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Mathematical Relationships Between Two Sets of Laser Anemometer Measurements for Resolving the Total Velocity Vector

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MATHEMATICAL RELATIONSHIPS BETWEEN TWO SETS OF LASER ANEMOMETER MEASUREMENTS FOR RESOLVING THE TOTAL VELOCITY VECTOR

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SUMMARY

The mathematical relations between the measured velocity fields for the same compressor rotor flow field resolved by two fringe type laser anemometers at different observational locations are developed in this report. The relations allow the two sets of velocity measurements to be combined to produce a total velocity vector field for the compressor rotor. This report presents the derivation of the mathematical relations, beginning with the specification of the coordinate systems and the velocity projections in those coordinate systems. The vector projections are then transformed into a common coordinate system. The transformed vector coordinates are then combined to determine the total velocity vector. A numerical example showing the solution procedure is included.

E-7511

INTRODUCTION

The laser anemometer, in it's various forms, has been used for over 15 years to nonintrusively measure flow in turbomachinery. This ability has made the laser anemometer ideal for measuring fluid velocities in the rotating components of turbomachines. Typically, the systems used can measure a maximum of two absolute velocity components; these, normally, are the axial throughflow component and the tangential component (fig. 1).

Since the third velocity component, the radial component, is small with respect to the two measured components, it has commonly been ignored. However, improved blade manufacturing techniques, improved computational analysis capabilities, and an increased emphasis on secondary flow losses have generated a greater interest in the radial velocity component.

As a result, the Army Propulsion Directorate has been engaged in an experimental program to measure the radial velocities in a typical axial flow compressor rotor. The experimental phase of this program has been completed, and the data are currently being reduced and processed for future publication. This report presents the mathematical derivation of the equations used to calculate all three components of the total velocity vector. To obtain the required information, the flow was observed from two different locations, and the compressor system geometry and both laser anemometer measurement system geometries were used.

SYMBOLS

A,B,C coefficients describing plane defined by $\overline{k}'$ unit vector and measured radial velocity projection

E,F,G coefficients describing plane defined by $\overline{i}''$ unit vector and the measured conventional velocity projection

$\overline{e}$ vector projection of unit vector $\overline{j}'$ onto the θ-axis

$\overline{g}$ vector projection of unit vector $\overline{j}'$ into the R-θ plane

h	distance along the axis of optical rotation from the R-θ plane to the radial velocity system reflecting mirror
$\bar{i},\bar{j},\bar{k}$	primary coordinate system unit vectors
N	unit vector in the direction of velocity vector
P	equation of plane
R	radial distance from compressor axis of rotation
R_F	line of intersection between R-θ plane and plane specified by axis of optical rotation and the probe volume
V_c	conventional velocity vector projection
V_R	radial velocity vector projection
X	radial velocity laser system bisecting line
γ	angle between the measured conventional velocity projection and the θ''-axis
Δ	angle between tangential velocity and total velocity
δ	angle of rotation about the Z-axis of the conventional laser anemometer system; offset angle
ε	$\xi - 90°$
κ	angle between the measured radial velocity projection and the θ'-axis
Λ	dot product of N_t and $\bar{i}$
λ,μ,ν	direction cosines of the radial coordinate system in the primary system
ξ	angle formed by line perpendicular to the axis of compressor rotation through the probe volume and line R_F
Φ	angle between h and X
ω	angle between the total velocity vector and $\bar{k}$

Subscripts

c	conventional
op	radial velocity laser anemometer axis of optical rotation
pv	probe volume
R,θ,Z	specifiers for R,θ,Z components of the appropriate coordinate system
t	total

Superscripts

$'$	radial laser anemometer coordinate system
$''$	conventional laser anemometer coordinate system

LASER ANEMOMETER THEORY

The theory underlying the optical operation of a laser anemometer system is relatively straightforward, and many previous publications have presented excellent descriptions (refs. 1 to 3). Therefore, this report will review only those features which are pertinent to the derivation of the total velocity vector.

