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KINEMATIC ANALYSIS OF THE ARID MANIPULATOR

PREPARED BY: Dr. Keith L. Doty

ACADEMIC RANK: Professor

UNIVERSITY AND DEPARTMENT: University of Florida
Department of Electrical Engineering

NASA/KSC

DIVISION: Mechanical Engineering Directorate

BRANCH: Special Projects (RADL)

NASA COLLEAGUE: Willis Crumpler
Bill Jones
Eduardo Lopez

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ABSTRACT

This paper develops the forward and inverse position kinematics of the ARID manipulator and also its forward velocity kinematics.

Keith L. Doty
NASA Faculty Fellow

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1. INTRODUCTION

The kinematic structure of the ARID manipulator lends itself to simple forward and inverse kinematics analysis. The purpose of this paper is to fully document and verify an existing analysis. The symbolic software package *MATHEMATICA* was used to produced and verify the equations presented here. In the analysis to follow, the standard Devenit-Hartenberg kinematic parameters of the ARID were employed.

2. ARID FORWARD KINEMATICS

Table 2.1 lists the Devenit-Hartenberg kinematic parameters for the ARID robot.

Table 2.1 : Kinematic Parameters for the ARID Robot

Joint	d	θ	a	α	Joint Limits
1 p	d_1	θ_1	a_1	0°	[0*inches*, 718*inches*]
2 r	0	θ_2	a_2	0°	[4° , 112°]
3 r	0	θ_3	a_3	0°	[102° , 148°]
4 r	0	θ_4	0	0°	[-16° , -117°]

Table 2.2 lists the nominal values of the link lengths and the fixed angle θ_1. The distance $1 + a_4$ computes the tool-end-point along the x-axis of the tool-frame whose origin is located at the flange.

From the DH-parameters of the ARID robot listed in Table 2.1, the four link transforms compute to

$$L_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1\,c_1 \\ s_1 & c_1 & 0 & a_1\,s_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2\,c_2 \\ s_2 & c_2 & 0 & a_2\,s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3\,c_3 \\ s_3 & c_3 & 0 & a_3\,s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad L_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{2-1}$$

Table 2.2 Nominal Values of ARID Kinematic Parameters

Parameter	Theoretical Value	Measured Value	Units
a_1	$\sqrt{66.37^2 + 48.28^2}$ $= 82.0727$	82.3	*inches*
a_2	45.00	45	*inches*
a_3	35.00	35	*inches*
$l + a_4$		35.75	*inches*
s_1	$\frac{48.28}{a_1} = 0.588259$		
c_1	$\frac{66.37}{a_1} = 0.808673$		
θ_1	36.0335°	35.2053°	Degrees

The forward kinematics transform of the ARID equals

$$^0T_4 = L_1\, L_2\, L_3\, L_4 \tag{2-2}$$

which reduces to

$$
{}^{0}\mathbf{T}_4 = \begin{bmatrix} c_{1234} & -s_{1234} & 0 & a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ s_{1234} & c_{1234} & 0 & a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2-3}
$$

If the ARID axes are not parallel, this model will produce erroneous results. A second paper will address the development of a error-model for the ARID kinematics.

3. ARID END FRAME JACOBIAN

The Jacobian of the ARID relates the joint-rates $\dot{\mathbf{q}} = [\, \dot{d}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \quad \dot{\theta}_4 \,]^\tau$ to the frame-velocity $\mathbf{V} = [v^\tau \;\; \omega^\tau \,]^\tau$ of the end-frame,

$$
\mathbf{V} = \mathbf{J}\, \dot{\mathbf{q}}. \tag{3-1}
$$

The Jacobian of the ARID computes to

$$
{}^{4,0}\mathbf{J}_4 = \begin{bmatrix} 0 & a_2 s_{34} + a_3 s_4 & a_3 s_4 & 0 \\ 0 & a_2 c_{34} + a_3 c_4 & a_3 c_4 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \tag{3-2}
$$

The leading superscript 4 means that this Jacobian is expressed in frame F_4 while the 0 indicates the motion is that of the end-frame relative to the base frame F_0 of the ARID.

