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# Stochastic Sensitivity Measure for Mistuned High-Performance Turbines

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# STOCHASTIC SENSITIVITY MEASURE FOR MISTUNED HIGH-PERFORMANCE TURBINES

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## ABSTRACT

A stochastic measure of sensitivity is developed in order to predict the effects of small random blade mistuning on the dynamic aeroelastic response of turbomachinery blade assemblies. This sensitivity measure is based solely on the nominal system design (*i.e.*, on tuned system information), which makes it extremely easy and inexpensive to calculate. The measure has the potential to become a valuable design tool that will enable designers to evaluate mistuning effects at a preliminary design stage and thus assess the need for a full mistuned rotor analysis. The predictive capability of the sensitivity measure is illustrated by examining the effects of mistuning on the aeroelastic modes of the first stage of the oxidizer turbopump in the Space Shuttle Main Engine. Results from a full analysis of mistuned systems confirm that the simple stochastic sensitivity measure predicts consistently the drastic changes due to mistuning and the localization of aeroelastic vibration to a few blades.

## INTRODUCTION

The current trend in the design of high performance propulsion turbomachinery has resulted in systems designed for finite service life. These systems produce high power, in compact and light-weight machines which require stringent safety factors and margins. In this environment, the design engineer is faced with the task of accurately predicting system performance and dynamics. When designing for

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specific life and reliability goals, the structural dynamic behavior of the turbomachine is of paramount importance.

The prediction of the dynamics of turbomachine rotors is further complicated by the presence of differences in structural and material properties among the blades, a phenomenon commonly referred to as *mistuning*. These differences are unavoidable because they arise from manufacturing deviations and in-service degradation. Such blade-to-blade discrepancies in structural properties are random by the nature of their origin. In particular, they result in random blade-to-blade variations in blade natural frequencies, known as frequency mistuning. Several studies investigated the influence of such mistuning on blade assembly dynamic behavior and concluded that the potential exists for the effects of mistuning to be significant and even drastic.<sup>1-3</sup> However, mistuning is routinely ignored in most current analyses of the dynamic response of turbomachinery, because (a) tremendous computational and analytical complexities are involved in dropping the assumption of tuned rotors (that is, those having identical blades) and more importantly, (b) the actual mistuning pattern and its standard deviation are generally not available until the design and manufacture of the rotor is complete. Although the tuned rotor assumption is extremely useful for the analysis of complex rotor systems such as high-performance turbomachinery, mistuned rotors may exhibit dynamic characteristics that are vastly different from those of tuned rotors.

In particular, previous studies have demonstrated that mistuning, while generally improving the aeroelastic stability of rotors<sup>4</sup>, results in larger forced response amplitudes.<sup>5,6</sup> Furthermore, it has recently been shown that mistuning can alter the overall dynamics of rotors in a qualitative fashion. Namely, the equally distributed vibration amplitudes that characterize tuned rotors have been shown to become *localized*<sup>2,3,7,8</sup> by mistuning to a few rotor blades, termed rogue blades. Thus, unlike the tuned rotor in which all blades equally distribute the excitation energy among themselves, in a mistuned rotor with localized modes, only a few blades may absorb most of the excitation energy, and therefore, experience much larger amplitudes of vibration. Previous studies of localization<sup>3,7</sup> have shown that the amplitudes of vibration of some mistuned blades can be several orders of magnitude larger than those obtained using the tuned rotor assumption, depending on the strength of interblade coupling, the excitation frequency and the number of blades. This has important implications in that the resulting energy confinement within a few blades indicates a possible cause for rogue blade failure in rotors.

Prior work by the authors has led to an understanding of the phenomenon of aeroelastic mode localization and the physical mechanisms behind it.<sup>8,9</sup> In these studies, perturbation theories were used to understand the effects of mistuning levels and various operating parameters, including unsteady aerodynamic forces, on the distribution of vibration energy around the rotor. General conclusions were reached regarding the sensitivity of turbomachinery rotors to blade frequency mistuning.

In the present paper, we introduce a sensitivity measure for predicting the potentially dangerous effects of random blade mistuning on the aeroelastic vibration characteristics of high-energy turbines. The measure of sensitivity we propose is formulated solely from the knowledge of the mean design properties of the turbomachinery blades, i.e., from the tuned assembly dynamics. It is extremely simple to calculate and does not require an aeroelastic eigenproblem solution. This measure allows us to predict mistuning effects essentially with a single scalar for each mode of vibration. Also, since the sensitivity measure we propose is stochastic, it will allow the designer to assess the potential for rogue blade (i.e., large amplitude) vibrations and drastic dynamic behavior changes due to mistuning, without specific knowledge of the actual mistuning pattern that would result from manufacture or in-service degradation. Thus, by evaluating the validity of the tuned rotor analysis and the need for a thorough mistuned rotor analysis, this measure has the potential to become a valuable design tool.

It is worth noting that the sensitivity methodology adopted in this paper differs from previous statistical studies of the effects of mistuning.<sup>10-13</sup> Typically in those studies, the free or forced response of mistuned assemblies is calculated and then statistically averaged over many realizations of mistuning. This necessitates solving for the dynamic response of many mistuned systems --- a very expensive (sometimes prohibitively so) procedure. In contrast, the present methodology offers a prediction of mistuning effects that is based solely on tuned system information, which makes the approach very simple and inexpensive. Indeed, the formulation developed here is conceptually similar to that used for estimating the sensitivity of control systems to plant uncertainties and design changes.<sup>14-16</sup> Although it is clear that our sensitivity measure cannot provide as detailed information about the mistuned system response as systematic Monte Carlo simulations do, its extremely low cost and simplicity make it a valuable alternative at the design stage.

The specific rotor we analyze in this paper is the first stage of turbine blades of the high pressure oxidizer turbopump (HPOTP) in the space shuttle main engine (SSME). The SSME rotor was selected because it exhibits many of the characteristics of modern high performance turbomachinery designs. These include high energy density, low blade aspect ratio, high aerodynamic loading and advanced superalloy materials. In addition, the SSME turbopump turbines have been plagued with in-service blade failures during development and operational experience. These blades have suffered both low-cycle and high-cycle fatigue, which leads to the obvious presumption that significant dynamic loading exists.

In this paper we first review the results of previous investigations regarding the extreme sensitivity of the aeroelastic behavior of turbomachinery rotors to small mistuning.<sup>9</sup> Perturbation theories that yield physical insights into the aeroelastic localization phenomena are then summarized and the proposed stochastic sensitivity measure is derived from these perturbation expressions. The performance of the sensitivity measure is tested by applying it to the SSME oxidizer turbopump turbine. The effectiveness of the sensitivity measure is illustrated by comparisons between the predictions by the sensitivity measure and the calculated localization.

## THE MODE LOCALIZATION PHENOMENON

In usual engineering practice, bladed disks of turbomachinery are designed to be composed of identically constructed blades that are uniformly placed on a disk, resulting in geometrically identical flow passages. Such rotors possess perfect cyclic symmetry and are referred to as tuned rotors. A well-known property of tuned rotors is that all the blades on such a rotor vibrate with identical amplitudes when subjected to a dynamic excitation which is also perfectly cyclic symmetric. Thus the energy of excitation is shared equally among all the blades, which obviously minimizes fatigue. Figure 1 schematically illustrates the vibration amplitudes of blades on a tuned rotor, a pattern that is generally the goal of the designer.

