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A COMPUTER SEARCH FOR ASTEROID FAMILIES

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Abstract - The improved proper elements of 4100 numbered asteroids have been searched for clusterings in a , e , i space using a computer technique based on the D-criterion. A list of 14 dynamical families each with more than 15 members is presented. Quantitative measurements of the density and dimensions in phase space of each family are presented.

Introduction - Large scale clustering in the proper elements of the minor planets was first noticed by Hirayama (1918), who coined the term "families" to indicate such clusterings. Hirayama (1928, 1933) added additional members to the previous families. Extensive lists of asteroid families have subsequently been compiled by Brouwer (1951), Arnold (1969), van Houten et al. (1970), Lindblad and Southworth (1971), Williams (1979), Kozai (1979a, 1983) and others. Unfortunately the criterion for family membership is not always stated, and it appears that the limits of a family in proper element space are largely a matter of personal judgement. A strict criterion of family membership based on the D-criterion of Southworth and Hawkins (1963) was introduced by Lindblad and Southworth (1971). The D-criterion has been used in numerous meteor stream studies and subsequently also for studying clustering amongst asteroid and comet orbits (Lindblad and Southworth 1971, Kresák 1982, Lindblad 1985). The D-criterion for orbital similarity in proper a , e , i space may be written

$$D(m, n)^2 = (e_n - e_m)^2 + (q_n - q_m)^2 + (2 \cdot \sin \frac{i_n - i_m}{2})^2 \quad (1)$$

where m and n represent two orbits to be compared. The search program computes $D(m, n)$ for all possible pairs in the sample. If $D(m, n)$ is less than a stipulated value D_s , the program accepts these two orbits as belonging to a cluster. During the continued comparison process more and more pairs are linked together to form a cluster. The program numbers the various groups and lists their members, computes the mean orbit M of the group and also the deviation $D(M, n)$ of each individual member from the mean orbit. Finally it computes the mean $D(M, n)$ for all members in a group.

In a search in three dimensional space the discriminant D_s should vary inversely as the cubic root of the sample size, if the samples are otherwise similar. Lindblad and Southworth (1971) noted that the best agreement with the classifications of Hirayama and Brouwer was obtained using $D_s = 0.020$ in a sample of 1697 numbered asteroids and $D_s = 0.013$ in a mixed sample of 2652 numbered asteroid and PLS orbits. Based on these results we may write

$$D_s = 0.24 \cdot N^{-1/3} \quad (2a)$$

$$D_s = 0.18 \cdot N^{-1/3} \quad (2b)$$

where equation (2a) is appropriate for asteroids with registration numbers less than about 2000 and equation (2b) for samples exhibiting a higher degree of concentration to the ecliptic plane.

Improved proper elements - Early computations of proper elements involved only low-order expansions in the eccentricities and inclinations. Subsequently Williams (1979), Yuaza (1973), Kozai (1979b), Knězevič (1986) and Knězevič and Milani (1989) have improved the theory of computing proper elements by developing methods to accurately handle high inclinations and/or large excentricities. A set of proper elements for 4100 minor planets computed by Knězevič and Milani, forms the data base for the present study.

Results of computer search - In the Knězević-Milani sample of 4100 orbits the asteroid population with serial numbers above 2000 showed a much stronger concentration to the ecliptic plane than the first 2000 numbered asteroids. It follows that a family search in this sample must be made at a rather strict rejection level. Eq. (2b) suggests a D_s value in the range 0.011 - 0.012. In order to take a conservative approach the search was made at $D_s = 0.011$. The search produced a list of 316 "families" totaling 2163 members, i.e. slightly more than 50 % of the asteroid population was grouped into families. However, 211 families had only 2 members, 54 had 3, 18 had 4, 6 had 5 members each, etc. Pending a detailed study of the possible significance of small families the present report focuses on the fourteen largest families only. A more detailed study will be published elsewhere.

Table 1 lists all asteroid families with 15 or more members together with their mean proper elements and mean $D(M, n)$ value. Families are named after the asteroid of lowest serial number. In a few cases a double name has been introduced so as to facilitate a comparison with the results of other workers. All the Hirayama families, except Phocaea, are detected. In addition nine "new" families with from 15 to 100 members appear. The three families: Eunomia, Nysa and Vesta have been previously mentioned by other investigators. The six new families in Table 1 represent small, but very compact groups.

Dimension and density of asteroid families in phase space - As a check on the consistency of the data we have for each family in Table 1 computed the product of the three standard deviations $\sigma(a)$, $\sigma(e)$ and $\sigma(i)$. This product is a measure of the extent of each family in phase space. In order to obtain suitable numerical values we define

$$\sigma = \sigma(a) \cdot \sigma(e) \cdot \sigma(i) \cdot 10^5 \quad (3)$$

The product σ is listed in column seven of Table 1. In Fig. 1 we have plotted $\log \sigma$ versus the equivalent radius $\overline{D(M, n)}$. A high correlation between $\overline{D(M, n)}$ and σ is evident, (correlation coefficient $r = 0.89$), i.e. both parameters are measures of the extent of an asteroid family in phase space. In column eight of Table 1 we list the number of family members per unit volume of phase space. Table 1 shows that Koronis, Dora and Adeona are extremely dense groups, whereas the other eleven families all have about the same density.

