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A Comparison of Refined Models for Flexible Subassemblies

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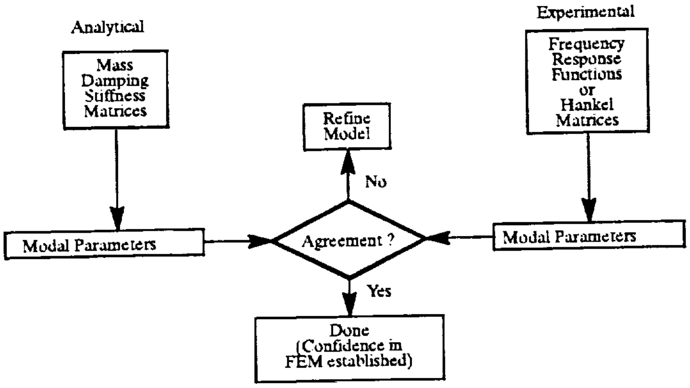
MODEL REFINEMENT – OVERVIEW



- **Model Refinement Necessary to Validate Analytical Models**
- **Comparison of Analytical and Experimental Eigensolution Used to Assess Accuracy of Models**
- **Several Approaches Have Been Developed in Recent Years, but Performance Comparisons Have Not Been Available**

Interactions between structure response and control of large flexible space systems have challenged current modeling techniques and have prompted development of new techniques for model improvement. Due to the geometric complexity of envisioned large flexible space structures, finite element models (FEMs) will be used to predict the dynamic characteristics of structural components. It is widely accepted that these models must be experimentally "validated" before their acceptance as the basis for final design analysis. However, predictions of modal properties (natural frequencies, mode shapes, and damping ratios) are often in error when compared to those obtained from Experimental Modal Analysis (EMA). Recent research efforts have resulted in the development of algorithmic approaches for model improvement [1], also referred to as system or structure identification.

MODEL REFINEMENT – METHODS

- **Optimal Update Approach**
  - **Methods are formulated as a constrained optimization problem**
  - **FEM property matrices (K,D, or M) are updated**

- **Eigenstructure Assignment Approach**
  - **Methods are formulated using pseudo–output feedback eigenstructure assignment**
  - **FEM property matrices (K,D, or M) are updated**

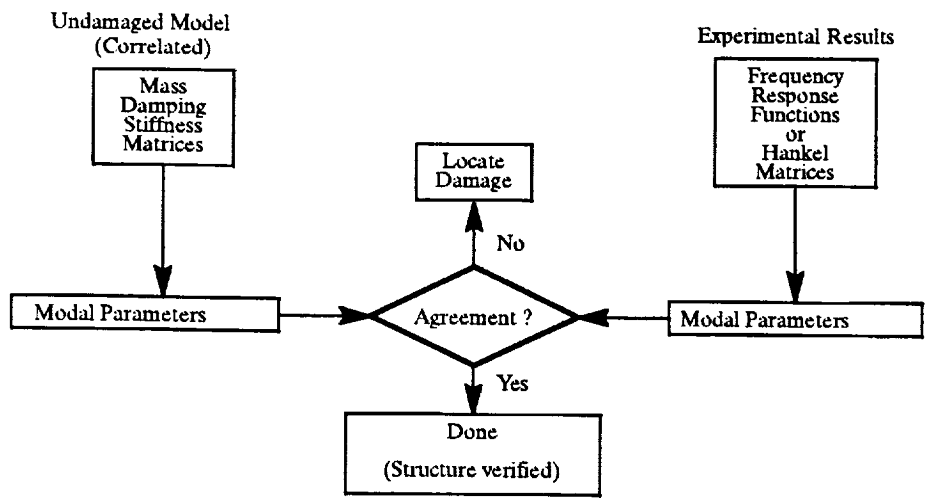
- **Model Sensitivity Approach**
  - **Methods are formulated using design sensitivities**
  - **Physical property parameters (areas, lengths, elastic moduli, etc.) are updated**

- **Other Approaches**

Among others, three approaches for linear-system identification are; optimal-update [2–5], eigenstructure assignment [6–9] and design sensitivity [10,11]. Optimal-update identification techniques produce, through the solution of a constrained optimization problem, updated property matrices (i.e., stiffness, damping or mass matrices) to more closely match the experimental modal properties. Eigenstructure assignment techniques for structure identification use a pseudo-output feedback formulation to update the structure property matrices. Design sensitivity techniques use parameter sensitivities from the initial model and use modal properties from the test structure to determine parameter adjustments. The adjusted parameters may represent physical or material properties, like cross-section area or elastic modulus. Comparisons between techniques and between general approaches are not readily available. In this work, two promising system identification approaches are examined and compared through a study of flexible components that are subassemblies of more complex structures.

