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RADIO AND OPTICAL INTERFEROMETRIC IMAGING

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Abstract

Since diffraction-limited imaging with a single aperture yields angular resolution ~λ /D, the attainment of high angular resolution with single apertures requires the construction of correspondingly large monolithic apertures, the whole surface of which must be figured to much less than a wavelength. At the longer wavelengths, it is impossible to build a sufficiently large single aperture: for example, at λ 21 cm, arcsec resolution requires an aperture of diameter ~ 50 km. At the shorter wavelengths, the atmosphere imposes a natural limit in resolution of about one arcsec. However, another route is possible; that is, using synthetic apertures to image the sky. Synthetic apertures are now in use in many fields, e.g., radio interferometry, radar imaging, and magnetic-resonance imaging. Radio-interferometric techniques developed in radio astronomy over the past 40 years are now being applied to optical and IR astronomical imaging by a number of groups. Furthermore, the problem of figuring synthetic apertures is considerably simpler, and can be implemented in a computer: new "self-calibration" techniques allow imaging even in the presence of phase errors due to the atmosphere.

Introduction

At the beginning of this century, Michelson investigated the use of interferometry for high resolution measurements of stellar diameters. Figure 1 shows a schematic of his interferometer. Light is collected from two mirrors, A and B, and interfered at a focal point. The contrast of the fringes yields information about the source structure. The position of the fringes (which is the

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fringe phase) also encodes source information but it is rather more sensitive to instrumental errors since, for example, a change in the position of mirror A will also shift the fringes. We will return to this point later. Imaging from coherence measurements relies on the van Cittert-Zernike theorem, which states that for an incoherent object, the coherence of the electric field far from the object is the Fourier transform of the sky brightness function. The complex coherence function of the electric field, E, between two points, Q_1,Q_2 is defined as:

$$\Gamma(Q_1,Q_2)=\langle E(Q_1,t)E^*(Q_2,t)\rangle_t \tag{1}$$

At radio wavelengths, this can be calculated directly using digitization of the received electric fields while, for optical wavelengths, it can be found from modulation of the position of a mirror such as A, by a distance corresponding to $\lambda/4$. The coherence function is the Fourier transformation of the sky brightness, I(x) (see Thompson et al. 1986).

$$\Gamma(\mathrm{u}_1,\mathrm{u}_2)=\int_C I(\mathrm{x})\,e^{\,i\frac{2\pi}{\lambda}(\mathrm{u}_1\cdot\,\mathrm{u}_2)\cdot\mathrm{x}}\,d\mathrm{x} \tag{2}$$

where **u** is the position vector from Q as projected on a plane perpendicular to the line of sight, x is an angular Cartesian coordinate system centered on the object, and C is the field of view of the array elements. Therefore, an interferometer measures a single Fourier coefficient of the sky brightness with a spatial frequency dependent on the separation and orientation of the interferometer elements as seen from the object. In 1960, Ryle and Hewish (1960) pointed out that one could synthesize a large aperture by collecting coherence samples with an interferometer using many different spacings of the elements, and then Fourier-transforming the resulting sampled coherence function in a computer to make an image. The concept of a synthetic aperture holds for all wavelengths but the technology required for the measurements differs considerably. These differences will be addressed in the following talks.

Interferometer and Array Design

While all imaging interferometric arrays measure the coherence function of the radiation from a celestial source, the details of instrumentation needed vary depending principally on the wavelength range and the maximum separation of elements in the array. Hence, I will concentrate on two typical cases: a radio interferometer designed to operate at centimeter wavelengths, and an optical interferometer. I will follow the signal through both systems.

Light collection: The light can be collected by simple mirrors or by telescopes in the optical, and by parabolic reflectors in the radio. The size of these is limited by the coherence size of the atmosphere in the optical, and by budget in the radio.

