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OPTIMAL TRAJECTORIES FOR ORBITAL TRANSFERS USING LOW AND MEDIUM THRUST PROPULSION SYSTEMS

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INTRODUCTION

For many problems it is reasonable to expect that the minimum time solution is also the minimum fuel solution. However, if one allows the propulsion system to be turned off and back on, it is clear that these two solutions may differ. In general, high thrust transfers resemble the well-known impulsive transfers where the burn arcs are of very short duration. The low and medium thrust transfers differ in that their thrust acceleration levels (10^{-4} earth g's for low thrust and 10^{-2} earth g's for medium thrust accelerations) yield longer burn arcs which will require more revolutions, thus making the low thrust transfer computational intensive. Here, we consider optimal low and medium thrust orbital transfers.

APPROACH

The formulation of the problem follows that of Horsewood et. al. (1), where the state of the spacecraft is given in terms of the slowly varying equinoctial elements, which are expressed in terms of the classical elements a, e, i, ω, Ω as

$$\mathbf{z} = (a, e\sin(\omega+\Omega), e\cos(\omega+\Omega), \tan(i/2)\sin\Omega, \tan(i/2)\cos\Omega)^T .$$

The spacecraft mass, m, is also a state variable. In addition, the position of the spacecraft within an orbit is given by the eccentric longitude, $F = E + \omega + \Omega$.

The equations of motion are

$$\dot{\mathbf{z}} = \frac{2P}{mc} M\hat{\mathbf{u}}, \dot{m} = -\frac{2P}{c^2} \qquad [1]$$

where $M = \frac{\partial \mathbf{z}}{\partial \mathbf{r}}$ is a 5×3 matrix calculated by treating F as an independent parameter, such that the variation of F with respect to the other state variables is zero; P is the power due to the thrusters, c is the exhaust velocity and $\hat{\mathbf{u}}$ is the unit vector in the direction of thrust.

It follows from the well-known maximum principle that the fuel optimal trajectory from a given state to some desired final state is found by thrusting at every point along the trajectory in the direction which maximizes the Hamiltonian function H. H can be written in terms of the state variables and their corresponding costate (adjoint) variables $\lambda_{\mathbf{z}}$ and λ_m as

$$H = \lambda_{\mathbf{z}}^T \dot{\mathbf{z}} + \lambda_m \dot{m}$$
$$H = \frac{2P}{mc}\left(\lambda_{\mathbf{z}}^T M\hat{\mathbf{u}} - \frac{m\lambda_m}{c}\right) \qquad [2]$$

where the costate variables satisfy a first order linear system of ordinary differential equations. Now, to maximize H we need only thrust in the direction given by $\hat{\mathbf{u}} = \frac{M^T\lambda_z}{|M^T\lambda_z|}$. Note that if the quantity in parenthesis in [2], call it σ, is negative, then H is negative and hence H is maximized by letting P = 0, which amounts to turning the propulsion system off , i.e. "coasting"; on the other hand, if σ is positive, then H is maximized by letting P take on its maximum value, i.e. "thrusting".

Because of the many orbit revolutions of a long duration transfer, the intensive computations can be reduced by an averaging technique. We can compute an "averaged" Hamiltonian function by holding the state and costate variables constant over an orbital period of duration τ, i.e. we assume Keplerian motion, and integrating the actual Hamiltonian function as follows:

$$\begin{aligned}\tilde{H} &= \frac{1}{\tau}\int_0^{\tau} H\left(\bar{\mathbf{z}}, \bar{\lambda}_{\mathbf{z}}, \bar{m}, \bar{\lambda}_m, F\right) dt \\ &= \int_0^{2\pi} H\left(\bar{\mathbf{z}}, \bar{\lambda}_{\mathbf{z}}, \bar{m}, \bar{\lambda}_m, F\right) s(\bar{\mathbf{z}}, F) dF\end{aligned}$$

where $s(\bar{\mathbf{z}}, F) = \frac{1}{\tau}\frac{dt}{dF}$. The "averaged" equations of motion can now be computed using this "averaged" Hamiltonian, and since the Hamiltonian and its derivatives are zero during coast phases, we need only integrate these equations over the predetermined thrust intervals.

NUMERICAL APPROACH

A FORTRAN program MINFUEL has been written to compute this minimum-fuel solution. The basic algorithm is:

(1) start iteration of the unknown initial costate values by making a first guess—for the simplified problem of a circle-to-circle coplanar transfer, an initial costate guess can be found by using the fact that it is fuel-optimal to thrust in the direction of motion, or tangential direction. This known direction can be used to help guess the initial costate values since $\hat{\mathbf{u}} = \frac{M^T\lambda_z}{|M^T\lambda_z|}$. Given any state, this optimal thrust direction can be calculated in terms of the costate values by running the program XFORM to compute the transpose of the matrix M and the velocity vector.

(2) compute a trajectory corresponding to an initial costate guess—one calls the Runge-Kutta routine to integrate the averaged equations of motion; the

switch function can be expressed in terms of the fast variable F around an orbit and the zeroes of the switch function $\sigma = \left[\lambda_z^T M M^T \lambda_z\right]^{\frac{1}{2}} - \frac{m\lambda_m}{c}$ are found, which defines the thrusting subintervals of $[0, 2\pi]$ for integration. Again for the circle-to-circle transfer we know that initially the switch function should be positive.

(3) compare the computed final state with the specified final state and the proper transversality conditions, such as $\lambda_{m_f} = 1$. The Newton method used for this iteration is very sensitive to the initial costate guess.

The medium thrust problem was investigated through the use of the Flight Mechanics Branch's program SCOOT (Simplex Computation of Optimal Orbital Transfers). SCOOT was known to converge to a quasi-optimum solution for high thrust levels and short burn times. Although low thrust transfers require long burn times, SCOOT was thought to be a viable option for guessing optimal low thrust trajectories by using the final output from the converged higher thrust cases as input to MINFUEL. After many runs of SCOOT, the fact that it calculates only a local optimal and is thus sensitive to coast and burn times guessed, makes it very difficult for comparisons of medium and low thrust transfers.

CONCLUSIONS

The results of SCOOT have indicated that lowering the thrust level "can" increase the amount of final propellant; increasing the ISP can increase the final propellant as well. The output of SCOOT, in particular the optimal thrust directions, from medium thrust level solutions can be used to help calculate an initial guess for low thrust transfers. The program MINFUEL, once properly debugged, should prove very capable of handling minimum-fuel low thrust transfers.

REFERENCES

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