The laser anemometer system used by the Propulsion Directorate is a single-channel, fringe type system. In this type of system, a laser beam is divided into two parallel beams which are then focused to a volume in space. A measurement volume, ellipsoidal in shape, is created by the intersection of the two laser beams (fig. 2(a)). In this measurement volume or "probe volume," an interference pattern is generated (fig. 2(b)). The fluid velocity component perpendicular to the interference pattern can be calculated by measuring the frequency of light reflected or emitted by a particle immersed in the fluid that enters the probe volume (fig. 2(c)).

Normally, two velocity components can be measured from one physical location with respect to the compressor passage of the focusing lens. This is accomplished by rotating the probe volume about a line that bisects the angle between the two intersecting laser beams (fig. 2(d)), thereby reorienting the interference pattern. This bisecting line runs through the center of the focusing lens of the system. It should be noted that the velocity component measured is determined by the orientation of the interference pattern and, thus, the location of the focusing lens. However, the light reflected from the particles can be observed from any physical location. Obviously, two additional velocity measurements can be made by moving the focusing lens and observing, from another position, the same physical location in the compressor. This was the approach used in the Propulsion Directorate experiments.

EXPERIMENTAL PROCEDURE

The experimental program was intended to measure the total velocity field inside an isolated transonic compressor rotor operating subsonically. The compressor rotor was designed at the Lewis Research Center as a transonic axial inlet stage. Anemometry data were acquired between September 1987 and May 1990 with a single component laser anemometer of the fringe type. Two sets of two velocities were measured at a given probe volume axial/radial location in the compressor rotor. The two sets were taken from two different physical locations of the focusing lens, thereby resulting in two different orientations of the probe volume and, thus, in the measurement of two different sets of velocity components. The first component measured consisted of velocities in the axial and the circumferential (tangential) direction. This is the conventional laser anemometer orientation. The focusing lens was located outside the rotor casing and radially over the compressor rotor (fig. 3(a)). The second measured component consisted of velocities nearly in the radial direction, and a component that was a combination of the axial-tangential velocity. The focusing lens of this radial-flow laser anemometer system was located upstream of the rotor and outside the rotor casing. The two beams entered the flow passage through a window positioned so that the bisecting line was oriented in approximately the same direction as the blade mean camber line (fig. 1). A schematic of the radial laser anemometer optical orientations, showing the relation of the focusing lens to the compressor rotor and the velocity measurement volume (probe volume), appears in figure 3(b).

The result of the experimental phase of this program was a pair of data sets, each consisting of a different measured velocity field. The successful combination of these two fields resolves the total velocity vector field. Currently, the data are being reduced in preparation for publication.

DERIVATION OF THE TOTAL VELOCITY VECTOR

Background

For each pair of data sets, the data acquisition software provides a velocity magnitude and an angle from the appropriate coordinate system axis; these are used for the resolution of the total velocity vector. The geometry of the compressor system and each laser system were also known.

Since a laser anemometer system measures a fluid particle's velocity component perpendicular to the interference pattern, it is measuring the particle velocity component in a plane perpendicular to a line bisecting the angle between the two laser beams that create the probe volume. Thus the observed velocity can be considered to be a projection of the total velocity vector onto an "observation plane" perpendicular to the bisecting line, as shown in figure 4.

It can be seen from figure 5 that the total velocity vector lies in a plane described by the bisecting line and the velocity projection on the observation plane. If the same velocity vector is observed from two different locations, it follows that the total velocity vector must lie along a line that defines the intersection of the two velocity planes.

Coordinate Systems

This section will define three coordinate systems that are convenient for describing the two measured velocity projections and the resolved total velocity components.

