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Computational Algorithms for Increased Control of Depth-Viewing Volume for Stereo Three-Dimensional Graphic Displays

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Abstract

Three-dimensional pictorial displays incorporating depth cues by means of stereopsis offer a potential means of presenting information in a natural way to enhance situational awareness and improve operator performance. Conventional computational techniques rely on asymptotic projection transformations and symmetric clipping to produce the stereo display. Implementation of two new computational techniques, an asymmetric clipping algorithm and a piecewise linear projection transformation, provides the display designer with more control and better utilization of the effective depth-viewing volume to allow full exploitation of stereopsis cuing. Asymmetric clipping increases the perceived field of view (FOV) for the stereopsis region. The total horizontal FOV provided by the asymmetric clipping algorithm is greater throughout the scene viewing envelope than that of the symmetric algorithm. The new piecewise linear projection transformation allows the designer to creatively partition the depth-viewing volume, with freedom to place depth cuing at the various scene distances at which emphasis is desired.

Summary

Three-dimensional (3-D) pictorial displays incorporating depth cues by means of stereopsis offer a potential means of presenting information in a natural way to enhance situational awareness and improve operator performance. Conventional computational techniques rely on asymptotic projection transformations and symmetric clipping to produce the stereo display. Implementation of two new computational techniques, an asymmetric clipping algorithm and a piecewise linear projection transformation, provides the display designer with more control and better utilization of the effective depth-viewing volume to allow full exploitation of stereopsis cuing.

Stereo displays are created by generating both left-eye and right-eye views of the visual scene. Conventional computational techniques utilized in stereopsis display generation rely on symmetric clipping algorithms and asymptotic projection transformations to provide the left-eye and right-eye stereo pair. Clipping algorithms are used to limit the computed view to the display screen boundaries. For a fixed screen distance and size, symmetric clipping dictates a smaller monocular field of view (FOV) for each eye than asymmetric clipping. Combining the monocular FOV for both eyes results in different stereo overlap regions and single-eye viewing regions at different scene distances for the two clipping approaches. The perceived FOV for the stereopsis region and also the total horizontal FOV provided by the asymmetric clipping algorithm are greater throughout the scene viewing envelope than those of the symmetric algorithm.

Conventional asymptotic projection transformations, which are used to map the visual scene depth into the stereo viewing volume, allow the display designer to fix a specific scene distance at the screen location (i.e., the display designer may decide to set up the projection transformations such that objects 150 ft away appear to be at the screen distance in the stereo depth-viewing volume, objects closer than 150 ft appear to be in front of the screen, and objects farther away than 150 ft appear to be behind the screen). The conventional asymptotic projection transformation also allows additional control by letting the designer specify a maximum distance any image is to appear from the viewer. This two-point control provided by the asymptotic projection transformation allows stereo emphasis to be placed only at one scene distance. However, the new piecewise linear approach allows the designer to creatively partition the depth-viewing volume, with freedom to place depth cuing at the various scene distances at which emphasis is desired.

The implementation of these computational techniques furnishes the display designer with more control and better utilization of the effective depth-viewing volume to allow full exploitation of stereopsis cuing in advanced flight display concepts that embody true 3-D images of synthetic (computer-generated) objects or scenes.

Introduction

Current electronic display technology can provide high-fidelity, "real world" pictorial displays (under flicker-free conditions) that incorporate true depth in the display elements. Advanced flight display concepts that embody true 3-D images of synthetic (computer-generated) objects or scenes are being conceived and evaluated at NASA Langley Research Center and at the Wright Research and Development Center (refs. 1–7). Innovative concepts are sought that exploit the power of modern graphics display generators and stereopsis, not only for situational awareness enhancements of pictorial displays, but also for the declutter of complex informational displays and to provide more effective alerting functions to the flight crew.

The intuitively advantageous use of a three-dimensional presentation of three-dimensional information, rather than the conventional two-dimensional presentation of such information, has been investigated for years within the flight display community (refs. 8–14). These efforts have been particularly intense for helmet-mounted head-up display applications, as stereopsis cuing is a feature of binocular helmet systems (refs. 5 and 8–11). Additional investigations utilizing electronic shutters or polarized filters, rather than helmet optics, to present separate left-eye and right-eye views have also been conducted (refs. 1–4, 6, 7, and 11–14).

Stereoscopic displays provide depth information by means of lateral disparity and the eye muscular cues associated with convergence. In these displays, the distance that affects eye accommodation (focus) is the viewer-screen distance, which remains constant. Thus, the major depth cue missing in synthetic generation of stereo 3-D displays is the change in eye accommodation with fixation point depth. And this is indeed a major deficiency, for accommodation and convergence are highly associative. For a fixed accommodation distance, there is a limited range of convergence conditions that will result in comfortable, clear, fused, single vision. Thus, for a given viewer-screen distance for a stereoscopic display, there are limits to the amount of lateral disparity that is usable. Recent experiments at NASA Langley Research Center (ref. 15) determined that the effective region of stereopsis cuing (the depth-viewing volume) for traditional directly viewed stereo 3-D display systems is on the order of 2 to 3 ft.

Stereo displays are created by generating both left-eye and right-eye views of the visual scene. Conventional computational techniques that are utilized in stereopsis display generation rely on symmetric clipping algorithms and asymptotic projection transformations to provide the left-eye and right-eye stereo pair. The goal of this effort was to expand upon these conventional techniques to provide the display designer with more complete control and better utilization of the effective depth-viewing volume to allow full exploitation of stereopsis cuing in advanced flight display concepts. This increased control is provided by the implementation of two new computational techniques, an asymmetric clipping algorithm and a piecewise linear projection transformation. Asymmetric clipping increases the perceived fields of view for the stereopsis regions. It also provides horizontal fields of view that are greater throughout the scene viewing envelope than those of the symmetric algorithm. The piecewise linear approach allows the designer to creatively partition the depth-viewing volume, with freedom to place depth cuing at the various scene distances at which emphasis is desired.

Generation of Stereo 3-D Displays

Modern computer graphics generators provide depth cues by creating for each eye a separate view of the visual scene by means of various hardware systems, such that the right eye sees only the right-eye scene and the left eye sees only the left-eye scene (stereopsis cues). These hardware systems include helmet-mounted displays, which depend on a direct presentation of each eye view, stereoscopes (refracting or reflecting), and systems that incorporate shutters (electronics or mechanical) (fig. 1), or filters (polarized or color).

Regardless of the display hardware system, graphics software is necessary to create the left-eye and right-eye stereo pair images. To perform this task, the graphics computer resolves the single-viewpoint visual data base into the desired stereo pair. In order to produce synthetic graphic objects that appear to have depth in 3-D space, it is necessary to displace the objects in each stereo pair image so that each eye will view the scene with the parallax required to achieve the desired depth. Figure 2 illustrates the parallax concept employed to produce objects that appear behind the monitor screen. Figure 3 illustrates the concept as it is employed to produce objects at various depths. To present an object that appears at the depth of the screen, both of the stereo-pair views are drawn in an overlapping manner so it appears that only one image is displayed. For objects to appear behind the screen, the object is displaced to the left for the left-eye view and to the right for the right-eye view, with the displacement reaching a maximum value to place an object at infinity. For objects to appear in front of the screen, the images are crossed so that the left-eye view is displaced to the right and the right-eye view is displaced to the left.

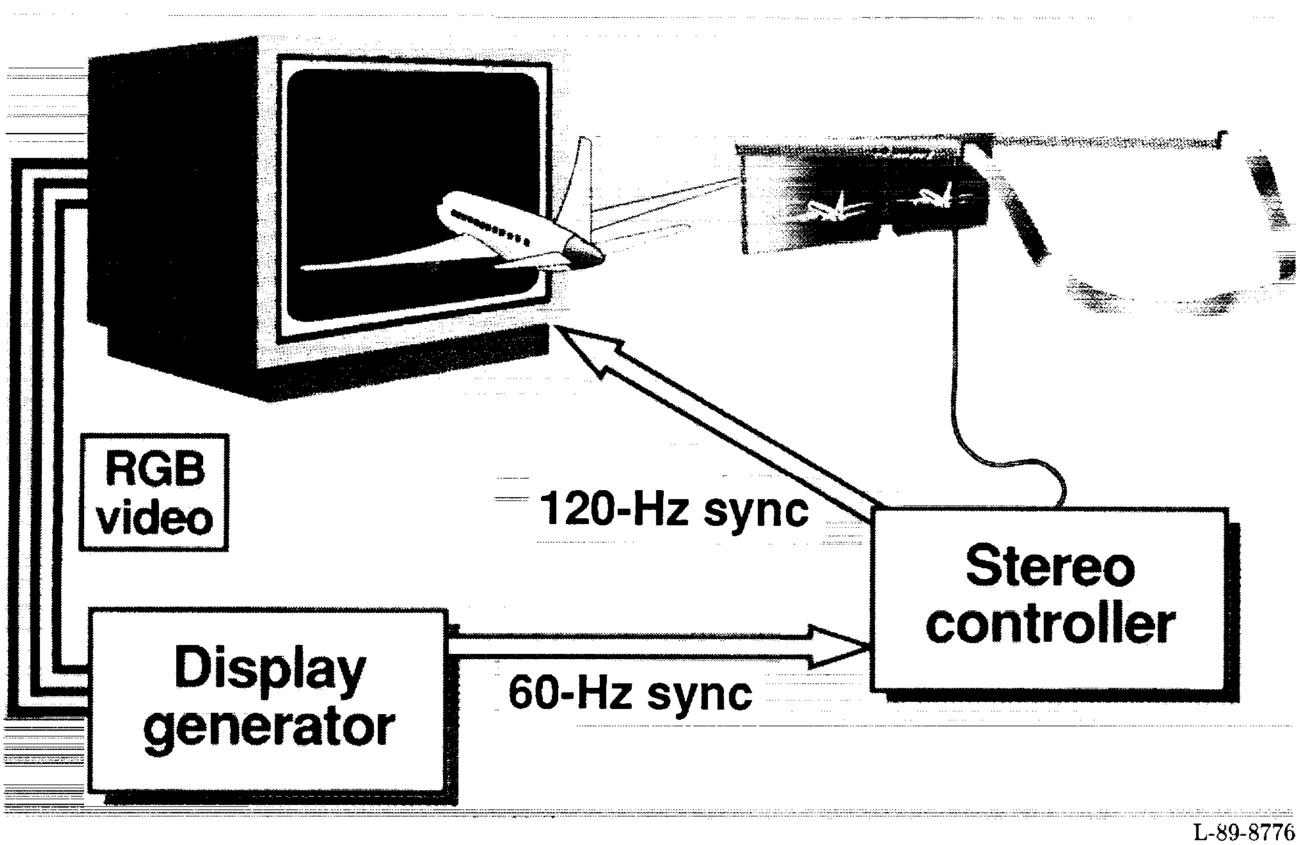

Figure 1. Stereo 3-D hardware system.