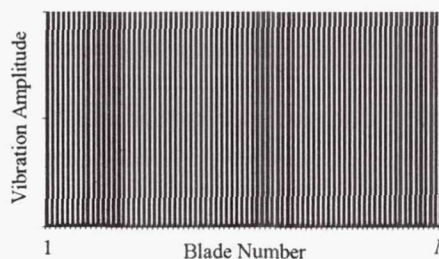
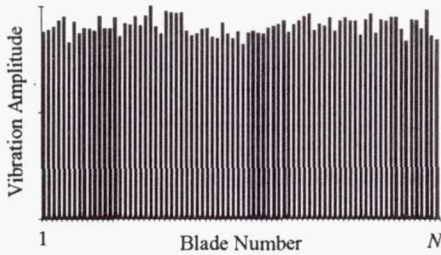


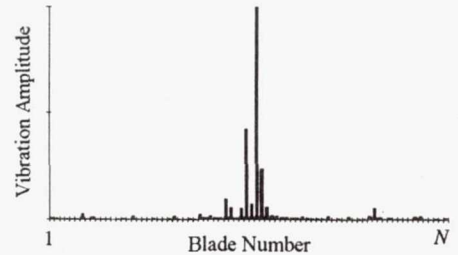
Figure 1 - Vibration amplitudes of blades on a tuned rotor.

Manufacturing tolerances and in-service degradation destroy the cyclic symmetry of the rotor and introduce blade-to-blade differences, known as mistuning. Such mistuning also results in blade-to-blade differences in the amplitudes of vibration. If the level of mistuning is sufficiently small, then all or most

blades still participate in the vibration, even though the amplitudes are not identical any longer, as shown in the schematic of Figure 2.



**Figure 2** - Vibration amplitudes of blades on a rotor with very low mistuning.



**Figure 3** - Localized vibration pattern of a mistuned rotor.

However, if the level of mistuning is sufficiently high, there could be a drastic change in the distribution of the vibration amplitudes. When this happens, vibration is constrained to very few blades of the rotor, as depicted (schematically) in Figure 3. Thus, most of the energy of the excitation is absorbed by a small number of blades, resulting in very high amplitudes which can lead to failure or shortened life. This phenomenon is known as mode localization.

The onset of mode localization is determined by the sensitivity of the assembly dynamics to blade mistuning. In general, this sensitivity is a function of the structural and aerodynamic characteristics of the tuned system and of the actual distribution of mistuning that is present. However, in the design phase, only the tuned system properties are known and the mistuning pattern and strength are unknown until the rotor is manufactured. Moreover, the mistuning distribution that results from in-service degradation is impractical to obtain in practice. Thus, it is very difficult to predict deterministically the onset of the damaging phenomenon of localization in practical propulsion systems, in a manner that is useful to the designers.

Can we then make any statement about the behavior of a mistuned system when knowing only the dynamics of the tuned system? In this paper we answer this question by treating mistuning stochastically rather than deterministically. Because mistuning is random and small in nature, a statistical and perturbative approach is chosen to develop a compact measure of sensitivity. The sensitivity formulation presented below is based on the singular behavior of the classical perturbation expansion of the aeroelastic eigensolution of bladed disks when mode localization takes place.

## DEVELOPMENT OF THE STOCHASTIC SENSITIVITY MEASURE

### Aeroelastic Equations of Motion

The bladed disk is modeled as a coupled system of  $N$  blades. Each blade's dynamics is described by a single in-vacuum (rotating) natural mode of vibration, say the  $n$ th natural mode. This simplified representation assumes that the coupling between the natural modes of an individual blade is negligible. Therefore, the rotor equations of motion form a system of  $N$  ordinary differential equations, each of which

corresponds to an individual blade on the rotor. (Note that for the SSME turbopump turbine, which is used as an example in this paper, we developed a general formulation and a computer program that allow for interactions between various blade modes; however, we found that the blade natural frequencies for the SSME turbopump are so well separated that inter-mode coupling is insignificant; this justifies the single-mode per blade assumption.)

For simplicity, we assume that the blades are coupled only aerodynamically, and that there is *no structural coupling* of the blades through the disk or at the shrouds. Moreover, we are examining the aeroelastic *free* vibration of the assembly, and thus include in our model only those aerodynamic forces that are motion-dependent. The application of component mode analysis to the  $N$ -blade assembly yields a set of  $N$  homogeneous, linear, ordinary differential equations in the modal amplitudes of the blades. We look for motions such that all the blade coordinates oscillate with the same frequency and/or decay or grow at the same rate. This yields the following aeroelastic eigenvalue problem of order  $N$ :

$$[\lambda^2 M + K - A(\omega_a)]u = 0 \quad (1)$$

where  $u$  is the complex eigenvector of the blade modal amplitudes,  $M$  and  $K$  are the real  $N \times N$  generalized inertia and stiffness matrices, respectively,  $A$  is the complex aerodynamic matrix, which depends on the assumed frequency,  $\omega_a$ , and  $\lambda^2$  is the complex eigenvalue.

Since there is no structural coupling, the rotor mass and stiffness matrices are diagonal and of the form  $M = I$  and  $K = \text{diag}(\omega_1^2, \omega_2^2, \dots, \omega_N^2)$ , where the blade modes have been normalized with a unit modal mass, and  $\omega_j$  is the natural frequency of the  $j$ th blade (rotating in a vacuum) for the blade mode considered. For a tuned assembly the diagonal elements of  $K$  are identical and equal to  $\omega_0^2$ , the nominal blade frequency squared. For a mistuned rotor the individual blade frequencies are generally distinct and the stiffness matrix is diagonal but not proportional to  $I$ .

The transformation of the aerodynamic influence coefficients between traveling wave coordinates and individual blade, or physical coordinates is defined by

$$A = E \tilde{A} E^* \quad (2)$$

where  $\tilde{A}$ , the aerodynamic matrix in traveling wave coordinates, is a diagonal matrix of complex elements  $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_N$ . The unitary transformation matrix  $E$  is given by

$$E = [e_1, \dots, e_j, \dots, e_N] \quad (3)$$

where

$$e_j = \frac{1}{\sqrt{N}} [1, e^{i\beta_j}, \dots, e^{i(N-1)\beta_j}] \quad \beta_j = \frac{2\pi}{N}(j-1) \quad j=1, \dots, N \quad (4)$$

where  $\beta_j$  is the *interblade phase angle*.

