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A Model to Compare Performance of Space and Ground Network Support of Low-Earth Orbiters

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This article compares the downlink performance in a gross average sense between space and ground network support of low-Earth orbiters. The purpose is to assess what the demand for DSN support of future small low-cost missions might be, if data storage for spacecraft becomes reliable enough and small enough to support the storage requirements needed to enable support only a fraction of the time. It is shown that the link advantage of the DSN over space reception in an average sense is enormous for low-Earth orbiters. The much shorter distances needed to communicate with the ground network more than make up for the speedup in data rate needed to compensate for the short contact times with the DSN that low-Earth orbiters have. The result is that more and more requests for DSN-only support of low-Earth orbiters can be expected.

I. Introduction

NASA's Tracking and Data Relay Satellite System (TDRSS) can support low-Earth orbiters, providing continuous coverage when necessary, for example, for piloted missions. Due to the synchronous altitude of TDRSS, the communication path length to a low-Earth orbiter is about equal to the synchronous altitude, or 36,000 km. The communication path from an Earth station such as the DSN to a low-Earth orbiter is much less, resulting in a huge

increase in the required data rate, to partially offset the gain from the shorter communication path.

For example, in a typical case of a 400-km circular orbit at a random inclination, ignoring station masks, the network sees the orbiter 8.8 percent of the time, for a required increase in image data rate of $1/0.088 = 11.36$, a 10.6-dB loss as accounted for in this article. This loss is much more than made up for by the gain due to shorter communica-

with only two rate changes. On average, data volume for 400-km orbits improves an average increased factor of 3.61, or 5.6 dB, as is shown below. The total average improvement for using an Earth station instead of one at a synchronous altitude for a random 400-km circular orbit is then $(23.9 + 5.6 - 10.6)$ dB $= 18.9$ dB.

The 18.9-dB average improvement does not consider that a ground station will typically have a greater aperture than a space relay spacecraft, and can also have a much lower noise temperature than that which could be provided by any space low-noise reception technology forseeable over the next decade. For example, a 26-m ground station such as in the DSN has an advantage in gain of 14.3 dB over a 5-m TDRSS-style space-based antenna, assuming equal antenna efficiencies, yet ignoring any noise temperature differences, which tend to favor ground-based antennas. Also, the space system may not use its highest gain antenna to support simpler missions, but rather its multiple-access capability. The multiple-access capability allows several spacecraft to be supported at once, which may allow the continuous support assumed here, but at a still greater advantage to the ground than the 14.3 dB quoted above.

The effect of frequency selection needs to be considered. Here, roughly the same frequency is assumed for the space network as for the ground network. The space network could ultimately make use of a relay frequency of 60 GHz, or even of an optical frequency. These choices could change some of the considerations. On the other hand, receiver improvements are much easier to make in a ground station than in a space relay station, once launched. In fact, servicing a synchronous spacecraft will be very difficult and is certainly not foreseen for at least 20 years. The rest of the article, therefore, assumes roughly the same frequencies for the space network as for the ground network.

Other factors ignored in this article, which is not intended to substitute for a system engineering trade-off study, are any costs to the using spacecraft of the access protocol to use a multiple-access capability on the space-based relay platform, implementation cost differences between space-based and ground-based reception capabilities and likewise any operations cost differences, the ability to upgrade a ground station during its lifetime, the number of spacecraft practically supportable by a space or ground network capability, the differential costs of onboard data storage by using the ground-based capability, and finally the costs of the using spacecraft communications package to produce the required effective isotropic radiated power (EIRP) in the two cases. The assumption of random circular orbits would also have to be replaced by realistic orbits and mission requirements in a more detailed analysis, which would also include realistic station masks.

Nevertheless, the potential gain of $(18.9 + 14.3)$ dB $= 33.2$ dB or more (a factor of 2100) will be attractive to using spacecraft as a cost, size, and weight reduction, assuming the availability of data storage that can overcome the low visibility from the ground. This will apply then to robotic missions that do not have very high average data acquisition rates. Many missions may be included in the set of potential users in coming years as spacecraft are downsized. Such spacecraft may use the 33.2-dB or more average decrease in required spacecraft EIRP in several ways, or use all these ways at once: lower spacecraft power, a lower gain or even an omnidirectional spacecraft antenna, less or no restrictions on spacecraft attitude, and a general relaxation of the constraints on the spacecraft radio. The mass reduction allows a smaller launch vehicle, which could open up whole new mission opportunities.

There are corresponding uplink and navigation improvements as well, which are not considered in this article. Overall, the result of support of low-Earth orbiters by the ground will be the provision of a communications capability that will result in smaller, cheaper, and lighter spacecraft of the same capability. Thus, it can be expected that in coming years there will be more and more low-Earth orbiters that are designed for support exclusively by the DSN.