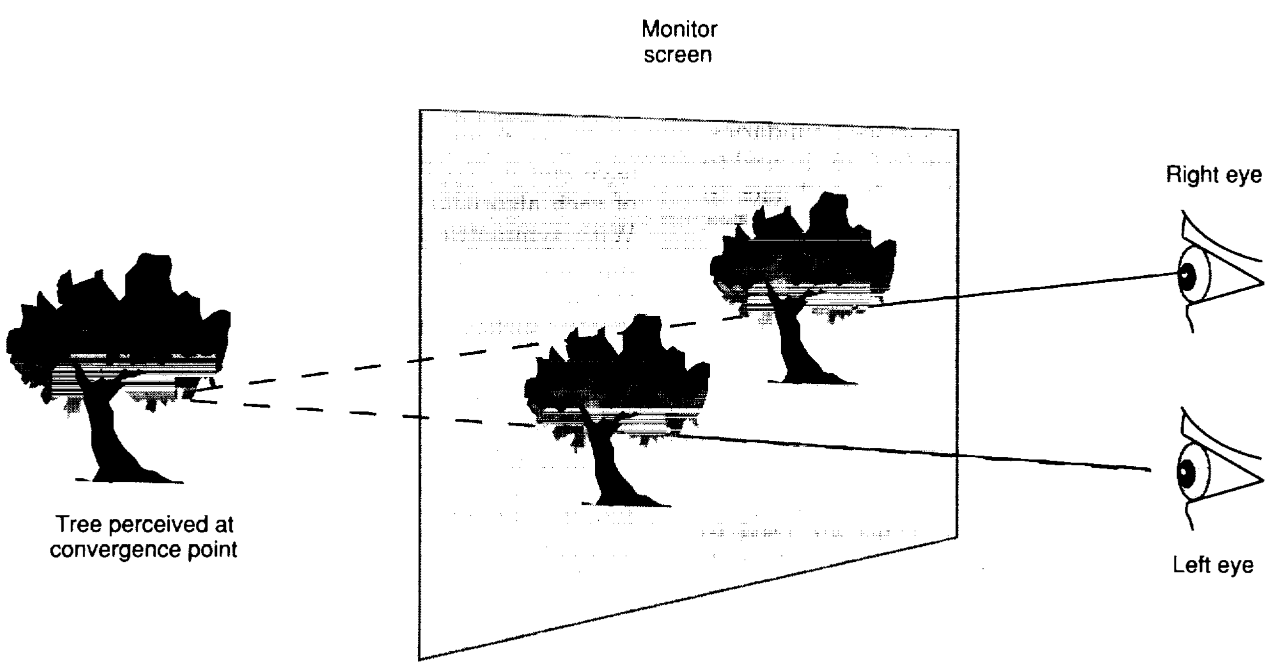

Figure 2. Concept for introducing depth.

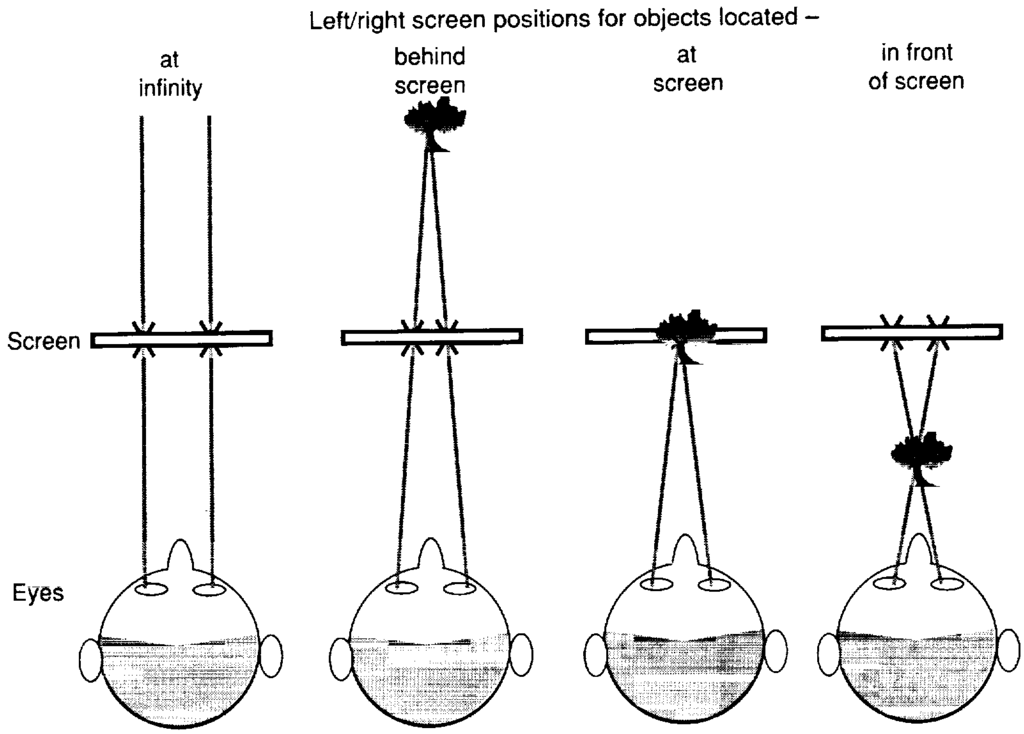

Figure 3. Top view of the geometric principle for producing left- and right-eye views.

Graphics software is used to generate the lateral displacement, which is known as lateral disparity (see fig. 4 for the geometric relationships involved). First, the left-eye and right-eye coordinate systems are created as offsets from the viewer coordinate system of the visual scene. Clipping is then employed to limit each eye view to the display surface boundaries. Finally, simple perspective division (ref. 16) is used to transform the three-dimensional viewing volumes to two-dimensional viewports, whose centers are offset from the center of the display screen by half of the maximum lateral disparity.

Enhanced Stereo 3-D Algorithms

Conventional computational techniques utilized in stereopsis display generation rely on symmetric clipping algorithms and asymptotic projection transformations to provide the left-eye and right-eye stereo pair images. The clipping algorithms are used to limit the computed view to the display screen boundaries, and the projection transformations are used to map the visual scene into the stereo viewing volume.

Implementation of two new computational techniques, an asymmetric clipping algorithm and a piecewise linear projection transformation, provides the display designer with more complete control and better utilization of the effective depth-viewing volume. This increased flexibility allows better exploitation of stereopsis cuing.

Stereo 3-D Clipping

Figure 5 presents an illustration of symmetric and asymmetric clipping, while table I presents the effects of each algorithm on the perceived horizontal FOV. With symmetric clipping (shown in the figure as a top view for the left eye), the left side of the left-eye viewing volume is determined by the left edge of the display surface. The right side of the left-eye viewing volume is determined by reflecting the left side about the sight vector, thus producing symmetric angles. In stereo 3-D displays, the left-eye and right-eye projections are offset from the screen centerline, producing unused blank spots on the display surface if symmetric clipping is employed. For a screen distance of 19 in. and a 40° horizontal FOV, these blank spots generate two 7° monocular regions on each edge of the monitor, with an overlap stereo 3-D region of 26°. This is not the most efficient use of the available screen width. Asymmetric clipping uses the entire display surface for each eye view. The left side of the left-eye viewing volume is determined by the

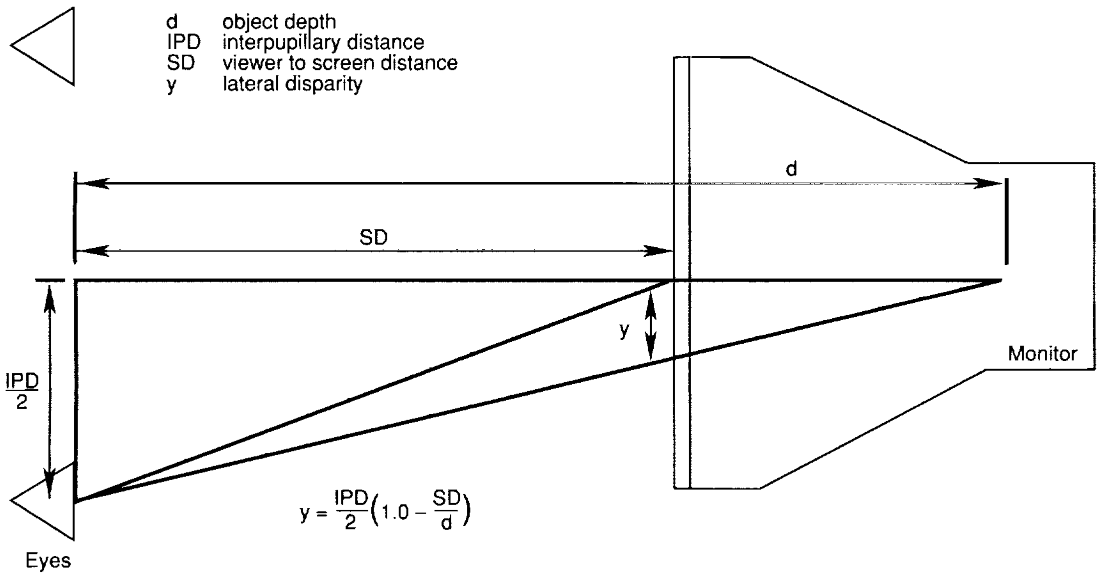

Figure 4. Relationship between depth and lateral disparity.

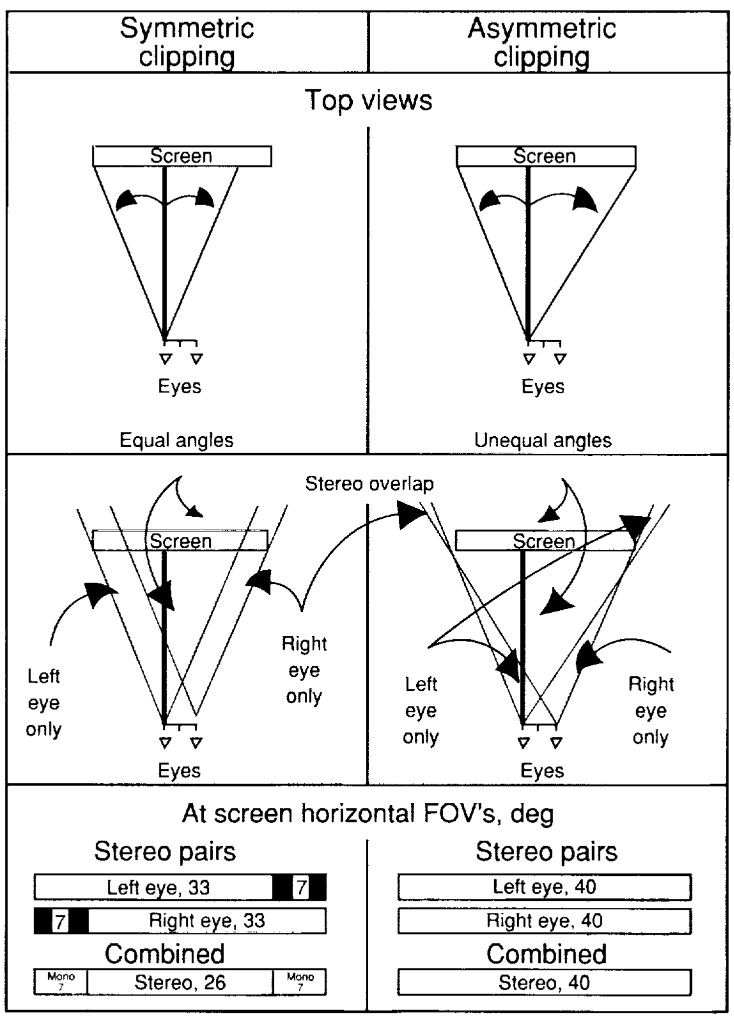

Figure 5. Stereo 3-D clipping algorithms.

