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A Two-Stage Magnetic Refrigerator for Astronomical Applications with Reservoir Temperatures Above 4 K

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Abstract

We propose a novel adiabatic demagnetization refrigerator (ADR) to produce temperatures as low as 100 mK starting from a high temperature reservoir between 4 and 8 K. The high temperature reservoir for the ADR can be provided by a mechanical cooler or an unpumped liquid helium bath. This refrigerator can be used to cool bolometric infrared detectors for low background astronomy from mountain tops, balloons or satellites as well as to cool cryogenic x-ray detectors. The two-stage ADR consists of a single magnet with a paramagnetic chromic-cesium-alum (CCA) salt pill to produce the low temperature and paramagnetic gadolinium-gallium-garnet (GGG) as the first stage to intercept heat from the high temperature reservoir. Thermal contact between the paramagnets and the reservoir during magnetization is made with a mechanical heat switch. The ADR is suspended with Kevlar

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chords under tension for high mechanical stiffness and low parasitic heat leak. In a single cycle, the ADR maintains a temperature of 100 mK for 10 to 100 hours. This time depends strongly on the magnetic field and reservoir temperature but not on the volume of the paramagnetic material as long as the heat leak is dominated by the suspension.

Keywords: space cryogenics;adiabatic demagnetization refrigerators

Introduction

Low background astronomical observations at millimeter and submillimeter wavelengths require very cold detectors for maximum sensitivity. Photoconductors perform poorly at wavelengths longer than $\approx 200 \mu\text{m}$, so bolometers are the detectors of choice. The sensitivity of bolometers increases rapidly with decreasing temperature and receiver development has progressed in that direction. ^3He cooled bolometers are widely used and magnetically cooled bolometers operating below 100 mK are beginning to be used. Bolometers working at 85 mK developed for the balloon-borne Millimeter Anisotropy Experiment (MAX) ¹ achieve noise equivalent powers (NEP's) of about $3 \times 10^{-17} \text{ W}/(\text{Hz})^{1/2}$ with time constants of order 20 ms.

Adiabatic demagnetization is the oldest technique for reaching milli-Kelvin temperatures ². It is a refrigeration cycle which is driven by a magnetic field and uses a paramagnetic material as the working substance. The

paramagnetic material has historically been called a salt pill because of the properties of the paramagnetic materials commonly used. The paramagnetic material is magnetized isothermally with an external field while it is in contact with a high temperature reservoir at temperature T . The degree of saturation of the magnetization depends on the ratio B/T . It is then isolated from the high temperature reservoir and demagnetized adiabatically by turning off the external field. The refrigeration cycle is efficient only when the magnetic working substance is in the paramagnetic state. Most paramagnetic materials have effective internal fields that cause a phase transition to an ordered magnetic state at some low temperature. The temperature of the phase transition sets the lowest temperature that can be reached.

Traditional single shot ADR's have been largely replaced by dilution refrigerators ³ in applications where continuous refrigeration and high cooling power are required. Infrared and x-ray bolometers however dissipate very little power so ADR's can be used. Such refrigerators are generally suitable for space applications since they do not rely on gravity and have no moving parts (except for mechanical heat switches). The efficiency is usually close to thermodynamic limits and is much better than for charcoal pumped ³He refrigerators. An ADR ^{4,5} operating at 100 mK was developed to provide long wavelength bolometric bands for NASA's Space Infrared Telescope Facility (SIRTF). A similar system ⁶ is under development for cooling x-ray micro-calorimeters on NASA's Advanced X-ray Astrophysics Facility (AXAF). Both ADR's require a superfluid ⁴He bath in order to reduce the heat flow from the high temperature reservoir into the paramagnetic material to an acceptable level. The SIRTF refrigerator is currently being used for a balloon

