Calculation of Stress Intensity Factors in an Isotropic Multicracked Plate: Part I - Theoretical Development

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Abstract

An essential part of describing the damage state and predicting the damage growth in a multicracked plate is the accurate calculation of stress intensity factors (SIF's). Here, a methodology and rigorous solution formulation for SIF's of a multicracked plate, with fully interacting cracks, subjected to a far-field arbitrary stress state is presented. The fundamental perturbation problem is derived, and the steps needed to formulate the system of singular integral equations whose solution gives rise to the evaluation of the SIF's are identified. This analytical derivation and numerical solution are obtained by using intelligent application of symbolic computations and automatic FORTRAN generation capabilities (described in the second part of this paper). As a result, a *symbolic/FORTRAN* package, named SYMFRAC, that is capable of providing accurate SIF's at each crack tip has been developed and validated.

Nomenclature

α	–	offset angle between inner tips of two parallel cracks
β_{lr}, β_{mz}	–	direction cosines between two local coordinate systems
ϵ_{jl}	–	strain tensor
μ_k	–	four roots of the characteristic equation
ν	–	Poisson's ratio
$\sigma^o_{jl}, \sigma^T_{jl}$	–	far-field and total stress field, respectively
$\sigma^o_{XX}, \sigma^o_{YY}, \sigma^o_{XY}$	–	components of stress in global coordinate system

φ_j	–	angle defining orientation of local coordinate system
ξ, τ	–	normalized real variables
$\Phi(s, y)$	–	Fourier transform of Airy stress function with respect to x variable
a_j	–	half crack length
a_{11}, a_{12}, a_{22}	–	coefficients of strain-stress relationship (compliant matrix)
$f_{\eta j}$	–	auxiliary functions
k_1, k_2	–	mode-I and mode-II stress intensity factors
$ker^{\alpha\beta}_{\gamma\delta}$	–	Fredholm kernels
p_j	–	normal traction at crack surface
q_j	–	shear traction at crack surface
s	–	Fourier variable
$\underline{r_j}$	–	position vector defining the origin of a local coordinate system
r_{jX}, r_{jY}	–	components of the position vector $\underline{r_j}$
u, v	–	displacement associated with x and y coordinates, respectively
w	–	weight function
x_j, y_j and X, Y	–	local and global coordinates
$[A]$	–	kernel matrix
C'_j, C_j	–	functions of s in Fourier space (i.e., constants in x, y-real space)
E	–	Young's modulus
$F_j(x_j, y_j)$	–	Airy stress function
$G_{\eta j}(\tau_p)$	–	discrete auxiliary function
$\{\mathcal{R}\}$	–	loading vector

1 Introduction

In a wide range of materials damage progression is strongly connected with microcrack nucleation and growth in solids. Hoagland et al. (1973), Leckie and Hayhurst (1974) and Meyers (1985), observed that a large part of applied energy is dissipated because the microcracks extend the fracture zone of a larger propagating crack. Similarly, Kachanov (1958) noticed that creep rupture is caused by the accelerated growth and localization of microcracks in the material. Therefore, it has been concluded that there must be a relationship between continuum damage mechanics and fracture mechanics concepts as applied to the problem of a multicracked plate.

Historically, the solution of a multicracked plane problem is typically solved using one of two techniques; the complex potential method (see Horii and Nemat-Nasser (1985)) or the singular integral equation approach (Erdogan (1978)). In this study we have selected the singular integral equation technique because of our familiarity with the method and its successful application to a number of plane problems; for example the work of Badaliance and Gupta (1976) on isotropic problems, Erdogan (1978) and Binienda et al. (1991) on anisotropic and homogeneous problems, and Delale (1985) on nonhomogeneous problems involving one or two independent cracks. Although the results for multicrack problems obtained using the singular integral equation approach in conjunction with the collocation technique are accurate, the required derivation of the system of singular integral equations is lengthy and tedious, even for the simplest case.

One way to avoid this tedious derivation (long hours of searching for possible errors and the inevitable "debugging" of a numerical program) is to design and develop a computer code that can derive the system of singular integral equations symbolically and can automatically generate the associated FORTRAN code required for the solution, by using the collocation technique. With such a code a new class of problems can be analyzed more rigorously and efficiently.

This subject is covered in two parts; Part I describes the general formulation and solution of the stress intensity factors (SIF's) at the tips of n number of straight microcracks embedded in an infinite isotropic plate that is subjected to in-plane loading and Part II (Arnold et al. (1992)), describes the symbolic computational aspects and numerical implementation. We expect the symbolic computational methodology established here and in Part II for n straight cracks in an isotropic plate to be easily adapted to an anisotropic material and, in the future, to more complicated crack geometries and nonhomogeneous materials. **Part I concludes with a comparison to validate the** accuracy of the results obtained with the present formulation and those reported in the literature, whereas other more complex examples, are presented in Part II (Arnold et al. (1992)).

2 Problem Formulation

Consider multiple cracks embedded in an infinite isotropic plate (Fig. 1(a)). The plate is under a far-field stress denoted by σ^o_{jl}, (in particular σ^o_{XX}, σ^o_{YY}, and σ^o_{XY}, where (X,Y) is the global coordinate system), and the cracks are defined in their local frames (x_j,y_j) (Fig. 1(b)). The origin of each local frame is defined by the position vector $\underline{r}_j$, and the orientation of the local frame with respect to the global frame is defined by the angle φ_j. Each crack is symmetrically situated within its own coordinate system and is $2a_j$ long, as shown in Fig. 1(b).

The general solution formulation can be outlined in four basic steps. The first step is to derive the local stress equations for each crack in its respective local coordinate system. This derivation, which need be done only once, is achieved by defining the fundamental problem; that is a single crack in an infinite isotropic plane (Fig. 1(b)). The fundamental problem is then decomposed into two subproblems: the problem of the undamaged plate containing an imaginary crack (Fig. 1(c)) and the perturbation problem (Fig. 1(d)) of a plate with a single crack subjected to the appropriate crack-surface tractions, which are found from the solution of the complementary undamaged problem. The analysis of the perturbation problem leads to singular stresses that govern local crack tip behavior.

The second step is to formulate the total perturbation stress field for each crack, which includes the interaction of all cracks through the summation of the transformed local stresses of all other cracks. In the third step of the formulation, the total stress equations are normalized. A set of Cauchy type singular integral equations, expressed in terms of unknown auxiliary functions, is obtained by subjecting the total perturbation stress equations to the crack-surface traction field at each crack location. The fourth and final step of the formulation is to express the SIF's in terms of the discrete auxiliary functions, $G_{\eta j}(\tau_p)$, evaluated at each crack tip. These discrete auxiliary functions are obtained through the implementation of the Lobatto-Chebyshev collocation technique. Now let us discuss each step in more detail.

