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3–D Orbital Evolution Model of Outer Asteroid Belt

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The evolution of minor planets in the outer part of the asteroid belt is considered. In the framework of the semi-averaged elliptic restricted three-dimensional three-body model, the boundary of regions of the Hill's stability is found. As was shown in our work (Gerasimov and Solovaya, 1989) the Jacobian integral exists.

The equations of the motion in the rotating coordinate system have the form:

$$\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= [U'_\xi] \\ \ddot{\eta} + 2n\dot{\xi} &= [U'_\eta] \\ \ddot{\zeta} \qquad &= [U'_\zeta] \end{aligned} \tag{1}$$

$$[U] = \frac{n^2}{2}(\xi^2 + \eta^2) + \frac{k^2 m_1}{2\pi}\int_0^{2\pi}\frac{n}{r_1}dt + \frac{k^2 m_2}{2\pi}\int_0^{2\pi}\frac{n}{r_2}dt\,, \tag{2}$$

where

$$\frac{n\,dt}{2\pi} = \frac{dm_1}{m_1} = \frac{dm_2}{m_2}\,.$$

The Jacobian integral is

$$\frac{1}{2}(\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2) = [U] - C\,.$$

After averaging, the Jacobian integral in the Sun-centered siderical coordinates will have the following form:

$$C = (\frac{1}{2a} + \sqrt{p}\cos i) + \frac{1}{2}\mu^2 + \mu(\frac{2K(\kappa_2)}{\pi\nu} - x) \tag{3}$$

$$\nu^2 = (\rho^2 + 3\alpha^2)^2, \quad \alpha = (1-\mu)e_j$$

$$x = R\cos\Theta, \quad \cos\Theta = \cos b\cos(l - l_j)$$

$$\cos\Theta_\pi = \cos\omega\cos l_{j_\pi} + \sin\omega\sin l_{j_\pi}\cos i\,. \tag{4}$$

We use a notation similar to the paper Gerasimov and Solovaya (1989), where R is the Sun–asteroid distance, ρ is the Jupiter–asteroid distance, Θ is the angle Jupiter–Sun–asteroid, μ is the ratio of Jupiter's mass to the sum of the masses of Sun and Jupiter, m_1 and m_2 are stationary masses on the rotating axes, e_j is the eccentricity of Jupiter's orbit, l and b are the longitude and latitude of an asteroid, l_j is the longitude of Jupiter, a, e, i and ω are the osculating elements

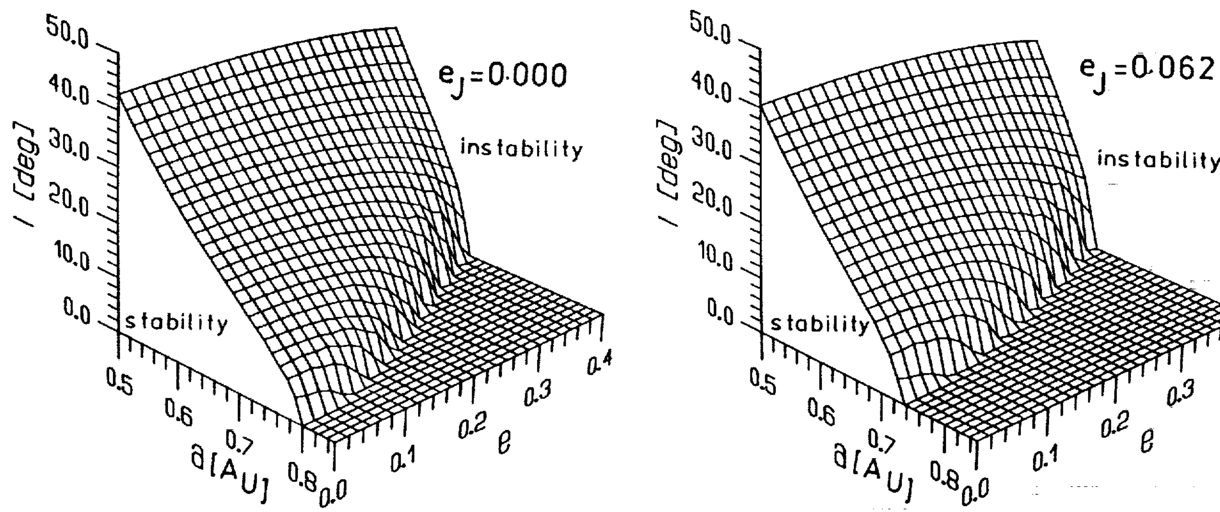

Figure 1. The regions of stability and instability of the motion.