There are three coordinate systems used for this derivation. They are (1) the primary coordinate system, in which the total velocity vector is described (fig. 6(a)); (2) the conventional coordinate system, in which the data acquired from the conventional laser anemometer system are described (fig. 6(b)); and (3) the radial coordinate system, in which the data from the radial-flow laser anemometer system are described (fig. 6(c)). For the purposes of this derivation, the two probe volumes are assumed to be collocated in space. Thus, the origins of all three coordinate systems are at the same position in space.

Primary coordinate system.—The primary coordinate system (fig. 6(a)) uses the turbomachine geometry to aid in it's specification. The R-axis ($\overline{i}$) is defined as positive and being radially outward along a line defined by the probe volume position and the axis of compressor rotation. The θ-axis ($\overline{j}$) is specified as a line in a plane defined as perpendicular to the axis of compressor rotation and perpendicular to the R-axis. Further, it is defined as positive in the direction of compressor rotation (clockwise viewed looking downstream). The third axis, the Z-axis ($\overline{k}$), is defined as parallel to the axis of compressor rotation and is positive in the direction of primary fluid flow.

Conventional coordinate system.—The conventional data coordinate system and its relationship to the primary coordinate system are shown in figure 6(b). Since a laser anemometer can only resolve a velocity component that is perpendicular to its bisecting line (line-of-sight), it is convenient to define the bisecting line as an axis of the conventional coordinate system. This results in the observed velocity vector being in a plane specified by the other two axes of the coordinate system.

The conventional laser anemometer system used for this program was capable of altering the line-of-sight only in a plane defined as perpendicular to the axis of compressor rotation. Therefore, the bisecting line for the conventional laser anemometer system is defined to lie in a plane that is perpendicular to the axis of compressor rotation, although it does not necessarily lie along a line radially outward from the axis of compressor rotation. The R″-axis ($\overline{i}''$) is defined as lying along the bisecting line and is positive in the direction opposite the observation direction.

The conventional coordinate system has it's Z″-axis ($\overline{k}''$) defined as parallel to the axis of compressor rotation. Therefore, the R″- and θ″-axes lie in the same plane as the R and θ of the primary coordinate system (fig. 6(b)). The Z″-axis is defined as positive in the direction of the primary fluid flow. For a right handed coordinate system, the θ″-axis ($\overline{j}''$) is positive in the direction of compressor rotation.

Radial coordinate system.—The radial velocity coordinate system and its relationship to the compressor geometry and primary coordinate system are shown in figure 6(c). The Z′-axis ($\overline{k}'$) is defined along the radial flow system bisecting line. This axis is defined as positive in the direction of a vector from the center of the focusing lens towards the probe volume.

The radial flow laser anemometer system was designed so that it positioned the probe volume by rotating about an axis that was outside of the compressor casing, but parallel to the compressor axis of rotation. A coordinate axis, θ′ ($\overline{j}'$), was defined to lie in the plane specified by this axis of optical rotation and the probe volume. This axis is perpendicular to the bisecting line and is positive in the approximate direction of compressor rotation. Notice in figure 6(c) that, for any Φ other than 0°, θ′ is not in the R-θ plane. Thus, the radial coordinate system has its θ′- and Z′-axes in a plane specified by the axis of optical rotation of the radial flow laser anemometer system and the location of the probe volume.

The third axis, R′ ($\overline{i}'$), is perpendicular to both the θ′- and Z′-axes and positive in an outward radial direction.

Description of the Measured Velocity Vectors in the Primary Coordinate System

To determine the total velocity vector, we must define the two observed velocity vectors in the same coordinate system, in this case, the primary coordinate system. Initially, the measured velocity vectors are referenced in their original coordinate systems. Then they are transformed into the primary coordinate system by coordinate transformation.