2.1 Local Stress Formulation

Consider the fundamental problem (Fig. 1(b)), which is defined as a single crack in an infinite isotropic plate, whose solution can be obtained by decomposing it into an undamaged problem (Fig. 1(c)) and a perturbation problem (Fig. 1(d)). The essence of this decomposition is that the traction forces applied along the crack surface in the perturbation problem are opposite to the obtained stress field of the undamaged plate at the particular location of the imaginary crack. As a result, this undamaged traction field can be defined in terms of the normal (p_j) and shear (q_j) stress components along the imaginary crack surface:

$$p_j(x_j) = \sigma_{y_j y_j}(x_j, 0) \tag{1}$$

$$q_j(x_j) = \sigma_{x_jy_j}(x_j, 0) \tag{2}$$

where

$$\sigma_{y_jy_j}(x_j, 0) = \sigma^o_{XX} \sin^2\varphi_j + \sigma^o_{YY} \cos^2\varphi_j - \sigma^o_{XY} \sin\ 2\varphi_j \tag{3}$$

$$\sigma_{x_jy_j}(x_j, 0) = -\frac{\sigma^o_{XX} - \sigma^o_{YY}}{2} \sin\ 2\varphi_j + \sigma^o_{XY} \cos\ 2\varphi_j \tag{4}$$

The mixed boundary conditions for the perturbation part of the fundamental problem (Fig. 1(d)) are expressed in terms of stresses

$$\sigma_{y_jy_j} = -p_j(x_j) \quad \text{and} \quad \sigma_{x_jy_j} = -q_j(x_j) \tag{5}$$

along the crack surface ($i.e.$, $y_j = 0\ and - a_j \leq x_j \leq a_j$), and in terms of continuity of displacements

$$v^+ = v^- \quad \text{and} \quad u^+ = u^- \tag{6}$$

outside of the crack ($i.e.$, $y_j = 0$ and $|x_j| > a_j$). Here + indicates the value of displacement at a point approached from the positive side of the plate, ($i.e.$, $y > 0$), whereas − indicates the same point approached from the negative side of plate, ($i.e.$, $y < 0$).

The governing equation for the preceding two-dimensional isotropic plate problem can be expressed in terms of the Airy stress function $F_j(x_j, y_j)$ as

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \tag{7}$$

Note: Any stress function that satisfies Eq. (7) and the mixed boundary conditions, Eqs. (5) and (6), constitutes the solution to this problem.

A rigorous solution for this stress function can be obtained by employing the following Fourier transformation relation

$$F(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(s, y) e^{-isx} ds \tag{8}$$

thereby transforming the partial differential Eq. (7) into a fourth-order ordinary differential equation with real constant coefficients:

$$s^4\Phi - 2s^2\frac{d^2\Phi}{dy^2} + \frac{d^4\Phi}{dy^4} = 0 \tag{9}$$

A known general solution of Eq., (9) is

$$\Phi(s, y) = C\ e^{\mu\ y\ s} \tag{10}$$

Differentiating Eq. (10) and substituting the results into Eq. (9), gives the characteristic equation

$$\mu^4 - 2\mu^2 + 1 = 0 \tag{11}$$

whose roots are $\mu_1 = \mu_2 = -\mu_3 = -\mu_4 = 1$. Therefore, the Airy stress function has the following form:

$$F(x, y) = \int_{-\infty}^{\infty} \left[(C_1' + C_2'y)e^{ys} + (C_3' + C_4'y)e^{-ys}\right] e^{-isx} ds \tag{12}$$

In order to satisfy the physical requirement that the stress function is finite throughout the domain of the plate, an alternative form is defined for the upper half plane (for $y > 0$)

$$F(x, y^+) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (C_1 + C_2 y) e^{-|s|y} e^{-isx} ds \tag{13}$$

and for the lower half plane (for $y < 0$)

$$F(x, y^-) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (C_3 + C_4 y) e^{|s|y} e^{-isx} ds \tag{14}$$

which are automatically bounded at infinity. Note: Constants C_j for j = 1, 2, 3, and 4 are functions of the Fourier variable s and are determined by using the local stress continuity conditions at the boundaries between the half planes ($y = 0$) and by using the perturbation boundary conditions subsequent to the determination of the total stresses at each crack location.

The continuity conditions for local stresses, σ_{yy} and σ_{xy}, are identically satisfied, given

$$F(x, 0^+) = F(x, 0^-) \tag{15}$$

and

$$\frac{\partial F(x, 0^+)}{\partial y} = \frac{\partial F(x, 0^-)}{\partial y} \tag{16}$$

respectively. Consequently, Eq. (15) gives

$$C_3 = C_1 \tag{17}$$

and Eq. (16), upon substitution of Eq. (17), leads to

$$C_4 = -2|s|C_1 + C_2 \tag{18}$$

Completion of the posed mixed boundary value problem requires the formulation of the local stresses and displacements. The local stresses in each half plane are obtained by taking the appropriate (Timoshenko 1969) second derivatives of the stress function given in Eqs. (13) and (14). Thus the stresses associated with the upper half plane are

$$\sigma_{xx}^+ = \frac{1}{2\pi} \int_{-\infty}^{\infty} (s^2 C_1 - 2|s|C_2 + s^2 y C_2) e^{-[|s|y + isx]} ds \tag{19}$$

$$\sigma_{yy}^+ = -\frac{1}{2\pi} \int_{-\infty}^{\infty} s^2 (C_1 + C_2 y) e^{-[|s|y + isx]} ds \tag{20}$$

$$\sigma_{xy}^+ = \frac{i}{2\pi} \int_{-\infty}^{\infty} s(-|s|C_1 + C_2 - |s| y C_2) e^{-[|s|y + ixs]} ds \tag{21}$$

and those associated with the lower half plane are

$$\sigma_{xx}^- = \frac{1}{2\pi} \int_{-\infty}^{\infty} (s^2 C_3 + 2|s|C_4 + s^2 y C_4) e^{[|s|y - isx]} ds \tag{22}$$

$$\sigma_{yy}^- = -\frac{1}{2\pi} \int_{-\infty}^{\infty} s^2 (C_3 + C_4 y) e^{[|s|y - isx]} ds \tag{23}$$

$$\sigma_{xy}^- = \frac{i}{2\pi} \int_{-\infty}^{\infty} s(|s|C_3 + C_4 + |s| y C_4) e^{[|s|y - ixs]} ds \tag{24}$$

The displacement components (u and v) associated with the x and y directions, respectively, are in turn obtained by integration of the appropriate strains,

$$u(x,y) = \int \epsilon_{xx}(x,y)dx \tag{25}$$

$$v(x,y) = \int \epsilon_{yy}(x,y)dy \tag{26}$$

which are related to the local stresses through the utilization of the generalized Hooke's law:

$$\epsilon_{xx} = a_{11}\sigma_{xx} + a_{12}\sigma_{yy} \tag{27}$$

$$\epsilon_{yy} = a_{12}\sigma_{xx} + a_{11}\sigma_{yy} \tag{28}$$

where

$$a_{12} = -\frac{\nu^*}{E^*}\ , \ a_{11} = \frac{1}{E^*} \tag{29}$$

and

$$\nu^* = \begin{cases} \nu & for\ \ \text{plane stress} \\ \dfrac{\nu}{1-\nu} & for\ \ \text{plane strain} \end{cases} \qquad E^* = \begin{cases} E & for\ \ \text{plane stress} \\ \dfrac{E}{1-\nu^2} & for\ \ \text{plane strain} \end{cases} \tag{30}$$

(E is Young modulus and ν is Poisson's Ratio).