For a tuned assembly the matrix  $E$  diagonalizes the aeroelastic problem, Eq. (1). This means that the eigenvectors of the system are the columns of  $E$ , hence the aeroelastic mode shapes of the tuned assembly are the  $e_j, j=1, \dots, N$ , given by Eq. (4). For a motion in the  $j$ th mode all blades vibrate with *equal* amplitudes but with a constant phase difference  $\beta_j$  between adjacent blades. We will refer to the modes of the tuned assembly as *constant interblade phase angle modes*. Physically, these normal mode motions are waves traveling through the assembly with a phase change  $\beta_j$  at each blade. To each backward traveling wave  $e_j$  corresponds a forward traveling wave  $e_{N-j+2}$  which has the same number of (traveling) nodal diameters. After diagonalization of Eq. (1) with the similarity transformation defined by  $E$ , the aeroelastic eigenvalues of the tuned assembly are readily given by:

$$\lambda_{0j}^2 = \tilde{A}_j - \omega_0^2 \quad j=1, \dots, N \quad (5)$$

For a mistuned assembly the constant interblade phase angle vectors  $e_j$  do not uncouple the system (1) and thus they are not the aeroelastic modes. A numerical or a perturbation solution of the aeroelastic eigenvalue problem is then required. In this work, we assume small random frequency mistuning: the mode shape of each blade is identical, the natural frequency of each blade is a small deviation from the nominal blade frequency, and the frequencies for the individual blades are generated using random numbers from a uniform probability distribution function.

### Perturbation Analysis

The most natural perturbation procedure to study the dynamics of a mistuned assembly is one that considers the tuned assembly as the unperturbed system and the blade mistuning as the small perturbation. This approach, which we refer to as the classical perturbation method, yields easily eigenvalue and eigenvector perturbations to any order in the mistuning (first- and second-order results are given below). However, this classical approach is inherently flawed in cases of high sensitivity and localization. This is because choosing the small mistuning as the perturbation parameter requires the eigensolution of the mistuned assembly to be a *small perturbation* of that of the tuned assembly. Clearly, this is not the case when the phenomenon of mode localization occurs, since in this case mistuning causes *qualitative* (i.e., very large) alterations in the assembly's dynamics (for instance, compare Figure 1 and Figure 3). Thus, the classical perturbation analysis cannot capture the drastic changes in the eigensolution caused by small mistuning. As we discuss below, however, the mere fact that the technique fails in the presence of localization can be used to predict the high sensitivity to mistuning.

Let us now explore the mechanisms of failure of the classical perturbation method. We denote the unperturbed stiffness matrix of the tuned assembly by  $K_0$  and the perturbation matrix due to small mistuning by  $\delta K$ , where the latter is a diagonal matrix of mistunings in the squares of the individual blade frequencies,  $\delta\omega_i^2$ , such that  $\delta\omega_i / \omega_0 \ll 1$  for  $i=1, \dots, N$ . The stiffness matrix of the perturbed (mistuned) assembly is therefore  $K = K_0 + \delta K$ . Since the unperturbed system is the tuned system, the modes of the unperturbed system are the constant interblade phase angle modes given by Eq. (4). Those of the perturbed system can be expanded in a truncated asymptotic series as

$$\left. \begin{aligned} \lambda_i^2 &= \lambda_{0i}^2 + \delta\lambda_i^2 + \delta^2\lambda_i^2 \\ u_i &= e_i + \delta u_i + \delta^2 u_i \end{aligned} \right\} \quad i=1, \dots, N \quad (6)$$

where  $\delta\lambda_i^2$  and  $\delta u_i$  (respectively  $\delta^2\lambda_i^2$  and  $\delta^2u_i$ ) are first-order (respectively second-order) perturbation terms in the small mistuning.

Applying perturbation theory to the eigenvalue problem,<sup>17</sup> one can show that the first-order perturbation of the eigenvalues is

$$\delta\lambda_i^2 = -\frac{1}{N} \sum_{k=1}^N \delta\omega_k^2 \quad (7)$$

We readily observe that for small mistuning this is always a small term. Hence the first-order eigenvalue perturbation **cannot** reveal high sensitivity to mistuning. Moreover, all eigenvalues are shifted by an identical amount and, for a mistuning pattern that averages to zero, the change in the eigenvalues equals zero. Another interesting remark is that the perturbation of the eigenvalue squared is real, therefore mistuning, to the first-order, does not affect the stability of the assembly (or very little). Here we reach the seemingly contradictory conclusion that if the average mistuning throughout the rotor is not zero, i.e., if we stiffen or soften **all** the blades on the average, the flutter boundary remains unchanged! The explanation of this paradox lies in the fact that the assumed frequency that was used for the aerodynamic computations is then no longer valid. If the effect of mistuning with nonzero mean on flutter is sought, the full mistuned eigenvalue problem must be solved in order to determine the correct assumed frequency. These remarks suggest that, although simple and cost-effective, perturbation methods must be used with care in a design environment.

Turning to the second-order perturbation of the eigenvalues, we can write this perturbation as:

$$\delta^2\lambda_i^2 = \sum_{\substack{k=1 \\ k \neq i}}^N \frac{|e_k^* \delta K e_i|^2}{\lambda_{0i}^2 - \lambda_{0k}^2} = \sum_{\substack{k=1 \\ k \neq i}}^N \frac{|e_k^* \delta K e_i|^2}{\tilde{A}_i - \tilde{A}_k} \quad i = 1, \dots, N \quad (8)$$

where \* denotes the complex conjugate and  $|\cdot|$  the modulus of a complex quantity. In Eq. (8), the numerator in the summation is a measure of the square of the mistuning (in the frequency squares), and the denominator is the spread among the constant interblade phase angle aerodynamic coefficients. (Equation (5) shows that this spread is simply the spread among the tuned eigenvalues.) For turbomachinery in general, and the HPOTP turbopump in particular, these aerodynamic coefficients are small because the unsteady aerodynamic forces are small compared to elastic and inertia forces, and provide *very weak coupling among the blades*. Thus, even if mistuning is small, the numerators and denominators in Eq. (8) may be of comparable magnitudes, in which case the second-order eigenvalue perturbation is of the order of unity, not second order. The fact that eigenvalue perturbations become large indicates the failure of the perturbation analysis and reveals the high sensitivity of the assembly dynamics to mistuning. This suggests the use of second-order eigenvalue perturbations in defining a sensitivity measure.

The first order eigenvector perturbation

$$\delta u_i = \sum_{\substack{k=1 \\ k \neq i}}^N \frac{e_k^* \delta K e_i}{\tilde{A}_i - \tilde{A}_k} e_k \quad i = 1, \dots, N \quad (9)$$



shows that the first-order aeroelastic mode shape perturbations behave similarly to the second-order eigenvalue perturbations, *i.e.*, the sensitivity of the eigenvectors is inversely proportional to the differences in aerodynamic coefficients. This is consistent with the observed occurrence of mode localization in Figure 3, where the drastic alteration of the mode shape from the tuned mode in Figure 1 is a direct consequence of the high sensitivity to mistuning. Although these perturbation results *cannot* be used to characterize the behavior of the mistuned assembly (*e.g.*, localization), as they are qualitatively in error, the failure of the perturbation analysis indicates high sensitivity to mistuning. However, note that the classical perturbation analysis does not always fail. For example, if mistuning is extremely weak, *e.g.*, 0.001% for the SSME turbopump, the ratios in Eq. (8) may be sufficiently small for the perturbation expression to be valid. This is also the case if the interblade coupling is strong, such that the aerodynamic coefficients, and thus the denominators in Eq. (8) are not small. In these cases mistuning has a small effect on the assembly's dynamics.