Table I. Perceived Horizontal Field-of-View for Symmetric and Asymmetric Clipping

$$\left[\begin{array}{c}\text{Screen distance} = 19 \text{ in.} \\ \text{Horizontal monitor width} = 12.4 \text{ in.} \\ \text{Interocular distance} = 2.5 \text{ in.}\end{array}\right]$$

Symmetric clipping

Scene object distance	Perceived horizontal FOV, deg			
	Left monocular	Stereo	Right monocular	Total
Nearest	30.8	0	30.8	61.6
Screen	7.0	26.0	7.0	40.0
Infinity	0	33.2	0	33.2

Asymmetric clipping

Scene object distance	Perceived horizontal FOV, deg			
	Left monocular	Stereo	Right monocular	Total
Nearest	36.0	0	36.0	72.0
Screen	0	40.0	0	40.0
Infinity	6.65	33.2	6.65	46.5

left edge of the display surface, just as in symmetric clipping. The right side of the viewing volume, however, is determined by the right edge of the display surface. This generates unequal, or asymmetric, angles about the sight vector. With asymmetric clipping, there are no unused portions of the display surface, with the stereo 3-D overlap region encompassing the entire 40° of the FOV.

Symmetric clipping thus dictates a smaller monocular FOV for each eye for a fixed screen distance and size. Combining the monocular fields of view for both eyes results in different stereo overlap regions and single-eye viewing regions at the screen and at different scene distances for the two clipping approaches. Table I presents perceived fields of view for the two algorithms. The horizontal FOV usually considered for nonstereo displays is that of the display surface, as measured from the midpoint between the eyes. However, for stereoscopic displays, the perceived FOV (e.g., with asymmetric clipping, the angle between the left boundary for the right eye and the right boundary for the left eye as measured from the midpoint between the eyes) varies with the depth of fixation within the viewing volume. For image planes in front of the screen, the total perceived FOV is larger than that at screen distance, while the total perceived FOV for image planes near infinity is smaller. Asymmetric clipping provides increased fields of view for both the stereo region and the total scene throughout the depth-viewing volume. As shown in the table, asymmetric clipping provides greater perceived fields of view for the stereopsis regions and greater total horizontal fields of view throughout the scene viewing envelope than those of the symmetric algorithm.

Stereo 3-D Projection Transformations

It is well known from psychophysical research that the effective range of human stereo vision is several hundred feet (ref. 17). In synthetically generated views of real-world scenes that are to be displayed stereoscopically, it is necessary to map the real-world depth cues of hundreds of feet into a virtual volume that is only several feet in depth (ref. 15). Figure 6 illustrates the mapping of a "real world" scene to the stereo viewing volume. Conventional asymptotic projection transformations allow the display designer to fix a specific scene distance at the screen location in the viewing volume. In figure 6, a real-world distance of 150 ft has been selected as the scene distance to map to the screen distance in the virtual volume. All objects closer than 150 ft from the viewer in the real-world scene will appear in front of the screen (objects such as the tree, located 100 ft away), whereas all objects farther away than 150 ft will appear behind the screen (e.g., the aircraft, located 200 ft away). The top portion of figure 7 illustrates the shape of the mapping curve as it is applied to a general case. Control of the real-world scene distance to screen location intercept and control of the maximum allowed virtual image distance allow some limited shaping of the curve, which can be used to place stereo emphasis over a single desired real-world range. If stereo emphasis is required over multiple real-world ranges, or over a large single range, the conventional asymptotic projection transformation will not be adequate. An example for this type of requirement could be a telerobotic arm operator performing a "peg in the hole" type of task. In this task, the telerobotic arm operator uses a remotely located robot arm to retrieve a peg from a hole and place it in another hole located farther away. If the operator

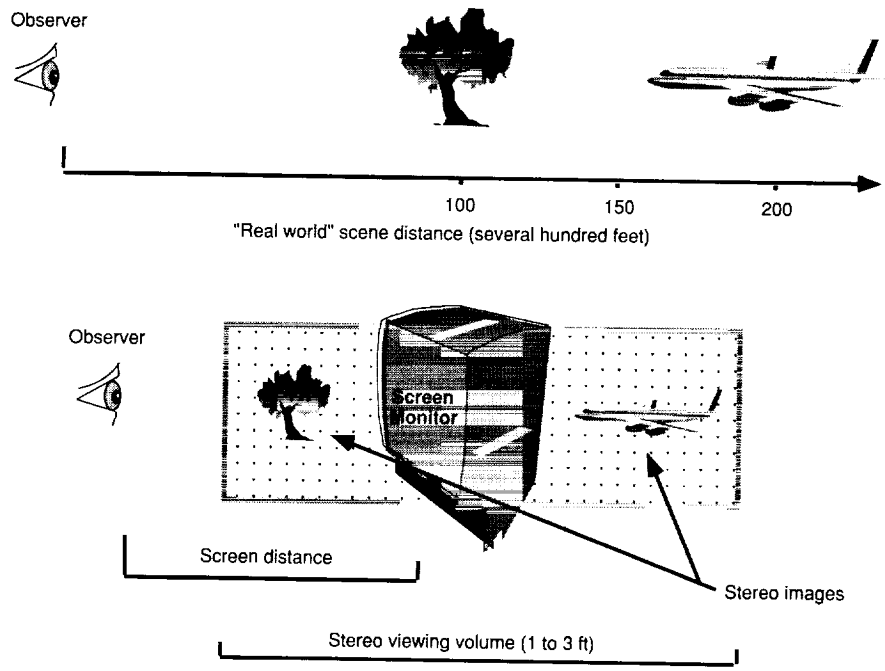

Figure 6. Scene-to-screen mapping problems with conventional stereo technology.

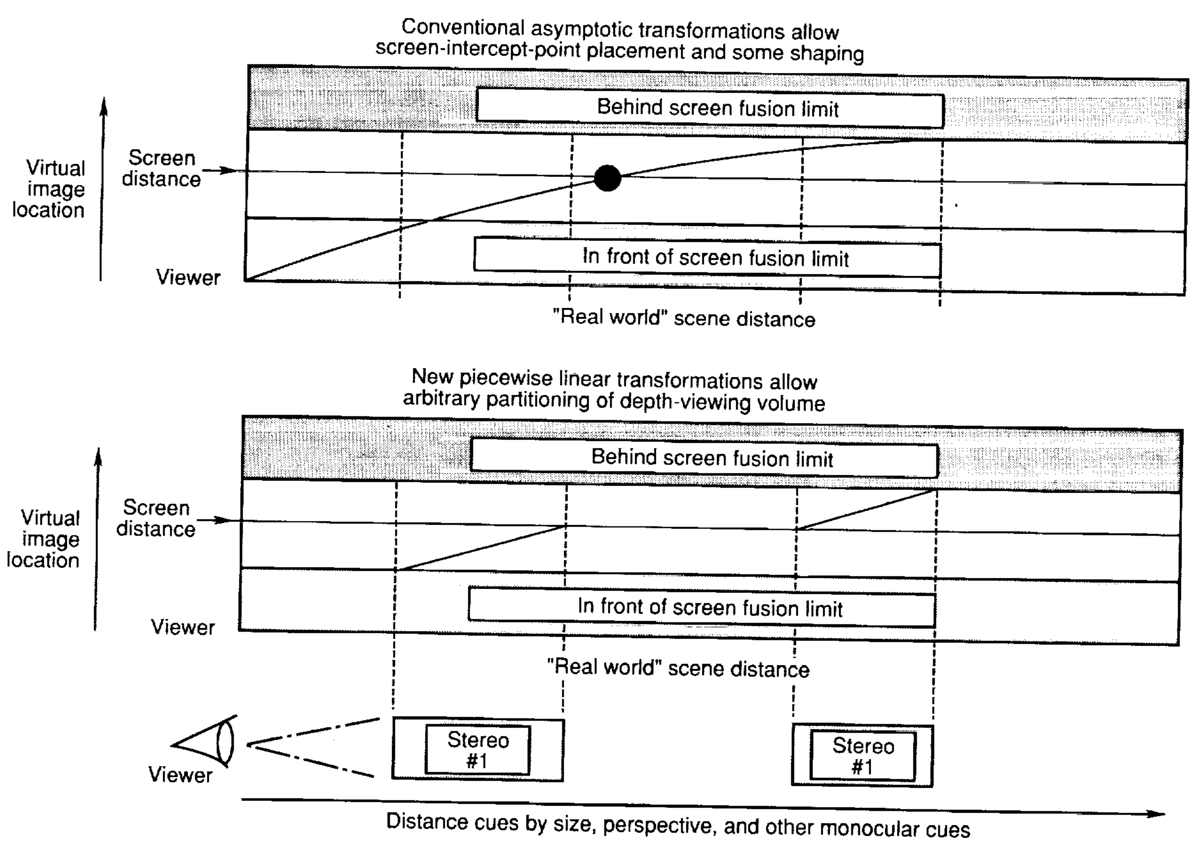

Figure 7. Visual scene mapping to stereo 3-D viewing volumes.

is viewing a stereo 3-D presentation of the scene, it could be advantageous to have stereo emphasis at the perceived depth of each of the two holes. With the conventional asymptotic projection transformation, stereo emphasis could be placed only at the depth of one of the holes. However, with the new piecewise linear approach, the stereo viewing volume could be partitioned into two regions where stereo emphasis could be applied. The bottom half of figure 7 illustrates this concept. Objects in stereo region 1 (the first hole, and the peg at the start of the task) and objects in stereo region 2 (the second hole, and the peg at the end of the task) would be presented with stereo depth cues, while objects outside these two regions would only contain monocular depth cues (i.e., perspective, and interposition). The piecewise linear projection transformation allows the display designer to creatively partition the depth-viewing volume into multiple regions where the stereo depth mapping can be independently controlled. This gives the designer the freedom to place depth cuing at the various scene distances at which emphasis is desired.

Implementation

Both of these new techniques could be implemented in true algorithmic form by passing every display vertex in the scene through a series of equations. However, because most modern graphics generators are designed to transform vertices with a 4×4 transformation matrix, the speed of the algorithms would be greatly increased if they were put into matrix form. In fact, since stereo 3-D depth mapping and clipping are so closely related, both can be implemented in a single projection transformation matrix. Appendix A shows the derivation and construction of the asymptotic/asymmetric projection matrix, and appendix B shows the derivation and construction of the piecewise linear/asymmetric projection matrix. Symmetric clipping is rarely desired and thus its implementation is not discussed. The mechanics of the transformation of the 3-D scene data base by using one of the above-mentioned projection transformation matrices is discussed in section 6.5 of reference 16. The generation of left-eye and right-eye images requires that the scene data base be rendered twice, once with a left-eye projection transformation matrix and once with a right-eye projection transformation matrix. When the two stereo pair images are presented to the appropriate eye (the left eye sees only the left-eye image, and the right eye sees only the right-eye image), the viewer will perceive a stereo 3-D representation of the scene data base.