Amplification: In the radio, the signals can be amplified without significant loss in signal-to-noise ratio (SNR) while, in the optical, such amplification is both technically impossible and theoretically unattractive (since it would introduce noise equivalent to a black body which peaks at optical wavelengths). The lack of amplification at optical wavelengths means that it is unattractive to divide the light into more than two or three interferometers simultaneously. This is in contrast to the radio regime, where there is no penalty for operating many interferometers simultaneously.

Heterodyne: Radio interferometers always operate as heterodyne systems; that is, the radiation after amplification is converted to some lower frequency for subsequent processing. For optical wavelengths, the fractional bandwidth accessible by heterodyne techniques is prohibitively small.

Light Relay: The light must be relayed to a central location for measurement of the coherence. In the optical, either propagation in an evacuated pipe or along an optical fiber is possible (although free space propagation is possible on the Moon). In the radio, there are many possibilities: cable, waveguide, microwave links, or tape recording and playback.

Digitization: In the radio, the signals from each element can be digitized, usually to one or two bits of precision. This enables the use of digital circuits for many subsequent steps. In the optical regime, this requires the use of low-bandwidth heterodyne systems and has not yet been attempted.

Delay Compensation: The geometry of an interferometer is usually such that the wavefront from a given object will reach one element before another. The light must, therefore, be delayed by the corresponding amount and, furthermore, this delay must be tracked continuously as the Earth rotates. This fringe acquisition and tracking is performed using moving mirrors in the optical, and digital delays and frequency synthesizers in the radio. Errors in the assumed geometry can result in the loss of coherence.

Correlation: At radio wavelengths, the coherence can be evaluated using a special-purpose digital computer to perform the multiplication and averaging required. This means that very high quality measurements of the coherence are possible. In the optical regime, analogue methods must be used. The light is brought together at one point and interfered. Modulation of the light path in one arm by $\lambda/4$ enables the phase to be measured. High-quality optical correlators are now being built using optical fibers for many of the steps.

Sampling of the coherence function over the synthesized aperture can be accomplished either by physically moving the interferometer elements or by allowing the rotation of the Earth to do so or by a combination of both approaches (see Thompson et al. 1986 for a detailed discussion of the design of radio interferometric arrays). For short baselines, up to some tens of kilometers, the signal transmission system may limit the layout of an array, as may the local geography, and the need to move the elements. Figure 2 shows a modern radio-interferometric array, the National Radio Astronomical Observatory (NRAO) Very Large Array (VLA) (see Napier et al. 1983) for which the instantaneous Fourier plane coverage is good and is improved by Earth rotation. In the case of the VLA, the elements are constrained to lie along straight lines by the waveguide used for signal transmission. As long as the light can be interfered coherently, the elements may be an arbitrarily large distance apart. As an example, figure 3 shows the NRAO Very Long Baseline Array (VLBA) now under construction. For the VLBA, signal transmission is accomplished using tapes to limit the layout principally by geography.

There are two major changes in array layout on going to optical wavelengths. First, as discussed above, it becomes advantageous to limit the number of interferometers operating simultaneously since the SNR degrades as the light is divided. The requirement to measure closure phase (see the next section) drives the optimum number of elements to about five or six. Second, since the atmospheric coherence time (~10 ms) is much shorter than the time for the source coherence to change due to Earth rotation (min), and since most imaging requires good SNR within an atmospheric coherence time, it becomes worthwhile to move the antenna every 10 min or more frequently to improve the sampling of the Fourier plane. At optical wavelengths, one therefore prefers a small number of easily movable elements.

Imaging

Once samples of the coherence function have been collected, edited, and calibrated, an image can, in principle, be formed by direct Fourier inversion. However, in practice, two generic problems afflict the measured coherence function: first, the sampling is often incomplete and, second, the calibration of the coherences may be uncertain because of the effects of the Earth's atmosphere or uncertainties in the geometry of the interferometer. The first problem may be addressed using deconvoluted algorithms, which can use a priori information about the sky's brightness to interpolate missing values of the coherence function. Examples of such algorithms are CLEAN (Hogbom 1974), the Maximum Entropy Method (Narayon and Nityananda 1986), and the Gerchberg-Saxton-Papoulis algorithm (Papoulis 1975).