By utilizing these perturbation ideas, we are able to develop a sensitivity measure that will allow the designer to predict, in a very simple way, the effects of mistuning on the various aeroelastic modes. Noting the close relationship between the first-order eigenvector perturbation and the second-order eigenvalue perturbation, we derive a sensitivity measure based on the second-order eigenvalue perturbation expression. Because mistuning is random in nature, a statistical approach is chosen to obtain a compact measure of sensitivity. This measure allows us to predict mistuning effects essentially with a single scalar for each mode. However, a possible drawback of this stochastic measure is that individual realizations of mistuning patterns may result in dynamic behaviors much different from the *average* behavior predicted by the sensitivity measure (for example, a "sinusoidal" mistuning pattern alters the dynamics much less than a truly random pattern). We further note that the sensitivity measure defined below does not require the mistuned system solution and thus is quite cost effective. Moreover, since the forced response of mistuned assemblies consists of linear combination of responses in the free modes of vibration, the sensitivity of the aeroelastic modes will provide useful information about that of the forced response. Thus, this measure has the potential to become a valuable design tool.

The basic idea is to define the sensitivity of the system by taking the statistical average of the second-order eigenvalue perturbation (where the mistunings are the perturbation parameters). This is motivated by the findings above, which showed that the mechanism for high sensitivity, *i.e.*, the closeness of the tuned eigenvalues, is embedded in the second-order perturbation, while the first-order eigenvalue perturbation always remains small. We rewrite Eq. (8) using the expression for the interblade phase angle mode shapes,  $e_p$ , given in Eq. (4). After some algebra, we obtain:

$$\delta^2(\lambda_i^2) = -\frac{1}{N^2} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{\tilde{A}_k - \tilde{A}_i} \left[ \sum_p \delta \omega_p^2 + \sum_{\substack{p,q \\ p \neq q}} \cos\left(\frac{2\pi}{N}(i-k)(p-q)\right) \delta \omega_p^2 \delta \omega_q^2 \right] \quad (10)$$

At this point we define the mistunings in the frequency squared,  $\delta \omega_p^2$ ,  $p=1, \dots, N$ , as independent and identical random variables of mean zero and standard deviation,  $\sigma$ . This simply means that the blades are chosen randomly from an (ideally) infinite population. Next we take the statistical average, over the random mistunings, of the second-order eigenvalue perturbation, given by Eq. (10). Denoting average quantities by  $\langle \cdot \rangle$ , this yields

$$\langle \delta^2(\lambda_i^2) \rangle = -\frac{1}{N} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{\tilde{A}_k - \tilde{A}_i} \sigma^2 \quad i = 1, \dots, N \quad (11)$$

because  $\langle (\delta\omega_p^2)^2 \rangle = \sigma^2$  and, for  $p \neq q$ ,  $\langle \delta\omega_p \delta\omega_q \rangle = 0$ .

Now consider the perturbation series of the  $i$ th eigenvalue in terms of mistuning, Eq. (6), and take its statistical average. Since the first-order perturbation is proportional to mistuning, it averages to zero and we obtain

$$\langle \lambda_i^2 \rangle = \lambda_{0i}^2 - \frac{1}{N} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{\tilde{A}_k - \tilde{A}_i} \sigma^2 \quad i = 1, \dots, N \quad (12)$$

This shows that to the second order, the locus of the average of an eigenvalue versus the mistuning standard deviation is a parabola. The curvature of this parabola determines the sensitivity of the associated aeroelastic mode to mistuning. We rewrite

$$\langle \lambda_i^2 \rangle = \lambda_{0i}^2 + S_i \sigma^2 \quad i = 1, \dots, N \quad (13)$$

where we have defined the stochastic sensitivity measure of the  $i$ th eigenmode to mistuning as

$$S_i = -\frac{1}{N} \left( \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{\tilde{A}_k - \tilde{A}_i} \right) \quad i = 1, \dots, N \quad (14)$$

We say a mode features a low, or normal sensitivity when the expansion (13) is valid. This occurs when the term  $S_i \sigma^2$  is second-order and therefore when the sensitivity measure  $S_i$  is of the order of unity. High sensitivity in a mode occurs when  $S_i \sigma^2$  is first-order or larger, implying the failure of the perturbation analysis. This happens when  $S_i$  is large.

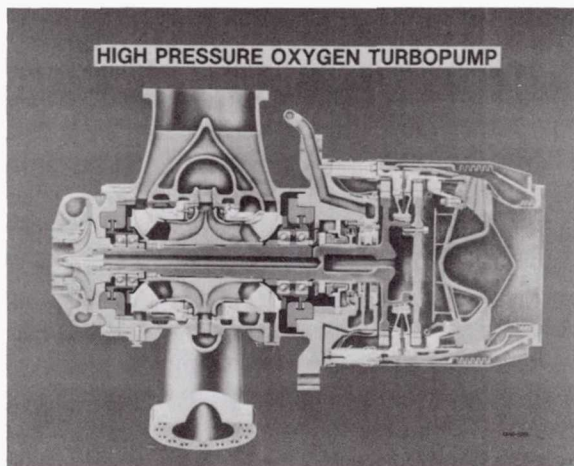
In order to interpret results, it is useful to make our sensitivity measure dimensionless. This is achieved first by expressing  $S_i$  in terms of the dimensionless mistuning standard deviation  $\epsilon$ , where  $\epsilon = \sigma/\omega_0^2$ . The second step is to divide  $S_i$  by a representative eigenvalue of the system, such that all sensitivities are referred to 1 rather than to the various  $\lambda_{0i}^2$ . Here we choose to nondimensionalize  $S_i$  by the eigenvalue corresponding to the tuned blade frequency squared,  $-\omega_0^2$  (ideally we should divide by  $\lambda_{0i}^2$  given in Eq. (5) but this would result in a complicated expression for  $S_i$ ; moreover, for small aerodynamic coupling, typical of turbomachinery, these two normalizations are nearly equivalent, because all tuned eigenvalues are close to each other). The selected normalization yields the dimensionless sensitivity measure:

$$\bar{S}_i = \frac{\omega_0^2}{N} \left( \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{\tilde{A}_k - \tilde{A}_i} \right) \quad i = 1, \dots, N \quad (15)$$

We observe readily that the sensitivity measure increases as the aerodynamic interblade coupling decreases. Note that the number of blades has opposite effects on the sensitivity measure through the factor  $1/N$  and through the number of terms in the summation. Generally, the sensitivity measure increases as the number of blades is increased.<sup>9</sup>

## RESULTS AND DISCUSSION

Here we apply our stochastic sensitivity measure to the SSME HPOTP turbopump rotor and investigate the effectiveness of this measure in predicting the occurrence of mode localization.



**Figure 4** - Space Shuttle Main Engine High Pressure Oxygen Turbopump Cross Section.

The SSME is a hydrogen-fueled liquid rocket engine which generates 512,000 lbf of thrust at full power. Each rocket engine is fed by two high pressure and two low pressure turbopumps. The high pressure turbopumps are driven by axial flow gas turbines which are powered by hydrogen-rich steam generated in individual preburners. The HPOTP supplies the oxidizer to the combustion chamber. The HPOTP gas turbine generates 30,000 hp at 28,000 rpm with each blade transferring approximately 300 hp. Figure 4 shows a cross-sectional view of the HPOTP turbopump. The pump impellers, shaft, gas turbine, preburner and the turbine blade coolant jet ring can be seen in the figure. The blades in the first stage of the turbine experienced frequent cracking in the shank region.