As a result the upper half plane displacements are

$$\begin{aligned} u^+ = & -\tfrac{a_{11}}{2\pi} \int_{-\infty}^{\infty} \tfrac{1}{is} \left[(s^2C_1 - 2|s|C_2 + s^2yC_2)\right] e^{-[|s|y+isx]}ds \\ & + \tfrac{a_{12}}{2\pi} \int_{-\infty}^{\infty} \tfrac{s^2}{is} \left[C_1 + C_2y\right] e^{-[|s|y+isx]}ds \end{aligned} \tag{31}$$

and

$$\begin{aligned} v^+ = & -\tfrac{a_{12}}{2\pi} \int_{-\infty}^{\infty} \left[|s|C_1 + C_2(|s|y - 1)\right] e^{-[|s|y+isx]}ds \\ & + \tfrac{a_{11}}{2\pi} \int_{-\infty}^{\infty} \left[|s|C_1 + C_2(|s|y + 1)\right] e^{-[|s|y+isx]}ds \end{aligned} \tag{32}$$

whereas the lower half plane displacements become

$$\begin{aligned} u^- = & -\tfrac{a_{11}}{2\pi} \int_{-\infty}^{\infty} \tfrac{1}{is} \left[(s^2C_3 + 2|s|C_4 + s^2yC_4)\right] e^{[|s|y-isx]}ds \\ & + \tfrac{a_{12}}{2\pi} \int_{-\infty}^{\infty} \tfrac{s^2}{is} \left[C_3 + C_4y\right] e^{[|s|y-isx]}ds \end{aligned} \tag{33}$$

and

$$\begin{aligned} v^- = & \tfrac{a_{12}}{2\pi} \int_{-\infty}^{\infty} \left[|s|C_3 + C_4(|s|y + 1)\right] e^{[|s|y-isx]}ds \\ & - \tfrac{a_{11}}{2\pi} \int_{-\infty}^{\infty} \left[|s|C_3 + C_4(|s|y - 1)\right] e^{[|s|y-isx]}ds \end{aligned} \tag{34}$$

It is important to note that the direct application of the stress and displacement boundary conditions to the above equations would lead to a set of dual integral equations defined for the stresses within the crack domain and for the displacements outside the crack domain. However, in this form such a set of dual integral equations is unsolvable for the unknown constants C_1 and C_2

- hence, the introduction of the following auxiliary functions, from which a set of solvable singular integral equations can be obtained. The typical form suggested in the literature (Badaliance and Gupta 1976; and Erdogan 1978) for these auxiliary functions is,

$$f_1(x) = \frac{\partial}{\partial x}[u^+(x,0) - u^-(x,0)] \tag{35}$$

$$f_2(x) = \frac{\partial}{\partial x}[v^+(x,0) - v^-(x,0)] \tag{36}$$

Upon substitution of Eqs. (31-34) and Eqs. (17-18) into Eqs. (35) and (36), respectively, the auxiliary functions become

$$f_1(x) = 4a_{11}\frac{1}{2\pi}\int_{-\infty}^{\infty} (s^2C_1 - |s|C_2)e^{-isx}ds \tag{37}$$

$$f_2(x) = -4a_{11}i\frac{1}{2\pi}\int_{-\infty}^{\infty} |s|s\ C_1e^{-isx}ds \tag{38}$$

Expressions for the unknown constants C_1 and C_2 can be determined in terms of the above auxiliary functions by inverting the Fourier transformations of Eqs. (37) and (38), where the condition that $f_1(t)$ and $f_2(t)$ are nonzero only within the crack region (*i.e.*, $-a < t < a$) has been applied. Otherwise, outside the crack region these functions are zero owing to the displacement perturbation boundary conditions; see Eq. (6). Therefore,

$$C_1 = \frac{1}{4a_{11}}\frac{i}{|s|s}\int_{-a}^{a} f_2(t)e^{ist}dt \tag{39}$$

$$C_2 = \frac{1}{4a_{11}}\left[\frac{-1}{|s|}\int_{-a}^{a} f_1(t)e^{ist}dt + \frac{i}{s}\int_{-a}^{a} f_2(t)e^{ist}dt\right] \tag{40}$$

Similarly, from Eqs. (17) and (18) C_3 and C_4 can also be expressed in terms of the auxiliary functions:

$$C_3 = \frac{1}{4a_{11}}\left[\frac{i}{|s|s}\int_{-a}^{a} f_2(t)e^{ist}dt\right] \tag{41}$$

$$C_4 = \frac{-1}{4a_{11}}\left[\frac{1}{|s|}\int_{-a}^{a} f_1(t)e^{ist}dt + \frac{i}{s}\int_{-a}^{a} f_2(t)e^{ist}dt\right] \tag{42}$$

Substituting expressions for the constants C_j into the local stress equations, for example Eqs. (19) to (21), results in the formulation of the set of double integral equations with respect to the Fourier variables s ($-\infty < s < \infty$) and t ($-a \le t \le a$). Integration of this set of double integrals with respect to s, will involve expressions having the following recursive form:

$$\begin{aligned} I_1 &= \textstyle\int_0^\infty e^{\rho s}ds \quad \text{where} \quad \rho = \rho(x,-y,t) \\ I_n &= \tfrac{\partial}{\partial y}I_{n-1} \qquad \text{for} \quad n = 2,3,.. \end{aligned} \tag{43}$$

The result of the preceding integration (Gradshteyn and Ryzhik 1980) is

$$I_n = \frac{(-1)^n}{\rho^n} \text{ for } n = 1,2\ 3,... \tag{44}$$

For the details of this integration process see Part II (Arnold et al. (1992)). Integrating with respect to s will give a set of singular integral equations with respect to t, which are valid for any j^{th} crack within its own local coordinate system (x_j, y_j) :

$$\sigma^{(j)}_{x_jx_j} = \frac{1}{4\pi a_{11}} \int_{-a_j}^{a_j} \left\{ f_{j2}(t_j)(t_j - x_j) \left[\frac{-2y_j^2}{[y_j^2 + (t_j - x_j)^2]^2} + \frac{1}{y_j^2 + (t_j - x_j)^2} \right] - y_j f_{j1}(t_j) \left[\frac{y_j^2 - (t_j - x_j)^2}{[y_j^2 + (t_j - x_j)^2]^2} - \frac{2}{y_j^2 + (t_j - x_j)^2} \right] \right\} dt_j \tag{45}$$

$$\sigma^{(j)}_{y_jy_j} = \frac{1}{4\pi a_{11}} \int_{-a_j}^{a_j} \left\{ f_{j2}(t_j)(t_j - x_j) \left[\frac{2y_j^2}{[y_j^2 + (t_j - x_j)^2]^2} + \frac{1}{y_j^2 + (t_j - x_j)^2} \right] + y_j f_{j1}(t_j) \frac{y_j^2 - (t_j - x_j)^2}{[y_j^2 + (t_j - x_j)^2]^2} \right\} dt_j \tag{46}$$

$$\sigma^{(j)}_{x_jy_j} = \frac{-1}{4\pi a_{11}} \int_{-a_j}^{a_j} \left\{ f_{j1}(t_j)(t_j - x_j) \left[\frac{2y_j^2}{[y_j^2 + (t_j - x_j)^2]^2} - \frac{1}{y_j^2 + (t_j - x_j)^2} \right] - y_j f_{j2}(t_j) \frac{y_j^2 - (t_j - x_j)^2}{[y_j^2 + (t_j - x_j)^2]^2} \right\} dt_j \tag{47}$$

This completes the formulation of the fundamental problem (or local stress state) for the j^{th} crack. Henceforth, the formulation of the multiple crack problem will be addressed.