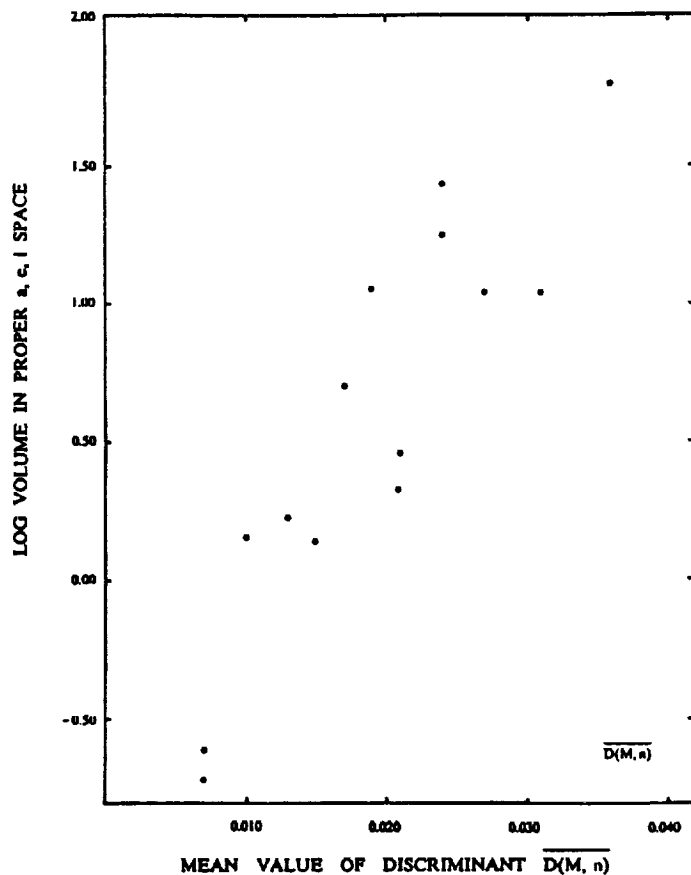
Statistical significance of asteroid families - The statistical significance of the families listed in asteroid studies is a matter of some concern. In most studies dynamical families are identified as groupings in proper a , e , i space and the boundaries are drawn rather subjectively. Since the agreement between the various lists of families is only partial, one is inclined to doubt the correctness of the used statistical significant tests. A different approach to the statistical significance problem was used by Lindblad and Southworth (1971). These authors constructed random samples of asteroid orbits and searched them at the same rejection level d_s as was used in the real sample. The random samples were created by scrambling the proper elements a and e of the asteroids under study. This procedure preserves the frequency distribution of each orbital element, but does not preserve any correlations which may exist between the proper elements a and e . Fortunately, this correlation is very low. In the present study a number of random samples are being searched at the rejection level $D_s = 0.011$. When these searches are completed one can directly derive the probability that a family of N members is due to a chance grouping in proper a , e , i space. In the previous study (Lindblad and Southworth 1971) it was found that in a sample of about 1700 orbits about 50% of the two- and three-member families and 30% of the four-member families were random groupings in the data.

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Family name	N	a	e	i	$\overline{D(M, n)}$	σ	N/ σ	Comments
Themis	212	3.144	0.156	1.4	0.027	10.9	19.5	
Eos	215	3.014	0.077	10.1	0.024	27.0	8.0	
Koronis	141	2.874	0.049	2.1	0.021	2.08	67.8	Compact
Maria	36	2.555	0.091	15.0	0.021	2.85	12.6	Compact
Flora	440	2.231	0.137	4.5	0.036	62.5	7.0	Loose assoc.
Eunomia	100	2.626	0.149	13.2	0.031	10.8	9.3	Brouwer 19-20
Nysa-Hertha	56	2.392	0.163	3.0	0.024	17.7	3.2	
Vesta	41	2.354	0.103	6.5	0.019	11.5	3.6	
Amalasantha	39	2.446	0.153	2.9	0.017	5.04	7.7	Rel. to Flora
Mildred-Beer	17	2.385	0.190	2.2	0.015	1.37	12.4	Rel. to Flora
Oppavia-Gefion	17	2.789	0.136	9.1	0.013	1.68	10.1	
Dora	16	2.787	0.196	7.8	0.007	0.19	83.3	Compact
Adeona	16	2.674	0.167	11.6	0.007	0.24	66.7	Compact
Goberta	15	3.174	0.135	1.1	0.010	1.45	10.3	

Table 1. Mean proper elements, mean discriminant $\overline{D(M,n)}$, volume σ and spatial density N/ σ of large families detected in search amongst 4100 asteroids at rejection level $D_s = 0.011$.

Fig. 1. Log σ versus equivalent radius $\overline{D(M, n)}$.



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