DAMAGE DETECTION

- **Some Model Refinement Techniques May Prove Useful For Nondestructive Damage Detection**
- **Damage Location is Analogous to Model Refinement with a Localized Error**
- **An Initial Model Correlated to the Undamaged System is Required**



With the approaches that have been developed for model refinement, a similar framework can be used to monitor structural integrity. For damage location, an initial model that has been accepted as an accurate representation of the undamaged structure is necessary. Predictions of dynamic characteristics from this model are compared to modal characteristics that are determined experimentally. Model refinement techniques employ differences between these characteristics to produce adjusted models that are then compared to the initial correlated model to indicate the location of possible damage. Structural damage is likely to occur at discrete locations, whereas modelling errors may be either discrete or "smeared" or both, due to uncertainties in the material properties, assumptions in the modeling process or errors in part fabrication, among others. Therefore, different mathematical formulations may be required for the different situations of model refinement and damage location.

OPTIMAL UPDATE APPROACH

- **For the Update, the Refined Model Must Satisfy A Constrained Optimization Problem**

Formulation 1:

$$\min \| A - A_a \|_F$$

subject to AS=Y, $A=A^t$ and other constraints

Formulation 2:

$$\min \| AS - Y \|_F$$

subject to $A=A^t$ and other constraints

- **Preservation of the Zero/Nonzero Pattern in the Update Reduces the Amount of Data Required**

- **Optimal–Update Techniques are Well–Suited for the Identification of Sparse Truss Models**

A recent work [4] separated techniques encompassed by the "optimal-update" classification into two formulation viewpoints. These are based on the cost function and the constraints of the minimization problem that is established to produce the update. The first view was used by Baruch and Bar Itzhack [2] and by Smith and Beattie [3] in their methods for stiffness matrix adjustment. Generally in this view, the cost function is formulated to minimize the distance from the initial model. Additional constraints preserve symmetry and represent the imposition of the measured modal data, among others. Here, A is the nxn adjusted property matrix, A_a is the nxn initial-model property matrix, and S and Y are the nxp matrices that define the constraints with the measured data. The matrix Frobenius norm is used for the distance measure. The second view for framing the optimal-update problem follows a slightly different formulation allowing for the probability of inconsistent data, modal data which cannot be matched exactly, due to noise and errors. Generally in techniques from this viewpoint, the cost function minimizes the residual between the updated property matrix and the measured data. Then additional constraints are imposed as well. Techniques in both viewpoints have been developed to preserve the zero/nonzero pattern of the property matrix which reduces the number of measured modes that are needed. These methods are especially suited for truss structures, which have considerable sparsity in the FEM mass and stiffness matrices.

EIGENSTRUCTURE ASSIGNMENT APPROACH

- **For Stiffness Update, the Refined Model Must Satisfy The Eigenproblem for Each Measured Mode**

  $$MV_d\Lambda_d + KV_d = \Delta K_d V_d$$

  $$\Lambda_d = \mathrm{diag}\left(\lambda_{d_1}^2, \lambda_{d_2}^2, \ldots, \lambda_{d_p}^2\right) \quad \textbf{(Measured Eigenvalues)}$$

  $$V_d = \left[\underline{v}_{d_1}, \underline{v}_{d_2}, \ldots, \underline{v}_{d_p}\right] \quad \textbf{(Measured Eigenvectors)}$$

- **Translate Perturbation Matrix into Pseudo−Output Feedback**

  $$MV_d\Lambda_d + KV_d = \Delta K_d V_d = (B_o G)V_d \quad G = FC = HB_o^T$$

  where $B_o = MV_d\Lambda_d + KV_d$**, the Control Influence Matrix Required for Perfect Eigenstructure Assignment (**B_o **also provides information concerning DOF damage)**

- **The Perturbation Matrix**

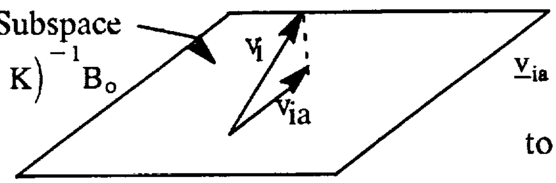
  $$\Delta K_d = B_o\left(B_o^T V_d\right)^{-1} B_o^T$$