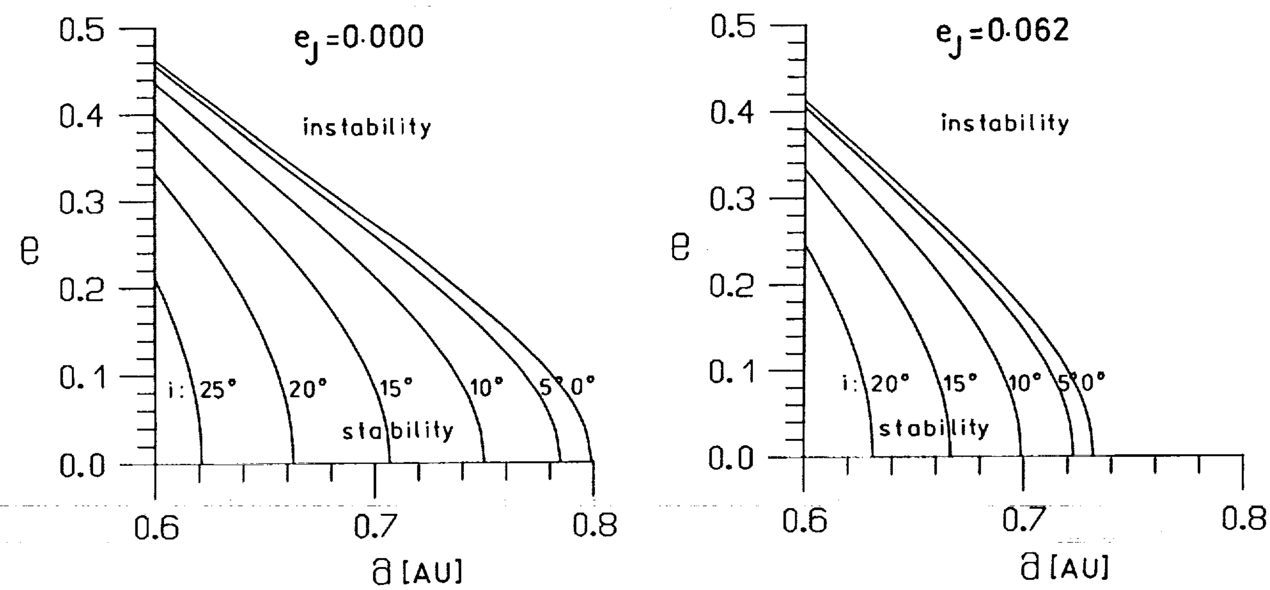

Figure 2. The regions of stability and instability of the motion projected to the (a,e) plane.

of an asteroid, and $K(\kappa_2)$ is the elliptic integral. When $e_j = 0$ then $K(\kappa_2) = \pi/2$. In our computation $\mu = 0.0009538$. We obtain the boundary of regions of possible stable motion from the equation (3) with a fixed value of C and changing elements of fictitious asteroids in the limits $0 \leq e \leq 0.4$ and $0^o \leq i \leq 40^o$.

For $e_j = 0$, the value of C is equal 1.51938 for the interior point L, for $e_j = 0.062$, $C = 1.538$, respectively. We decided to use the equation (3) for a model, in which the heliocentric elongation of an asteroid from Jupiter is equal 180^o. The regions of the Hill's stability and instability are shown for the three-dimensional case in Figure 1. On the (a,e) plane these regions are plotted in Figure 2.