The Tool-Center-Point

The tool-center-point vector (**tcp**), expressed in the end-effector-frame F_4 for this robot, equals, by definition,

$$ {}^4\mathbf{tcp} = [24\ \textit{inches} \quad 0 \quad 0\,]^{\tau} = 24\ \mathbf{x}. \qquad (3\text{-}3)$$

Hence, the base-frame expression of the tool-center-point vector equals

$$\begin{bmatrix} {}^0\mathbf{tcp} \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} {}^4\mathbf{tcp} \\ 1 \end{bmatrix}. \qquad (3\text{-}4)$$

The ARID base-frame expression of the **tcp** position, therefore, equals

$$ {}^0\mathbf{tcp} = \mathbf{p} + \mathbf{R}\ {}^4\mathbf{tcp}. \qquad (3\text{-}5)$$

The ARID base-frame velocity of the **tcp**, therefore, equals

$$\mathbf{v}_{tcp} = \dot{\mathbf{p}} + \omega \times \mathbf{R}\ {}^4\mathbf{tcp} = \mathbf{R}(\ {}^{4,0}\mathbf{v}_4 + {}^{4,0}\omega_4 \times 24\ \ \mathbf{x})\ , \qquad (3\text{-}6)$$

where

$$ {}^{4,0}\mathbf{V}_4 = \begin{bmatrix} {}^{4,0}\mathbf{v}_4 \\ {}^{4,0}\omega_4 \end{bmatrix} = {}^{4,0}\mathbf{J}_4\ \dot{\mathbf{q}}. \qquad (3\text{-}7)$$

In general, the middle expression in (3-6) can be used to compute the velocity of any point fixed in the end-frame F_4.

4. ARID INVERSE POSE KINEMATICS

Specify the pose of the end-effector frame at the ARID flange by the homogeneous matrix **H**,

$$\mathbf{H} = \begin{bmatrix} n_x & b_x & t_x & p_x \\ n_y & b_y & t_y & p_y \\ n_z & b_z & t_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-1}$$

where $\mathbf{p} = [p_x \quad p_y \quad p_z]^\tau$ is the position of the flange center-point and

$$\mathbf{R} = \begin{bmatrix} n_x & b_x & t_x \\ n_y & b_y & t_y \\ n_z & b_z & t_z \end{bmatrix} \tag{4-2}$$

is the rotation matrix that rotates the base-frame of the robot into a frame parallel to the tool-frame. Equivalently, **R** transforms a vector expressed in the end-effector-frame coordinates into the same vector expressed in the robot base-frame coordinates. The inverse position kinematics problem is to solve for the joint variables given **H**. This means solving the matrix equation

$$^0\mathbf{T}_4 = \mathbf{H} \tag{4-3}$$

Solving for the Sum of the Three Revolute Joint Angles

Unless **R** a rotation about the *z-axis* , $t_z = 1$, no joint variable set will satisfy (4-3). Given that $t_z = 1$, $^0\mathbf{T}_{4[1,1]} = \mathbf{H}_{[1,1]}$, $^0\mathbf{T}_{4[2,1]} = \mathbf{H}_{[2,1]}$ implies

$$c_{1234} = n_x \qquad \text{and} \qquad s_{1234} = n_y . \tag{4-4}$$

Hence,

$$\theta_2 + \theta_3 + \theta_4 = atan2[n_y, n_x] - \theta_1. \tag{4-5}$$

The known value of θ_1 in Table 2.2 provides the additional information to compute the sum of the last three joint angles.

Solving for the Prismatic Displacement of Joint One

The value of the prismatic joint follows immediately from ${}^0\mathbf{T}_4[3,4] = \mathbf{H}[3,4]$,

$$p_z = d_1. \tag{4-6}$$

Next, we solve for θ_3.