In the present model, the inertia and stiffness matrices,  $M$  and  $K$ , are obtained from a MSC/NASTRAN normal modes analysis of the HPOTP first-stage turbine blade. This stage has 78 blades. The finite element model for MSC/NASTRAN was generated by converting the original ANSYS model provided by the SSME contractor, Rocketdyne Division of Rockwell International Corporation. The model consists of 10,014 nodal points and 7758 solid hexahedron elements, as shown in Figure 5. The first three modes of this blade are well-separated: at 28000 rpm, mode 1 (bending) is at 4748 Hz, mode 2 (edgewise) at 9950 Hz and mode 3 (torsion) at 16580 Hz. An unsteady aerodynamic analysis reveals that aerodynamic forces introduce little coupling among the three modes. Thus, the single-degree-of-freedom-per-blade theoretical model presented above is suitable to the study of the motion of the blades on the rotor. The aeroelastic stability analysis of this blade row was presented in a previous study.<sup>18</sup> Here we employ the same procedure in order to illustrate the effectiveness of the sensitivity measure.

The aerodynamic matrix  $A$  is a fully populated complex matrix which is evaluated using a linearized unsteady aerodynamic theory that accounts for the effects of camber, thickness and incidence. Verdon's method<sup>19,20</sup> is employed to calculate the unsteady forces on the blades due to a particular natural mode of motion for a (tuned) cascade of identical blades. This results in a traveling wave representation of the aerodynamic forces for the tuned cascade. A detailed description of the unsteady force calculation using this theory is given in references 18 and 21.

The sensitivity measure we derived above is a complex number that characterizes the sensitivity of both the frequency and damping of a tuned mode to random mistuning. We illustrate the effectiveness of the measure by considering the real part of this complex number. We do not focus on the imaginary part of the sensitivity measure because the aerodynamic damping is generally small and because we have consistently observed that the sensitivity of the frequency is the one that governs mistuning effects and the occurrence of mode localization.<sup>8</sup> Also, unless otherwise stated, the results we present are for the second blade mode group, as this mode group was shown to have the least damping<sup>18</sup>.

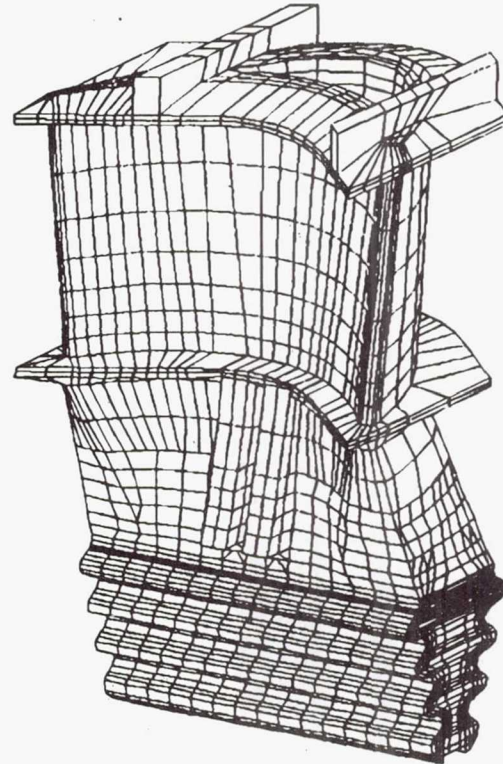


Figure 5 - Finite element model of the SSME HPOTP turbine blade.

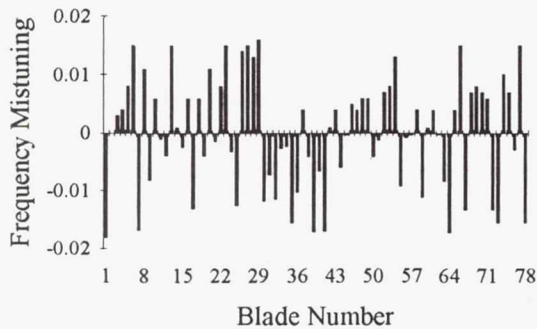


Figure 6 - The relative individual blade frequency mistuning distribution as a function of blade number. The mistuning depicted is for 1% standard deviation.

The sensitivity measure is validated by correlating the magnitude of its real part with the localization observed in the mode shapes. These mode shapes are calculated using *one* random mistuning distribution with various values of standard deviation. The mistuning distribution used for the results is depicted in Figure 6. It was obtained by generating random numbers having a uniform probability distribution between  $-\epsilon\sqrt{3}$  and  $\epsilon\sqrt{3}$ , where  $\epsilon$  is the dimensionless standard deviation of the blade stiffness mistuning. Although it would have been desirable to validate the sensitivity measure against Monte Carlo simulations of the mode shapes (because  $\bar{S}_i$  is stochastic), such simulations are almost impossible to perform because of the difficulties involved in the tracking of eigenvalues and eigenvectors as the mistuning distribution varies. Running Monte Carlo simulations without accurate tracking is meaningless because then the mode shape localization is lost in the averaging process.

Figure 7 displays the variation of the real part of the sensitivity measure for the modes of the 78-bladed rotor when these modes are sorted by increasing frequency. We first observe that the sensitivities of all the modes are very large (much larger than 1; note the scale). We also note that the largest sensitivity corresponds to the frequency index 60 and that the smallest sensitivity corresponds to the frequency index 26.

The two mode shapes of the bladed disk with highest and lowest sensitivity are compared in Figure 8 for various values of mistuning. Each plot displays the amplitude of the 78 blades in the corresponding mode at a given level of mistuning. As mistuning increases, the equal amplitude nature of the mode shapes breaks down and both modes rapidly display the tendency to localize. For a mistuning of standard deviation  $\epsilon = 0.2\%$  (unavoidable in practice), the character of the modes has changed drastically to become severely localized to a relatively small group of blades, while for  $\epsilon = 0\%$  the modes are as in Figure 1. This alteration is in full agreement with the large values of the sensitivity measure obtained for all modes in Figure 7. Moreover, the mode with the least sensitivity, which is shown on the right in Figure 8, displays somewhat less tendency to localize than the mode predicted to be most sensitive, which is shown on the left. This is consistent with Figure 7. Nevertheless, the difference in behavior between these two modes is not stark and could lead to the conclusion that, although the sensitivity measure is effective in predicting mode localization, it does not do very well at differentiating between the sensitivities of various modes in the same blade mode group.

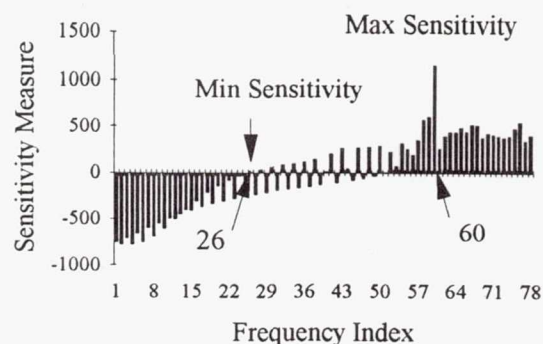


Figure 7 - Real part of the sensitivity measure for the aeroelastic modes in the second blade mode group. The modes are sorted in order of increasing frequency.