measurement of the cosmic microwave background (MAX). In this application the cold stage warms to ≈ 4 K on the launching pad where it is not convenient to use a mechanical pump on the ^4He bath. A charcoal pumped ^4He refrigerator is used to provide a stable reservoir temperature. In a similar refrigerator designed for astronomical observations at the south pole, a ^3He refrigerator is used for the same purpose ⁷. The European Space Agency (ESA) is planning to launch the FIRST satellite early in the next century. This satellite will employ SIS heterodyne mixers for doing millimeter wave molecular line spectroscopy. These receivers require temperatures ≈ 4 K, which can in principle be provided by mechanical refrigerators. The scientific mission of FIRST would be greatly enhanced by the availability of sensitive bolometric detectors for continuum measurements which, however, require much lower temperatures. The SIRTf ADR would not be suitable for this mission because of the excessive heat leak from a high temperature reservoir at ≈ 4 K, and the efficiency of charcoal pumped refrigerators is not high enough to be attractive for this application. The purpose of this paper is to describe a way in which space ADR technology can be extended so that it is compatible with reservoir temperatures above 2 K. An ADR such as the one designed for SIRTf can be modified by adding an intermediate temperature stage using a second magnetic working substance to intercept the heat leak from the high temperature reservoir to the lowest temperature region. Thermal guards of this type, usually called guard salt pills, have occasionally been used in magnetic refrigerators for producing milli-Kelvin temperatures ⁸. The guard stage uses a magnetic material with a high heat capacity and spin density and consequently a high ordering temperature

while the low temperature second stage uses a material with low heat capacity and ordering temperature. Several variations of this technique exist. In one the stage containing the guard pill is demagnetized first thereby pre-cooling the low temperature stage. The advantage of this method is that a smaller magnetic field is needed for the second stage to achieve a particular B/T ratio before demagnetization. It has the disadvantage that the magnet must be moved (or two magnets must be used). Also, a heat switch is required between the two stages. Another method demagnetizes both stages simultaneously. In this paper we will present a design for a refrigerator of the second kind which is more compact, has fewer parts and seems to be more easily space qualifiable.

Construction

The basic building blocks of the ADR are a superconducting magnet, a mechanical heat switch and two thermally isolated magnetic materials. The two stages are simultaneously magnetized while in contact with the high temperature reservoir and then adiabatically demagnetized until the second stage reaches the desired operating temperature $T \approx 100$ mK. This refrigeration cycle which can last ≈ 0.5 hour is then followed by a feedback regulated isothermal demagnetization mode lasting many hours in which the magnet current is slowly reduced in response to the external heat leak in order to maintain a constant temperature. A schematic diagram of a particular compact version of the ADR is shown in Figure 1. Both stages are suspended

in the magnet bore by Kevlar chords under tension. Thermal intercepts from the guard stage are attached to the Kevlar chords midway between the 100 mK stage and the high temperature reservoir. The basic building blocks of this system are a straightforward extension of the single stage SIRTf design ^{4,5}. The heat leak into the 100 mK stage in regulation mode is dominated by the parasitic leak through the suspension. In the SIRTf ADR, which has a reservoir temperature of 1.6 K (see Table 1), the heat leak through the Kevlar suspension is about 0.25 μ W. The resonance frequency of the ADR is proportional to $(AE/mL)^{1/2}$, where E is the elastic modulus of Kevlar, A and L its cross-sectional area and length respectively and m is the mass of the 100 mK stage. The hold time t of the ADR at its operating temperature is

$$t = \Delta Q / P \quad (1)$$

where ΔQ is the heat which the ADR can pump and P is the heat leak. Since ΔQ is proportional to the mass of the paramagnetic material and P is proportional to A/L, the hold time is independent of ADR size for a fixed resonance frequency ⁹.

The leak due to \approx 20 electrical leads in the SIRTf ADR has been reduced to $\approx 3 \times 10^{-8}$ W by the use of specially designed ribbon cables ⁵. This heat leak can be further reduced by using superconducting wire. The heat loads due to detector bias and infrared signals are generally small but there can be very significant infrared loading of the detectors when ambient temperature optics are used. Assuming telescope mirror temperatures of 200 K, a total mirror emissivity of 2 %, a 10 % bandwidth, and a conservative throughput of

$A\Omega = 10 \lambda^2$, we obtain a black body power loading which is linear in frequency. For example, at a frequency of 50 cm^{-1} ($\lambda = 200 \text{ }\mu\text{m}$) the power loading would be $1.6 \times 10^{-10} \text{ W}$ for a single pixel, in addition to the conduction heat leak. Thus an array of 100 detectors would have an unavoidable heat leak of $1.6 \times 10^{-8} \text{ W}$, which is 6 % of the parasitic heat leak for the salt pill suspension discussed above.

