N93-

STABILITY OF GENERALLY STIFFENED ANISOTROPIC NONCIRCULAR CYLINDERS

BY

NAHIL ATEF SOBH
OLD DOMINION UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
NORFOLK, VA 23508–0247
sobh@mem.odu.edu

Continuous filament grid-stiffened structure [1] is a stiffening concept that combines structural efficiency and damage tolerance. However, finite element design of such structures against buckling is expensive due to the complexities of the structure. An analytical model of such a structure is developed using a penalty method (artificial springs) with a First Order Shear Deformation theory. The buckling analysis under combined loading is done using Energy method with a penalty/Rayleigh-Ritz technique. The penalty/Rayleigh-Ritz approach is computationally less demanding when compared to the finite element solution and mesh generation.

Apart from the published research works on buckling of stiffened plates and shells by finite element and finite strips, research works on buckling of stiffened plates and shells utilize three different approaches, smeared, column, and discrete approaches. The discrete approach [2] considers the discrete effects of the stiffeners in the buckling behavior by modeling stiffeners as line of bending (EI) and torsion (GJ) stiffnesses on panel skin. Some local deformations are lost when stiffeners are modeled as (EI) and (GJ) stiffeners. This approach becomes difficult in the case of plate stiffened in more than two directions. Most of the work done using the discrete approach involved the Classical Plate Theory (CLPT) rather than the FSDT. In the remaining part of this abstract we report on our formulation of a discrete approach coupled with a penalty formulation and FSDT.

The displacement field for a cylindrical shell according to First Order Shear Deformation theory (FSDT, also called Reissner-Mindlin Plate Theory) are

$$u = u_0 + z\phi_x, \quad v = v_0 + z\phi_y, \quad w = w_0(x,y)$$

where u_0 is the membrane axial displacement. v_0 is the membrane circumferential displacement. w is the out of plane displacement. ϕ_x, ϕ_y are the cross-sectional rotations.

The critical loading is determined on the basis of the principle that during buckling the elastic strain energy stored in the structure is equal to the work done by the applied loading. The internal strain energy, the external work done and the total potential energies are expressed as

$$U_e = \frac{1}{2}\int_A (N_x\epsilon_x^o + N_y\epsilon_y^0 + N_{xy}\gamma_{xy}^0 + M_x\kappa_x + M_y\kappa_y + M_{xy}\kappa_{xy} + Q_x\gamma_{xz} + Q_y\gamma_{yz})\,dA$$

$$W_d = \frac{1}{2}\int_A \left(\bar{N}_x\frac{\partial w}{\partial x}\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\frac{\partial w}{\partial y} + 2\bar{N}_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)dA$$

$$\Pi = U_e + W_d$$

The following approximate representation of the displacement field are used.

$$\{\vec{U}\} = \begin{Bmatrix} u \\ v \\ w \\ \phi_x \\ \phi_y \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos\frac{m\pi x}{L}\sin\frac{n\pi y}{S} \\ V_{mn}\sin\frac{m\pi x}{L}\cos\frac{n\pi y}{S} \\ W_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{S} \\ (\Phi_x)_{mn}\cos\frac{m\pi x}{L}\sin\frac{n\pi y}{S} \\ (\Phi_y)_{mn}\sin\frac{m\pi x}{L}\cos\frac{n\pi y}{S} \end{Bmatrix}$$

The approximating functions satisfy simply supported boundary conditions. General boundary conditions are realized by introducing a low order global finite element shape functions.

The discrete form of the internal elastic energy and external work done are expressed as

$$U_e = \frac{1}{2}\left(\frac{LS}{4}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \mathbf{U}_\mathbf{A}^\mathrm{T}\mathbf{K}_{mn}\mathbf{U}_\mathbf{A}$$

$$W_d = \frac{1}{2}\left(\frac{LS}{4}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\bar{N}_x\left(\frac{m\pi}{L}\right)^2 + \bar{N}_y\left(\frac{n\pi}{S}\right)^2\right]\mathbf{U}_\mathbf{A}^\mathrm{T}\mathbf{\Delta}_{(3,3)}\mathbf{U}_\mathbf{A}$$

$$+ \quad \mathbf{U}_\mathbf{A}^\mathrm{T}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\bar{N}_{xy}\left(\frac{LS}{4}\right)I_{mnpq}\mathbf{U}_\mathbf{A}^\mathrm{T}\mathbf{\Delta}_{(3,3)}\mathbf{U}_\mathbf{B}$$

$$I_{mnpq} = \begin{cases} \frac{32}{LS}\frac{mnpq}{(p^2-m^2)(q^2-n^2)} & m \pm p, n \pm p \ \ odd \\ 0 & otherwise \end{cases}$$

We verified our curved panel code by analyzing the stability of an anisotropic cylindrical panel. The cylindrical panel is not stiffened. The panel is 30.0 in, long and 28.82 in, wide in the circumferential direction with a radius of 40.0 in. This panel was sized by VICON to carry 1000 labs/in axial load. The analysis of this panel using our code gave a buckling load of 900.12 labs/in while the stability analysis by VICON predicted a 984 lbs/in for the buckling load. Our conservative estimate stems for the fact that we have included shear deformation in out energy formulation above.

We introduce the penalty formulation to connect generally oriented stiffeners to the skin of a given fuselage. The internal elastic energy of each stiffener is added to the internal elastic energy of the skin of the fuselage. The skin and stiffeners compatibility is enforced by using stiff springs (penalty functions). The total potential energy of a generally stiffened structure can be written symbolically as

$$\Pi = \sum \Pi_i + \sum \alpha_i \mathbf{f}_i$$

where Π, Π_i, α_i, and $\mathbf{f}_i$ are the total potential energy, potential energy of each structure, penalty functions (artificial springs), and functional constraints, respectively. The total potential energy is then minimized for a specific value of the α_i's. The approximate stability load is obtained by minimizing the total potential energy after choosing a suitable functional expansion of each component of the displacement field.

REFERENCES

1. Rouse M., Ambur D. R., "*Damage Tolerance of a Geodesically Stiffened Advanced Composite Structural Concept for Aircraft Structural Applications*," Presented at the 9th DoD/NASA/FAA Conference on Fibrous Composite in Structural Design, Lake Tahoe, Nevada, November 4–7, 1991

2. Wang J. T. S., Hsu T. M., "*Discrete Analysis of Stiffened Composite Cylindrical Shells*," AIAA Journal, Vol 23, No 11, Nov 1985