- **Results in a Rank p Update of Model**

Several Eigenstructure Assignment based approaches have recently been investigated. The simplest, in both equation and computational complexity; involves the problem of an undamped multi-degree-of-freedom (MDOF) system in which the mass matrix is assumed to be correct and it is desired to determine a symmetric stiffness adjustment such that the updated model matches the p measured eigenvalues/vectors [8]. The technique uses the mathematical framework of a pseudo-output feedback eigenstructure assignment where the pseudo-outputs are the structural positions. The control influence matrix B_o is chosen such that perfect eigenvector assignment is achieved [9]. In Ref. 8, it is proven that: (i) the update is symmetric if the assigned eigenvectors are mass orthogonal and (ii) that if the exact perturbation matrix (which is essentially what model refinement procedures are attempting to estimate) is a rank p matrix, the calculated perturbation is the exact matrix if p correct (i.e. no measurement errors) eigenvalues/vectors are measured. This makes this technique especially well-suited for discrete model errors/damage. Techniques to incorporate damping and mass changes have also been developed.

EIGENVECTOR EXPANSION

- **Optimal Least Squares Technique Involves Both Expansion and Projection Into Achievable Subspace**



$$L_i = \left(M\lambda_i^2 + D\lambda_i + K\right)^{-1} B_o$$

$$\underline{v}_{ia} = L_i\left[\tilde{L}_i^*\tilde{L}_i\right]^{-1}\tilde{L}_i^*\underline{v}_i$$

to measured DOF's

- **Orthogonal Procrustes Expansion**

$$v = \begin{bmatrix} u_e \\ d_e \end{bmatrix} = \begin{bmatrix} u_a \\ d_a \end{bmatrix} P_{op}$$

u – measured DOF's eigenvectors
d – unmeasured DOF's eigenvectors
$(\ )_e$ – experimental
$(\ )_a$ – analytical(FEM)

- **Optimal Rotation of Analytical Into Experimental**

$$\min_{\substack{\text{wrt} \\ P_{op}}} \| u_e - u_a P_{op} \|_F \qquad \text{subject to } P_{op}^T P_{op} = I_p$$

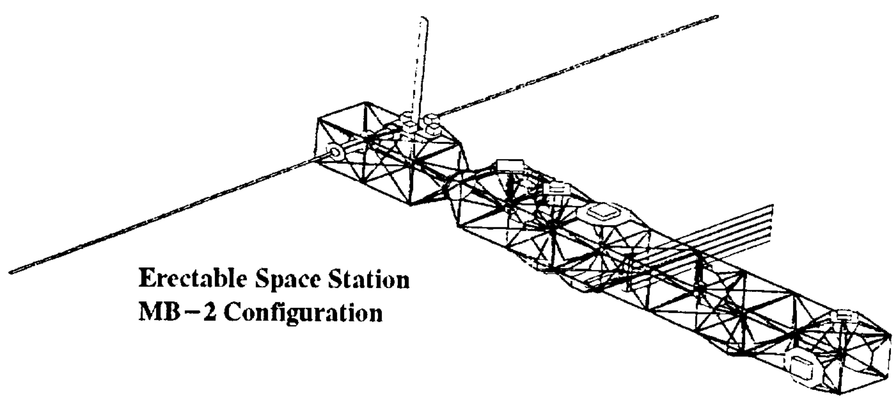
- **Two Possible Expansions**

$$v = \begin{bmatrix} u_a \\ d_a \end{bmatrix} P_{op} \qquad v = \begin{bmatrix} u_e \\ d_a P_{op} \end{bmatrix}$$

Common to all model refinement algorithms, the dimension of the experimentally measured eigenvectors is usually much less than that of the FEM eigenvectors due to practical EMA testing limitations. One solution to this problem is to employ a model reduction technique so that the reduced dimension and DOF's of the analytical model match that of the experimentally measured eigenvector. An alternative approach, which is employed in this work, is to expand the measured eigenvector to the size of the analytical eigenvector [1]. An examination of the eigenvalue problem reveals that the expanded eigenvector must lie in the space spanned by the columns of L_i, which depends both on the original FEM, the measured eigenvalue, and an arbitrary matrix B_o. Two techniques have been investigated: one involves the expansion and projection of the eigenvectors into the achievable subspace. An alternate approach uses the mathematical framework of the classic Orthogonal Procrustes Problem to rotate a portion of the analytical modal matrix into the experimental modal matrix. It is then assumed that this rotation matrix can be used to rotate the unmeasured components to provide an estimate of the complete eigenvector [12].