During magnetization, heat must be extracted from the paramagnetic material. Several types of heat switch are possible, including a superconducting heat switch ¹⁰, a gas gap heat switch ¹¹ and a mechanical heat switch ¹². Superconducting heat switches are generally used only below 1 K, because of the relatively large heat leak in the 'off' state at higher temperatures. The gas gap heat switch has also a finite heat leak in the 'off' state and a failure mode associated with gas leakage. We have had very good success with a mechanical switch. It has no parasitic heat leak in the 'off' state and is very reliable. The SIRTf switch has passed a room temperature shake test, where it was subjected to the vibration spectrum expected during a rocket launch. It has also been cycled more than 9000 times while held continuously at $T = 4 \text{ K}$ without loss in thermal conductance. Figure 2 shows the switch which was developed for SIRTf, but with modified jaws to allow simultaneous thermal contact between the reservoir and both stages. The switch is activated by passing current through the superconducting coil. The yoke is pulled into the magnet and forces the two jaws together thus clamping the two cold fingers extending from the ADR. The thermal conductance of the SIRTf switch in the 'on' state at $T = 4 \text{ K}$ is approximately 15 mW/K at 100 mA of solenoid current.

The magnet used in the SIRTF ADR is a ultra low current superconducting solenoid 8 cm long with a clear bore of 2.5 cm. The central field is 2.3 T at a current of 1.6 A. We propose a similar low current magnet for the two stage ADR since the heat load from the magnet leads on the high temperature reservoir must be kept small. A magnetic shield is used to protect field sensitive instruments in the vicinity of the magnet and to avoid damaging voltages in case of a quench. The induced voltage in a loop of area A in a fringing field B_f is AB_f / τ , where τ is the quench time which is typically of order 0.1 sec for the class of magnets proposed. A passive shield made of a single layer of high permeability, high saturation Vanadium Permendur (49% Fe, 49% Co, 2% V) was developed for SIRTF. This shield ⁴ reduced the fringing field to less than 0.1 mT at distances of more than 3 cm outside the shield. Fields at this level will not significantly influence the performance of SIS mixers. They can easily be further reduced by light-weight ferromagnetic shielding if desirable. The largest values of induced emf during quench would be $\leq 1\mu\text{V}$. Another approach to the problem of magnetic shielding is a magnet with additional windings ¹³ to cancel out the field outside the bore.

Paramagnetic Materials

The paramagnetic material gadolinium-gallium-garnet (GGG) which has the chemical formula $\text{Gd}_3\text{Ga}_5\text{O}_{12}$, is an attractive working substance for a guard stage in the liquid helium temperature range. It is nearly magnetically

isotropic ¹⁴ with $g = 2$, angular momentum $J=7/2$, has a Gd ion density of $1.27 \times 10^{22} \text{ cm}^{-3}$ and a density of 7.1 g cm^{-3} . It orders magnetically near $T = 1 \text{ K}$. The resulting large zero field heat capacity shown in Figure 3 makes GGG an effective thermal buffer at this temperature. The thermal conductivity ¹⁵ of GGG is $\approx 0.1 \text{ W/cm-K}$ at 4 K , which is comparable to that of OFHC copper. This facilitates the extraction of heat during magnetization. Thermal contact to GGG can easily be made by gluing copper strips to the crystal surface. Garnets have the additional advantage of being chemically and thermally stable. Rod shaped single crystals of GGG are commercially available.

The material of choice for the second stage is the hydrated paramagnetic salt chromic-cesium-alum (CCA), which has the chemical formula $\text{CsCr}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$. It is magnetically similar to chromic-potassium-alum (CPA) and about 30 % less effective per unit mass than ferric-ammonium-alum (FAA), but dehydrates at a higher temperatures than these alternatives ¹⁶. This is important for satellite application, since large space cryogenic vacuum systems typically must be baked at temperatures approaching 30°C for periods of up to 2 weeks ¹⁷. The magnetic Cr^{3+} ions in this salt have $g=2$, $J=3/2$ and a density of $2.1 \times 10^{21} \text{ cm}^{-3}$. The quadruplet is split by the crystal electric field into two doublets ¹⁸ separated by an energy of 0.19 K and the remaining degeneracy is lifted due to magnetic interactions at temperatures near 10 mK . The thermal conductivity of CCA is much lower than of GGG. Good thermal contact is made in the SIRTf ADR by growing the crystals directly on a skeleton of gold wires in an aqueous solution ¹⁶. The pill is afterwards sealed in a stainless steel can to prevent dehydration. Attention must be paid to sources of eddy current heating. The general formula for

power dissipation per unit length in a ring of conductivity σ , radius r , and thickness dr with axis parallel to B is given by

$$dP = \pi \sigma (dB/dt)^2 r^3 dr / 2. \quad (2)$$

Materials with high electrical conductivity shaped in form of closed loops with large radii should be avoided. The eddy current heating for the SIRTf design is $\leq 1 \mu\text{W}$ during adiabatic demagnetization.