2.2 Total Stress Formulation

The total stress state ${}_j(\sigma^T_{rz})$ for the j^{th} crack is defined as the local stress state of the j^{th} crack (σ^j_{rz}) plus the contribution to that stress state of all remaining cracks. This may be represented mathematically as

$${}_j\sigma^T_{rz}(x_j, y_j) = \sigma^j_{rz}(x_j, y_j) + \sum_{p=1}^{n-1} \sigma'^p_{rz}[x_p(x_j, y_j), y_p(x_j, y_j)] \tag{48}$$

for $j = 1, ..., n$. Here standard tensor transformation is incorporated, $\sigma'_{rz} = \beta_{lr}\beta_{mz}\sigma_{lm}$ and β_{lr}, β_{mz} are the direction cosines between the (x_j, y_j) and (x_p, y_p) coordinates with p identifying the remaining cracks (see Part II). Note: This statement does not imply that the concept of superposition has been invoked, since the stress perturbation boundary conditions (see Eqs. (5)) have not yet been utilized to determine the unknown auxiliary functions.

For functional compatibility within Eq. (48), coordinate transformations must be simultaneously applied to all remaining (p^{th}) crack coordinate variables. As a result, the dominant term (i.e., the first term of Eq. (48)) possesses a singularity whereas terms within the summation lose their original singularities and yet still contribute to the total stress state, as one might expect. The coordinate transformation between the (x_j, y_j) and (x_p, y_p) systems is determined from the following geometric relationship (see Fig. 2):

$$r_{jX} + x_j cos\varphi_j - y_j sin\varphi_j = r_{pX} + x_p cos\varphi_p - y_p sin\varphi_p \tag{49}$$

$$r_{jY} + x_j sin\varphi_j + y_j cos\varphi_j = r_{pY} + x_p sin\varphi_p + y_p cos\varphi_p \tag{50}$$

where r_{jX}, r_{jY} are the rectangular components of the j^{th} crack position vector referred to the global coordinate system $X - Y$, and φ_j is the angle of rotation between the global and local systems (see Fig. 2). By rearranging Eqs. (49) and (50), one obtains

$$x_p = (r_{jY} - r_{pY})sin\varphi_p + (r_{jX} - r_{pX})cos\varphi_p \\ +x_j \cos(\varphi_j - \varphi_p) - y_j \sin(\varphi_j - \varphi_p) \tag{51}$$

$$y_p = (r_{jY} - r_{pY})cos\varphi_p - (r_{jX} - r_{pX})sin\varphi_p \\ +x_j \sin(\varphi_j - \varphi_p) + y_j \cos(\varphi_j - \varphi_p) \tag{52}$$

2.3 Singular Integral Equations Formulation

Since the total stress state at each crack location has been formulated, the appropriate stress boundary conditions (Eqs. (5)) can be applied. As a result, 2n singular integral equations are obtained with 2n unknown auxiliary functions. With the collocation technique, these unknown auxiliary functions can be determined in a discrete sense. The collocation technique, however, requires that the interval of integration be from -1 to 1 and not from -a to a, as in Eqs. (45-47). In order to accomplish this normalization, the following variable substitution, involving the actual j^{th} crack length a_j, is employed:

$$x_j = a_j \xi \tag{53}$$

$$t_j = a_j \tau \tag{54}$$

where the ξ and τ are between -1 and 1.

The formulation of this system of singular integral equations is complete once the single-value conditions for the auxiliary functions, $f_{\eta j}$, are chosen. In the case of straight cracks, this single-value condition (Erdogan 1978) is:

$$\int_{-1}^{1} f_{\eta j}(\tau) d\tau = 0 \tag{55}$$

where j stands for the j^{th} crack and η takes on the value of 1 or 2.

For example, for the case of an isotropic plate with two cracks, the system of four singular integral equations normalized by Eqs. (53 and 54) is obtained by using the σ_{yy}^T and σ_{xy}^T stress components of Eqs. (48) so that

$$_1\sigma_{xy}^T = \frac{1}{4a_{11}} \left\{ \frac{1}{\pi} \int_{-1}^{1} \frac{f_{11}}{\tau - \xi} d\tau + \int_{-1}^{1} \ker{}_{21}^{11} f_{21} d\tau + \int_{-1}^{1} \ker{}_{22}^{11} f_{22} \right\} \tag{56}$$

$$_1\sigma_{yy}^T = \frac{1}{4a_{11}} \left\{ \frac{1}{\pi} \int_{-1}^{1} \frac{f_{12}}{\tau - \xi} d\tau + \int_{-1}^{1} \ker{}_{21}^{12} f_{21} d\tau + \int_{-1}^{1} \ker{}_{22}^{12} f_{22} \right\} \tag{57}$$

and

$$_2\sigma_{xy}^T = \frac{1}{4a_{11}} \left\{ \frac{1}{\pi} \int_{-1}^{1} \frac{f_{21}}{\tau - \xi} d\tau + \int_{-1}^{1} \ker{}_{11}^{21} f_{11} d\tau + \int_{-1}^{1} \ker{}_{12}^{21} f_{12} \right\} \tag{58}$$

$$_2\sigma_{yy}^T = \frac{1}{4a_{11}} \left\{ \frac{1}{\pi} \int_{-1}^{1} \frac{f_{22}}{\tau - \xi} d\tau + \int_{-1}^{1} \ker{}_{11}^{22} f_{11} d\tau + \int_{-1}^{1} \ker{}_{12}^{22} f_{12} \right\} \tag{59}$$

where $\ker{}_{\gamma\delta}^{\alpha\beta}$ are the Fredholm kernels (Erdogan 1978) and $\alpha, \beta, \gamma, \delta = 1, 2$. The above eight kernel functions can be represented by four independent relations, given that $\theta = -\theta$ (where $\theta = \varphi_2 - \varphi_1$). For example, $\ker{}_{21}^{11} = \ker{}_{11}^{21}$, $\ker{}_{22}^{11} = \ker{}_{12}^{21}$, $\ker{}_{21}^{12} = \ker{}_{11}^{22}$, and $\ker{}_{22}^{12} = \ker{}_{12}^{22}$.