CASE STUDIES

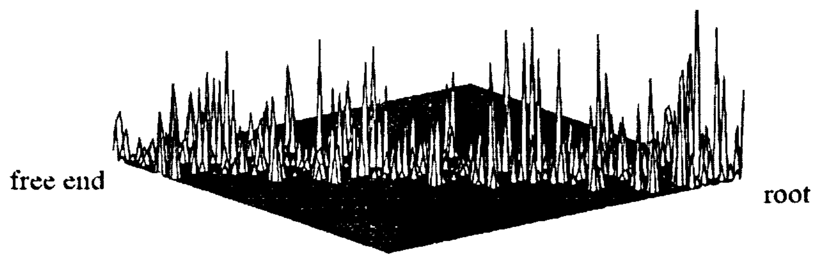
- **8-Bay Laboratory Truss Structure**
  - **a subassembly of the Dynamic Scale Model Technology (DSMT) program at NASA Langley Research Center**
  - **96 dof model; 6 measured frequencies, 96 measured dofs**
- **CASE I**
  - **Model refinement for the undamaged truss**
- **CASE II**
  - **Damage location of a missing member**



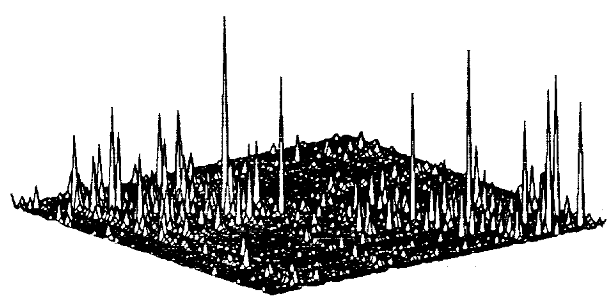
The 8-bay hybrid-scaled truss structure used for this investigation is part of a series of structures designed for research in dynamic scale model ground testing of large space structures. This truss, with the same number of bays as the primary structure in the erectable Space Station MB-2 configuration, is a focus structure in an ongoing effort to examine damage detection [13]. For testing, the truss was cantilevered and instrumented with 96 accelerometers to measure three translational DOF's at each node. The number of acceleration measurements (at all degrees of freedom of the model) is unusual, but provides an opportunity to select subsets of the measurements in future studies of instrumentation placement schemes. Three simultaneous excitation sources were used. Six frequencies and corresponding mode shapes were extracted using the Polyreference complex exponential technique. Each truss member was modeled as a rod element. Concentrated masses were added at the nodes to represent the nodes and instrumentation. Tests were conducted for an undamaged situation and a damaged situation, with truss element number 35 removed.