Performance

We have calculated the hold time of the ADR shown in Figure 1 as a function of magnetic field and reservoir temperature with the results shown in Figure 4. The weight and suspension parameters were adopted from the SIRTf ADR and are listed in Table 1. The complete CCA salt pill including 0.1 moles of CCA, the gold wires, copper rod and stainless steel can weigh 100 g, and an additional 100 g is assumed for the weight of the bolometer stage. The mass of the GGG stage is assumed to be dominated by the garnet and is fixed at 200 g, corresponding to 0.2 moles of GGG. The entropy reduction in GGG during magnetization was calculated from published data^{14,19}. For CCA, the entropy was calculated using the Brillouin function for an ideal paramagnet plus the lattice entropy²⁰. Following demagnetization to 100 mK, the residual field is of order 100 mT and the entropy data¹⁴ are used to calculate the GGG temperature.

The heat leak into the GGG is the combination of the flow through its own suspension and through the thermal intercepts on the CCA suspension. The attachment point of the intercepts is fixed to be midway between the CCA pill and the bath which is close to optimum for maximum hold time. The heat leak is calculated using the thermal conductivity ²¹ of Kevlar $\kappa = 20 \text{ T}^2 \mu\text{W/cm K}^3$ for temperatures near 1 K. The warm-up rate of the GGG is calculated using the zero field heat capacity data from Figure 3, which is justified for the small residual field strength. The leak P into the 100 mK stage is determined by the GGG temperature. The hold time at $T = 100 \text{ mK}$ is given by $t = 0.1\Delta S / \langle P \rangle$, where $\Delta S = S(B=0T, T=0.1K) - S(B_0, T_0)$ is the available entropy of the CCA pill at 100 mK, and $\langle P \rangle$ is the average heat leak. In an actual refrigerator, ΔS will be further reduced by the entropy lost in cooling the various parts of the 100 mK stage. For the assumptions given above this will reduce the available entropy by 10-20 % for $B_0/T_0 = 1 \text{ T/K}$. The zero field entropy was taken from the entropy curve for CPA ²². As can be seen in Figure 4, hold times of 24 hours or longer with reservoir temperatures around 4 K are achievable for a modest field of 4 Tesla. It would be possible to increase the hold time at 100 mK by increasing the mass of GGG while strengthening its suspension. This will asymptotically approach the limiting case when the heat flow into the GGG is entirely through its own suspension. An increase of order 50 % in hold time would be achievable by increasing the amount of GGG by a factor of 3. Assuming $G = 15 \text{ mW/K}$ from the SIRTIF heat switch, the time constant for isothermal magnetization is limited by the heat capacity of GGG to $C/G = 15 \text{ min}$. The duty cycle of the ADR at 100 mK for these parameters can thus be more than 95 %.

Small temperature fluctuations in the high temperature reservoir associated with mechanical coolers are not critical to the performance of the ADR. Because of the large heat capacity of GGG and the small thermal conductance of the Kevlar suspension, the time constant will be more than 10 days and will filter out thermal oscillations. The thermal loading from the ADR on the high temperature reservoir will be dominated by the magnet leads. This can be much reduced by use of superconducting wire up to some intermediate temperature stage. Nb₃Sn wire can be used to T= 12 K and it is hoped that the new YBa₂Cu₃O₇ materials can be used up to T= 80 K.

Conclusion

The two-stage ADR presented here will be useful for cooling infrared and x-ray detectors to temperatures near 100 mK in several situations. Future space missions employing mechanical coolers can use it for astronomical observations. It will also be convenient for ground based observations since it can cool detectors to 100 mK without pumping on the liquid helium bath. Finally it would permit a balloon launch of detectors at 100 mK without pumped liquid helium.

Acknowledgements

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Table 1: Mass and suspension parameter of the SIRTf ADR.