The decisions of which projection to use (asymptotic or piecewise linear) and how to manipulate the projection, are task dependent. The asymptotic projection will produce a more natural looking presentation of a real-world scene than the piecewise linear projection. A simple rule to follow could be if the purpose of the stereo 3-D presentation is to produce a more natural or realistic representation of a scene, or if the task involved can be accomplished with stereo emphasis in one region (i.e., a formation-flying or station-keeping type of task where the asymptotic screen intercept value is set to the desired formation or station distance), use an asymptotic/asymmetric projection. Although the piecewise linear projection technique is more flexible and can make better use of the stereo 3-D depth-viewing volume, the resulting stereo 3-D display will have an artificial feel. As implemented in this paper, the asymptotic projection can be used successfully in many applications. The piecewise linear projection should be reserved for those occasions when the limits of an asymptotic projection preclude the ability to easily or safely perform the desired task.

Concluding Remarks

The implementation of two new computational techniques, an asymmetric clipping algorithm and a piecewise linear projection transformation, provides the display designer with more complete control and better utilization of the effective stereo 3-D depth-viewing volume. These enhancements allow full exploitation of stereopsis cuing. Asymmetric clipping increases the perceived fields of view for the stereopsis regions, and also, the total horizontal fields of view provided by the asymmetric clipping algorithm are greater throughout the scene viewing envelope than those of the symmetric algorithm. The new piecewise linear projection transformation allows the designer to creatively partition the depth-viewing volume, with freedom to place depth cuing at the various scene distances at which emphasis is desired.

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Appendix A

Asymptotic/Asymmetric Projection Transformation Matrix

The asymptotic/asymmetric projection transformation matrix is a 4×4 element matrix that is used to convert a 3-D scene data base into a 2-D perspective representation for the left-eye and right-eye images of a stereo pair. The matrix is built based on a right-handed viewer coordinate system with the origin located at the viewpoint in the scene data base. The x-axis has positive direction to the right, the y-axis is positive up, and the z-axis is positive behind the viewer. All vertices in the data base are represented in homogeneous (x, y, z, w) form. The homogeneous coordinate w is a scale parameter and is initially set to 1. If, after any of the steps in the projection transformation, the w coordinate is a value other than 1, it is forced to unity by dividing the vertex coordinates (x, y, z, and w) by the nonunity w coordinate value. For a more complete discussion of the homogeneous coordinate system and 3-D geometric transformations, viewing, and projections, please refer to chapters 5 and 6 in reference 16.

To generate the two eye views, left-eye and right-eye matrices are constructed. The scene data base is then rendered twice, once after being transformed by the left-eye projection matrix and once after being transformed by the right-eye projection matrix. When the two stereo pair images are presented to the appropriate eye (the left eye sees only the left-eye image, and the right eye sees only the right-eye image), the viewer will perceive a stereo 3-D representation of the 3-D scene data base that encompasses the advantages and properties of the asymptotic projection transformation and the asymmetric clipping algorithm. This matrix is derived from eight parameters, four of which describe the physical aspects of the presentation and four that determine the asymptotic mapping and depth-viewing volume desired.

Physical Parameters

Interpupillary distance. The interpupillary distance (IPD) is the distance between the eyes of the person viewing the stereo 3-D presentation.

Screen distance. The screen distance (SD) is the distance from the stereo 3-D viewer to the display surface.

Width. The width (W) is the physical width of the display surface.

Height. The height (H) is the physical height of the display surface.

Asymptotic Mapping and Depth-Viewing Volume Parameters

Minimum view distance. The minimum view distance (MIND) is the minimum distance a virtual image is to appear from the viewer. As objects are presented closer and closer to the viewer, eventually a limit is reached at which the viewer can no longer comfortably fuse the stereo 3-D presentation into a single image (ref. 15). Setting the minimum view distance parameter to within this limit ensures that no objects will be too close to fuse.

Maximum view distance. The maximum view distance (MAXD) is the maximum distance a virtual image is to appear from the viewer. As objects are presented farther and farther away from the viewer, eventually a limit is reached at which the viewer can no longer comfortably fuse the stereo 3-D presentation into a single image (ref. 15). Setting the maximum view distance parameter to within this limit ensures that no objects will be too far away to fuse. As objects in the real-world scene approach an infinite distance away from the viewer, they will approach the maximum view distance in the virtual image.

Screen intercept. The screen intercept (SI) is the distance in the real-world scene that is to appear at the screen distance in the virtual image. The region in the real world centered about this distance will have the most pronounced stereo effect.

Far clipping plane. The far clipping plane (FCP) is the distance in the real-world scene beyond which objects will not be drawn. This is used to help speed up the drawing time of the display. Most objects get too small to see at great distances and the far clipping plane is used to eliminate these objects from the rendering process.

Matrix Construction

The asymptotic/asymmetric projection transformation matrix is built in eight steps by combining basic geometric transformation matrices. Each of the transformations is based on values calculated from one or more of the aforementioned parameters.

Projection offset translation. The first step in the construction is to offset the centerline of the projection horizontally to generate either a left-eye or right-eye coordinate system. The amount of offset is dependent on the viewer's eye separation, the screen distance, the maximum view distance, and the screen intercept value. The projection offset translation (POT) is the major transformation controlling what real-world scene distance is to appear at the depth of the display surface in the stereo 3-D virtual image (screen intercept value). The relationship is described by the following equation:

$$\mathrm{POT} = \frac{\mathrm{IPD}}{2} \times \left(1.0 - \frac{\mathrm{SD}}{\mathrm{MAXD}}\right) \times \frac{\mathrm{SI}}{\mathrm{SD}}$$

The derivation of this equation is discussed in a later section. The projection offset equation above is for the generation of the left-eye coordinate system. To generate the right-eye coordinate system, the projection offset translation is the negative of the left-eye value. The translation transformation matrix used for the projection offset is shown as matrix (A1):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{POT} & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A1)}$$

***z* invert scale.** In the right-handed coordinate system used for this construction, the positive z-axis extends behind the viewer. In order for the 3-D to 2-D conversion (perspective division) to work correctly, the coordinate system must be a left-handed system with the positive z-axis extending in front of the viewer. This transformation is achieved by inverting the z-axis with a z-scale of -1. The scale transformation matrix used for this operation is shown as matrix (A2):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A2)}$$

Screen distance scale. When perspective division occurs, the 3-D data base is projected onto a 2-D plane. To maintain the correct stereo 3-D presentation, this 2-D plane must be the same distance away as the display screen. To accomplish this, the x-y plane of the coordinate system is scaled by the screen distance. The scale transformation matrix used in this step is shown as matrix (A3):

$$\begin{bmatrix} \mathrm{SD} & 0 & 0 & 0 \\ 0 & \mathrm{SD} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A3)}$$

Perspective division. The perspective division step projects the 3-D scene data base onto a 2-D plane. This projection is achieved by dividing the x- and y-coordinates of the data base vertices by the vertex z-coordinate value. The screen distance scale step above has established the 2-D plane to be coincident with the display screen. To accomplish this division with a transformation matrix, a matrix that transposes the homogenous coordinate w with the vertex z-coordinate is used. This matrix is shown as matrix (A4):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \text{(A4)}$$

***z*-clip translation.** After the perspective divide operation, the x- and y-coordinate values of the scene data base vertices represent the 2-D projection. The z-coordinate value has been transformed into its reciprocal. Two more transformations are necessary to bring the z-coordinate value within the z clipping range. The first of these is a translation. This z-clip translation (ZCT) is based on the near clipping plane and far clipping plane values and is the negative average of their reciprocals. The far clipping plane value is one of the eight parameters used in defining the projection; the near clipping plane (NCP) value must be calculated so that it is aligned with the minimum view distance:

$$\mathrm{NCP} = \frac{(\mathrm{SI} \times \mathrm{MIND} \times \mathrm{MAXD}) - (\mathrm{SI} \times \mathrm{SD} \times \mathrm{MIND})}{(\mathrm{SD} \times \mathrm{MAXD}) - (\mathrm{SD} \times \mathrm{MIND})}$$

$$\mathrm{ZCT} = -\left(\frac{\mathrm{FCP} + \mathrm{NCP}}{2.0 \times \mathrm{FCP} \times \mathrm{NCP}}\right)$$

The derivation of the NCP equation is discussed in a later section. For a discussion of the effect of the near and far clipping planes on the z-clip translation, please refer to chapter 6 of reference 16. The z-clip translation transformation matrix is shown as matrix (A5):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \mathrm{ZCT} & 1 \end{bmatrix} \qquad \text{(A5)}$$

***z*-clip scale.** The second of the z-coordinate transformations necessary to bring the z-coordinate value into the z clipping range is a scale transformation. Like the z-clip translation, the z-clip scale (ZCS) is based upon the near and far clipping plane values:

$$\mathrm{ZCS} = -\left(\frac{2.0 \times \mathrm{FCP} \times \mathrm{NCP}}{\mathrm{FCP} - \mathrm{NCP}}\right)$$

For a discussion on how the near and far clipping planes affect the z-clip scale, please refer to chapter 6 in reference 16. The z-clip scale transformation matrix is shown as matrix (A6):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{ZCS} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A6)}$$

Viewport separation. The separation between the left-eye and right-eye viewports controls the maximum positive lateral disparity. As an object in the real-world scene approaches an infinite distance away from the viewer, the object approaches the center of the projection, and the center of the projection appears in the center of the viewport. Thus, the viewport separation controls where objects at infinity in the real-world scene appear in the virtual image (maximum view distance). The relationship between maximum view distance and viewport separation translation (VPT) is as follows:

$$\mathrm{VPT} = -\frac{\mathrm{IPD}}{2} \times \left(1.0 - \frac{\mathrm{SD}}{\mathrm{MAXD}}\right)$$

The derivation of this equation is discussed in a later section. The equation above gives the viewport separation translation amount for the left-eye image. The right-eye value is the negative of the left-eye value. The viewport separation translation transformation matrix is shown as matrix (A7):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{VPT} & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A7)}$$

Field-of-view scale. To maintain the proper aspect ratio and fields of view, the image plane that contains the 2-D projection of the 3-D scene data base must be scaled based on the physical constraints of the presentation. The three physical parameters that determine FOV and aspect ratio are the width and height of the display surface, and the viewer-to-screen distance. The screen distance effect has already been accounted for in the screen distance scale step. The display surface width and height must now be accounted for. This is accomplished by scaling the x-coordinate by the reciprocal of one-half the width, and by scaling the y-coordinate by the reciprocal of one-half the height:

$$\mathrm{FOVX} = \frac{2}{\mathrm{W}}$$

$$\mathrm{FOVY} = \frac{2}{\mathrm{H}}$$

The FOV scale transformation matrix is shown as matrix (A8):

$$\begin{bmatrix} \mathrm{FOVX} & 0 & 0 & 0 \\ 0 & \mathrm{FOVY} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A8)}$$

Asymptotic/asymmetric projection transformation matrix. Sequentially multiplying the eight matrices presented above will generate the asymptotic/asymmetric projection transformation matrix. This matrix is shown as matrix (A9). Using this matrix as the projection matrix to convert the 3-D scene data base to the 2-D perspective view used for the left-eye or right-eye image of the stereo pair is discussed in section 6.5 of reference 16:

$$\begin{bmatrix} (\mathrm{SD})(\mathrm{FOVX}) & 0 & 0 & 0 \\ 0 & (\mathrm{SD})(\mathrm{FOVY}) & 0 & 0 \\ -(\mathrm{VPT})(\mathrm{FOVX}) & 0 & -(\mathrm{ZCT})(\mathrm{ZCS}) & -1 \\ -(\mathrm{POT})(\mathrm{SD})(\mathrm{FOVX}) & 0 & -(\mathrm{ZCS}) & 0 \end{bmatrix} \qquad \text{(A9)}$$

Asymptotic Projection Mapping Conversions and Geometry

The mapping of real-world distances to a stereo 3-D virtual volume with the asymptotic/asymmetric projection technique requires three major real-world to virtual volume conversions. The first conversion is the mapping of the real-world infinite distance to the maximum view distance (MAXD) of the stereo 3-D virtual volume, the second is that of mapping the real-world screen intercept (SI) distance to the virtual image screen distance (SD), and the third is mapping of the stereo 3-D virtual volume minimum view distance (MIND) to a real-world near clipping plane (NCP) value.