The remainder of this article is organized as follows. Section II presents the simple model of random circular orbits that is being considered. Section III calculates the average visibility of a low-Earth orbiter to the DSN, and thus the amount of data storage that must be provided on board the using spacecraft. Section IV finds the average improvement in the data volume received when using the ground network as opposed to the space network. Finally, Section V summarizes the results and computes the average advantage of ground versus space network support for low-Earth orbiters, considering both communication distance and spacecraft visibility.

II. Model of Random Circular Orbits

Here random circular orbits are assumed. Real orbits are only sometimes circular. Moreover, certain inclinations are often preferred, due for example to the requirement of keeping the line of apsides fixed or to provide Sun synchronism. The location of a single tracking station on Earth may as well be assumed fixed, given random orbits. The rotation of the Earth can also then be ignored, since

one is in effect assuming a random phase of the orbiter in its orbit. The DSN has its communication complexes far enough apart so that there is never simultaneous visibility of low-Earth orbiters. Thus, visibilities from a single station, when multiplied by 3, the number of complexes, give the long-term average visibility to the entire DSN with its present complex locations.

The radius of the Earth, r, is taken in calculations as 6400 km. The altitude above Earth of the circular orbit is h; in calculations, h is taken as 400 km. The fraction of time that a spacecraft in random circular orbit at distance h above the Earth is visible to a given ground station on a spherical Earth of radius r is found in the next section.

III. Average Visibility of a Low-Earth Orbiter

Since the spacecraft is in a random circular orbit, random phase and inclination, it is at a given time at a random (uniform) point on a sphere of radius $r + h$ centered at the Earth's center. A given tracking station sees a certain fraction of the surface area of the larger sphere, and that fraction is clearly the average visibility from a single station. The single station may as well be random too, since the average visibility is clearly independent of the station's location. By the same reasoning, the spacecraft may as well be considered fixed. Thus, the visibility of a spacecraft from a single Earth station is just the fraction of the Earth's area seen by a fixed spacecraft.

The fraction of the area on the Earth seen by a spacecraft at height h above the Earth is now easily found (see Fig. 1). A tangent is drawn from the spacecraft to the Earth, and the area of the spherical cap is sought. Let θ be the angle subtended by the spacecraft tangent, so that

$$\theta = \arccos\left(\frac{r}{r + h}\right)$$

Let ϕ be the angle of integration between a point on the cap and the center of the Earth. This angle goes from 0, the angle corresponding to the spacecraft location at zenith, to the edge of Earth visibility at angle θ. The perpendicular radius of the circle at ϕ is $r \sin \phi$, and its circumference is $2\pi r \sin \phi$. The slant length of the differential ring corresponding to angle $d\phi$ around ϕ is $rd\phi$. Thus, the area dA of the differential ring is

$$dA = 2\pi r \sin \phi \cdot rd\phi = 2\pi r^2 \sin \phi d\phi$$

The area of the cap A is then

$$A = \int_{\phi=0}^{\theta} 2\pi r^2 \sin \phi d\phi$$

$$= 2\pi r^2 \left(-\cos \phi\right)_{\phi=0}^{\theta}$$

$$= 2\pi r^2 \left(1 - \cos \theta\right)$$

$$= 2\pi r^2 \left(1 - \frac{r}{r + h}\right)$$

so that

$$A = 2\pi r^2 \left(\frac{h}{r + h}\right)$$

The area of the surface of the Earth is $4\pi r^2$, so the visibility V from a single station is

$$V = 2\pi r^2 \left(\frac{h}{r + h}\right) / 4\pi r^2$$

$$V = \frac{h}{2(r + h)}$$

With $h = 400$ km and $r = 6400$ km, $V = 1/34$. The visibility from the entire network of three complexes is 3 times this, or $3/34 = 8.8$ percent. This is a long-term average. It does not of course mean that the DSN will see 8.8 percent of every or any orbit.

Assuming the orbiter is gathering data even while transmitting, it must transmit $34/3 = 11.33$ times as fast (10.5 dB more link is needed) as a spacecraft that can always be seen and is continually serviced by a synchronous relay satellite system. Such a data rate is supportable even by present technology for satellites whose data acquisition rate is up to several megabits per second. This is because all the data can be dumped if during a pass there is an average data rate of tens of megabits per second, which is compatible with current technology. Future data acquisition digital equipment in the signal processing centers of the DSN is expected to be able to accommodate even higher peak rates.

What about data storage requirements? Maximum storage requirements cannot be found from the averaging techniques used here. Nevertheless, the following discussion is of some interest. The orbital period for a low-Earth

orbiter is about 90 minutes (5400 sec). This means, on average, a pass every 1800 sec or so, with a three-complex network, assuming every pass is supported as in the assumption here. If the spacecraft transmits while also acquiring and recording more data, then 1800 sec, say 2000 sec, of data must be stored. If the storage on board is limited to 10 gigabits, today's technology, a maximum spacecraft data acquisition rate of 5 megabits per second can then be supported. Future terabit onboard storage could support 100 times this data acquisition rate, or 500 megabits/sec downlink. The link, not the storage, would then become the limiting factor. The storage and dump mode is clearly feasible for small low-Earth orbiters from the standpoint of data storage technology.