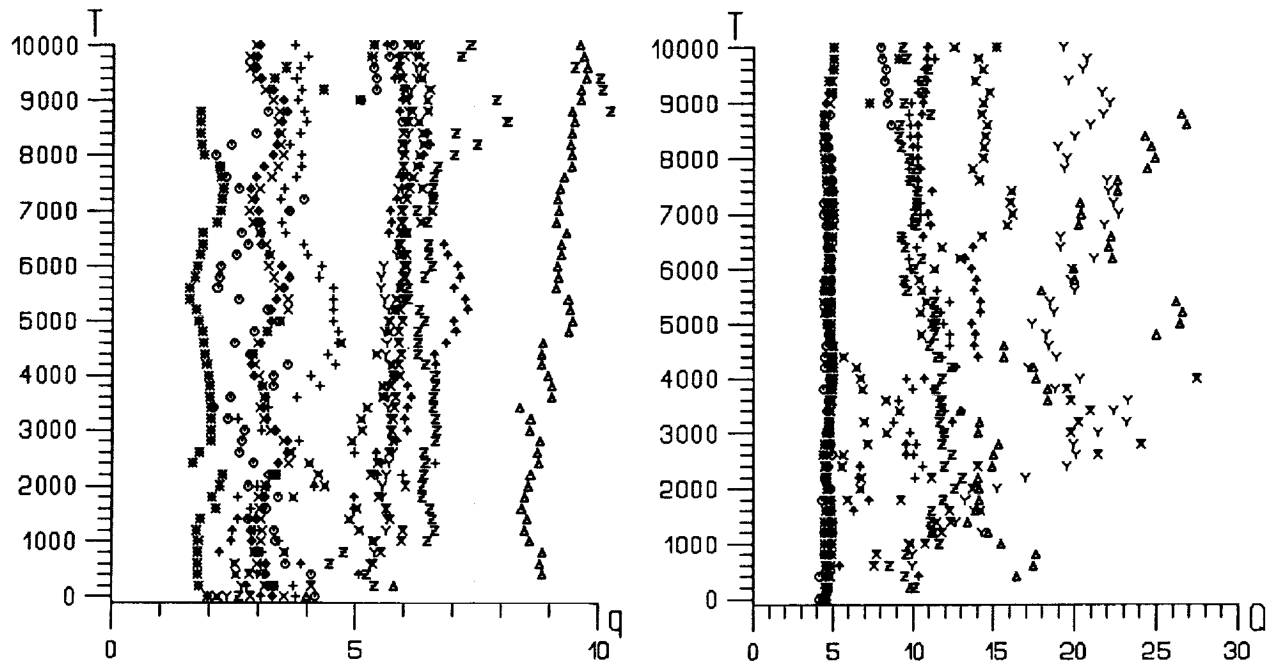

Figure 3. The evolution of the perihelion distance q [AU] and the aphelion distance Q [AU] of model asteroids within the period of 10 000 years for the first variant of e_j and the commencing asteroid inclination $i = 0^o$. Asteroids: ⦶ — 0.00 (the eccentricity), Δ — 0.04, + — 0.08, × — 0.12, ◇ — 0.16, ✢ — 0.20, ⨯ — 0.24, Z — 0.28, Y — 0.32, ⌘ — 0.36, ✳ — 0.40.

In the outer part of the asteroid belt Hill's stable asteroid orbits cannot exist with high inclinations. Among real asteroids of the outer asteroidal belt we found only a few asteroids from Ephemerides of Minor Planets for 1991 (Batrakov, 1990), which were above the surface represented by Figure 1. They are asteroid 153 Hilda ($a = 0.766$, $e = 0.145$, $i = 7.85^o$), asteroid 361 Bononia ($a = 0.757$, $e = 0.125$, $i = 12^o.67$), asteroid 1212 Francette ($a = 0.760$, $e = 0.110$, $i = 7^o.58$), and asteroid 1529 Oterma ($a = 0.769$, $e = 0.190$, $i = 9^o.03$). According to Milani and Nobili (1985) and Schubart (1991) asteroids may exist within the outer asteroid belt if there are librations of the argument of perihelion. Then their aphelions will always remain far from Jupiter. Numerical investigations in the framework of the restricted three-body problem have been developed by many authors. They show that asteroids will be ejected by Jupiter in only a few thousand years, with the exception of the stable librators.