Real-world infinite distance to maximum view distance conversion. This mapping conversion is achieved by the viewport translation step in the matrix construction. The viewport translation value (VPT) is derived by a direct application of the lateral disparity equation shown in figure 4. Figure A1 shows the geometry of the VPT derivation. With interpupillary distance IPD, screen distance SD, and desired maximum view distance MAXD, the VPT derivation is as follows:

From similar triangles ABC and EDC, the relationship

$$\frac{\text{IPD}}{2.0 \times \text{MAXD}} = \frac{\text{VPT}}{\text{MAXD} - \text{SD}}$$

can be determined. Solving the above relationship for VPT, with appropriate reductions, will generate the VPT equation of

$$\text{VPT} = \frac{\text{IPD}}{2} \times \left(1.0 - \frac{\text{SD}}{\text{MAXD}}\right)$$

For a left-eye projection, the viewport needs to be translated to the left, which requires a negative translation value for the coordinate system used. Translating the viewport centers horizontally by amount VPT ensures that the maximum positive lateral disparity will never exceed VPT and that no image will be presented farther away than MAXD.

Screen intercept to screen distance mapping. The screen intercept mapping conversion is controlled by the projection offset translation step in the matrix construction. The geometry for the derivation of the projection offset translation (POT) value that will result in the desired screen intercept distance is shown in figure A2. Although the center of projection for the left-eye view is translated from the center of the viewer coordinate system (point B) by amount POT to point A, the result of the projection is presented to the left eye from point D, which is only offset by amount VPT (this is done to ensure that no object will have a positive lateral disparity greater than VPT and no image will appear farther away than MAXD). Presenting the image to the eye with a horizontal offset less than that from which the image was generated produces a foreshortening effect that places objects in the stereo 3-D virtual image closer to the viewer than they are in the real world. This is the basis for the screen intercept mapping conversion. An object at point C, which is located SI distance in the real-world scene directly in front of the viewer, will be shifted to point F in the virtual scene when the image is presented (lines AC and DF are parallel). The object will appear on the screen where line DF intersects the screen distance plane, with the lateral disparity equal to the distance between this intersection point and the center of the

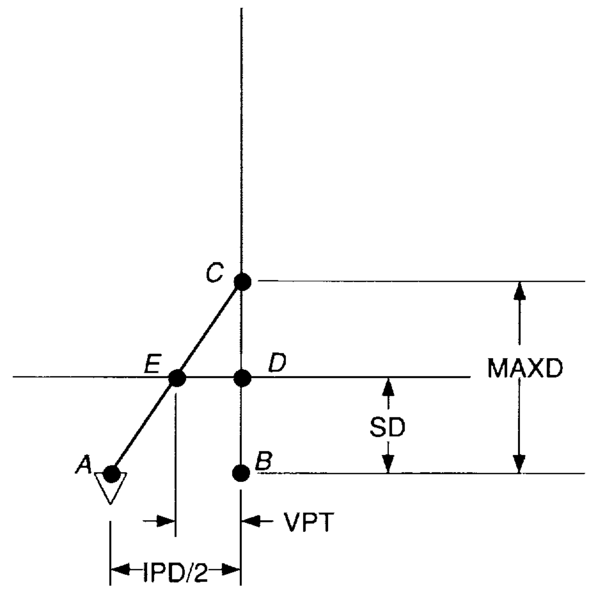

Figure A1. Geometry of VPT derivation.

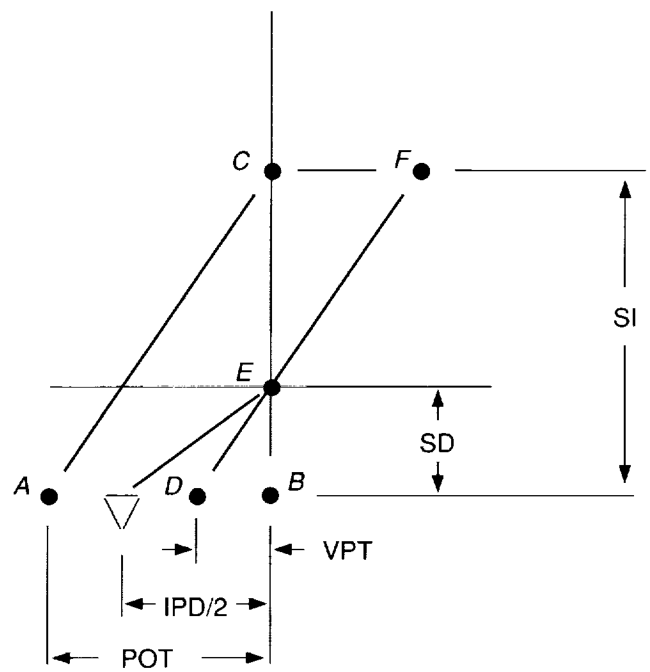

Figure A2. Geometry of POT derivation.

screen, point E. To make the object appear at the screen distance (SD) in the virtual image, it must be presented with no lateral disparity between the two eye-views. Thus, line DF must intersect the screen at point E. From this geometry, triangles ABC and DBE are similar, and the relationship of

$$\frac{\text{POT}}{\text{SI}} = \frac{\text{VPT}}{\text{SD}}$$

can be determined. Solving the above equation for POT results in

$$\mathrm{POT} = \mathrm{VPT} \times \frac{\mathrm{SI}}{\mathrm{SD}}$$

Inserting the equation generated for VPT earlier produces

$$\mathrm{POT} = \frac{\mathrm{IPD}}{2} \times \left(1.0 - \frac{\mathrm{SD}}{\mathrm{MAXD}}\right) \times \frac{\mathrm{SI}}{\mathrm{SD}}$$

The POT value for the left-eye translation is a positive (to the right) amount. This seemingly reversed sign is due to the fact that in the construction of projection matrices, the point of projection is considered fixed and the relative motion of the world is to the right for the left-eye offset.

Minimum view distance to near clipping plane conversion. With the VPT and POT values calculated, a real-world distance to virtual image distance mapping conversion can be applied with a guarantee that no virtual image distance will exceed MAXD and that objects with real-world distances of SI will appear at the plane of the screen in the virtual image. The remaining mapping conversion is to ensure that no object in the virtual image is presented closer than the minimum view distance (MIND). This is controlled by setting the real-world near clipping plane (NCP) value to a distance that corresponds to the MIND value in the virtual presentation. Figure A3 shows the geometry involved in the NCP derivation. The lateral disparity y of an object perceived at distance MIND can be calculated with the lateral disparity equation from figure 4 with MIND substituted for object depth d. The equation would take the form

$$y = \frac{\mathrm{IPD}}{2} \times \left(1.0 - \frac{\mathrm{SD}}{\mathrm{MIND}}\right)$$

and the object would be drawn at point E when displayed at screen distance SD. With the viewport center at point D, which is offset by amount VPT, line DE represents the projection of an object appearing at the MIND distance in the stereo 3-D virtual image. To determine where the MIND distance appears in the real-world scene, line DE is translated to the center of projection at point A, which is offset by the POT amount. The NCP value is determined where this new line AC (parallel to DE) intersects the center of the viewer coordinate

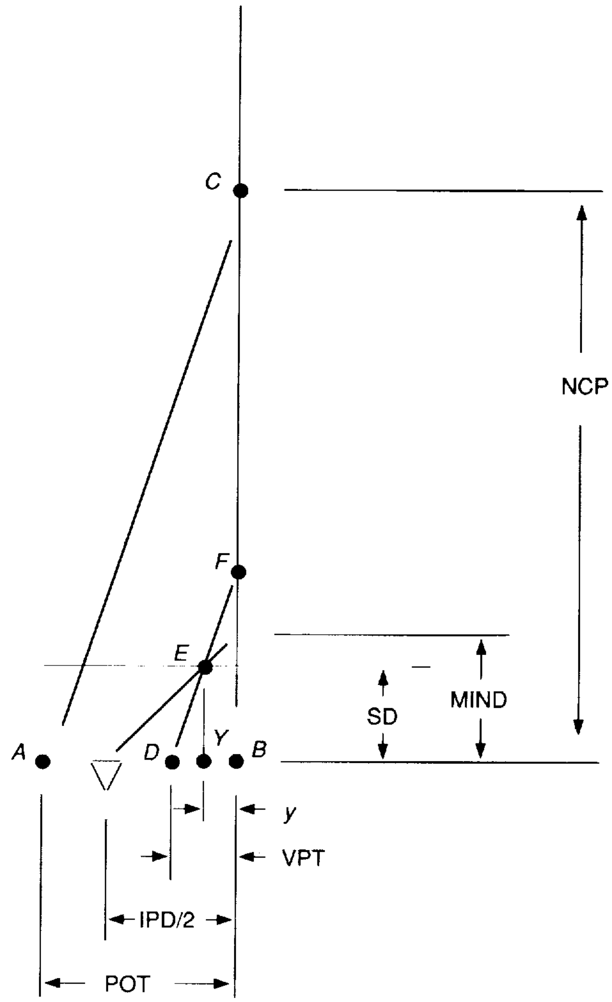

Figure A3. Geometry of NCP derivation.

system. From the similar triangles ABC and DYE, the relationship

$$\frac{\mathrm{NCP}}{\mathrm{FCP}} = \frac{\mathrm{SD}}{\mathrm{VPT} - y}$$

can be derived. By solving the above equation for NCP and inserting the known relationships for POT, VPT, and y, the NCP relationship can be reduced to

$$\mathrm{NCP} = \frac{(\mathrm{SI} \times \mathrm{MIND} \times \mathrm{MAXD}) - (\mathrm{SI} \times \mathrm{SD} \times \mathrm{MIND})}{(\mathrm{SD} \times \mathrm{MAXD}) - (\mathrm{SD} \times \mathrm{MIND})}$$

Overall geometry and distance mapping. One final useful derivation is the equation for the conversion from real-world scene distance to that of stereo 3-D virtual volume distance. Figure A4 represents the combination of figures A1 to A3 and shows the overall general geometry of the asymptotic/asymmetric projection for the left-eye view.

