at 100 Pa drops from 1.0 ms to 0.5 ms, then rises to 1.0 ms at a distance of 5 km. The development of the rise function with distance for this case is shown in Fig. 6. The 1.0 ms risetime at 5.0 km is determined principally by the foot of the wave. This portion of the wave is due to velocity dispersion caused by molecular relaxation. This time development of the wave shape should be contrasted with Fig. 2, where the final waveform approximates the hyperbolic tangent function.

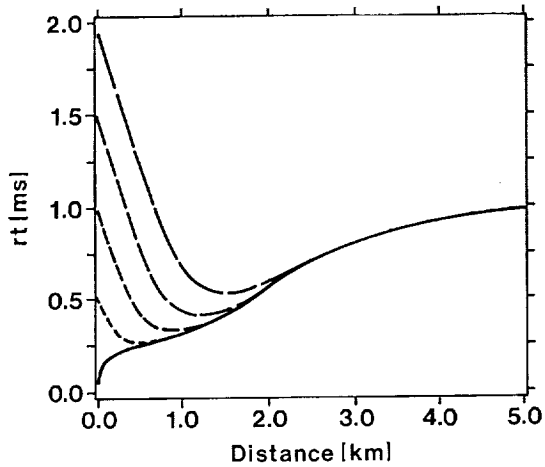


Figure 5. Development of risetime for a 100 Pa overpressure shock wave for  $T=295^{\circ}\text{K}$  and a relative humidity of 10%.

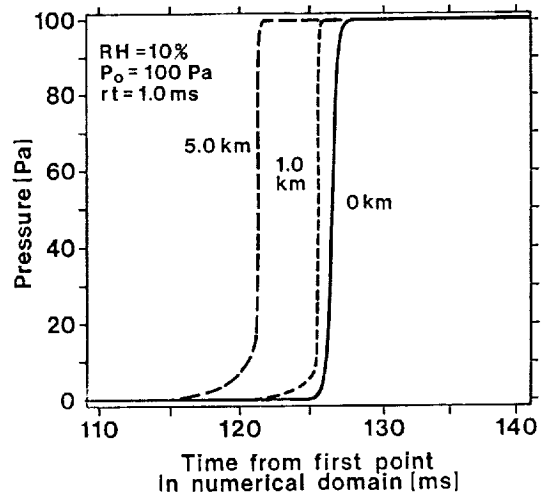


Figure 6. Development of the shock front with distance for a square wave with 100 Pa overpressure at  $295^{\circ}\text{K}$  and a relative humidity of 10%.

Table II contains a chart of the approximate healing distance for a shock risetime perturbed by 0.5 ms to return to within 0.1 ms of a shock started with the correct risetime (but not the final rise shape). These distances are an average of the distance for a pulse started with the equilibrium risetime plus 0.5 ms and a pulse started with the equilibrium risetime minus 0.5 ms.

Table II. Approximate healing distance for different shocks perturbed by .5 ms in risetime

Relative Humidity	Pressure	Distance (km)
10%	100	0.8
	30	10.0
	10	70.0
30%	100	0.8
	30	3.0
	10	35.0

## CONCLUSION

The enhanced Pestorius algorithm applied to square pulses agrees closely with steady shock calculations of risetimes. The use of the square pulse is convenient for comparisons.

The square pulses were also used to examine the finite wave healing of perturbed pulses. This rough calculation shows that if scattering in the planetary boundary layer induces a longer risetime in a sonic boom, the longer risetime will still be present at the ground. Finite wave calculations on the scattered waveforms predicted by Yao<sup>10</sup> will be performed in the near future.

Another implication of the large healing distance is that atmospheric conditions at higher altitudes may determine quiet atmosphere risetimes at the ground.

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