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# RECOGNITION OF PARTIALLY OCCLUDED THREAT OBJECTS USING THE ANNEALED HOPEFIELD NETWORK

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This project is being supported by:  
NASA Center of Research Excellence  
(NASA-CORE)  
School of Engineering  
North Carolina A&T State University

Research Memorandum # HMSEL-CORE-0193

OCTOBER 1992

# Recognition of partially occluded threat objects based on the annealed Hopfield network<sup>1</sup>

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## ABSTRACT

Recognition of partially occluded objects has been an important issue to airport security because occlusion causes significant problems in identifying and locating objects during baggage inspection. Neural network approach is suitable for the problems in the sense that the inherent parallelism of Neural Networks pursues many hypotheses in parallel resulting in high computation rates. Moreover, they provide a greater degree of robustness or fault tolerance than conventional computers. The annealed Hopfield network which is derived from the mean field annealing(MFA) has been developed to find global solutions of a non-linear system. In the study, it has been proven that the system temperature of MFA is equivalent to the gain of sigmoid function of Hopfield network. In our early work, we developed the hybrid Hopfield network(HHN) on the purpose of fast and reliable matching[1]. However, HHN doesn't guarantee global solutions and yields false matching under heavily occluded conditions because HHN is depending on initial states by its nature. In this paper, we present the annealed Hopfield network(AHN) for occluded object matching problems. In AHN, the mean field theory is applied to the hybrid Hopfield network in order to improve computational complexity of the annealed Hopfield network and provide reliable matching under heavily occluded conditions. AHN is slower than HHN. However, AHN provides near global solutions without initial restrictions and provides less false matching than HHN. In conclusion, a new algorithm based upon a Neural Network approach was developed to demonstrate the feasibility of the automated inspection of threat objects from X-ray images. The robustness of the algorithm is proved by identifying occluded target objects with large tolerance of their features.

## 1. INTRODUCTION

Pattern recognition and computer vision theory has been considerably improved during the last decade such that the appearance of an automated vision system seems very close to our future[2,3]. However, because of the higher computational burden of image understanding algorithms, use of object recognition from an image is still limited to the restricted environment. In the mean time researchers developed a new idea of computation which imitates human brain structure, called a neural computing[4,5]. Techniques in neural computing are based on a new concept of distributed parallel computation, and applicable to any number crunching objectives. Hopfield network, one of the neural computations is very popular in real world application due to simple architecture and well defined time domain behavior[5-9]. The Hopfield network is composed of single-layer neurons with fully connected feedback connections. The neurons have the sigmoid gain characteristic, while the connectivity matrix corresponding to the connection is symmetric and the diagonal terms of the matrix are zero. Such networks always move in the direction of decreasing the energy of the networks and get stable states at the local minimum of the energy. Since the energy function of a Hopfield network has many local minima, the resultant network output is usually the closest local minimum to initial states. This nature of the Hopfield network must be the demerit in solving an optimization problem. In our early work, we developed the hybrid Hopfield network algorithm to improve deficiency of the original Hopfield network. The method yields a good solution by adding an adjusting procedure for the output neuron states of the

<sup>1</sup>This research has been supported by FAA under Contract DTF A01-87-c-00043, NSF under Grant ECD-8212696, and the NASA Center for Research Excellence of North Carolina A&T State University under Grant No. NAGW-1924.

Hopfield network. However, the method still does not guarantee global solutions. Simulated annealing is one heuristic technique to help escape the local minima by perturbing the energy function with the annealing temperature and artificial noise[10]. It is proven that the solution obtained by the simulated annealing is independent of the initial condition of the network and is usually very close to the global minimum[9]. Since the network should settle down at each temperature and the temperature decrement is very small, extraordinarily long time is required in the software computation. The mean field theory(MF) has been applied to the simulated annealing in the effort on reducing the computational time and many impressive outputs in image processing area have been reported. The MF has the analogy to the Hopfield networks[11]. It was proven that the system temperature of MF is equivalent to the neural gain. D. E. Van den Bout *et al.* also developed a new algorithm, the mean field annealing(MFA) which merges many features of simulated annealing and Hopfield networks[12]. They do not use a sigmoid function but use the normalization technique. However the normalization technique can be applied to the case in which the sum of a normalized subgroup is equal to 1. An occluded object matching problem is the one which can be cast into an optimization problem when the graph theory is applied to the problem. MFA can be used to solve the occluded object matching problem. Unfortunately, the problem is not suitable to normalization technique because the sum of normalized subgroups is zero or one. Thus, we use the sigmoid function, which is one of the important characteristic of Hopfield network. We can do a hardware implementation as well as an algorithm approach. We call this technique as the annealed Hopfield network(AHN).