The second problem is of varying importance in different applications. A good rule of thumb is that for wavelengths shorter than about 30 cm (including IR and optical), imaging at better than arcses resolution requires some countermeasures to the neutral atmosphere (Woolf 1982). In other regimes, countermeasures are necessary for high-quality imaging. The geometric uncertainties are worst for long baselines (note the similarity to the problem of figuring a single aperture). Most of the effective techniques are related to the concept of closure phase introduced by Jennison about 30 years ago (Jennison 1958). Since calibration errors are predominantly associated with the interferometer elements, rather than pair of elements, a sum of the observed coherence phase around any closed loop of interferometers will be invariant to those errors. To clarify this, note that the coherence phase measured between elements i and j, $\theta_{i,j}$, is related to the true coherence phase, $\hat{\theta}_{i,j}$:

$$\theta_{i,j} = \hat{\theta}_{i,j} + \phi_i - \phi_j \tag{3}$$

where ϕ_i is the phase error associated with the i th element, and I have ignored additive noise. Jennison's sum of the phase around a loop, the "closure phase" Φ_{ijk}, is defined as:

$$\Phi_{ijk} = \theta_{i,j} + \theta_{j,k} + \theta_{k,i} \tag{4}$$

The true closure phase follows a similar definition:

$$\hat{\Phi}_{ijk} = \hat{\theta}_{i,j} + \hat{\phi}_{j,k} + \hat{\phi}_{k,i} \tag{5}$$

Hence we have that the true and observed closure phrases must be equal, no matter what values may be taken on by the phase errors ø.

$$\Phi_{ijk} = \hat{\Phi}_{ijk} \tag{6}$$

A similar observable can be derived for the coherence amplitudes (Smith 1952; Twiss et al. 1960). High-resolution imaging, therefore, uses these closure quantities rather than the observed coherences as constraints on the final image. As a result, high-quality imaging of complex objects is possible even in the presence of severe phase errors due to the atmosphere (Pearson and Readhead 1984) or in the case where the interferometer geometry is not accurately known (Schwab and Cotton 1983). While these techniques were first developed in the radio regime, they have recently been demonstrated in high resolution optical imaging (Haniff et al. 1987). Indeed, while the details of imaging vary with wavelength, the general principles remain the same, so much so that one software package will suffice for most interferometric imaging.