Transformation of the radial coordinate system velocity vector.—As shown in figure 6(c), the Z′-axis lies along the bisecting line and in a plane described by the center of the probe volume and the axis of optical rotation. For the purposes of data analysis, it was assumed that the probe volume is a point in space. The axis of optical rotation, a line that is parallel to the axis of compressor rotation, is a distance of R_F from the axis of compressor rotation and intersects the bisecting line. The optical system is rotated about this axis to vary the distance between the probe volume and the center of compressor rotation. The lengths of all three sides of the triangle (R_F, R_{pv}, and R_{op}) are known from the compressor geometry and the optical geometry, as shown in figure 7. Therefore, applying the law of cosines

$$\xi = \cos^{-1}\left(\frac{R_{pv}^2 + R_F^2 - R_{op}^2}{2R_F R_{pv}}\right) \tag{1}$$

where R_{pv} is the distance from the probe volume to the center of the compressor, and R_{op} is the distance from the probe volume to the axis of rotation of the radial velocity laser anemometer. Subtracting $\pi/2$ gives

$$\epsilon = \xi - 1.5708 \tag{2}$$

Figure 8 shows the plane defined by the axis of optical rotation (h) and the probe volume. From this figure

$$h = R_F \cot \Phi \tag{3}$$

$\bar{X}$ is defined relative to the primary coordinate system as

$$\bar{X} = (-R_F \sin \varepsilon)\bar{i} + (-R_F \cos \varepsilon)\bar{j} + (R_F \cot \Phi)\bar{k} \tag{4}$$

The unit vector $\bar{k}'$ is defined to be in the same direction as the $\bar{X}$ vector, and relative to the primary coordinate system, it is

$$\bar{k}' = (-\sin \Phi \sin \varepsilon)\bar{i} + (-\sin \Phi \cos \varepsilon)\bar{j} + (\cos \Phi)\bar{k} \tag{5}$$

In figure 8, the projection of $\bar{j}'$, a unit vector along the θ'-axis, onto the R-θ plane along R_F has a magnitude of

$$|\bar{g}| = \cos \Phi \tag{6}$$

and $\bar{g}$ can be described in the primary coordinate system as

$$\bar{g} = (\cos \Phi \sin \varepsilon)\bar{i} + (\cos \Phi \cos \varepsilon)\bar{j} \tag{7}$$

The magnitude of the projection of $\bar{j}'$ onto the Z-axis in the primary coordinate system is

$$|\bar{e}| = \sin \Phi \tag{8}$$

From equations (7) and (8), the unit vector $\bar{j}'$ in the primary coordinate system is

$$\bar{j}' = (\cos \Phi \sin \varepsilon)\bar{i} + (\cos \Phi \cos \varepsilon)\bar{j} + (\sin \Phi)\bar{k} \tag{9}$$

The third unit vector, relative to the primary coordinate system $\bar{i}'$, can be obtained simply by taking the crossproduct of $\bar{j}'$ and $\bar{k}'$:

$$\bar{i}' = (\cos \varepsilon)\bar{i} + (-\sin \varepsilon)\bar{j} + (0)\bar{k} \tag{10}$$

Thus, the transformation from the radial flow coordinate system to the primary coordinate system is defined by equations (10), (9), and (5) as

$$\begin{aligned}
\bar{i}' &= (\cos \varepsilon)\bar{i} + (-\sin \varepsilon)\bar{j} + (0)\bar{k} \\
\bar{j}' &= (\cos \Phi \sin \varepsilon)\bar{i} + (\cos \Phi \cos \varepsilon)\bar{j} + (\sin \Phi)\bar{k} \\
\bar{k}' &= (-\sin \Phi \sin \varepsilon)\bar{i} + (-\sin \Phi \cos \varepsilon)\bar{j} + (\cos \Phi)\bar{k}
\end{aligned}$$

In these equations, both Φ and ε can be calculated from the compressor and optical system geometries.