## 2. ANNEALED HOPFIELD NETWORK

### 2.1 Feature extraction and graph formation

In boundary based approaches, corner points are important since the information of the shape is concentrated at the points having high curvatures. From the corner points, we can extract useful features such as a local feature of an angle between neighboring corners and relational features of distances between the corners. These two features which are invariant under translational and rotational changes are used for the robust description of shape of the boundary. Corner points are usually detected in a curvature function space by capturing the points whose curvature values are above a certain threshold value. We developed a new corner detection algorithm which provides reliable and invariant corners for a matching procedure in the early study[9]. A graph can be constructed for a model object using corner points as nodes of the graph. Each node has a local feature as well as relational features with other nodes. For the matching process, a similar graph is constructed for the input image which may consist of one or several overlapped objects. Each model graph is then matched against the input image graph to find the best matching subgraph.

### 2.2 Hopfield Network versus Mean field theory

The continuous Hopfield network(CHN) is a deterministic model which retains the significant characteristics of the discrete Hopfield network. The discrete network(DHN) uses binary states. However, real neurons and real physical circuits have integrative time delays due to capacitance, and the time evolution of the state of such systems is represented by a differential equation, so called the equation of motion. The continuous network has flow of neuron states in a continuous domain while the discrete network has flow of neuron states in a discrete domain. It means that CHN is better than DHN for the optimization problems since CHN has a smooth energy function surface. A two dimensional array is constructed to apply a matching problem into the neural networks. The columns of the array label the nodes of an object model, and the rows indicate the nodes of an input object[14,15]. The number of column  $n$  is the number of nodes of a model object and the number of rows  $m$  is the number of nodes of input image. Therefore, the state of each neuron represents the measure of match between two nodes from each graph. The matching process can be characterized as minimizing the following energy function:

$$E = -\frac{1}{2} \sum_i \sum_j \sum_k \sum_l C_{ijkl} V_{ik} V_{jl} + \frac{q}{2} \left( \sum_i \sum_k \sum_{l \neq k} V_{ik} V_{li} + \sum_k \sum_l \sum_{j \neq l} V_{lk} V_{jl} \right) \quad (1)$$

where  $V_{ik}$  is a variable which converges to "1" if the  $i$ th node in the input image matches the  $k$ th node in the object model; otherwise, it converges to "0". The first term in Eq.(1) is a compatibility constraint. Local and relational feature which have different measures are normalized to give tolerance for ambiguity of the features as follows:

$$C_{ijkl} = W_1 \times F(f_i, f_k) + W_2 \times F(f_j, f_l) + W_3 \times F(r_{ij}, r_{kl}) \quad (2)$$

The fuzzy function  $F$  has a value 1 for a positive support and -1 for a negative support. The value of  $F(x, y)$  is defined such that if the absolute value of the difference between  $x$  and  $y$  is less than a threshold  $\theta$ , then  $F(x, y)$  is set to 1, otherwise  $F(x, y)$  is set to -1. The sum of the weights is 1. In our early work[], we used two features such as angle and distance. Angle helps us to recognize the shape of object. However, false segmentation cause to generate different angles from those of original segmentation. In this paper, relational features are more emphasized than local features. AHN even works well without local features. The last two terms are included to enforce the uniqueness constraint so that each node in the object model eventually matches only one node in the input image and the summation of the outputs of the neurons in each row or column is no more than 1. Some papers concerning a matching problem with the Hopfield style neural network have used  $\sum \sum (1-V_{ik})^2$  as an uniqueness constraint. This term implies global restriction. However, matching of occluded objects will not guarantee that every row or every column has only one active neuron. Thus the energy function of the occluded matching problem excludes the global restriction condition in Eq.(1). In a traveling salesman problem, uniqueness coefficient  $q$  is more weighted than the coefficient of the compatibility term because  $q$  contributes yielding valid solutions. However, conditions of valid solutions in the matching of occluded objects are indefinite, so the coefficient  $A$  is supposed to be more weighted in the matching problem. Eq.(1) can be cast into the discrete Hopfield energy function(DHN) as follows:

$$E = -\frac{1}{2} \sum_i \sum_j \sum_k \sum_l T_{ijkl} V_{ik} V_{jl} - \sum_i \sum_j I_{ik} V_{ik} \quad (26)$$

$$T_{ijkl} = C_{ijkl} - q (\delta_{ij} + \delta_{kl} - \delta_{ij} \delta_{kl})$$

where  $\delta_{ij} = 1$  when  $i = j$ , otherwise  $\delta_{ij} = 0$ . Hopfield proved that the energy function is a Liapunov function. Thus the energy function converges to a local minimum when the states of neurons converge to stable states[13]. Unlike the other application[14,15], the constraint that  $\sum \sum V_{ik}$  is equal to the number of column can not be used in the occluded object matching problem since occluded objects can lose a lot of segments of the original. The matching process of CHN can be characterized by the same energy function as that of DHN. Only an integral term is added to the energy function as follows:

$$\sum_i \sum_k (1/R_{ik}) \int_{V_{ik}}^{V_{ik}^*} g^{-1}(V) dV \quad (4)$$

Where  $g$  is a sigmoid function and  $R_{ik}$  is the input resistance of a neuron. This term comes from the point of view that neural input state  $u_{ik}$  will lag because of the existence of capacitance in an analog electrical circuit. Thus, there is a resistance-capacitance charging equation, called the equation of motion that determines the rate of change of  $u_{ik}$ [14]. It is the first order differential equation. The equation of the motion is as follows[9]:

$$\frac{du_{ik}}{dt} = -u_{ik}/\lambda + \sum_j \sum_l T_{ijkl} V_{jl} + I_{ik} \quad (5)$$

where

$$g(u_{ik}) = \frac{1}{1 + \exp[-u_{ik}/\lambda]} \quad (6)$$

Now, let us consider MFA application. A motion equation is shown in Eq.(5) with the sigmoid function  $g$ . Our energy function of the matching problem is organized as Eq.(1). The output of each neuron for the matching problem has the value of 0 or 1 to represent measure of similarity. We will call output of each neuron a *spin* for the mean field annealing

approach. It was assumed that the *spin* interactions  $T_{ijk}$  are symmetric and have no self-interaction (i.e.,  $T_{iik} = 0$ ). The state space of each *spin* is:

$$s_{ik} \in \{0, 1\} \text{ for } 1 \leq i, k \leq N \quad (7)$$

where  $N^2 = m \times n$ . In simulated annealing, random perturbations move the system towards its thermal equilibrium at the current temperature. Assuming that all the *spins* are at equilibrium, one can determine the equilibrium spin average of the *ith spin*  $\langle s_{ik} \rangle$  from the Boltzmann distribution and the change in the average system energy as  $s_{ik}$  flips from 0 to 1. To illustrate, let  $H_0 = \langle H(s) \rangle |_{s_{ik}=0}$ ,  $H_1 = \langle H(s) \rangle |_{s_{ik}=1}$ . Since the system is Boltzmann distributed, the equilibrium value of  $\langle s_{ik} \rangle$  is calculated as follows:

$$\begin{aligned} \langle s_{ik} \rangle &= Pr\{s_{ik}=0\} \times 0 + Pr\{s_{ik}=1\} \times 1 \\ &= \frac{\exp(-\frac{H_1}{T})}{\exp(-\frac{H_0}{T}) + \exp(-\frac{H_1}{T})} \\ &= \{1 + \exp[-\frac{(H_0-H_1)}{T}]\}^{-1} = \{1 + \exp[\frac{u_{ik}}{T}]\}^{-1} \end{aligned} \quad (8)$$