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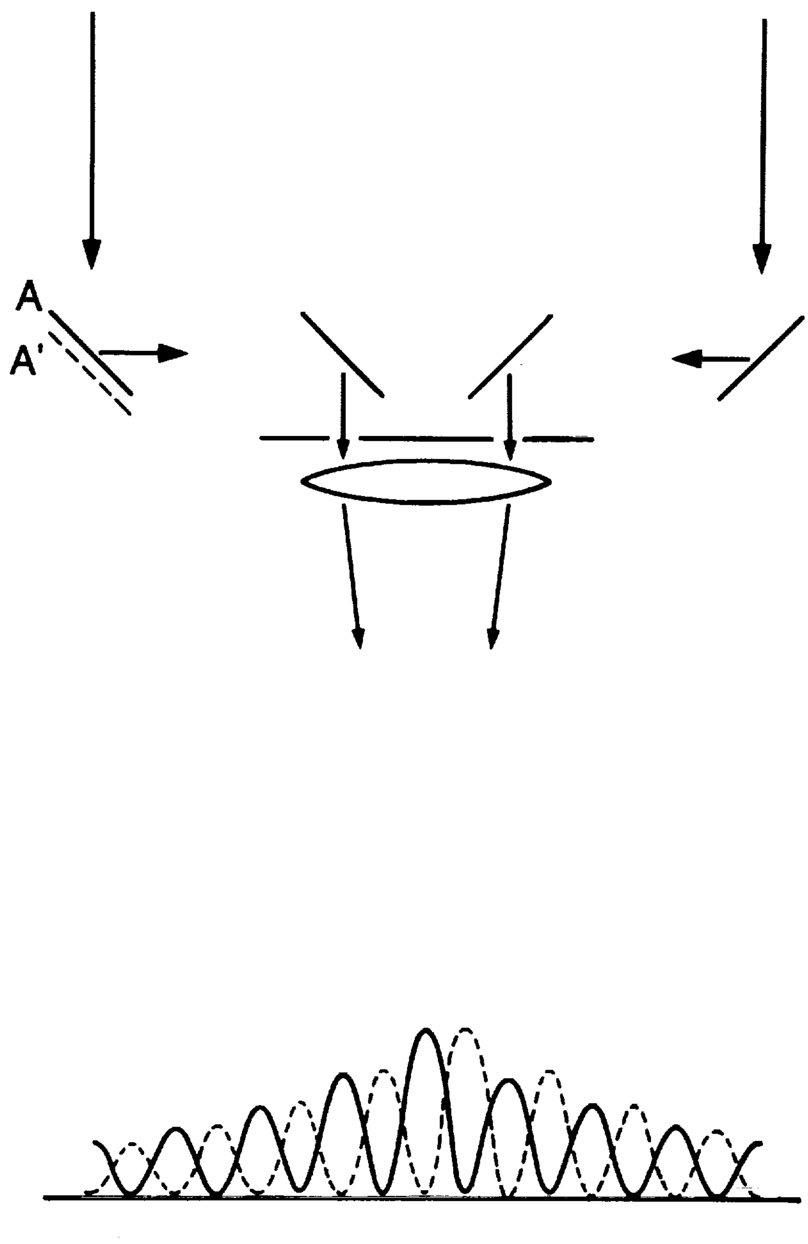

Figure 1: Schematic of a Michelson interferometer. The light reflected from the two outer mirrors produces interference fringes at the focus. The contrast and position of the fringes yield information about the source structure. The fringe position is best measured at optical wavelengths by modulating the position of one mirror (A to A').