With screen distance SD, interpupillary distance IPD, and maximum screen distance MAXD, an object at point C, located distance d' from the viewer in the real-world scene, will appear at distance d in the stereo 3-D virtual presentation. From the geometry in figure A4, by using similar triangles ABC and DYE and substituting the VPT, POT, and y (from fig. 4) relations, the stereo 3-D virtual distance d can be derived as a function of the real-world scene distance d'. This function is

$$d = \frac{d' \times \mathrm{SD} \times \mathrm{MAXD}}{(d' \times \mathrm{SD}) + (\mathrm{SI} \times \mathrm{MAXD}) - (\mathrm{SI} \times \mathrm{SD})}$$

As a proof of the validity of this equation, four tests may be made: (1) The limit may be taken as real-world scene distance d' approaches infinity. When this is done, d is seen to approach MAXD. (2) If the screen intercept value SI is inserted for d', d becomes the screen distance SD. (3) If the relationship for the NCP real-world scene distance is substituted for d', the equation reduces to MIND. (4) If the screen intercept value SI is set to the screen distance SD, the function reduces to

$$d = \frac{d' \times \mathrm{MAXD}}{d' + \mathrm{MAXD} + \mathrm{SD}}$$

and the limit of this function as MAXD approaches infinity is d'. Thus, if the stereo 3-D virtual volume mapping parameters are set to equal their real-world counterparts (the screen intercept SI is set equal to the screen distance SD, and the maximum view distance MAXD is set to infinity), the stereo 3-D virtual volume distances match the real-world scene distances precisely.

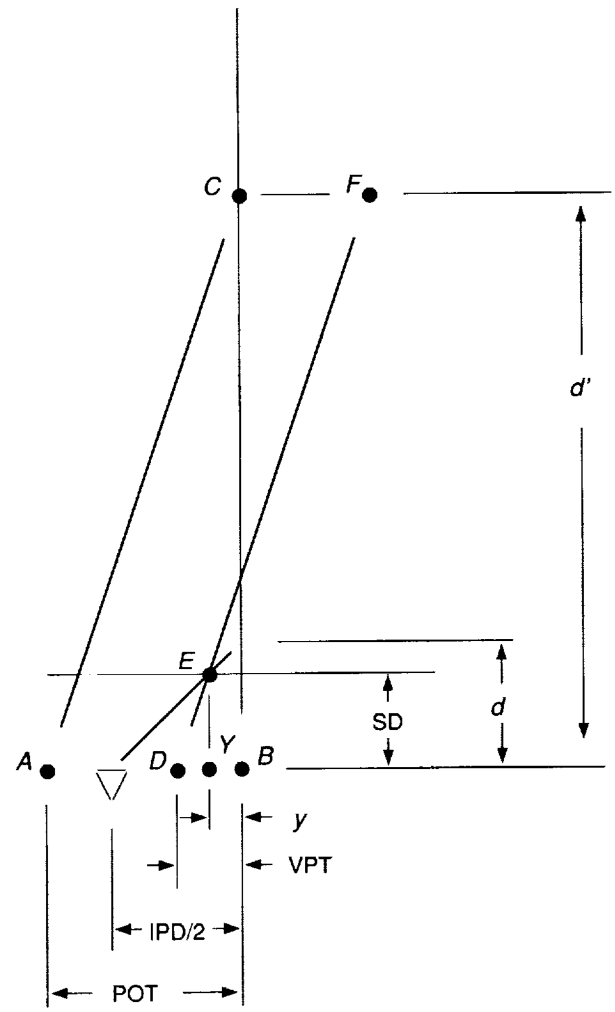

Figure A4. Overall geometry for asymptotic projection.

Appendix B

Piecewise Linear/Asymmetric Projection Transformation Matrix

The piecewise linear/asymmetric projection transformation matrix is a 4×4 element matrix that is used to convert a 3-D scene data base into a 2-D perspective representation for the left-eye and right-eye images of a stereo pair. The matrix is built based on a right-handed viewer coordinate system with the origin located at the viewpoint in the scene data base. The x-axis has positive direction to the right, the y-axis is positive up, and the z-axis is positive behind the viewer. All vertices in the data base are represented in homogeneous (x, y, z, w) form. The homogeneous coordinate w is a scale parameter and is initially set to 1. If, after any of the steps in the projection transformation, the w coordinate is a value other than 1, it is forced to unity by dividing the vertex coordinates (x, y, z, w) by the nonunity w coordinate value. For a more complete discussion of the homogeneous coordinate system, and 3-D geometric transformations, viewing, and projections, please refer to chapters 5 and 6 in reference 16.

To generate the two eye views, a left-eye matrix and a right-eye matrix are constructed for each partition of the piecewise linear projection. The example on the bottom half of figure 7 contains 4 partitions. The first is stereo region 1, the second is the area bounded by the end of stereo region 1 and the beginning of stereo region 2, the third is stereo region 2, and the fourth extends from the end of stereo region 2 to infinity. In this particular mapping example, partitions 2 and 4 will not contain stereo depth cues and will appear as flat postcard-like perspective images at discrete planes in the virtual volume. Objects in partition 2 will appear at the screen distance in the virtual volume, and objects in partition 4 will appear at the maximum view distance of the virtual image. To render a stereo 3-D image with a multiple partition piecewise linear projection, the entire scene data base must be rendered once for each partition for each eye. The piecewise linear mapping example in figure 7 would require 8 scene renderings (4 partitions $\times$ 2 eye views). During the construction of the matrices and the scene renderings for the separate partitions, the near and far clipping planes are used to limit the scene to the bounds of each partition. The near clipping plane scene distance is mapped to the virtual distance located at the minimum view distance. The far clipping plane scene distance is mapped to the virtual distance located at the maximum view distance. Objects in the scene data base whose depths fall between the near clipping plane and the far clipping plane will be linearly projected in stereo 3-D space in the virtual image between the minimum view distance and the maximum view distance. In the piecewise linear mapping example in figure 7, the first partition projection matrix would be built with mapping parameters of the near clipping plane equal to the beginning of stereo region 1, the minimum view distance equal to the in-front screen fusion limit (or to the in-front depth limit determined in ref. 15), the far clipping plane equal to the end of stereo region 1, and the maximum view distance equal to the screen distance. The second partition projection matrix would use a near clipping plane value equal to the end of stereo region 1, and minimum view distance equal to the screen distance, a far clipping plane value equal to the beginning of stereo region 2, and a maximum view distance set equal to the screen distance. The third partition projection matrix would be built with a near clipping plane distance set equal to the beginning of stereo region 2, a minimum view distance value set equal to the screen distance, a far clipping value equal to the end of stereo region 2, and a maximum view distance set equal to the behind screen fusion limit (or to the behind depth limit determined in ref. 15). The fourth and final partition projection matrix would use a near clipping plane value set equal to the end of stereo region 2, a minimum view distance equal to the behind screen fusion limit, a far clipping plane value set to near infinity, and a maximum view distance set equal to the behind screen fusion limit.

The piecewise linear projection matrix is also constructed with four parameters that define the physical aspects of the projection. These four physical parameters combined with the four mapping parameters discussed above give the display designer a total of eight parameters to control the presentation of a piecewise linear/asymmetric projection.

Physical Parameters

Interpupillary distance. The interpupillary distance (IPD) is the distance between the eyes of the person viewing the stereo 3-D presentation.

Screen distance. The screen distance (SD) is the distance from the stereo 3-D viewer to the display surface.

Width. The width (W) is the physical width of the display surface.

Height. The height (H) is the physical height of the display surface.

Piecewise Linear Mapping and Depth-Viewing Volume Parameters

Minimum view distance. The minimum view distance (MIND) is the minimum distance a virtual image in the current partition is to appear from the viewer. Care must be taken so that the minimum view distance of the closest partition does not exceed the in front of screen fusion limit.

Maximum view distance. The maximum view distance (MAXD) is the maximum distance a virtual image in the current partition is to appear from the viewer. Care must be taken so that the maximum view distance of the farthest partition does not exceed the behind screen fusion limit.

Near clipping plane. The near clipping plane (NCP) is the distance in the real-world scene of the current partition in front of which objects will not be drawn. The NCP is used in piecewise linear projection as the distance in the real world to be mapped to the minimum view distance in the stereo 3-D virtual image.

Far clipping plane. The far clipping plane (FCP) is the distance in the real-world scene of the current partition beyond which objects will not be drawn. The FCP is used in piecewise linear projection as the distance in the real world to be mapped to the maximum view distance in the stereo 3-D virtual image.

Matrix Construction

The piecewise linear/asymmetric projection transformation matrix is built in eight steps by combining basic geometric transformation matrices. Each of the transformations is based on values calculated from one or more of the aforementioned parameters.

Linear projection offset translation. The first step in the construction is to offset the centerline of the projection horizontally to generate either a left-eye or a right-eye coordinate system. This translation, in conjunction with the linear viewport separation translation later on, is one of the two transformations controlling the slope and intercept of the stereo 3-D mapping for the current partition. The linear projection offset translation (LPOT) is calculated based on five of the eight control parameters:

$$\mathrm{LPOT} = \frac{\mathrm{IPD}}{2} \times \left[\frac{\mathrm{FCP} \times \mathrm{NCP} \times (\mathrm{MAXD} - \mathrm{MIND})}{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP})}\right]$$

The derivation of the LPOT equation is discussed in a later section. The linear projection offset translation equation above is for the generation of the left-eye coordinate system. To generate the right-eye coordinate system, the linear projection offset translation is the negative of the left-eye value. The translation transformation matrix used for the linear projection offset is shown as matrix (B1):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{LPOT} & 0 & 0 & 1 \end{bmatrix} \qquad \text{(B1)}$$

z invert scale. In the right-handed coordinate system used for this construction, the positive z-axis extends behind the viewer. In order for the 3-D to 2-D conversion (perspective division) to work correctly, the coordinate system must be a left-handed system with the positive z-axis extending in front of the viewer. This transformation is achieved by inverting the z-axis with a z-scale of -1. The scale transformation matrix used for this operation is shown as matrix (B2):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(B2)}$$

Screen distance scale. When perspective division occurs, the 3-D data base is projected onto a 2-D plane. To maintain the correct stereo 3-D presentation, this 2-D plane must be the same distance away as the display screen. To accomplish this, the x-y plane of the coordinate system is scaled by the screen distance. The scale transformation matrix used in this step is shown as matrix (B3):

$$\begin{bmatrix} \mathrm{SD} & 0 & 0 & 0 \\ 0 & \mathrm{SD} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(B3)}$$

Perspective division. The perspective division step projects the 3-D scene data base onto a 2-D plane. This projection is achieved by dividing the x- and y-coordinates of the data base vertices by the vertex z-coordinate value. The screen distance scale step above has established the 2-D plane to be coincident with the display screen. To accomplish this division with a transformation matrix, a matrix that transposes the homogenous coordinate w with the vertex z-coordinate is used. This matrix is shown as matrix (B4):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \text{(B4)}$$

z-clip translation. After the perspective division operation, the x- and y-coordinate values of the scene data base vertices represent the 2-D projection. The z-coordinate value has been transformed to its reciprocal. Two transformations are necessary to bring the z-coordinate value within the z clipping range. The first of these is a translation. This translation is based on the near clipping plane and far clipping plane values and is the negative average of their reciprocals:

$$\mathrm{ZCT} = -\frac{\mathrm{FCP} + \mathrm{NCP}}{2.0 \times \mathrm{FCP} \times \mathrm{NCP}}$$