We define  $u_{ik}$  to represent the quantity  $H_0 - H_1$ , which is the mean or effective field experienced by the *ikth spin*. Unfortunately, it is in general difficult to compute  $u_{ik}$  for large  $N$ :

$$\begin{aligned} \langle H(s) \rangle &= \langle \sum_i \sum_j \sum_k \sum_l T_{ijkl} s_{ik} s_{jl} + \sum_i \sum_k I_{ik} s_{ik} \rangle \\ &= \sum_i \sum_k \sum_j \sum_l T_{ijkl} \langle s_{ik} s_{jl} \rangle + \sum_i \sum_k I_{ik} \langle s_{ik} \rangle \end{aligned} \quad (9)$$

The difficulty arises from the fact that  $s_{ik}$  and  $s_{jl}$  are not independent, so that their expected values are not separable in the above equation. However, when the number of interacting spins is large enough that the effect of any single *spin* on any other *spin* is very small in comparison to the total field, then the mean field approximation can be used:

$$\langle H(s) \rangle = \sum_i \sum_k \sum_j \sum_l T_{ijkl} \langle s_{ik} \rangle \langle s_{jl} \rangle + \sum_i \sum_k I_{ik} \langle s_{ik} \rangle \quad (10)$$

The Eq.(8) and Eq.(10) has the same structure as Eq.(3) and Eq.(6). In addition, random perturbation to move the system towards its thermal equilibrium in simulated annealing is the same as updating rule of the Hopfield network. The only difference is that  $\lambda$  in eq. (6) is replaced with temperature  $T$ . It means that given  $T$ , flow to thermal equilibrium in MFA is the same as the flow of Hopfield network given  $\lambda$ . Therefore, if we find the stable points of states by slowly lowering  $\lambda$  from the high value, then we will find global solutions or near global solution of the network without initial restriction. We call this algorithm as the annealed Hopfield network(AHN).

### 2.3 The Critical Temperature( $T_c$ ) and Uniqueness Coefficient( $q$ )

Setting the operating parameters for the annealed network significantly affects a final solution. Starting at too high a temperature above the critical temperature is just time-consuming since no progress is made toward a solution until the critical temperature is reached. Starting at too low a temperature can quench the system and quickly force it into a poor solution. In addition, neural networks often enforce problem constraints through penalty functions which must be weighted in importance against the remaining cost components of the objective function. Weighting penalties too heavily leads to valid but poor solutions, while reducing the penalties permits infeasible solutions to arise. In this section, techniques for estimating  $T_c$  and  $q$  are explained. D. E. Van Den Bout *et al.* use normalization technique to improve solutions of TSP[16]. However, the technique is hard to implement hardware since it is not natural flow in biological neural networks or the analog Hopfield

network model. In this paper, we use the sigmoid function and derive critical temperature from the function. The *spin* perturbations near  $T_c$  are small enough so that all the spins remain near their high temperature average of  $1/N$ . With this assumption, the effect mean field changes have on the *spins* are found from the sigmoid function in Eq.(8) to be

$$\frac{\partial s_{ik}}{\partial u_{ik}} = \frac{(N-1)}{N^2 T} \quad (11)$$

From Eq.(5),

$$\frac{\partial u_{ik}}{\partial s_{jl}} = \begin{cases} 0, & ik = jl \\ T_{ijkl}, & ik \neq jl \end{cases} \quad (12)$$

The change of  $s_{ik}$ ,  $\Delta s_{ik}$  cause the change of inputs of the other neurons  $\Delta u_{jl}$  as follows:

$$\begin{aligned} \Delta u_{jl} &= \frac{\partial u_{jl}}{\partial s_{ik}} \Delta s_{ik} + \sum_{j'l \neq ik} \frac{\partial u_{jl}}{\partial s_{j'l}} \Delta s_{j'l} \\ &= T_{ijkl} \Delta s_{ik} \end{aligned} \quad (13)$$