We took a dynamic model of the solar system, consisting of six major planets and eleven massless fictitious asteroids located at the similar to the four asteroids as mentioned above. We traced the orbital evolution of these asteroids using the numerical integration program with the integrator RA15 (Everhart, 1985). The input data a, e and i were taken from the equation (3) for the eccentricity of Jupiter $e_j = 0$ — the first variant, and $e_j = 0.062$ — the second variant. The starting value of the argument of perihelion was selected from the equation (4) at $\Theta = 180^o$. The starting epoch of the numerical integration is March 25, 1991 UT. The results obtained by the numerical integration for the groups of asteroids with started values of i equal 0^o and 10^o and for the same starting values of a and e are plotted in Figures 3–4.

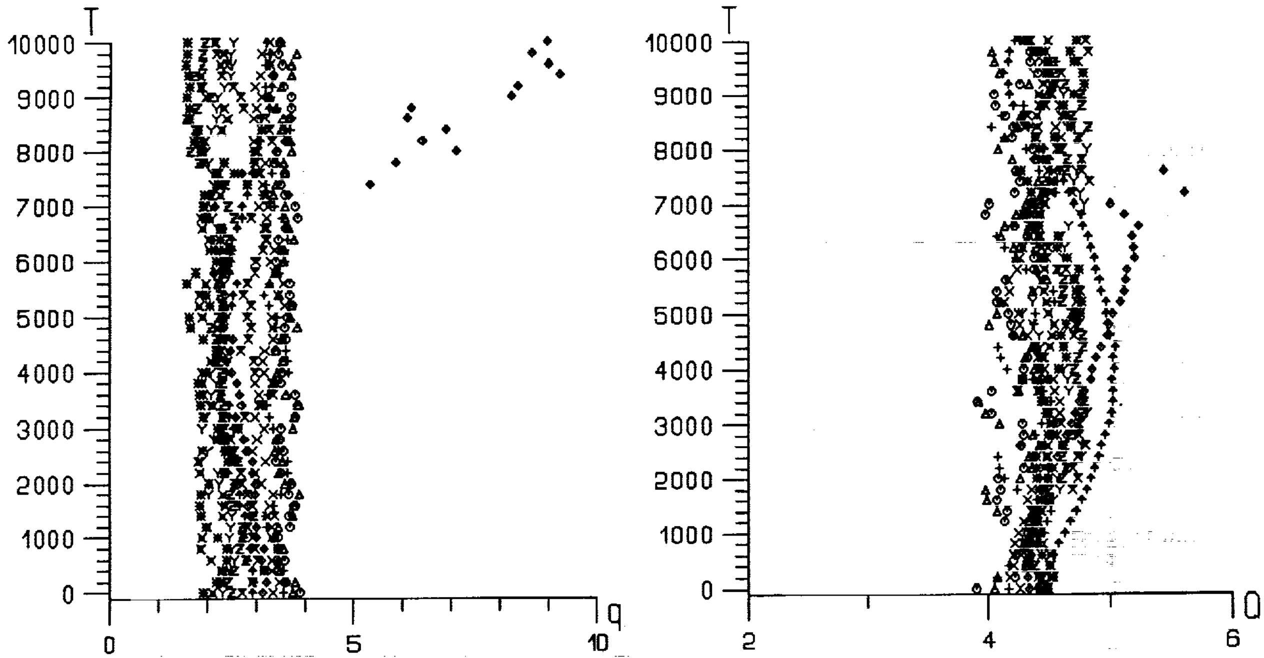

Figure 4. The evolution of the perihelion distance q [AU] and the aphelion distance Q [AU] of model asteroids within the period of 10 000 years for the first variant of e_j and the commencing asteroid inclination $i = 10^o$. Asteroids: ⦶ — 0.00 (the eccentricity), △ — 0.04, + — 0.08, × — 0.12, ◇ — 0.16, ✢ — 0.20, ⊼ — 0.24, Z — 0.28, Y — 0.32, ⌘ — 0.36, ✳ — 0.40.

Asteroids with $i = 0^o$ are ejected to Saturn after close encounters with Jupiter. Asteroids having the inclination $\sim$ 10–20° and the same commencing values of a and e as of the previous case are practically all stable. We see, that the inclination of orbits of asteroids from the outer belt has the same role in the three-dimensional model as the libration of the argument of perihelion in the two-dimensional model. When the inclinations of asteroids are $\sim$ 10–20°, aphelions of their orbits will always remain far from Jupiter.

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