For the case when n cracks exist, each singular integral equation would contain more integral terms and their associated kernels, thereby giving rise to a system of singular equations of dimension 2n. Note that all of the required kernels for a multiple crack problem will differ from the four kernels found for the case of two cracks only by a position vector. Therefore, we can rewrite the total stresses (σ_{yy}^T and σ_{xy}^T) for n cracks as follows

$$\begin{aligned} {}_n\sigma_{xy}^T = \tfrac{1}{4a_{11}} \Big\{ &\textstyle\int_{-1}^{1} \ker_1 f_{11} d\tau + \int_{-1}^{1} \ker_2 f_{12} d\tau \\ &+ ... + \textstyle\int_{-1}^{1} \ker_1 f_{(n-1)1} d\tau + \int_{-1}^{1} \ker_2 f_{(n-1)2} d\tau + \frac{1}{\pi}\int_{-1}^{1} \frac{f_{n1}}{\tau-\xi} d\tau \Big\} \end{aligned} \tag{60}$$

$$\begin{aligned} {}_n\sigma_{yy}^T = \tfrac{1}{4a_{11}} \Big\{ &\textstyle\int_{-1}^{1} \ker_3 f_{11} d\tau + \int_{-1}^{1} \ker_4 f_{12} d\tau \\ &+ ... + \textstyle\int_{-1}^{1} \ker_3 f_{(n-1)1} d\tau + \int_{-1}^{1} \ker_4 f_{(n-1)2} d\tau + \frac{1}{\pi}\int_{-1}^{1} \frac{f_{n2}}{\tau-\xi} d\tau \Big\} \end{aligned} \tag{61}$$

(see the appendix for the four independent kernels ($\ker_i$ for $i = 1, ..., 4$)), and we can associate $\ker_1 = \ker_{21}^{11}$, $\ker_2 = \ker_{22}^{11}$, $\ker_3 = \ker_{21}^{12}$, and $\ker_4 = \ker_{22}^{12}$ for the case of two cracks. An equivalent association, with differing position vectors, can be done for the case of n cracks. This completes the formulation of the system of singular integral equations.

2.4 Solution for the Stress Intensity Factors

The integral equations obtained are of the Cauchy type; thus, for sharp cracks the stresses and strains will have a square-root singularity and the classic definition of SIF may be used (see Badaliance and Gupta 1976; Delale and Erdogan 1977 and 1979; and Delale et al. 1977). Therefore, the modes I and II SIF's for the j^{th} crack are

$$k_1^j(1) = \lim_{\xi \to 1} [2(\xi - 1)]^{\frac{1}{2}} \left\{ {}_j\sigma_{yy}^T(\xi, 0) \right\} \tag{62}$$

$$k_2^j(1) = \lim_{\xi \to 1} [2(\xi - 1)]^{\frac{1}{2}} \left\{ {}_j\sigma_{xy}^T(\xi, 0) \right\} \tag{63}$$

$$k_1^j(-1) = \lim_{\xi \to -1} [-2(1 + \xi)]^{\frac{1}{2}} \left\{ {}_j\sigma_{yy}^T(\xi, 0) \right\} \tag{64}$$

$$k_2^j(-1) = \lim_{\xi \to -1} [-2(1 + \xi)]^{\frac{1}{2}} \left\{ {}_j\sigma_{xy}^T(\xi, 0) \right\} \tag{65}$$

where the normal and shear stresses, Eqs. (60 and 61), are used.

It is well known (Erdogan 1978) that the auxiliary functions (f) can be expressed as a product of the unknown bounded functions (G) and the known singular weight functions w:

$$f(\tau) = G(\tau) w(\tau) \tag{66}$$

The singular weight function w for a sharp crack is

$$w(\tau) = (\tau^2 - 1)^{-\frac{1}{2}} \tag{67}$$

Erdogan (1978) found, for example, that in the case of a Cauchy-type singular integral equation (Eqs. (60) and (61)), the dominant part can be expressed in terms of the function G evaluated at the tips of the j^{th} crack:

$$\frac{1}{\pi} \int_{-1}^{1} \frac{f_{\eta j} d\tau}{\tau - \xi} = G_{\eta j}(-1) \frac{e^{i\pi/2}}{\sqrt{2}} (\tau + 1)^{-\frac{1}{2}} - G_{\eta j}(1) \frac{1}{\sqrt{2}} (\tau - 1)^{-\frac{1}{2}} + O(\tau) \tag{68}$$

where η is 1 or 2 and $O(\tau)$ is the higher order term, which in subsequent calculations is neglected. The substitution of Eqs. (68) for the dominant parts (the last term in Eqs. (60) and (61)) of the normal and shear components of the total stresses in Eqs. (62-65), and subsequent evaluation of the limits at the crack tips, results in the redefinition of the SIF's (normalized with respect to $\sqrt{a_1}$ and σ^o_{jp}) expressed in terms of the functions $G_{\eta j}$:

$$k_1^j(1) = -\frac{1}{4a_{11}} G_{2j}(1)\sqrt{a_j/a_1} \tag{69}$$

$$k_2^j(1) = -\frac{1}{4a_{11}} G_{1j}(1)\sqrt{a_j/a_1} \tag{70}$$

$$k_1^j(-1) = \frac{1}{4a_{11}} G_{2j}(-1)\sqrt{a_j/a_1} \tag{71}$$

$$k_2^j(-1) = \frac{1}{4a_{11}} G_{1j}(-1)\sqrt{a_j/a_1} \tag{72}$$

The Lobatto-Chebyshev collocation integration technique is known to provide excellent results when dealing with Cauchy-type singular integral equations and so it was used. The unknown function $G_{\eta j}$ is determined at a discrete set of points τ_1, $\tau_2, ..., \tau_m$ called abscissas. In this way, each integral equation is reduced to a set of algebraic equations with unknowns $G_{\eta j}(\tau_1)$, $G_{\eta j}(\tau_2), ...,$ $G_{\eta j}(\tau_m)$, which are the discrete values of the functions $G_{\eta i}$; hence its name, a discrete auxiliary function. Each of the singular integral equations subjected to the stress boundary conditions (Eqs. 5), is replaced (see Erdogan 1978) by $m-1$ algebraic equations with 2nm unknown parameters:

$$\frac{1}{\pi}\sum_{r=1}^{m}\frac{G_{\eta j}(\tau_r)w_r}{\tau_r - \xi_z} + \sum_{\delta=1}^{2}\sum_{r=1}^{m} k_{\gamma\delta}^{\alpha\beta}(\tau_r, \xi_z)G_{\gamma\lambda}(\tau_r)w_r + Rem(\xi_z) = 4a_{11}\rho_\alpha(a_\alpha\xi_z) \tag{73}$$

where $z, \lambda = 1, ..., n$; $\alpha, \beta, \gamma, \delta, \eta = 1, 2$; $\rho_\alpha = \begin{cases} q_\alpha & if \quad \eta=1 \\ p_\alpha & if \quad \eta=2 \end{cases}$; $\xi_z(z = 1, 2, ..., m-1)$ are collocation points chosen in such a way that $\xi_z \neq \tau_r$; $w_r =$ the weighting constants of the related integration formula and $Rem =$ the remainder. For m sufficiently large, Rem can be made as small as necessary for the desired accuracy and consequently can be neglected.