Now that the average visibility is known, one can find the link improvements, using the method given in the next section.

IV. Average Link Improvement

The average link improvement is readily calculated from the average inverse squared distance to the Earth tracking station from the low-Earth orbiter. Recall that it is being assumed that the distance to a space relay station is essentially constant at 36,000 km. Thus, the decibel improvement due to the distance effect is correctly calculated in terms of data volume received as the product of 36,000 km squared times the average inverse squared distance. This gain must be de-rated by the visibility factor, i.e., the fraction of time that the low-Earth orbiter is visible to the ground tracking network. The resulting product is the average link improvement taking both communication distance and visibility into account.

The calculation to follow assumes that the data rate is varied by an onboard stored sequence during a pass as the spacecraft distance to the Earth station changes. The goal is to make the data rate as large as possible at all times. This is similar to deep space support, where the rate can be varied during a pass, but with more frequent changes during a near-Earth pass. Rate changing results in a maximum to minimum data rate in an overhead 400-km orbit of $(2298/400)^2 = 33.0$ or 15.2 dB. If the minimum data rate is 2 megabits/sec, the maximum rate when the spacecraft is directly overhead would be 66 megabits/sec. So the high maximum–minimum ratio is not an insurmountable problem, except perhaps at the highest average data acquisition rates.

The calculation shall proceed by calculating the average improvement factor F relative to distance squared at spacecraft rise or set:

$F = [(r + h)^2 - r^2] \times$ average inverse squared distance

This is used for ease of correctness testing later. Referring to Fig. 2, it is seen, as in the previous section, that with

$$\theta = \arccos\left(r/(r + h)\right)$$

as before, and normalizing over the area of the cap,

$$F = h(2r + h) \int_{\phi=0}^{\theta} \frac{2\pi r^2 \sin\phi d\phi / \left(2\pi r^2 h/(r + h)\right)}{l^2} \quad (1)$$

Here l is the distance from the spacecraft to what is now thought of as the randomly situated tracking station, found from a vector diagram (Fig. 3) to be

$$l^2 = (r + h)^2 + r^2 - 2r(r + h)\cos\phi \quad (2)$$

$$l^2 = 2r^2 + 2rh + r^2 - 2r(r + h)\cos\phi \quad (3)$$

Substituting Eq. (2) into Eq. (1) and using $\sin\phi d\phi = -d\cos\phi$, Eq. (1) becomes

$$F = (2r + h)(r + h) \int_{\phi=0}^{\theta} \frac{-d\cos\phi}{2r^2 + 2rh + h^2 - 2r(r + h)\cos\phi} \quad (4)$$

Using $\cos\theta = r/(r + h)$, Eq. (4) becomes, with $u = \cos\phi$,

$$F = (2r + h)(r + h) \int_{u=\frac{r}{(r+h)}}^{1} \frac{du}{2r^2 + 2rh + h^2 - 2r(r + h)u} \quad (5)$$

and so

$$F = \frac{(2r + h)(r + h)}{2r^2 + 2rh + h^2}$$

$$\times \int_{u=\frac{r}{(r+h)}}^{1} \frac{du}{1 - 2r(r + h)u/(2r^2 + 2rh + h^2)} \quad (6)$$

Make the substitution

$$v = 2r(r + h)u/\left(2r^2 + 2rh + h^2\right) \qquad (7)$$

so that Eq. (6) becomes

$$(2r + h)(r + h) \quad \left(2r^2 + 2rh + h^2\right)$$

$$\frac{x - \ln(x + 1)}{x^2}$$

whose numerator is obviously and well-known to be positive for $x > 0$, as required.

$$G = \frac{d^2}{h(2r+h)} \left(1 + \frac{h}{2r}\right) \ln \left(1 + \frac{2r}{h}\right) \frac{3h}{2(r+h)} \quad (13)$$

using Eq. (12) and the equation for the visibility V from a single Earth station derived in Section II. The factor of 3 occurs because expected values add together, and no two DSN stations ever see the same low-Earth orbiter at the same time. Equation (13) simplifies to

$$G = \frac{3}{4} \frac{d^2}{r(r+h)} \ln \left(1 + \frac{2r}{h}\right) \quad (14)$$

Here d is synchronous altitude, 36,000 km, r is the radius of the Earth, 6400 km, and h is the height of the circular orbit in kilometers. This G is easily seen to be decreasing as h increases, as is reasonable.