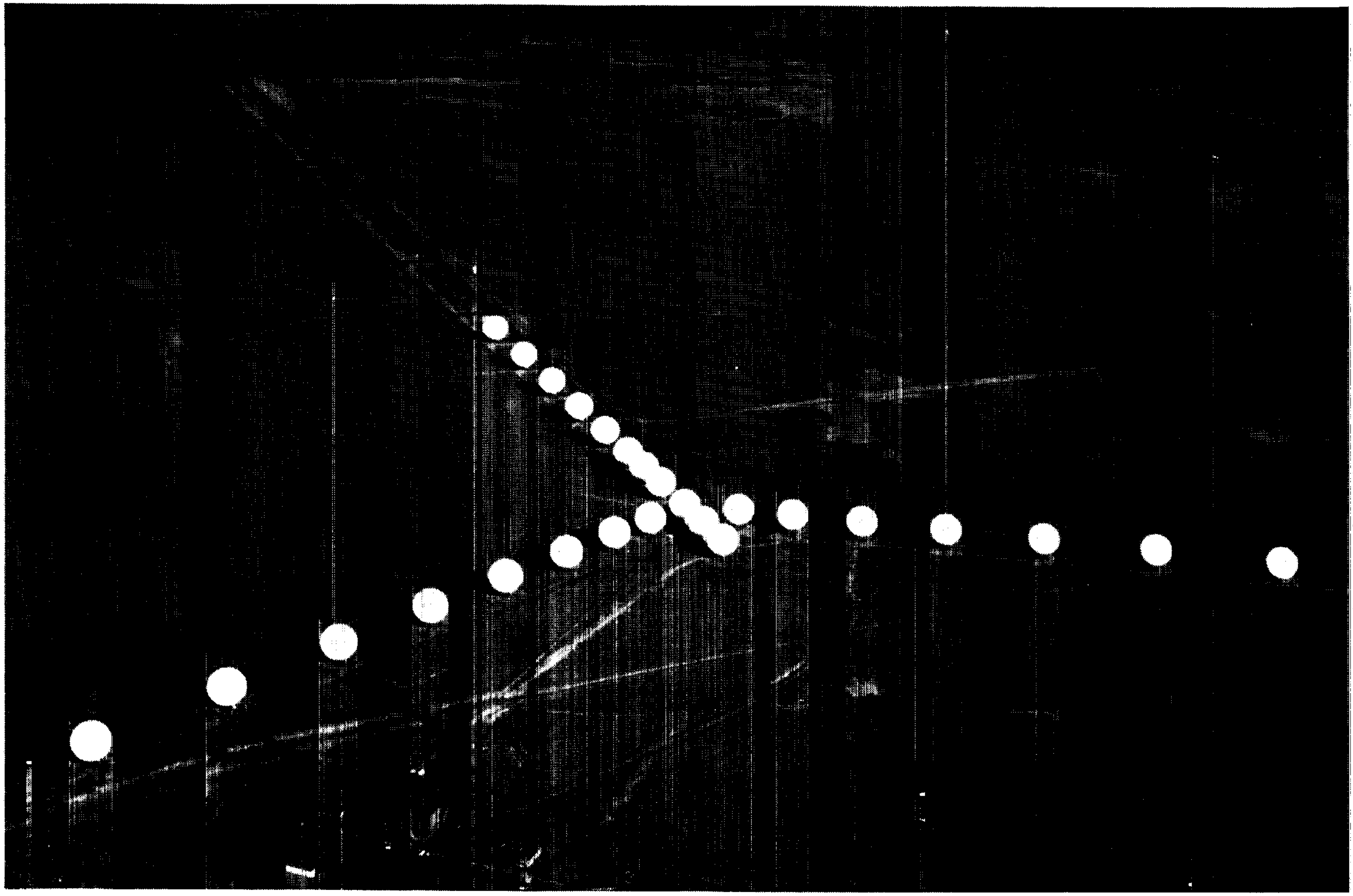

Figure 2: The National Radio Astronomy Observatory Very Large Array near Socorro, New Mexico. The VLA consists of 27 antennas located on a Y-shape which is reconfigurable to give baselines ranging from 40 m to 35 km. Observing wavelengths are 4 m, 90 cm, 21-18 cm, 6 cm, 3.6 cm, 2 cm and to 1.3 cm.