In the Lobatto-Chebyshev method, the abscissas are calculated according to

$$\tau_r = \cos\frac{(r-1)\pi}{m-1} \quad \text{for} \quad r = 1, ..., m \tag{74}$$

with the corresponding weights given by

$$w_1 = w_m = \frac{\pi}{2(m-1)} \quad \text{and} \quad w_r = \frac{\pi}{m-1} \quad \text{for } r = 2, 3, ..., m-1 \tag{75}$$

the collocation points are then found by using the formula

$$\xi_z = \cos\frac{(2z-1)\pi}{2m-2} \quad \text{for } z = 1, 2, ..., m-1 \tag{76}$$

In order to have the complete system of 2nm algebraic equations, the single-value conditions (Eqs. (55)) are also expressed by using the collocation technique:

$$\sum_{r=1}^{m} G_{\eta j}(\tau_r)w_r = 0 \tag{77}$$

Thus, the resulting system of algebraic equations can be written in the form

$$[A]\{G\} = \{\mathcal{R}\} \tag{78}$$

where [A] is a fully populated $2nm \times 2nm$ matrix of coefficients and $\{\mathcal{R}\}$ is the loading function vector; see Part II (Arnold et al. (1992)) for details.

The unknown parameter vector $\{G\}$ can be determined through inversion of the [A] matrix; thus,

$$\{G\} = [A]^{-1}\{\mathcal{R}\} \tag{79}$$

although only the appropriate values (*i.e.*, $G_{\eta j}(\pm 1)$) are used to calculate the SIF's for the j^{th} crack (see Eqs. (69-72)).

Automatic generation of the associated FORTRAN code (see Part II, Arnold et al. (1992)) for the evaluation of Eq. (79) completes the development of the solution for any multicrack problem. This FORTRAN program was utilized to obtained the following results, which are compared with results obtained from other methods existing in the literature.

3 Numerical Application

In order to validate the results obtained using the SYMFRAC code, the well-known problem of two parallel interacting cracks is considered here. The plate with two cracks of length 2a is subjected to a normal stress field (σ°_{YY}) as shown in Fig. 3. The location of the cracks are defined by the distance r_d between inner crack tips and the offset angle α.

This problem has been solved for mode I SIF's and $\alpha > 0$ by Isida (1976), who used a Laurent series expansion that gave less than 2-percent error; by Yokobori et. al. (1971), who used a continuously distributed model that gave results within 5-percent error; and by Yijun and Atilla (1991), who used an approximate stress field model that gave less than 10-percent error. For the case of the collinear cracks ($\alpha = 0$), an exact solution was obtained by Erdogan (1962) and later by Horii and Nemat-Nasser (1985) using the approximate method of pseudo-tractions.

In Table I the SIF's at the inner crack tips from Isida (1976), Yokobori et al. (1971), and Yijun and Atilla (1991) are compared to those of the present method for the noncollinear case (i.e., $\alpha > 0$). Similarly, in Table II the results for the SIF's from Horii and Nemat-Nasser (1985) and Erdogan (1962) are compared to those of the present singular integral technique for the collinear case (i.e., $\alpha = 0$) at both the inner and outer crack tips.

Table II shows that the exact results given by Erdogan (1962) and the results from using the pseudo-traction method (Horii and Nemat-Nasser 1985) agree exactly with the results given by the present singular integral technique. This implies that the error introduced by solving the system of singular integral equations with the collocation technique is minimal.

Also, in the case of noncollinear cracks the SIF's calculated by the present method are closest to the most accurate results of Isida (1976) for the specific cases addressed. We strongly believe that the present method is the most accurate because of the minimum number of assumptions required for the solution and its favorable comparisons. This fact is further borne out by the observation that the most approximate method of Yijun and Atilla (1991) gives the smallest SIF's for the closest crack tip distance, whereas the present method gives the highest values.

4 Concluding Remarks

A rigorous formulation has been presented and validated for calculating the SIF's of a multicracked isotropic medium. The size, orientation, and distribution of all cracks were considered to be independent parameters of the solution. The formulation was constructed in such a way that a *symbolic/numeric* system (named SYMFRAC, see Part II (Arnold et al. (1992))), capable of automatically formulating the required system of singular integral equations for a multicrack boundary value problem and generating the associated FORTRAN code, can be developed and utilized. Extension to anisotropic and/or nonhomogeneous materials, as well as more complex crack geometries, would require modifications of the governing equations, boundary/continuity conditions, constitutive relations, and such. Although the final results would be significantly different, the *symbolic/numeric* procedure developed here would remain the same. Because of symbolic implementation, the solution of the more complex problems would be a straightforward and automated procedure. With this capability, numerous parametric studies could easily be performed to analyze the contribution of each parameter on the local stress field as well as the characteristics of the damage progression in a material.

Future work will attempt to generate such solutions and to generate the required data base necessary to establish a link between continuum damage mechanics and linear elastic fracture mechanics by homogenizing the effects of interacting distributed cracks into macrodamage models for isotropic and anisotropic materials and for more complex crack geometries . These macrodamage models can than be efficiently coupled with the finite element method in order to analyze structural problems.

Appendix - Analytical Form of the Fredholm Kernels

$$
\begin{aligned}
\ker{}_1 = \frac{a'}{\pi}\Bigg\{ & \frac{-sin2\theta(p_2 + a\xi sin\theta)[(a'\tau - p_1 - a\xi cos\theta)^2 - (p_2 + a\xi sin\theta)^2]}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& \frac{2cos2\theta(a'\tau - p_1 - a\xi cos\theta)(p_2 + a\xi sin\theta)^2}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& + \frac{-sin2\theta(p_2 + a\xi sin\theta) - cos2\theta(a'\tau - p_1 - a\xi cos\theta)}{(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2}\Bigg\}
\end{aligned}
\qquad \text{(A1)}
$$

$$
\begin{aligned}
\ker{}_2 = \frac{a'}{\pi}\Bigg\{ & \frac{cos2\theta(p_2 + a\xi sin\theta)[(a'\tau - p_1 - a\xi cos\theta)^2 - (p_2 + a\xi sin\theta)^2]}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& \frac{2sin2\theta(a'\tau - p_1 - a\xi cos\theta)(p_2 + a\xi sin\theta)^2}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2}\Bigg\}
\end{aligned}
\qquad \text{(A2)}
$$