Now, the change of  $s_{jl}$ ,  $\Delta s_{jl}$  from the change of  $u_{jl}$  is:

$$\Delta s_{jl} = \frac{\partial s_{jl}}{\partial u_{jl}} \Delta u_{jl} = \frac{N-1}{N^2 T} T_{ijkl} \Delta s_{ik} \quad (14)$$

From the change of  $s_{jl}$ ,  $\Delta s_{jl}$ , the new input of  $ik$ th neuron  $\Delta u_{ik}^b$  is calculated:

$$\begin{aligned} \Delta u_{ik}^b &= \frac{\partial u_{ik}}{\partial s_{ik}} \Delta s_{ik} + \sum_{j'l \neq ik} \frac{\partial u_{ik}}{\partial s_{j'l}} \Delta s_{j'l} \\ &= \sum_{j'l \neq ik} T_{ijkl} \frac{(N-1)}{N^2 T} T_{ijkl} \Delta s_{ik} = \Delta s_{ik} \frac{(N-1)}{N^2 T} \sum_{j'l \neq ik} T_{ijkl}^2 \end{aligned} \quad (15)$$

Finally, we get the new perturbation  $\Delta s_{ik}^b$  from the Eq.(15)

$$\begin{aligned} \Delta s_{ik}^b &= \frac{\partial s_{ik}}{\partial u_{ik}} \Delta u_{ik} = \frac{(N-1)}{N^2 T} \frac{(N-1)}{N^2 T} \Delta s_{ik} \sum_{j'l \neq ik} T_{ijkl}^2 \\ &= \frac{(N-1)^2}{N^4 T^2} \Delta s_{ik} \sum_{j'l \neq ik} T_{ijkl}^2 \end{aligned} \quad (16)$$

In fact, the Hopfield network in the object matching problem is a fully connected network and the flow of the output change of neurons are very complicated. The result is based on the assumption that output changes of all the other neurons caused by the change of the  $ik$ th neuron  $\Delta s_{ik}$  are fed back to the  $ik$ th neuron and force the change of the  $ik$ th neuron to be accelerated when a temperature is near  $T_c$ . Therefore, we ignore the effect of other neuron outputs to simplify this problem. Let an average of connection strength be  $w$ . At the critical temperature, the spin perturbation must persist so that  $\Delta s_{ik}^b = \Delta s_{ik}$ . This results in:

$$\Delta s_{ik}^b = \frac{(N-1)^2}{N^4 T^2} \Delta s_{ik} N^2 w^2 = \frac{(N-1)^2}{N^2 T^2} \Delta s_{ik} w^2 \quad (17)$$

At  $T = T_c$ ,  $\Delta s_{ik}^b = \Delta s_{ik}$

$$T_c = \frac{N-1}{N} |w| \approx |w| \quad (18)$$

for  $N \gg 1$ .  $q$  is not emphasized because valid solution is not quite definite in this problem. Therefore  $q$  has relatively small

value as it does in the TSP problem.  $q$  is set to the unit value in the energy function.

### 3. EXPERIMENTAL RESULTS

Several models are obtained and used in the matching procedure to test the new algorithm. Figure-1 shows images of model objects and occluded images. Once images are obtained, boundary is extracted by the chain code method. After extracting the boundary, corner points are obtained by using the optimal boundary smoothing method based on the constrained regularization technique. Figure-2 shows the boundary and corner points of the model objects. The number of segments of the models and occluded images ranges from 6 to 25. From each segment, features are extracted: an angle as local feature and the distance between nodes as relational feature. The boundary segmentation algorithm is very reliable in the sense that it is not noise dependable and it keeps detecting the same corner points from an object in different scenes. However, some models in occluded images are occasionally oversegmented or lose some corner points under the same threshold value. They affect matching procedure as occluded parts does. A matching algorithm should be tolerable for the false segmentation occurred in preprocessing stage as well as occlusion. AHN shows good performance in the above situation. Fig.-2(c) and Fig.-3(d) shows robustness of the algorithm under over-segmentation as well as occlusion. The number of the model is 8 but in the occluded image, the model has 14 segments. 8 nodes of the model are exactly matched with those of images. Figure-3 shows output plots of AHN. Star signs indicate matched nodes between model and occluded images. The results show the desired matchings are successfully obtained.