CASE I
Undamaged 8–Bay Truss Structure

• Optimal–Update Approach



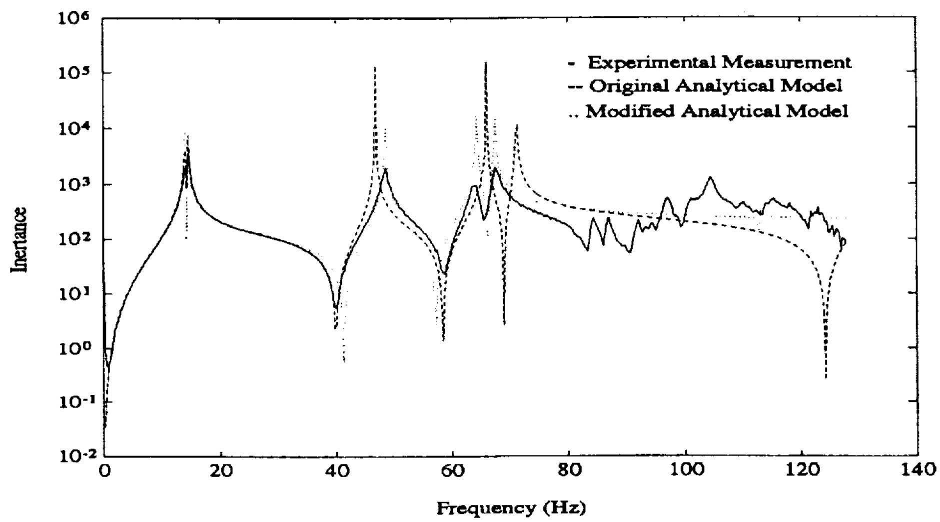
• Eigenstructure Assignment Approach



A mesh plot of the difference matrix ($\Delta K_d = K_{final} - K_{initial}$) provides a visual representation of the stiffness update. Here, the absolute values of the difference matrix are plotted for the two approaches. An iterative, first–viewpoint, optimal–update approach [5] produced an adjusted stiffness model which had the largest changes at the cantilevered end of the truss, but numerous changes at the free end. The sparsity preserving eigenstructure assignment approach used 10 iterations to achieve its refined stiffness model. As can be seen from the mesh plot of the perturbation matrix (ΔK_d), this algorithm clearly focuses the majority of the changes at the cantilever end and the free end. The model refinement is obviously correcting for the imperfect cantilever condition. In addition, it should be noted that test shakers, which were not included in the analytical model, were mounted near the free end.

CASE I
Undamaged 8–Bay Truss Structure

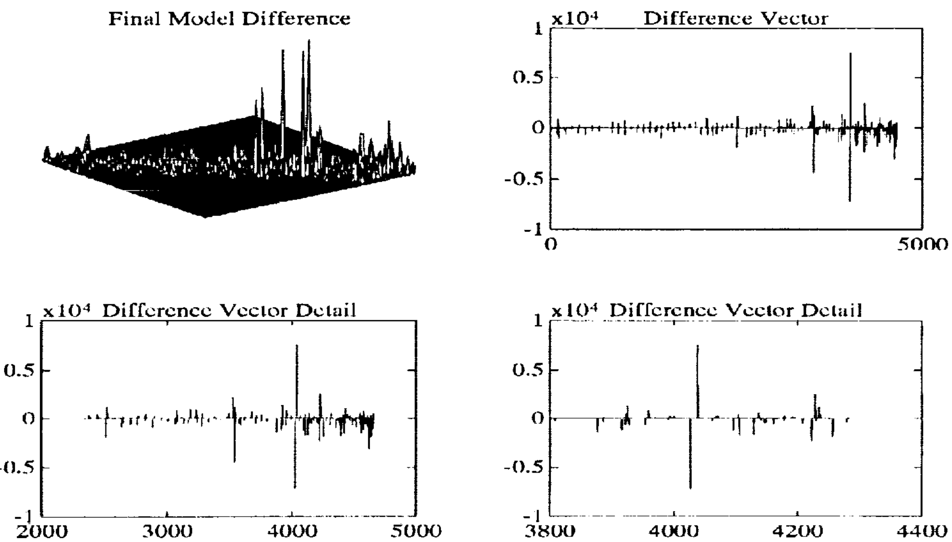
- **Eigenstructure Assignment Approach**



A frequency response function for one driving point of the truss is shown in the figure above. In the figure, the solid line corresponds to the experimental measurement, the dashed line corresponds to the initial FEM, and the dotted line corresponds to the refined FEM from the eigenstructure assignment approach. Note that both FEM's have a zero damping matrix. The importance of including a damping model is seen in Case Study III. This comparison of the results shows some discrepancy between the experimental frf and the updated model version. The measured frequencies used for the update do not correspond exactly to the peaks of this function. Even with that, the updated model is considerably improved.

CASE II
"Case H" Damage for the 8–Bay Truss

- **Optimal–Update Approach**



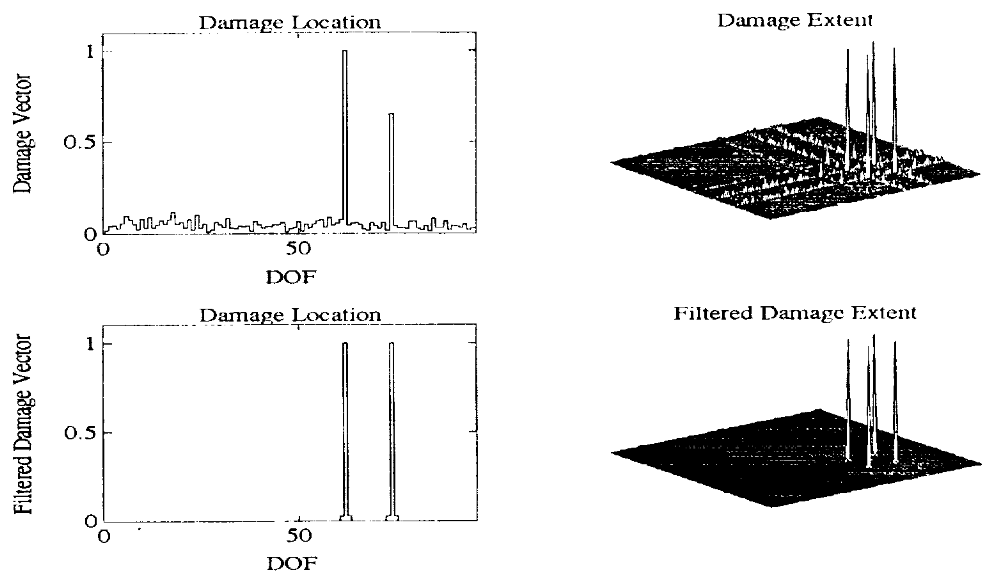
- **Model Difference is ΔK_d ; Difference Vector is Upper Triangle of ΔK_d**
- **Typical Damage Pattern is Evident for Element from dof 62 to dof 74 – Member 35**