$$
\begin{aligned}
\ker{}_3 = \frac{a'}{\pi}\Bigg\{ & \frac{-2sin2\theta(p_2 + a\xi sin\theta)^2(a'\tau - p_1 - a\xi cos\theta)}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& - \frac{cos2\theta(p_2 + a\xi sin\theta)[(a'\tau - p_1 - a\xi cos\theta)^2 - (p_2 + a\xi sin\theta)^2]}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& + \frac{2sin^2\theta(p_2 + a\xi sin\theta) + sin2\theta(a'\tau - p_1 - a\xi cos\theta)}{(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2}\Bigg\}
\end{aligned}
\qquad \text{(A3)}
$$

$$
\begin{aligned}
\ker{}_4 = \frac{a'}{\pi}\Bigg\{ & \frac{-sin2\theta(p_2 + a\xi sin\theta)[(a'\tau - p_1 - a\xi cos\theta)^2 - (p_2 + a\xi sin\theta)^2]}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& + \frac{2cos2\theta(a'\tau - p_1 - a\xi cos\theta)(p_2 + a\xi sin\theta)^2}{[(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2]^2} \\
& + \frac{(a'\tau - p_1 - a\xi cos\theta)}{(p_2 + a\xi sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2}\Bigg\}
\end{aligned}
\qquad \text{(A4)}
$$

where

$$p_1 = (r_y - r_{y'})sin\varphi' + (r_x - r_{x'})cos\varphi' \qquad \text{(A5)}$$

$$p_2 = (r_y - r_{y'})cos\varphi' - (r_x - r_{x'})sin\varphi' \qquad \text{(A6)}$$

and $\theta = \varphi' - \varphi$

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Table I: Mode-I Normalized SIF's for Inner Tips of Noncollinear Cracks

Offset angle, α, degrees	$\frac{r_d}{a}$	From literature			Present method
		Yijun and Atilla (1991)	Yokobori et al. (1991)	Isida* (1976)	
45	1.41	1.0305	1.0606	1.12	1.1254
45	2.82	1.0135	1.0533	1.04	1.0489
45	4.24	1.0051	1.0350	1.02	1.0254
38.66	1.28	1.0360	1.0920	1.13	1.1317
38.66	2.56	1.0184	1.0603	1.05	1.0551
38.66	3.84	1.0046	1.0355	1.03	1.0302

* Results taken from graph in literature

Table II: Mode-I Normalized SIF's at Inner and Outer Crack Tips of Collinear Cracks

$\frac{r_d}{a}$	From literature				Present method	
	Horii and Nemat-Nassr (1985)		Erdogan (1962)			
	Inner	Outer	Inner	Outer	Inner	Outer
0.220	---	---	1.45387	1.11741	1.45387	1.11741
0.500	1.2289	1.0811	1.22894	1.08107	1.22894	1.08107
0.857	1.1333	1.0579	1.13326	1.05786	1.13326	1.05786

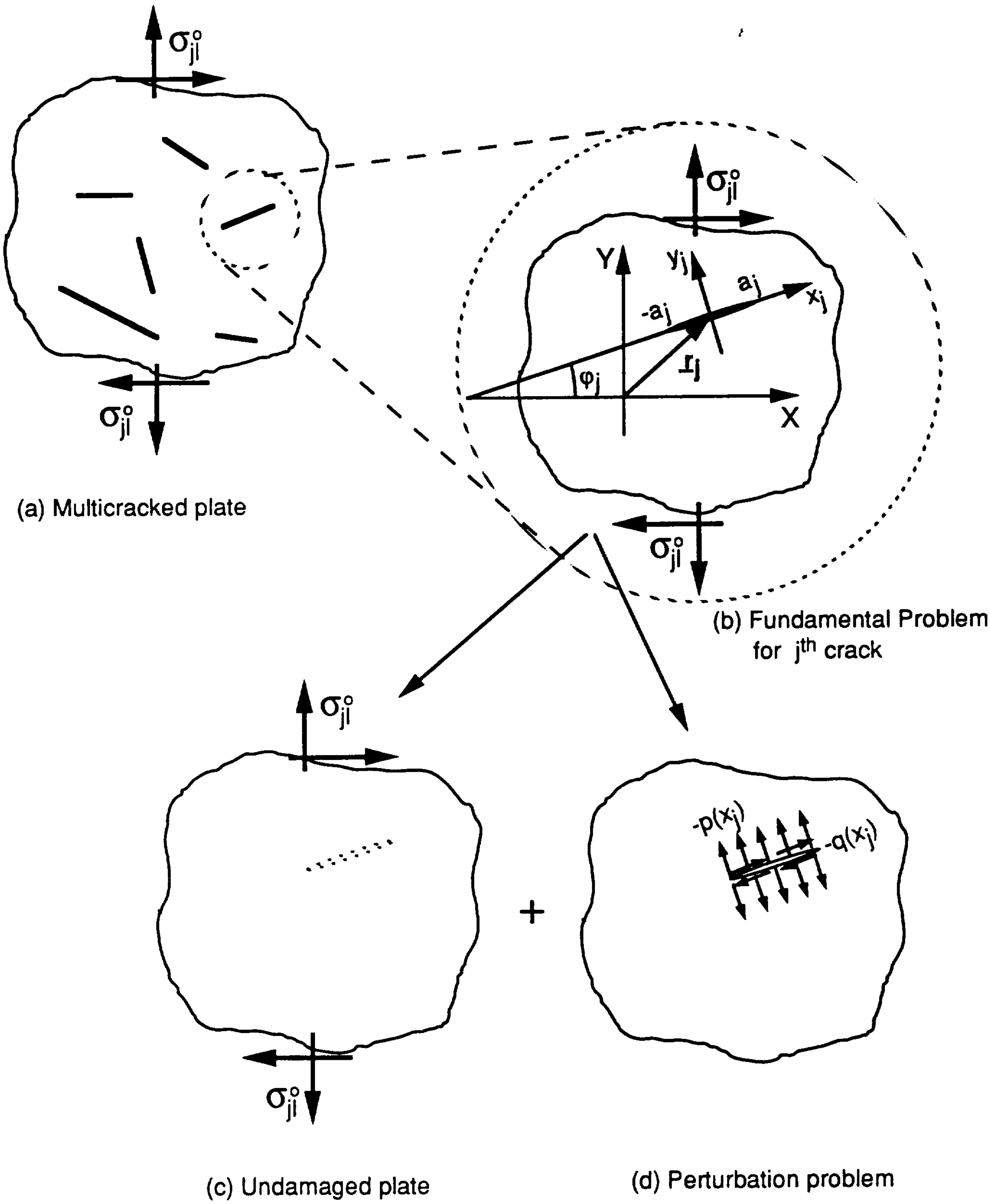

Fig. 1. Multicracked plate geometry and the method of solution; (a) Multicracked plate, (b) Fundamental problem for j^{th} crack, (c) Undamaged plate, (d) Perturbation problem.