CCA amount	0.1 moles
mass of 100 mK stage	200 g
resonance frequency	200 Hz
Kevlar cross section	10^{-3} cm^2
total A/L of suspension	$5 \times 10^{-3} \text{ cm}$
suspension heat leak	$0.25 \text{ } \mu\text{W}$
reservoir temperature	1.6 K

Figure Captions

Fig.1: Schematic diagram of 2-stage ADR with CCA salt pill (a), GGG crystal (b), superconducting magnet (c), magnetic shield (d), Kevlar suspension (e), and thermal intercepts (f).

Fig.2: Mechanical heat switch with superconducting coil (a), ferromagnetic core (b), gold plated copper jaws (c), flex pivots (d), cold fingers (e) and restoring spring (f). The flex pivots correct for small misalignments of the cold fingers extending from the two stages.

Fig.3: Specific heat of GGG per mole of Gd^{3+} in zero magnetic field and the entropy at various magnetic fields. From reference 14.

Fig.4: ADR hold time at $T=100$ mK as a function of magnetic field and reservoir temperature with 0.2 mole of GGG and 0.1 mole of CCA. The entropy lost in cooling the various components of the 100 mK stage is not included. The parasitic heat leak is assumed to be dominated by that of the Kevlar suspensions. For both suspensions, $A/L=0.005$ cm.