"Case H" damage of the 8-bay truss is the removal of longeron member number 35. The iterative optimal update approach produced an adjusted stiffness model for the truss using the six measured frequencies and mode shapes. The mesh plot of the difference between the refined model and the model representing the undamaged truss indicates the location of the damaged member. The maximum difference occurs for the matrix off-diagonal elements (62,74) and (74,62), indicating the truss member that connects these two DOF's – member number 35. A vector that stores the upper triangle of the difference matrix, row by row, is plotted to examine the magnitude of the damage. Here the maximum difference is of the same order as the stiffness of the removed member, indicating the considerable loss of stiffness in this case. Detail plots show a typical damage pattern for a damaged longeron or batten. At the root and free ends, the relatively small effects represent the update for the undamaged situation, which was not incorporated prior to this case study.

CASE II

"Case H" Damage for the 8−Bay Truss

- **Eigenstructure Assignment Approach**



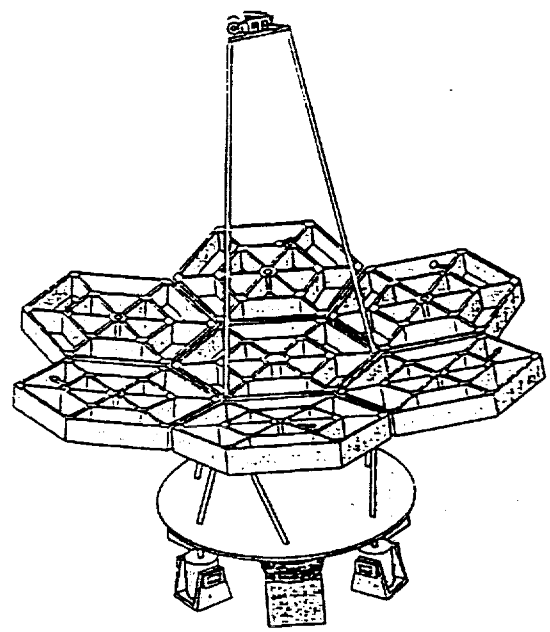
- **Upper Left is B_0 the Damage Vector; Upper Right is ΔK_d**

- **Filtering of Experimental Measurements Making Use of the Damage Vector Enhances ΔK_d Estimate**

In the eigenstructure assignment approach for damage detection, damage location and extent can be decoupled. Inspection of the B_o vector (or matrix if more than one eigenvector is measured) gives a direct indication of which degrees of freedom have been most affected by damage. In fact, when B_o is calculated using noise-free measurements (perfect eigenvalue/eigenvector information), degrees of freedom that are not directly influenced by damage will have a corresponding zero element in B_o. From the upper left plot, which has the elements of B_o plotted versus degree of freedom, it is clear that two degrees of freedom (62 and 74) have been most affected by damage. These are exactly the degrees of freedom that were coupled before the truss member was removed. The small numerical elements at all other degrees of freedom can be attributed to experimental measurement errors. Note that the damage location problem is performed independent of the damage extent problem. The upper right figure shows ΔK_d using the B_o of the upper left figure. To improve the damage extent estimate, a filtering algorithm for B_o has been developed which sets to zero those elements of B_o that are below a specified threshold (related to the maximum element of B_o). The results of applying the filtering algorithm are shown in the lower two plots. Details of this decoupled damage location and extent algorithm can be found in Zimmerman and Kaouk [9].