We also experiment on the critical temperature to see if the parameter estimation is correct. As shown in Figure-5, annealing through the higher temperature is wasted work since it has no effect on the energy function. Instead, most of the optimization occurs near the critical temperature. There is a precipitous drop in the energy function around the critical temperature where a coagulation starts. Annealing through the low temperature does not improve the solution but serves only to saturate the neurons at 1 or 0. As shown in Figure-4, the experiment result of Figure-3(a) was 0.45 of  $T_c$  while the estimate of  $T_c$  was 0.7. This discrepancy may result from the small number of neurons of the example since we assume that the number of neurons are very large for mean field approximation. Therefore we have the  $T_c$  be the half of the estimated  $T_c$  to get a valid solution.

### 4. CONCLUSION

Issues related to the reliable matching have been discussed in this paper. Annealing the network allowed convergence to begin close to the critical temperature such that good solutions were found. By estimating the critical temperature, we can get a near optimal solution by few steps decreasing temperature. In conclusion, AHN gives a reliable matching of the corresponding segments between two objects. The method eliminates possibility for a part of an object to be matched to similar segments in a different object. We conclude that AHN is a robust approach to solve the two-dimensional occlusion problems.

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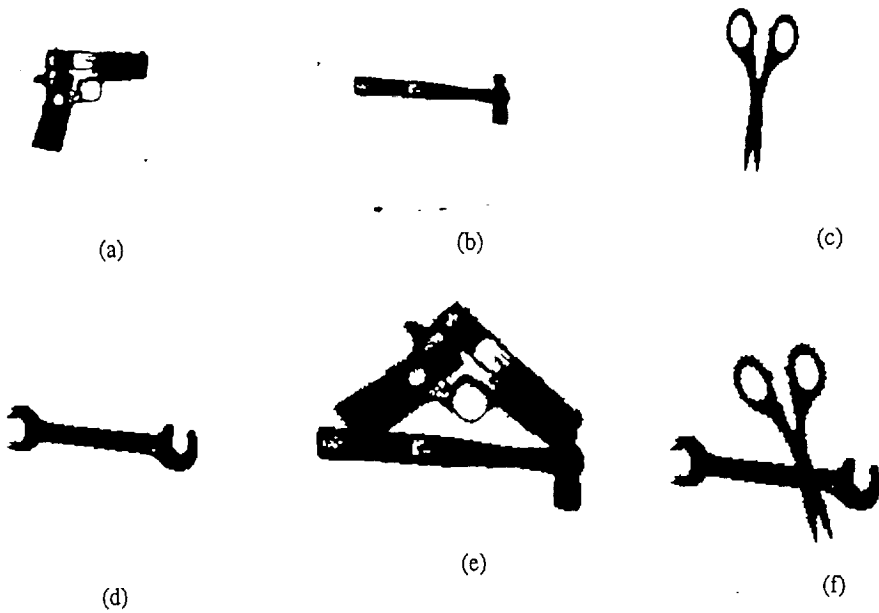