In order to investigate the reason for this behavior, the results were further examined and it was found that the apparently disappointing prediction by the sensitivity measure is due to the phenomenon of mode switching, which is known to occur in systems with closely-spaced modes as a parameter (here the mistuning) is varied. We first note that the sensitivity measure is a continuous function of the system eigenvalues, which themselves depend continuously on the unsteady aerodynamic coefficients. These unsteady aerodynamic coefficients are also continuously dependent on the interblade phase angle (with some exceptions when aerodynamic resonances occur). Hence the sensitivity measure ought to vary relatively smoothly with the interblade phase angle. This is depicted in Figure 9, which displays the real part of the sensitivity measure of the modes of the 78-bladed rotor as a function of the interblade phase angle index. We note that the largest sensitivity corresponds to phase index 11 and the smallest sensitivity corresponds to the phase index 52. The frequency indices corresponding to these are 60 and 26 respectively, and their mode shapes are shown in Figure 8. Notice that although the sensitivity measure is a smooth function of the interblade phase angle over most of the range, it changes extremely rapidly at the point of maximum sensitivity, *i. e.*, near phase index 11. Thus, some of the mistuned mode shapes displayed in the left half of Figure 8 probably correspond to modes whose phase indices are close to the phase index of the mode with maximum sensitivity, but whose sensitivity is much lower. This provides an explanation for the behavior shown in the left half of Figure 8, as follows. For very small mistuning of 0.01%, the mode shape features the onset of localization, which is consistent with the calculated *highest* sensitivity measure for that mode. As mistuning increases to 0.02%, however, it is obvious that the mode has become *less* sensitive to mistuning, this being due to a switching with a neighboring, less sensitive mode. As mistuning keeps increasing, the onset of localization is thus governed by the sensitivity of the *switched* mode, not any longer by the *highest* sensitivity of the original tuned mode of phase index 11. This explains the apparently unsatisfactory prediction of localization by the sensitivity measure. To summarize, the near discontinuity of the sensitivity function at the interblade phase index corresponding to the highest sensitivity causes mode switching for very small mistuning values, and this results in the

Frequency Index = 60  
Real Sensitivity = 1157.1

Frequency Index = 26  
Real Sensitivity = -1.14

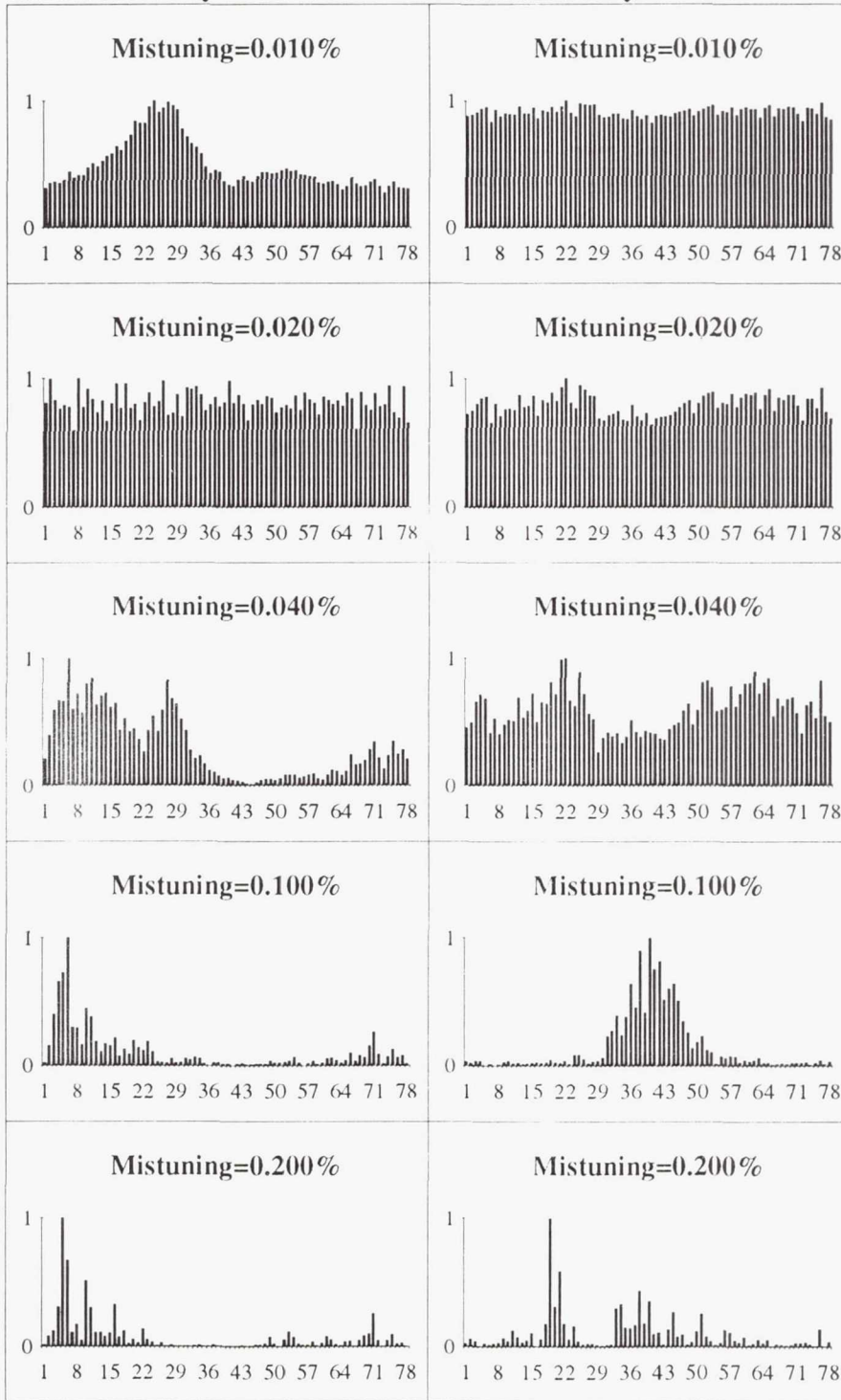


Figure 8 - Amplitudes (vertical axis) vs blade number (horizontal axis) for mode shapes having maximum and minimum magnitude of sensitivity measure.

apparent disappointing performance of the sensitivity measure in differentiating the tendencies for localization in the two modes of Figure 8.

In order to further check whether the sensitivity measure is a good predictor of mode localization, we compared the modes having the highest and lowest sensitivity amplitudes, but by ignoring the mode whose sensitivity measure changes abruptly, *i.e.*, the mode with frequency index 60 and phase index 11. The next most sensitive mode has the interblade phase index 41 and the frequency index 4. That is, we compare the modes with frequency indices 4 (marked H in Figure 9) and 26. This comparison is shown in Figure 10. As is readily apparent, the mode on the left localizes much more rapidly than that on the right, which is in complete agreement with the prediction made by the sensitivity measure. It is also clear that no mode switching occurs in Figure 10, except perhaps between  $\epsilon = 0.1\%$  and  $0.2\%$ .