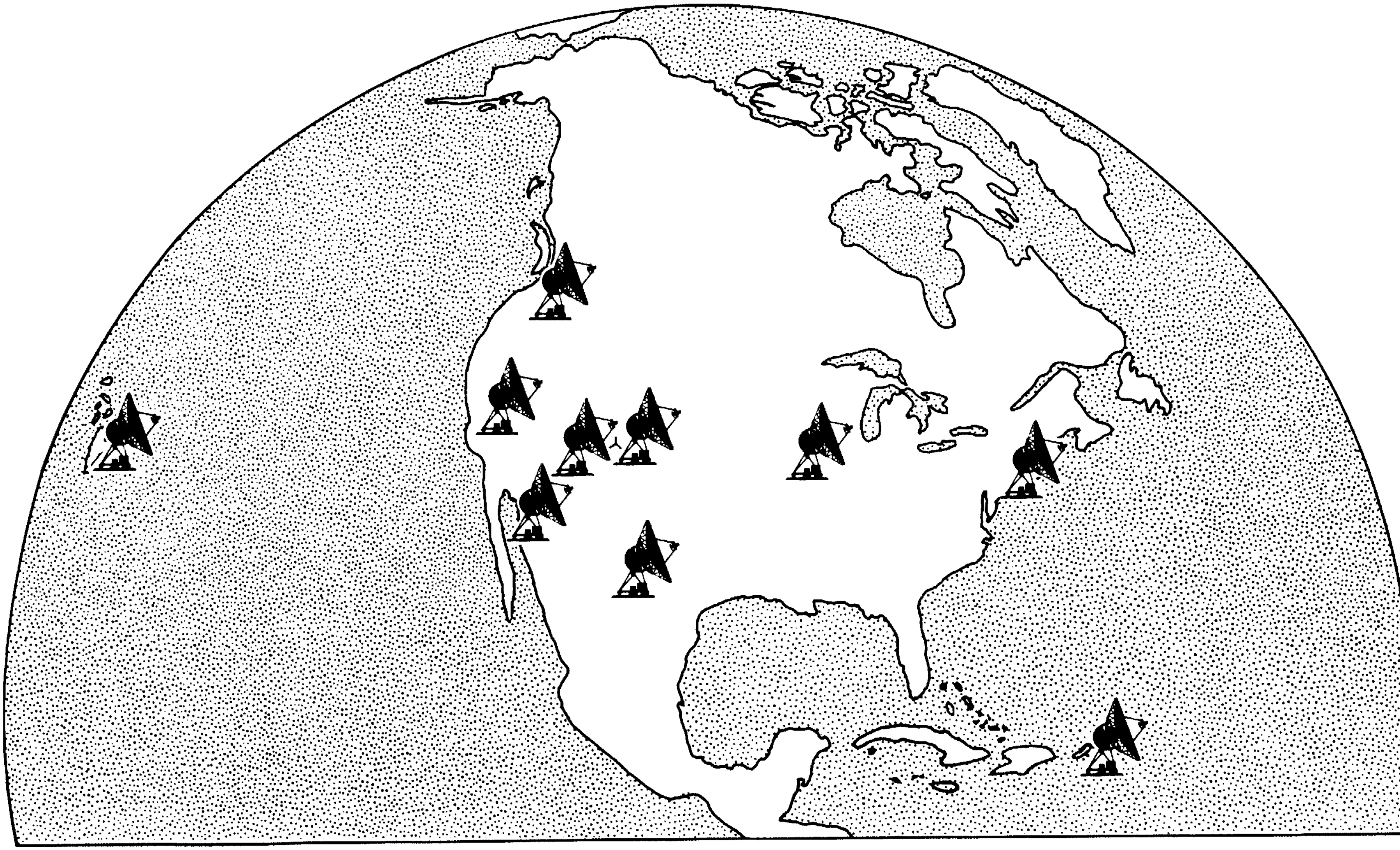

Figure 3: The NRAO Very Long Baseline Array will provide a dedicated array for very high-resolution imaging at radio wavelengths. The maximum baseline is fixed at about 8000 km.

PART II

GROUND-BASED OPTICAL AND INFRARED INTERFEROMETERS

The papers in this section describe the impressive advances made in recent years in ground-based optical and near-IR interferometers. The technologies described in these papers are keys to the successful establishment of a synthetic aperture telescope on the Moon.

H.A. McAlister begins by describing the CHARA Optical Array constructed and operated by Georgia State University. K.J. Johnston and colleagues then discuss the technical status and recent astrometric measurements from the Mount Wilson Optical Interferometer run by NRL. A. Labeyrie presents a discussion of the Optical Very Large Array currently under development in Europe and its possible extension to a lunar-based interferometer. S.R. Kulkarni next describes high-resolution imaging at Mt. Palomar using Non-Redundant Masking and Weigelt's Fully Filled Aperture methods. An infrared (9-12 microns) spatial interferometer using Earth rotation aperture synthesis techniques developed at Berkeley is described by W.C. Danchi and colleagues. The final paper in this section, by S. Prasad, discusses the shot-noise limits to sensitivity of optical interferometry, an important topic debated extensively at the workshop.