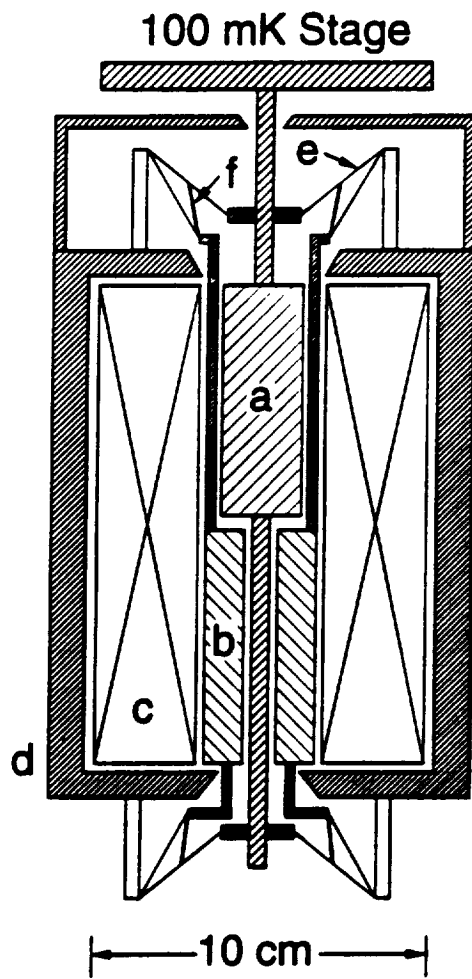


FIGURE 1

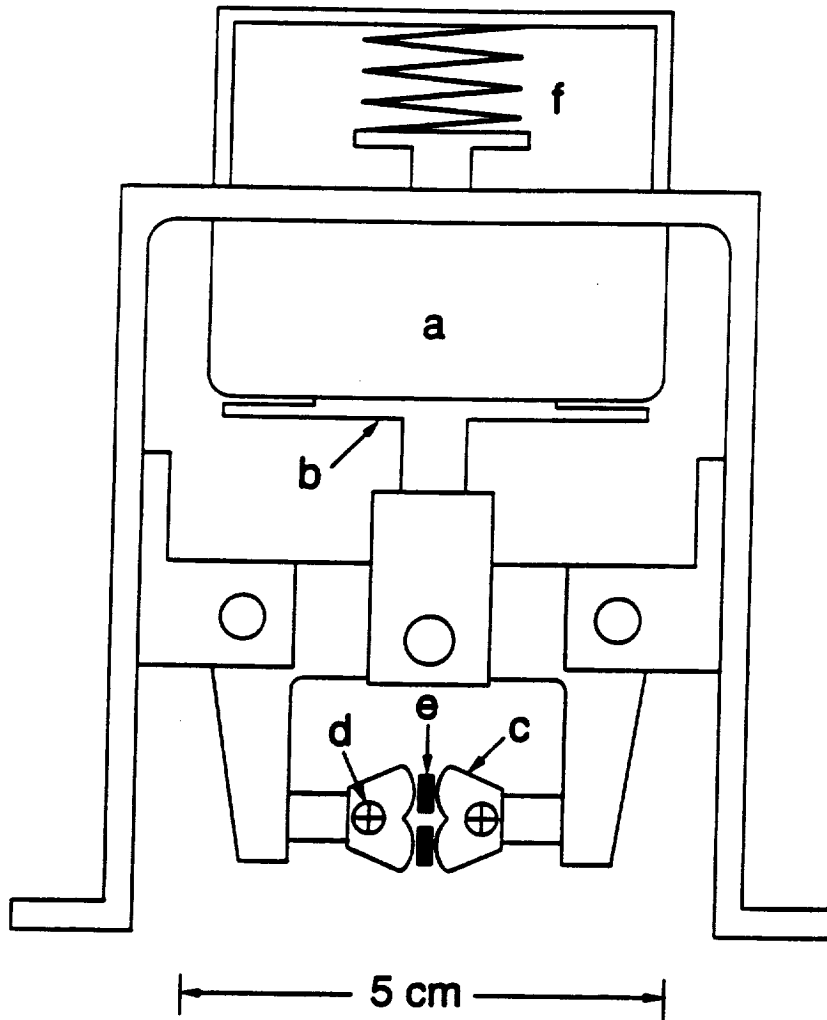


FIGURE 2