For a discussion of the effect of the near and far clipping planes on the z-clip translation, please refer to chapter 6 of reference 16. The z-clip translation transformation matrix is shown as matrix (B5):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \mathrm{ZCT} & 1 \end{bmatrix} \qquad \text{(B5)}$$

z-clip scale. The second of the z-coordinate transformations necessary to bring the z-coordinate value into the z clipping range is a scale transformation. Like the z-clip translation, the z-clip scale is based upon the near and far clipping plane values, but it is equal to the negative reciprocal of one-half of the distance between their reciprocals:

$$\mathrm{ZCS} = -\frac{2.0 \times \mathrm{FCP} \times \mathrm{NCP}}{\mathrm{FCP} - \mathrm{NCP}}$$

For a discussion of the effect of the near and far clipping planes on the z-clip scale, please refer to chapter 6 of reference 16. The z-clip scale transformation matrix is shown as matrix (B6):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{ZCS} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(B6)}$$

Linear viewport separation. The separation between the left-eye and right-eye viewports and the real-world scene distance far clipping plane value controls the maximum positive lateral disparity for the current partition. As an object in the real-world scene approaches infinity, the object approaches the center of the projection, and the center of the projection appears in the center of the viewport. Thus, the viewport separation controls where objects near infinity in the real-world scene would appear in the current partition of the piecewise linear projection. In a piecewise linear projection, the far clipping plane value is used as the farthest distance in the real-world scene that is to appear in the current partition and at the maximum view distance MAXD. In most partitions of a piecewise linear projection, the farthest objects drawn are not near infinity, so the linear viewport translation amount LVPT must be calculated such that the maximum positive lateral disparity is generated by objects at real-world scene distances equal to the far clipping plane value, and the lateral disparity generated must produce objects perceived at MAXD. The linear viewport translation, along with the linear projection offset translation discussed earlier, is one of the two transformations controlling the slope and intercept of the stereo 3-D mapping of the current partition:

$$\mathrm{LVPT} = -\frac{\mathrm{IPD}}{2} \times \left[\frac{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP}) + \mathrm{SD} \times (\mathrm{NCP} \times \mathrm{MAXD} - \mathrm{FCP} \times \mathrm{MIND})}{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP})}\right]$$

The derivation of this equation is discussed in a later section. The equation above gives the linear viewport separation translation amount for the left-eye image. The right-eye value is the negative of the left-eye value. The linear viewport separation translation transformation matrix is shown as matrix (B7):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{LVPT} & 0 & 0 & 1 \end{bmatrix} \tag{B7}$$

Field-of-view scale. To maintain the proper aspect ratio and fields of view, the image plane that contains the 2-D projection of the 3-D scene data base must be scaled based on the physical constraints of the presentation. The three physical parameters that determine FOV and aspect ratio are the width and height of the display surface, and the viewer-to-screen distance. The screen distance effect has already been accounted for in the screen distance scale step. The display surface width and height must now be accounted for. This is accomplished by scaling the x-coordinate by the reciprocal of one-half the width, and by scaling the y-coordinate by the reciprocal of one-half the height:

$$\mathrm{FOVX} = \frac{2}{\mathrm{W}}$$

$$\mathrm{FOVY} = \frac{2}{\mathrm{H}}$$

The FOV scale transformation matrix is shown as matrix (B8):

$$\begin{bmatrix} \mathrm{FOVX} & 0 & 0 & 0 \\ 0 & \mathrm{FOVY} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{B8}$$

Piecewise linear/asymmetric projection transformation matrix. Sequentially multiplying the eight matrices presented above will generate the piecewise linear/asymmetric projection transformation matrix. This matrix is shown as matrix (B9). The conversion of the 3-D scene data base to the 2-D perspective view used for the current partition in the left-eye and right-eye images of the stereo pair with this matrix as the projection matrix is as discussed in section 6.5 in reference 16:

$$\begin{bmatrix} (\mathrm{SD})(\mathrm{FOVX}) & 0 & 0 & 0 \\ 0 & (\mathrm{SD})(\mathrm{FOVY}) & 0 & 0 \\ -(\mathrm{LVPT})(\mathrm{FOVX}) & 0 & -(\mathrm{ZCT})(\mathrm{ZCS}) & -1 \\ -(\mathrm{LPOT})(\mathrm{SD})(\mathrm{FOVX}) & 0 & -(\mathrm{ZCS}) & 0 \end{bmatrix} \tag{B9}$$

Piecewise Linear Projection Mapping Conversions and Geometry

Figure B1 represents the geometry for an object at the far clipping plane distance for one partition of a left-eye piecewise linear projection. The center of projection for the left-eye image is translated from the center of the viewer coordinate system (point B) by the linear projection offset translation step in the matrix construction, by amount LPOT, to point A. For an object at point C, which is FCP distance from the viewer in the real-world scene, line AC represents the projection of the objects as seen from the center of projection. The image is then presented to the viewer in a viewport that is offset, by the linear viewport translation step in the matrix construction, from the center of the viewer coordinate system by amount LVPT to point D. Thus, when the object is viewed it is shifted to point F (lines AC and DF are parallel) and displayed on the

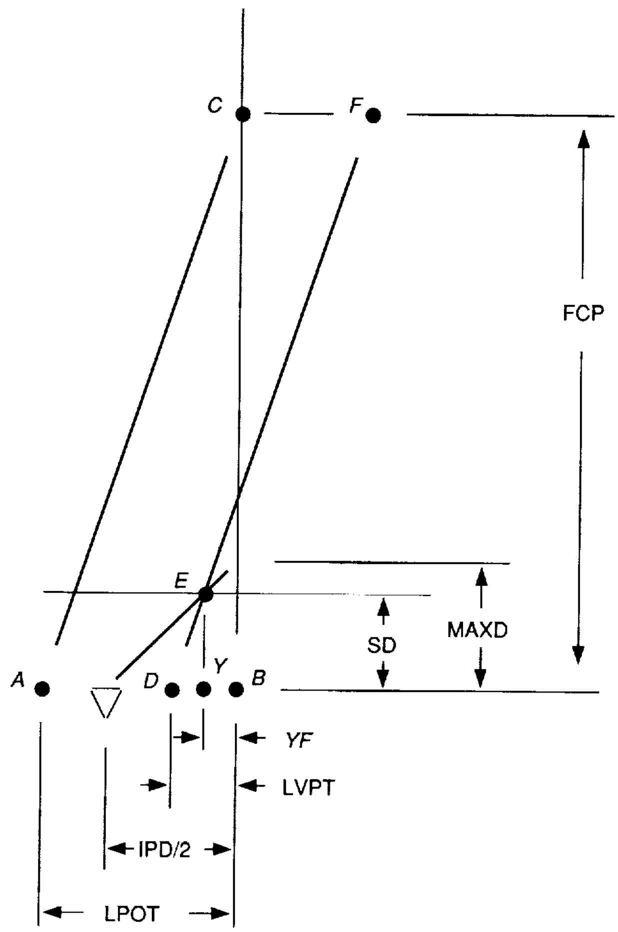

Figure B1. Geometry at far clipping plane.

screen where line DF intersects the screen plane, which is SD distance from the viewer, at point E. The left eye then views the object with a lateral disparity of amount YF and the object appears MAXD distance from the viewer in the stereo 3-D virtual image. From the lateral disparity equation in figure 4, the equation for the object lateral disparity YF can be determined:

$$YF = \frac{\text{IPD}}{2} \times \left(1.0 - \frac{\text{SD}}{\text{MAXD}}\right)$$

From similar triangles ABC and DYE, the relationship

$$\frac{\text{LPOT}}{\text{FCP}} = \frac{\text{LVPT} - YF}{\text{SD}}$$

can be determined. Solving this equation for LPOT produces

$$\text{LPOT} = \frac{\text{FCP} \times (\text{LVPT} - YF)}{\text{SD}}$$

Figure B2 represents the geometry of an object at the near clipping plane distance for one partition of a left-eye piecewise linear projection. Like the far clipping plane geometry of figure B1, the center of projection has

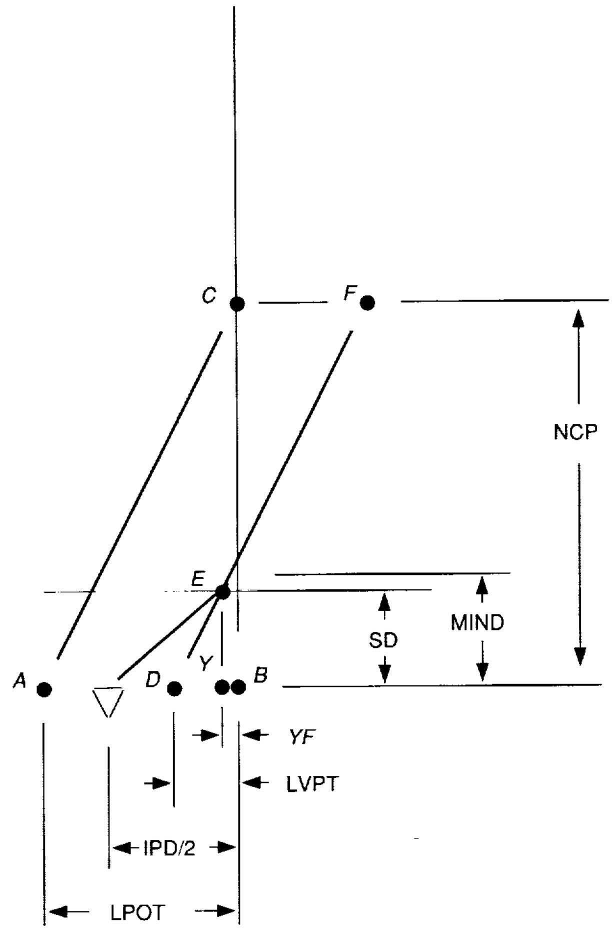

Figure B2. Geometry at near clipping plane.

been translated to point A, and the viewport center has been translated to point D. Assuming the geometries in figures B1 and B2 are from the same piecewise linear projection, the amounts LPOT, IPD, LVPT, and SD are the same between the figures. For figure B2, line AC represents the projection of an object located at point C, which is distance NCP from the viewer in the real-world scene. When the object is presented to the viewer from the center of the viewport at point D, the object is shifted to point F and is displayed on the screen where line DF intersects the screen plane, point E. The left eye then views the object with a lateral disparity of amount YN and the object appears distance MIND from the viewer in the stereo 3-D virtual volume. From the lateral disparity equation in figure 4, the equation for the object lateral disparity YN can be determined:

$$YN = \frac{\mathrm{IPD}}{2} \times \left(1.0 - \frac{\mathrm{SD}}{\mathrm{MIND}}\right)$$

From similar triangles ABC and DYE, the relationship

$$\frac{\mathrm{LPOT}}{\mathrm{NCP}} = \frac{\mathrm{LVPT} - \mathrm{YN}}{\mathrm{SD}}$$

can be determined. Solving this equation for LPOT produces

$$\mathrm{LPOT} = \frac{\mathrm{NCP} \times (\mathrm{LVPT} - \mathrm{YN})}{\mathrm{SD}}$$

Substituting the LPOT equation derived from figure B1 for LPOT in the equation derived from figure B2 generates

$$\frac{\mathrm{FCP} \times (\mathrm{LVPT} - YF)}{\mathrm{SD}} = \frac{\mathrm{NCP} \times (\mathrm{LVPT} - YN)}{\mathrm{SD}}$$

Substituting the equations for YF and YN into the above relationship and solving for LVPT will produce the linear viewport translation equation:

$$\mathrm{LVPT} = \frac{\mathrm{IPD}}{2} \times \left[\frac{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP}) + \mathrm{SD} \times (\mathrm{NCP} \times \mathrm{MAXD} - \mathrm{FCP} \times \mathrm{MIND})}{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP})}\right]$$

For a left-eye projection matrix, the linear viewport translation is to the left, which requires a negative value for the coordinate system used. Substituting the LVPT equation above into either of the two LPOT equations, with appropriate reductions, will generate the linear projection offset equation

$$\mathrm{LPOT} = \frac{\mathrm{IPD}}{2} \times \left[\frac{\mathrm{FCP} \times \mathrm{NCP} \times (\mathrm{MAXD} - \mathrm{MIND})}{\mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP})}\right]$$

The linear projection offset translation value for a left-eye projection matrix is to the right, a positive value. This seemingly reversed sign is due to the fact that in the construction of projection matrices, the point of projection is considered fixed and the relative motion of the world is to the right for the left eye.