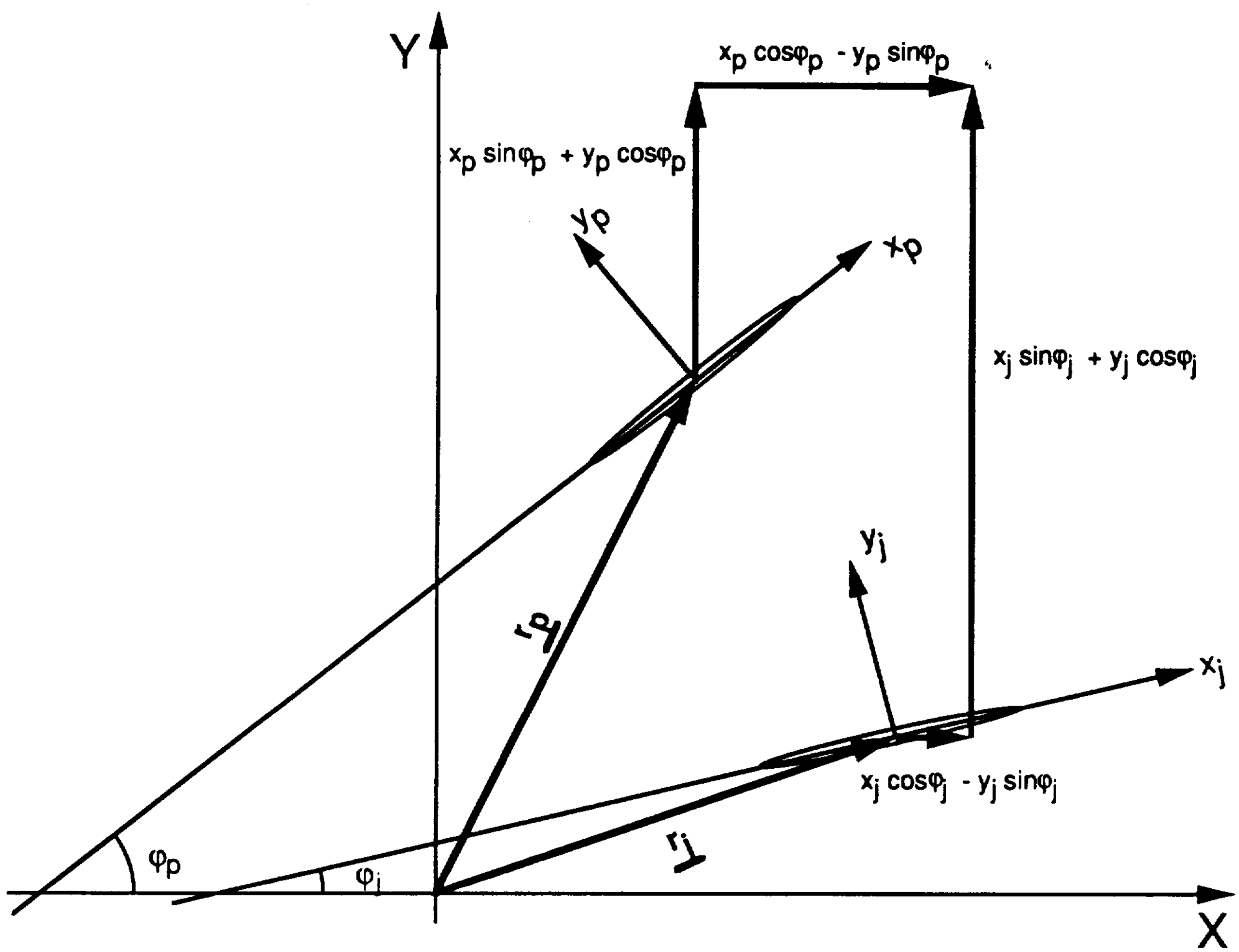

Fig. 2. Geometric relations between a pair of cracks and their local variables.

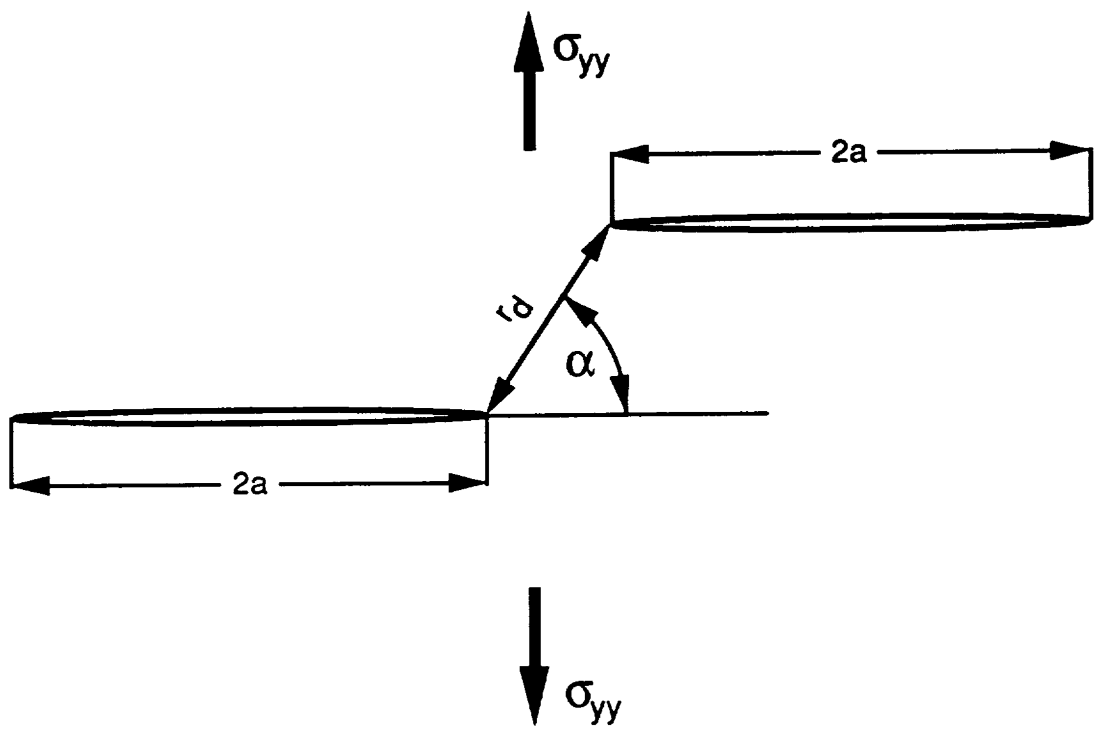

Fig. 3. Problem of two crack under normal stress field conditions.

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13. ABSTRACT ***(Maximum 200 words)***

An essential part of describing the damage state and predicting the damage growth in a multicracked plate is the accurate calculation of stress intensity factors (SIF's). Here, a methodology and rigorous solution formulation for SIF's of a multicracked plate, with fully interacting cracks, subjected to a far-field arbitrary stress state is presented. The fundamental perturbation problem is derived, and the steps needed to formulate the system of singular integral equations whose solution gives rise to the evaluation of the SIF's are identified. This analytical derivation and numerical solution are obtained by using intelligent application of symbolic computations and automatic FORTRAN generation capabilities (described in the second part of this paper). As a result, a symbolic/FORTRAN package, named SYMFRAC, that is capable of providing accurate SIF's at each crack tip has been developed and validated.

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