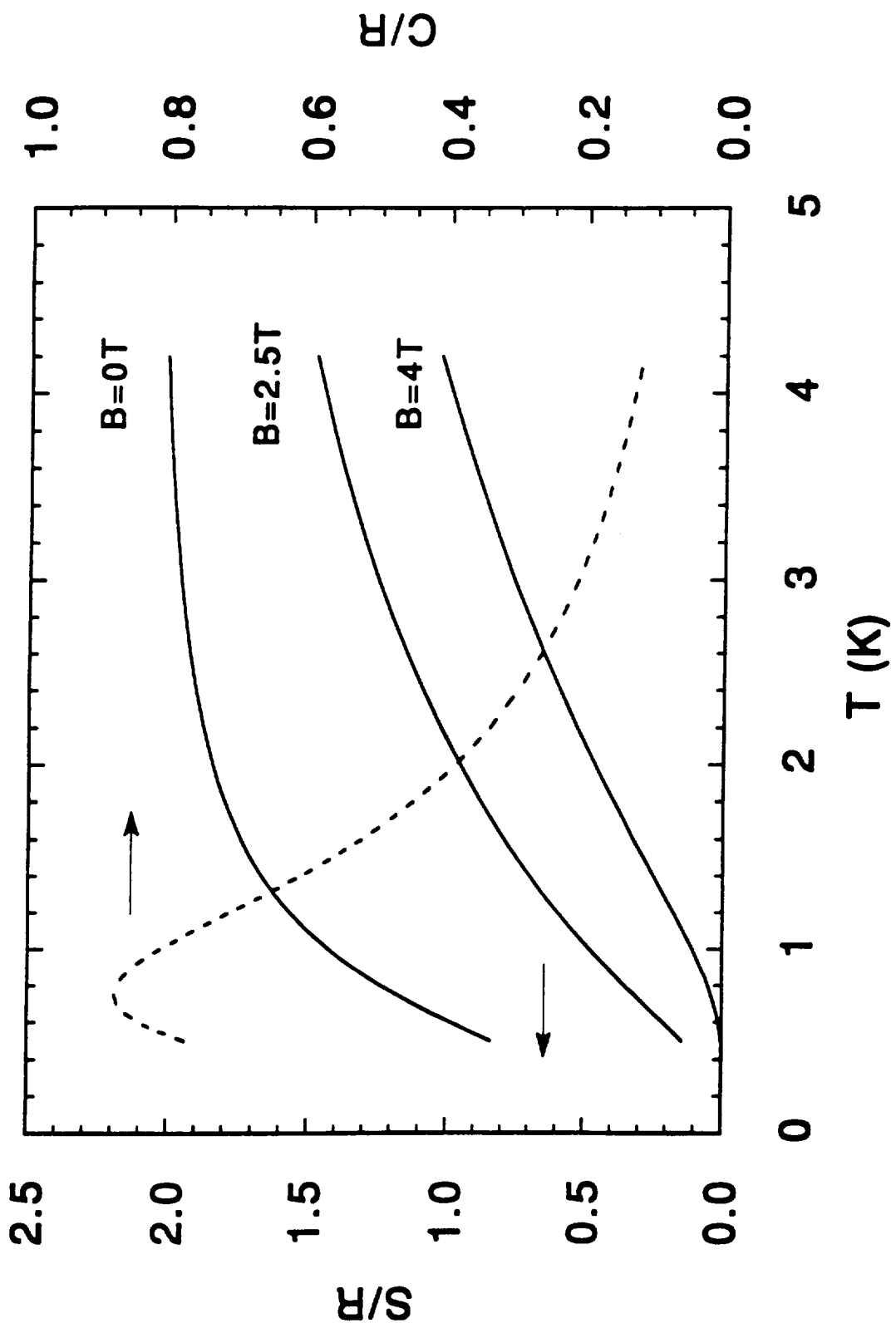


FIGURE 3

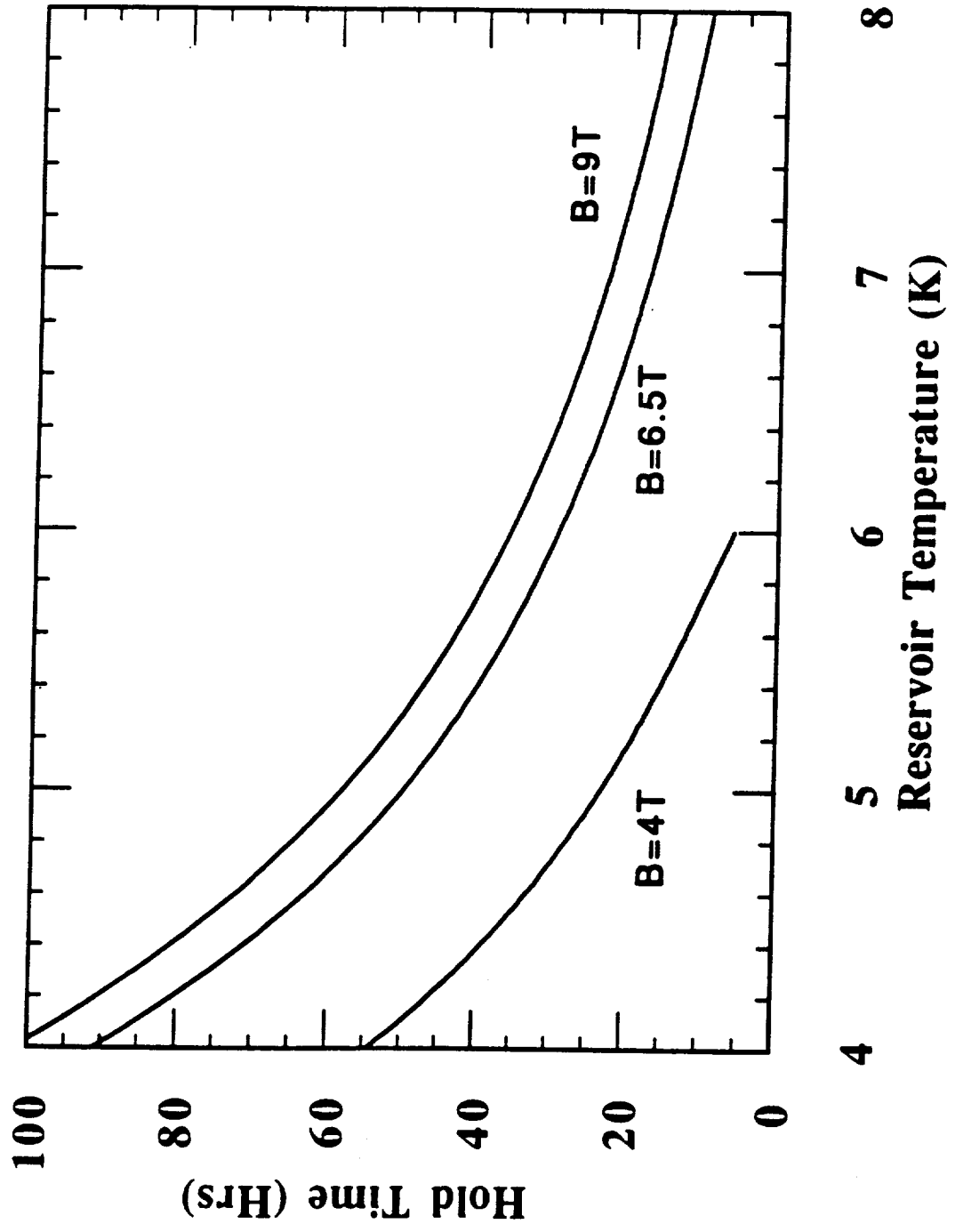


FIGURE 4