As a final derivation, the equation for the mapping of real-world scene distances to stereo 3-D piecewise linear partition distances can be determined from the geometry in figure B3 and from the LPOT, LVPT, and the figure 4 lateral disparity equation. From the geometry in figure B3, an object located at point C, which is distance d' from the viewer in the real-world scene, will appear to the viewer at a distance of d in the stereo 3-D virtual presentation. With similar triangles ABC and DYE, and the LPOT, LVPT, and y (from fig. 4) equations, the stereo 3-D virtual distance d can be derived as a function of the real-world scene distance d'. This function is

$$d = \frac{d' \times \mathrm{MAXD} \times \mathrm{MIND} \times (\mathrm{FCP} - \mathrm{NCP})}{d' \times (\mathrm{FCP} \times \mathrm{MIND} - \mathrm{NCP} \times \mathrm{MAXD}) + \mathrm{FCP} \times \mathrm{NCP} \times (\mathrm{MAXD} - \mathrm{MIND})}$$

As a proof of the validity of this equation, two tests may be made: (1) If the real-world scene value of FCP is substituted for the real-world scene distance d', the equation reduces to MAXD. (2) If the real-world scene value of NCP is substituted for the real-world distance d', the equation reduces to MIND.

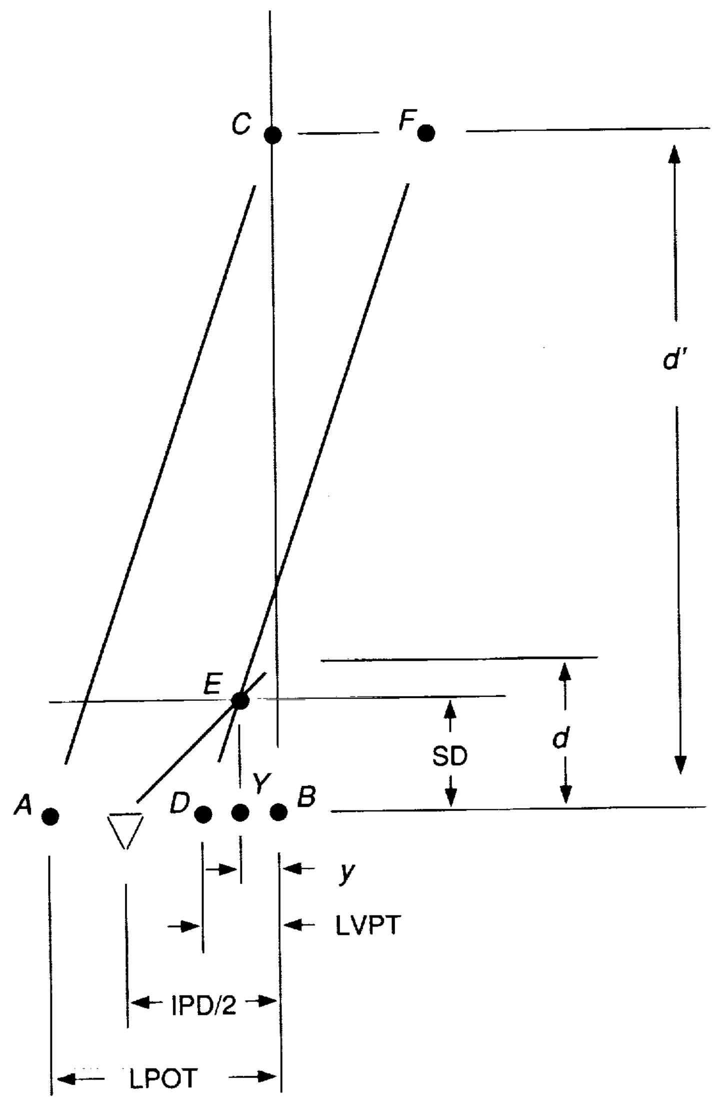

Figure B3. Geometry for piecewise linear projection.

References

1. Nataupsky, Mark; Turner, Timothy L.; Lane, Harold; and Crittenden, Lucille: Development of a Stereo 3-D Pictorial Primary Flight Display. *Spatial Displays and Spatial Instruments*, Stephen R. Ellis, Mary K. Kaiser, and Arthur Grunwald, eds., NASA CP-10032, 1989, pp. 39-1–39-8.

2. Nataupsky, Mark; and Crittenden, Lucille: Stereo 3-D and Non-Stereo Presentations of a Computer-Generated Pictorial Primary Flight Display With Pathway Augmentation. *A Collection of Technical Papers—AIAA/IEEE 8th Digital Avionics Systems Conference*, Oct. 1988, pp. 552–557. (Available as AIAA-88-3965-CP.)

3. Williams, S. P.; and Parrish, R. V.: Stereopsis Cueing Effects on a Simulated Precision Rotorcraft "Hover-in Turbulence" Task. *A Collection of Technical Papers—AIAA Flight Simulation Technologies Conference and Exhibit*, Aug. 1989, pp. 195–212. (Available as AIAA-89-3289-CP.)

4. Parrish, Russell V.; and Williams, Steven P.: *Stereopsis Cueing Effects on Hover-in-Turbulence Performance in a Simulated Rotorcraft.* NASA TP-2980, AVSCOM TR-90-B-002, 1990.

5. Martin, Wayne L.; and Warner, Debra A.: *The Use of a Virtual Cockpit in a Simulated Helicopter Attack Mission (Preliminary Observations).* AFAMRL-TP-85-301, U.S. Air Force, Feb. 1985.

6. Bridges, Alan L.; and Reising, John M.: Three-Dimensional Stereographic Displays. *NAECON 1987—IEEE 1987 National Aerospace and Electronics Conference, Volume 1,* IEEE Catalog No. 87CH2450-5, Inst. of Electrical and Electronics Engineers, Inc., 1987, pp. 233–240.

7. Zenyuh, John P.; Reising, John M.; Walchli, Scott; and Biers, David: A Comparison of a Stereographic 3-D Display Versus a 2-D Display Using an Advanced Air-to-Air Format. *Proceedings of the Human Factors Society 32nd Annual Meeting, Volume 1*, 1988, pp. 53–57.

8. Mountford, S. Joy; and Somberg, Ben: Potential Uses of Two Types of Stereographic Display Systems in the Airborne Fire Control Environment. *Proceedings of the Human Factors Society 25th Annual Meeting*, Robert C. Sugarman, A. Stephen Baum, Jan L. Ditzian, Douglas J. Funke, Valerie J. Gawron, and K. Ronald Laughery, eds., 1981, pp. 235–239.

9. Setterholm, Jeffrey M.; Mountford, S. Joy; and Turner, Paul N.: *Assessment of Stereographics for Fire Control and Navigation in Fighter Aircraft.* AFWAL-TR-82-3008, U.S. Air Force, Mar. 1982. (Available from DTIC as AD A115 414.)

10. Woodruff, Robert R.; Hubbard, David C.; and Shaw, Alex: *Advanced Simulator for Pilot Training and Helmet-Mounted Visual Display Configuration Comparisons.* AFHRL-TR-84-65, U.S. Air Force, May 1985. (Available from DTIC as AD A155 326.)

11. Kruk, Ronald; and Longridge, Thomas M.: Binocular Overlap in a Fiber Optic Helmet Mounted Display. *The 1984 IMAGE Conference III*, AFHRL-TR-84-36, U.S. Air Force, Sept. 1984, pp. 363–378. (Available from DTIC as AD P004 331.)

12. Kim, Won S.; Ellis, Stephen R.; Tyler, Mitchell E.; Hannaford, Blake; and Stark, Lawrence W.: Quantitative Evaluation of Perspective and Stereoscopic Displays in Three-Axis Manual Tracking Tasks. *IEEE Trans. Syst., Man, & Cybern.*, vol. SMC-17, no. 1, Jan./Feb. 1987, pp. 61–72.

13. Eshow, Michelle M.; Aiken, Edwin W.; and Hindson, William S.: Preliminary Results of a Flight Investigation of Rotorcraft Control and Display Laws for Hover. *Proceedings—National Specialists' Meeting on Rotorcraft Flight Controls and Avionics*, American Helicopter Soc., Oct. 1987, pp. 1–13.

14. Hess, Ronald A.; and Gorder, Peter James: Design and Evaluation of a Cockpit Display for Hovering Flight. AIAA-88-4495, Sept. 1988.

15. Parrish, Russell V.; and Williams, Steven P.: *Determination of Depth-Viewing Volumes for Stereo Three-Dimensional Graphic Displays.* NASA TP-2999, AVSCOM TM-90-B-016, 1990.

16. Foley, James D.; Van Dam, Andries; Feiner, Steven K.; and Hughes, John F.: *Computer Graphics—Principles and Practice*, Second ed. Addison-Wesley Publ. Co., Inc., c.1990.

17. Boff, Kenneth R.; and Lincoln, Janet E., eds.: *Engineering Data Compendium—Human Perception and Performance, Volume II.* Harry G. Armstrong Aerospace Medical Research Lab., Wright-Patterson Air Force Base, 1988.

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13. ABSTRACT *(Maximum 200 words)*
Three-dimensional pictorial displays incorporating depth cues by means of stereopsis offer a potential means of presenting information in a natural way to enhance situational awareness and improve operator performance. Conventional computational techniques rely on asymptotic projection transformations and symmetric clipping to produce the stereo display. Implementation of two new computational techniques, an asymmetric clipping algorithm and piecewise linear projection transformation, provides the display designer with more control and better utilization of the effective depth-viewing volume to allow full exploitation of stereopsis cuing. Asymmetric clipping increases the perceived field of view (FOV) for the stereopsis region. The total horizontal FOV provided by the asymmetric clipping algorithm is greater throughout the scene viewing envelope than that of the symmetric algorithm. The new piecewise linear projection transformation allows the designer to creatively partition the depth-viewing volume, with freedom to place depth cuing at the various scene distances at which emphasis is desired.

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Stereo displays; Transformation metrics; Field of view; Stereopsis cuing; Computer graphics; Viewing transformations; Clipping algorithms; Depth-viewing volume

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