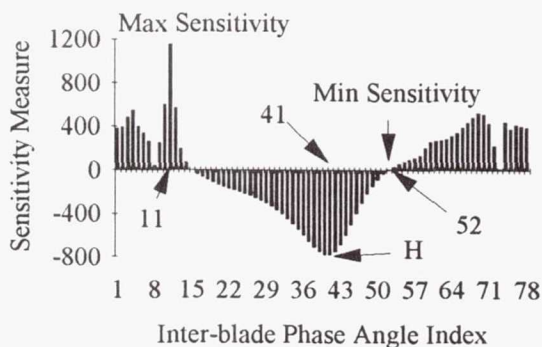


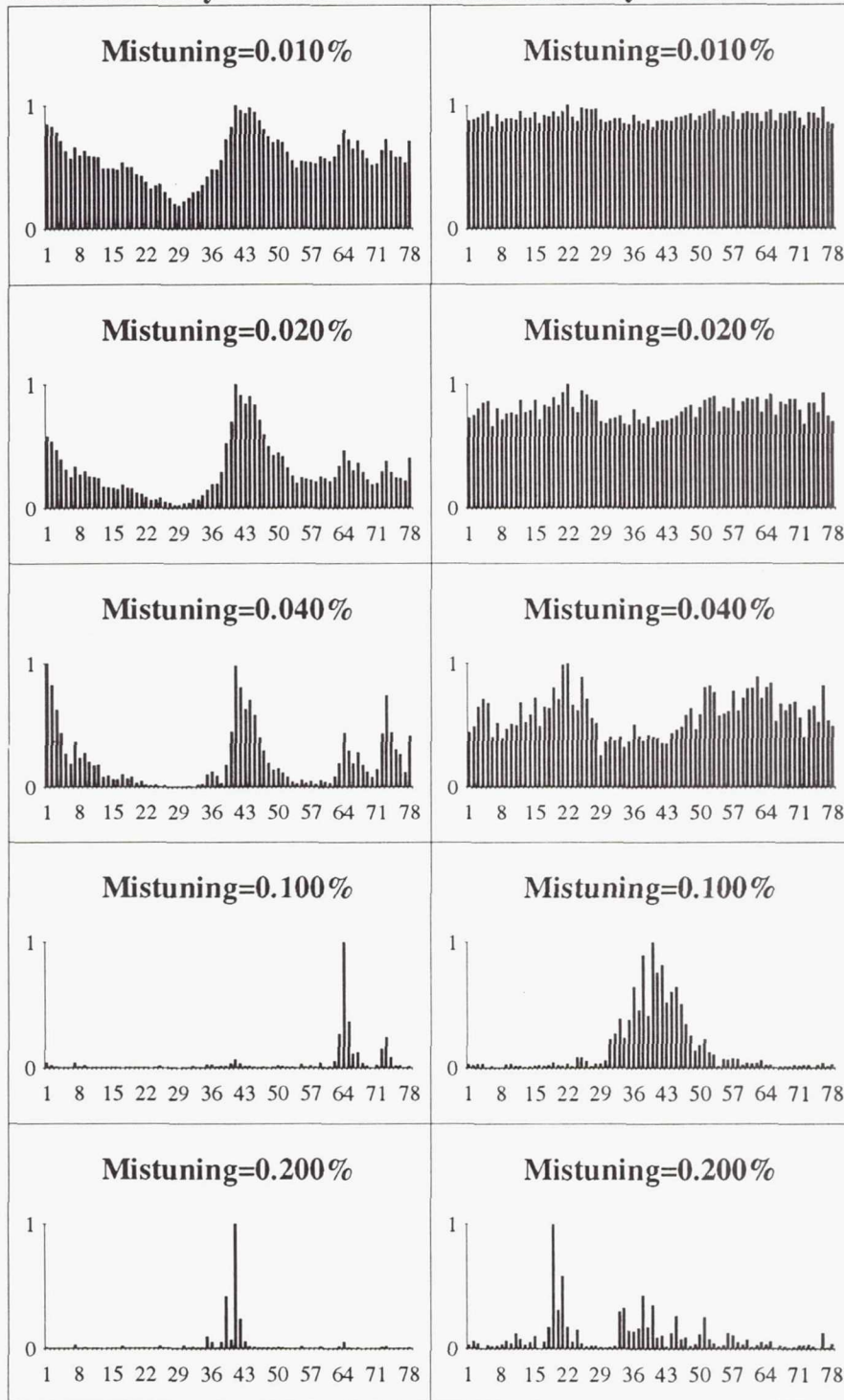
Figure 9 - Real part of the sensitivity measure as a function of interblade phase index, for the second blade mode group.

We also examined the sensitivity to mistuning of the first group of modes of the SSME turbopump. Recall that in the first cluster of 78 modes, each blade vibrates nearly in its first normal mode, in this case a first bending mode. Here we are interested in determining whether our sensitivity measure is able to predict the relative strength of mistuning effects for the first two blade mode groups. Figure 11 displays the real part of the sensitivity measure for the first mode group versus the interblade phase index. Comparing it with Figure 9 for the second group, we immediately observe that most modes in the first group are more sensitive than those in the second group. This suggests that mode localization occurs more rapidly in the first mode group than in the second, the kind of information which is important to the designer. To confirm this, the most sensitive mode shapes in each of the two blade mode groups (frequency index 2 for the first group, bending, and 4 for the second group, edgewise) are compared in Figure 12 for various mistuning values. Observe that the mode in group 1 becomes localized more rapidly than that in group 2 as mistuning increases. This confirms the validity of the sensitivity measure as a practical tool for the prediction of mode localization.

Finally, to illustrate one practical usefulness of the sensitivity measure, we compared the localization behavior of the above SSME rotor to that of a hypothetical rotor having blades with natural frequencies ten times smaller, with all other parameters and aerodynamic forces unchanged. From Eq. (15), we note that all sensitivities for the hypothetical rotor are 100 times smaller than for the SSME rotor. To confirm that the mode shapes of the hypothetical rotor are correspondingly less sensitive to mistuning, we compare the mode shapes with the highest sensitivity for both rotors in Figure 13. Note that the hypothetical rotor mode shape is not appreciably altered by mistuning levels which have significant effect on the mode shape of the SSME rotor --- as predicted by the calculated sensitivities given in Figure 13. Furthermore, the most sensitive mode shape of the hypothetical rotor with 1% mistuning is almost identical to the corresponding mode shape of the SSME rotor with 0.01% mistuning (a hundred times smaller). This clearly illustrates the strong correlation between the sensitivity measure and the mistuned rotor mode shapes, because the predicted sensitivity for the SSME rotor is exactly a hundred times more than that of the hypothetical rotor.

**Frequency Index = 4**  
**Real Sensitivity = -787.3**

**Frequency Index = 26**  
**Real Sensitivity = -1.14**



**Figure 10** - Amplitudes (vertical axis) vs blade number (horizontal axis) for mode shapes having maximum and minimum magnitude of sensitivity measure, when rapidly changing regions are ignored.