Solving for the Angle of Joint Three

Define the known, translated position vector $\tilde{\mathbf{p}} = [\tilde{p}_x \;\; \tilde{p}_y \;\; \tilde{p}_z]^\tau$,

$$\tilde{p}_x := p_x - a_1 c_1, \qquad \tilde{p}_y := p_y - a_1 s_1, \qquad \text{and} \qquad \tilde{p}_z := p_z. \tag{4-7}$$

The equalities ${}^0\mathbf{T}_4[1,4] = \mathbf{H}[1,4]$ and ${}^0\mathbf{T}_4[2,4] = \mathbf{H}[2,4]$ imply

$$\tilde{p}_x = a_3 c_{123} + a_2 c_{12} \tag{4-8}$$

$$\tilde{p}_y = a_3 s_{123} + a_2 s_{12} \tag{4-9}$$

The previous two equations constitute the well-known elbow equations. Squaring both equations and adding yields the kinematic equation

$$c_3 = \frac{\tilde{p}_x^2 + \tilde{p}_y^2 - a_2^2 - a_3^2}{2\, a_2\, a_3}, \tag{4-10}$$

which has two solutions,

$$\theta_3 = \pm atan2[\sqrt{1 - c_3^2}, c_3], \tag{4-11}$$

or, equivalently,

$$\theta_3 = \pm\cos^{-1}\left[\frac{\tilde{p}_x^2 + \tilde{p}_y^2 - a_2^2 - a_3^2}{2\, a_2\, a_3}\right], \tag{4-12}$$

as long as the constraint

$$-1 \leq \frac{\tilde{p}_x^2 + \tilde{p}_y^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3} \leq 1, \quad (4\text{-}13)$$

or, equivalently,

$$(a_2 - a_3)^2 \leq \tilde{p}_x^2 + \tilde{p}_y^2 \leq (a_2 + a_3)^2, \quad (4\text{-}14)$$

is satisfied.

Equation (4-14) expresses the geometric constraints that the manipulator cannot reach out past the sum of its link lengths and no closer than the difference. The reachable *work space*, is contained in an envelope consisting of an annular ring with inner radius of $|a_2 - a_3|$ and outer radius of $a_2 + a_3$. When the end-point is on the inner radius, $\cos\theta_3 = -1$ and on the outer boundary, $\cos\theta_3 = 1$. On these boundaries there is only one solution for $\cos\theta_3$ in (4-12). The constraints on θ_3 prevents the ARID robot from reaching either the inner or outer boundaries. No position in the ARID work space violates either conditions. In the workspace on the orbiter, only the positive solution $\theta_3 > 0$ is considered.

Solving for the Angle of Joint Two

With joint angle θ_3 determined, θ_2 can be computed from (4-8) and (4-9) by expanding the trigonometric terms using the sum of angles formulas and collecting the s_2 and c_2 terms to form the simultaneous equations:

$$\begin{bmatrix} \tilde{p}_x \\ \tilde{p}_y \end{bmatrix} = \begin{bmatrix} -a_3 \cdot s_3 & a_2 + a_3 \cdot c_3 \\ a_2 + a_3 \cdot c_3 & a_3 \cdot s_3 \end{bmatrix} \cdot \begin{bmatrix} s_2 \\ c_2 \end{bmatrix}. \quad (4\text{-}15)$$

Relation (4-15) is a linear algebraic system of equations in s_2 and c_2. Solutions exist to (4-15) provided the determinant of the matrix is non-zero,

$$\begin{vmatrix} -a_3 \cdot s_3 & a_2 + a_3 \cdot c_3 \\ a_2 + a_3 \cdot c_3 & a_3 \cdot s_3 \end{vmatrix} = -[a_2^2 + a_3^2 - 2 \cdot a_2 \cdot a_3 \cos(\pi - \theta_3)] = -[\tilde{p}_x^2 + \tilde{p}_y^2] \neq 0$$