In summary, small Earth orbiters will more and more find it advantageous to be supported by the ground network rather than by the space network. The advantage of lowered communications package mass, size, and power consumption, resulting from smaller communication distances capitalized upon by available onboard data storage, is amplified by the greater figure of merit (gain divided by noise temperature) of ground stations compared to space relay stations. Overall, NASA costs are expected to be lower as well, but this article does not attempt a formal trade-off study. Nevertheless, the DSN can expect to be supporting more and more low-Earth orbiters.
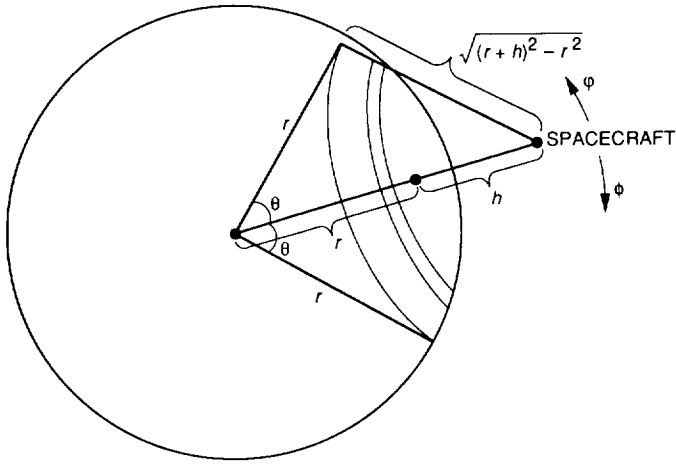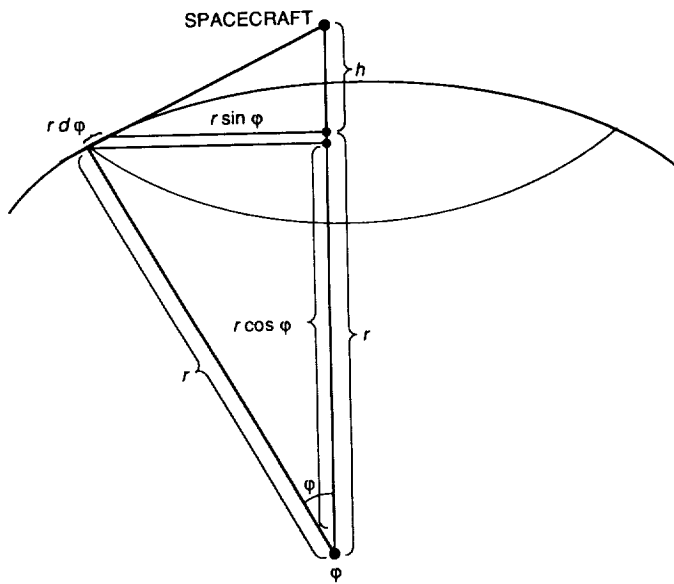
Fig. 1. The geometry of visibility.

$$\sqrt{(r+h)^2 - r^2}$$

Fig. 2. The average inverse squared distance.

$$\| \bar{x} \| = r, \| \bar{y} \| = r + h,$$
$$\| \bar{y} - \bar{x} \| = \ell$$
$$\ell^2 = \| \bar{y} \|^2 + \| \bar{x} \|^2 - 2(\bar{y} \cdot \bar{x})$$

AND

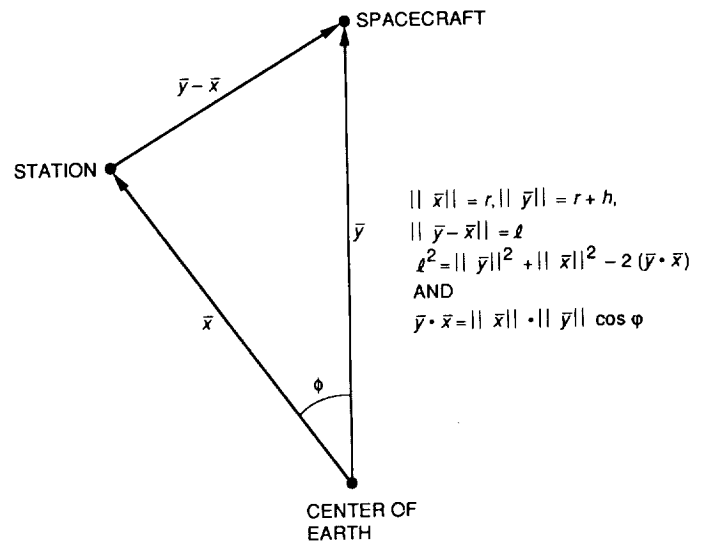$$\bar{y} \cdot \bar{x} = \| \bar{x} \| \cdot \| \bar{y} \| \cos \varphi$$

Fig. 3. The distance between the spacecraft and the tracking station.

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