CASE STUDIES

- **Tower Substructure**
  - **a subassembly of the Multi-Hex Prototype Experiment at Harris Corporation**
  - **57 dof model; 8 measured eigenvalues, analytical eigenvectors**
- **CASE III**
  - **Model refinement for the tower structure**

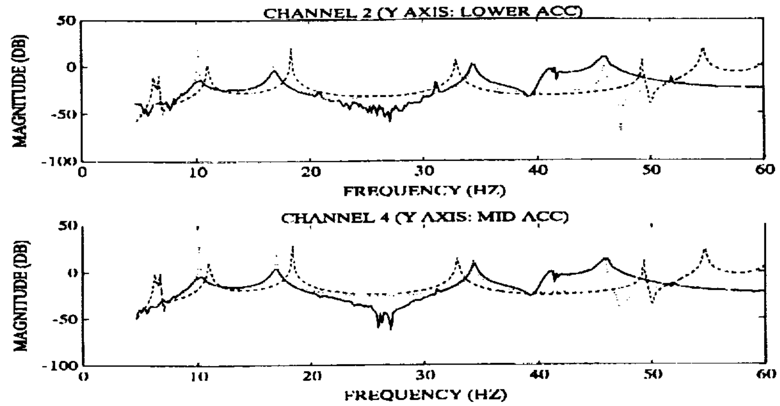


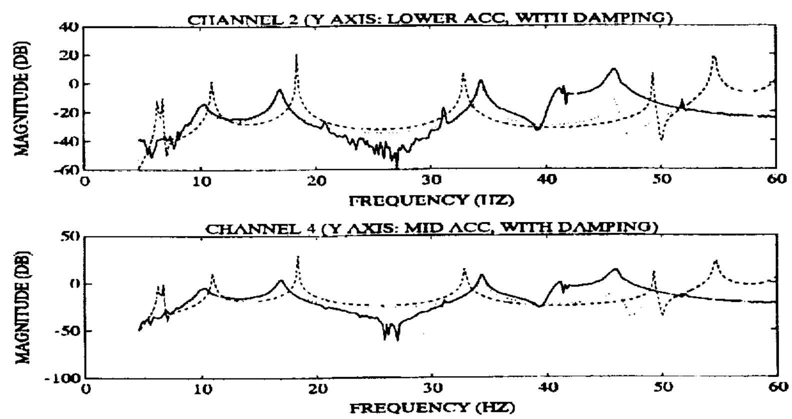
The Harris Multi-Hex Prototype Experiment (MHPE) test structure design incorporates many of the features and technology of the Harris Solar Dynamic Concentrator. The major structural subsystems include the reflector surface (the seven panel array), the secondary tower (tripod) and the base plate/support system. In this particular study, the secondary tower was removed from the reflector surface and hard mounted to ground. Three linear precision actuators (LPACTS) were used as excitation sources in performing the modal survey of the tower. One leg of the tower was instrumented with 6 translational accelerometers. Velocity estimates at the LPACT locations served as 3 additional outputs. The three tower struts were modeled as 12 beam elements (4 per strut). The tower top was modeled as 9 beam elements. Fittings connecting the tower struts to the tower top and to the center panel were included as point masses, as was the stationary part of the LPACT. The moving proof-mass of each tower LPACT was modeled as a point mass attached to the tower top by springs. The Eigensystem Realization Algorithm (ERA) was used to estimate the experimental eigenvalues and eigenvectors. Because of the limited sensor spatial resolution, only the experimental eigenvalues were used. It has been assumed that the original analytical eigenvectors match those that would have been measured.