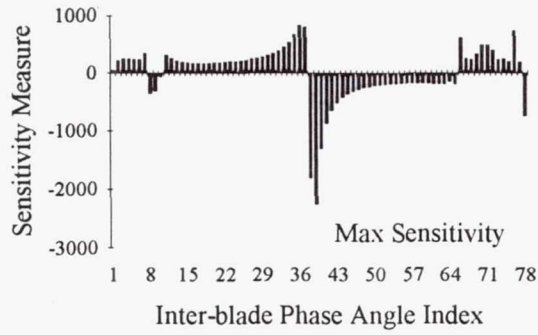


Figure 11 - Real part of the Sensitivity Measure as a function of interblade phase index for blade mode group 1.

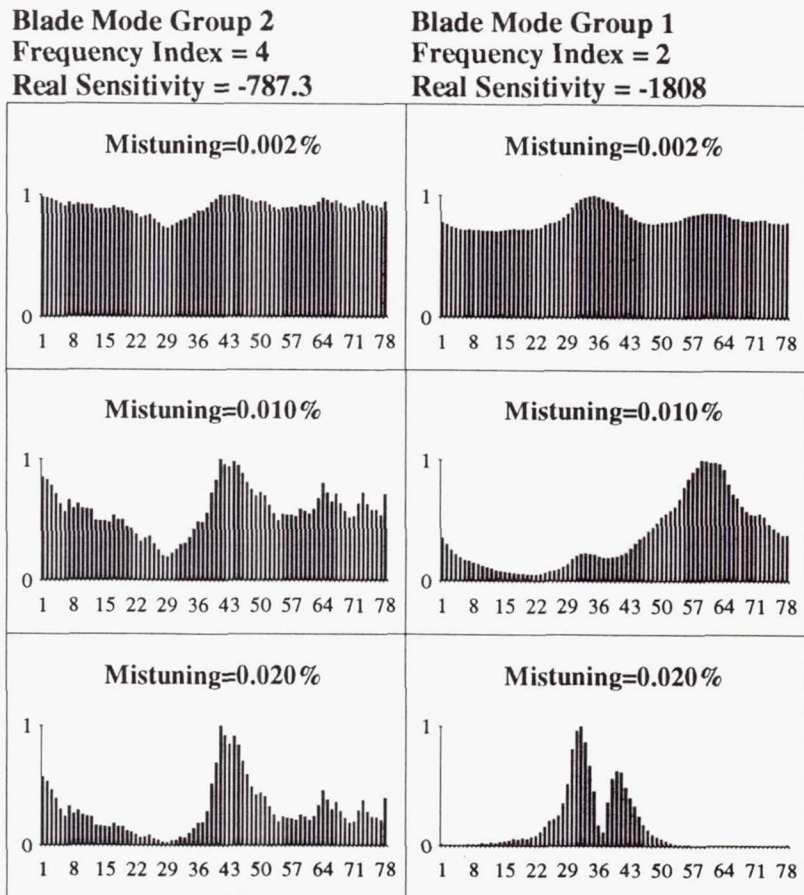


Figure 12 - Comparison of the most sensitive mode shapes from blade mode groups 1 and 2 (Amplitudes on the vertical axis and blade number on the horizontal axis).

SSME HPOTP Rotor  
 Frequency Index = 4  
 Real Sensitivity = -787.3

Hypothetical Rotor  
 Frequency Index = 5  
 Real Sensitivity = -7.87

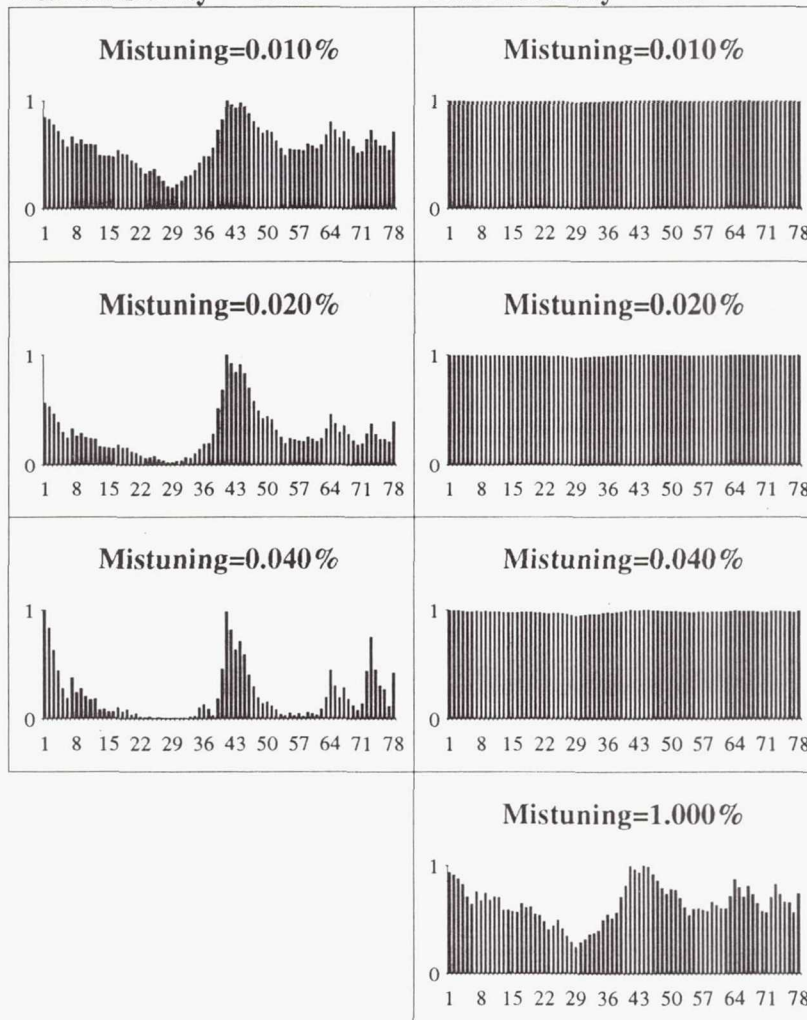


Figure 13 - Comparison of the most sensitive mode shapes for the SSME HPOTP rotor and a hypothetical rotor. (Amplitudes on the vertical axis and blade number on the horizontal axis).

## CONCLUSIONS

A sensitivity measure that indicates the potential for the occurrence of mode localization in practical turbomachinery rotors is presented. This measure is derived by statistically averaging the second order perturbation expression for the system eigenvalues and appropriately non-dimensionalizing the resulting expression.

This measure is very inexpensive to compute compared to high-cost Monte Carlo simulations of mistuned systems. Because it is based on statistical modeling of mistuning, it does not require knowledge of the actual mistuning distribution. Thus, it is suitable for estimating the potential for the occurrence of mode localization during the design phase.

The effectiveness of the sensitivity measure is demonstrated by application to the first stage of the Space Shuttle Main Engine's High Pressure Oxidizer Turbopump turbine. The sensitivity measure is calculated for various modes and then is validated by comparison to the degree of localization obtained by an exact calculation which uses a randomly generated mistuning pattern. The predictions made by the sensitivity measure correlate well with the localization observed. The sensitivity measure is thus found to be an effective tool for predicting the potential for mode localization.

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