Thus, as long as the position vector $\tilde{p}$ of the end-frame has at least one of its planar components non-zero, $\tilde{p}_x^2 + \tilde{p}_y^2 \neq 0$, the system of equations (4-15) is invertible,

$$\begin{bmatrix} s_2 \\ c_2 \end{bmatrix} = \frac{-1}{\tilde{p}_x^2 + \tilde{p}_y^2} \begin{bmatrix} a_3 \cdot s_3 & -a_2 - a_3 \cdot c_3 \\ -a_2 - a_3 \cdot c_3 & -a_3 \cdot s_3 \end{bmatrix} \cdot \begin{bmatrix} \tilde{p}_x \\ \tilde{p}_y \end{bmatrix} \qquad (4\text{-}16)$$

One cannot be sure that (4-16) is valid unless, $s_2^2 + c_2^2 = 1$. This constraint can be proven to be satisfied whenever a solution to θ_3 exists. Finally, to obtain θ_2, compute

$$\theta_2 = \operatorname{atan2}\left[-a_3 \cdot s_3 \cdot \tilde{p}_x + (a_2 + a_3 \cdot c_3) \cdot \tilde{p}_y,\ (a_2 + a_3 \cdot c_3) \cdot \tilde{p}_x + a_3 \cdot s_3 \cdot \tilde{p}_y\right]. \qquad (4\text{-}17)$$

Each value of θ_3 in (4-17) determines a unique value of θ_2.. In the ARID robot only the positive solution for θ_3 is used, hence, only one value of θ_2 requires computing.

ARID Robot Inverse Pose Kinematic Summary

Table 4.1 summarizes the inverse kinematics analysis for the ARID robot. The first column supplies the equations needed to solve for the joint variables in terms of the configuration variables and the DH-parameters. The second column states the *natural kinematic constraints* which must be satisfied in order for the solution to be valid. Natural kinematic constraints indicate geometric relationships between and amongst the DH-parameters and configuration variables that must be satisfied, independent of joint range limits or link interference. For purposes of theoretical analysis, therefore, joint variables range over all the real numbers. In the ARID, for example, the solution $d_1 = p_z$ is always theoretically valid, even though the solution is not *realizable* when p_z exceeds the physical length of the robot track. The inverse solutions to the ARID manipulator that satisfy the natural kinematic constraints must be checked to determine if the computed values of the joint variables fall within the physical joint limits dictated for the ARID. Those solutions which do satisfy the joint limits are said to be *realizable* by the manipulator. Refer to Table 2.2 for joint limits of the ARID robot.

Table 4.1 Inverse Solution to the ARID Manipulator

Joint Variable Solution	Theoretical Kinematic Constraints for Solution
$d_1 = p_z$	None
$\theta_3 = \pm\cos^{-1}\left[\dfrac{\tilde{p}_x^2 + \tilde{p}_y^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right]$	$(a_2 - a_3)^2 \leq \tilde{p}_x^2 + \tilde{p}_y^2 \leq (a_2 + a_3)^2$
$\theta_2 = \text{atan2}\,[-a_3 \cdot s_3 \cdot \tilde{p}_x + (a_2 + a_3 \cdot c_3) \cdot \tilde{p}_y,\ (a_2 + a_3 \cdot c_3) \cdot \tilde{p}_x + a_3 \cdot s_3 \cdot \tilde{p}_y]$	$(a_2 - a_3)^2 \leq \tilde{p}_x^2 + \tilde{p}_y^2 \leq (a_2 + a_3)^2$
$\theta_4 = \text{atan2}\,[n_y, n_x] - (\theta_1 + \theta_2 + \theta_3)$	$t_z = 1$

5. CONCLUSION

The nominal ARID forward position and velocity kinematics have been developed, as well as the forward velocity kinematics. A follow-up paper will develop a kinematics error-model for calibrating the ARID.

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