Figure-1: Model Images and Occluded Images

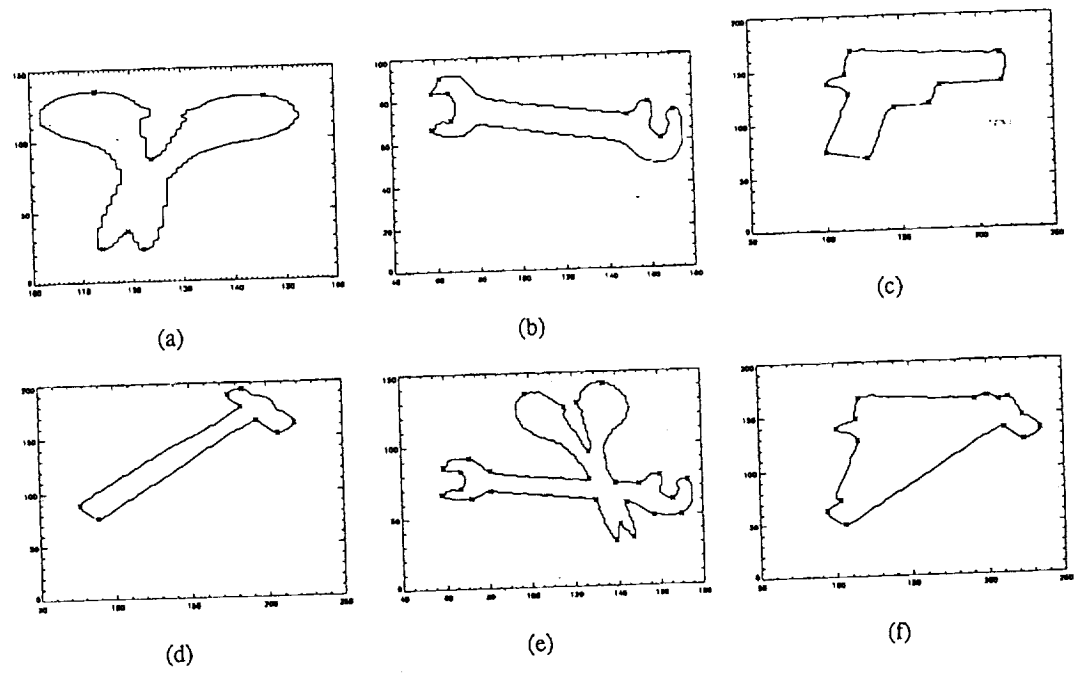


Figure-2: Boundary of Models and Occluded Images

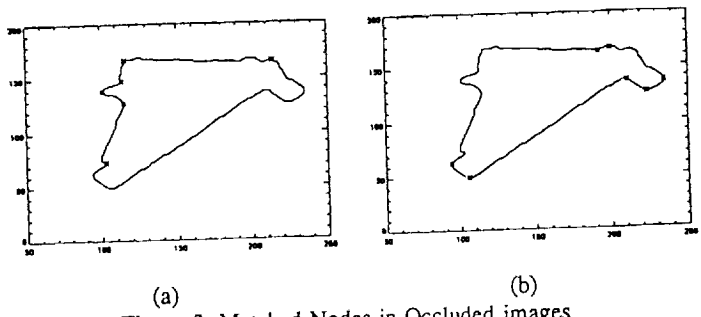


Figure-3: Matched Nodes in Occluded images

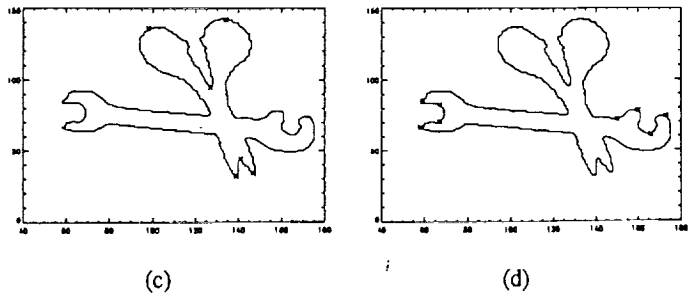


Figure-3: Matched Nodes in Occluded images(continued)

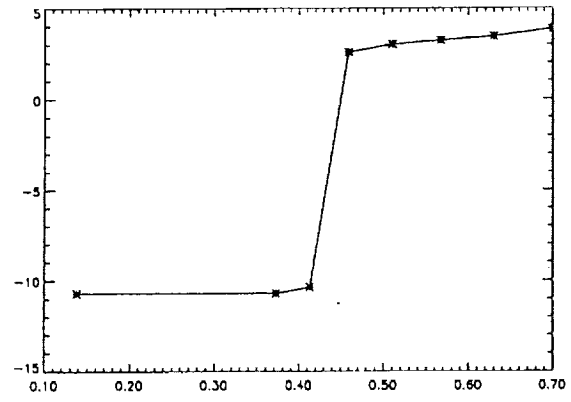


Figure-4: Energy vs. critical temperature