CASE III

Model Refinement for the Tower Structure

- **Optimal–Update Approach**



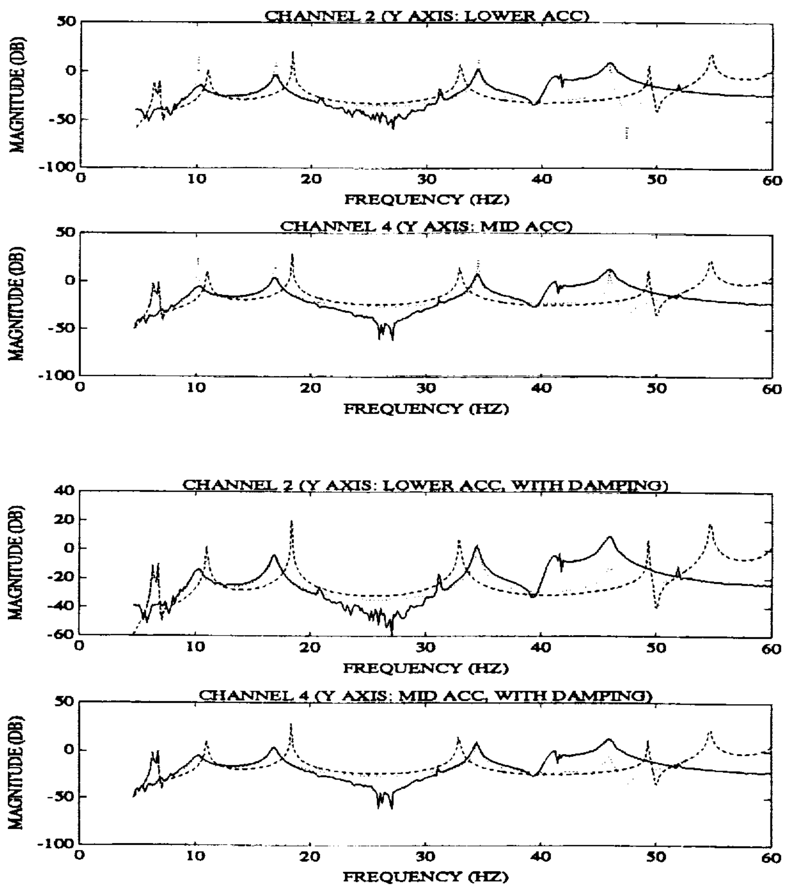


In each figure above (showing results of the optimal–update approach) and for the next slide (showing results of the eigenstructure assignment approach) the solid line corresponds to the experimental measurement, the dashed line corresponds to the initial FEM (provided by Harris), and the dotted line corresponds to the refined FEM. In the top pair of plots, both FEM's have a zero damping matrix; the finite peaks at the resonant frequencies are only due to the frequency spacing at which the frequency response functions have been calculated. It is obvious that there is improvement in frequency matching; however the amplitude mismatch between experiment and refined FEM is of some concern. There are two main causes of this mismatch (i) using the original mode shapes and (ii), not including the effects of damping. It should be noted that for both approaches, the magnitude of the elements of the perturbation stiffness matrix was quite small. In fact, the maximum element was of the order 10, whereas the original stiffness elements are several orders of magnitude higher. This result is not surprising in that it is known from control theory that moving eigenvalues requires less control effort than moving both eigenvalues *and* eigenvectors.

CASE III

Model Refinement for the Tower Structure

• Eigenstructure Assignment Approach



In the second test for both approaches, a "modal" damping matrix was calculated using the measured eigenvalue ($2\zeta\omega_n$) and the original eigenvectors. With the original eigenvectors mass orthogonalized, a physical damping matrix was approximated by the following:

$$C = MU \ \mathrm{diag}(2\zeta_i\omega_{ni}) \ U^T M,$$

where M is the mass matrix, U is the orthonormal eigenvector matrix, and diag ($2\zeta_i\omega_{ni}$) is a diagonal "modal" damping matrix. The effect of introducing this damping matrix in the frequency response calculation is shown in the bottom pair of plots for each approach. Here, the solid line corresponds to the experimental measurement, the dashed line corresponds to the initial FEM, and the dotted line corresponds to the FEM in which both the stiffness matrix has been updated and the experimental damping matrix has been included. In comparing the two figures for both approaches, it is clear that introduction of the damping model greatly enhances the amplitude matching of the resonant peaks, as well as providing better matching throughout the frequency response.

SUMMARY

- **Two Promising Approaches for Model Refinement were Examined and Compared with Data from Real Structures**

- **Optimal–Update Methods are a Viable Approach for Model Improvement and Damage Location**
  - **Refined analytical results agree with experiment**
  - **Techniques are well–suited for sparse models**

- **Output Feedback Approach Produces Excellent Agreement Between Analytical and Experimental Results**
  - **With p measured modes, produces a rank p update**
  - **Algorithm computationally feasible for large FEM's (inverse of a pxp matrix)**

- **Continuing Studies Will Examine Other Structure Types, Sensor Placement, and Model Refinement for Assemblies**

To date, new techniques for model refinement have most often been presented with an application to a simulated example, without a basis for comparison of different methods and approaches. In this work, performance of techniques from two approaches were compared through studies with real data via the updated stiffness matrix results, frequency response of the improved models with respect to experimental measurements, and physical interpretation of the refinements. Optimal-update and eigenstructure assignment approaches both demonstrate their viability for model refinement and damage location. Differences in the approach formulations have been examined. With these results, strengths and weaknesses of approaches and specific techniques are more readily available for CSI applications of model improvement.

Acknowledgements

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