The nine coefficients shown in the preceding equations are the direction cosines of the radial flow coordinate system with respect to the primary coordinate system. Thus,

$$
\begin{aligned}
\lambda_1 &= (\cos\ \varepsilon) & \mu_1 &= (\cos\ \Phi\ \sin\ \varepsilon) & \nu_1 &= (-\sin\ \Phi\ \sin\ \varepsilon) \\
\lambda_2 &= (-\sin\ \varepsilon) & \mu_2 &= (\cos\ \Phi\ \cos\ \varepsilon) & \nu_2 &= (-\sin\ \Phi\ \cos\ \varepsilon) \\
\lambda_3 &= 0 & \mu_3 &= (\sin\ \Phi) & \nu_3 &= (\cos\ \Phi)
\end{aligned}
\tag{11}
$$

Therefore, any vector defined in the radial flow coordinate system can be transformed into the primary coordinate system by using the following relationships (ref. 4):

$$
\begin{aligned}
R &= \lambda_1 R' + \mu_1 \theta' + \nu_1 Z' \\
\theta &= \lambda_2 R' + \mu_2 \theta' + \nu_2 Z' \\
Z &= \lambda_3 R' + \mu_3 \theta' + \nu_3 Z'
\end{aligned}
\tag{12}
$$

Figure 9 shows the projection of the velocity vector that was measured by the radial flow laser anemometer. The measured projection of the velocity vector in the radial flow coordinate system is

$$
\overline{V}_R = |V_R|\left[(\sin\ \kappa)\bar{i}' + (\cos\ \kappa)\bar{j}' + (0)\bar{k}'\right] \tag{13}
$$

where κ is the angle between the θ'-axis and the velocity vector projection. Normalizing the vector $\overline{V}_R$ and substituting the calculated direction cosines from equation (12) gives

$$
\overline{N}' = (\lambda_1\ \sin\ \kappa + \mu_1\ \cos\ \kappa)\bar{i} + (\lambda_2\ \sin\ \kappa + \mu_2\ \cos\ \kappa)\bar{j} + (\lambda_3\ \sin\ \kappa + \mu_3\ \cos\ \kappa)\bar{k} \tag{14}
$$

where $\overline{N}'$ is a unit vector in the primary coordinate system with the same direction as the projection of the total velocity vector in the radial flow coordinate system. The measured projection of the velocity vector is the product of $\overline{N}'$ and the measured velocity magnitude $|V_R|$.

Transformation of the conventional coordinate system velocity vector.—The same procedure is used to transform the velocity projection measured in the conventional coordinate system to the primary coordinate system. The transformation is simplified because the Z- and Z″-axes are congruent. This equality exists because the optical system for this research always has the bisecting line in the R-θ plane of the primary system. Therefore in figure 6, which shows the conventional coordinate system and the primary coordinate system,

$$
\bar{k}'' = \bar{k} \tag{15}
$$

From figure 6(b)

$$
\bar{i}'' = (\cos\ \delta)\bar{i} + (\sin\ \delta)\bar{j} \tag{16}
$$

and

$$\bar{j}'' = (-\sin\ \delta)\bar{i} + (\cos\ \delta)\bar{j} \tag{17}$$

Thus, by using equations (16), (17), and (15), the conventional coordinate system can be described in the primary coordinate system as

$$\begin{aligned}
\bar{i}'' &= \quad (\cos\ \delta)\bar{i} + (\sin\ \delta)\bar{j} + (0)\bar{k} \\
\bar{j}'' &= (-\sin\ \delta)\bar{i} + (\cos\ \delta)\bar{j} + (0)\bar{k} \\
\bar{k}'' &= \qquad\quad (0)\bar{i} + \qquad (0)\bar{j} + (1)\bar{k}
\end{aligned}$$

As in the radial velocity coordinate system, a velocity vector projection in the conventional coordinate system is described as

$$\bar{V}_c = |V_c| \left[(\cos\ \gamma)\bar{j}'' + (\sin\ \gamma)\bar{k}\right] \tag{18}$$

where γ is the angle between the velocity vector projection and the θ''-axis (fig. 10). By substituting equation (17) into equation (18) and normalizing, we obtain a unit vector in the direction of the velocity vector projection, relative to the primary coordinate system:

$$\bar{N}'' = (-\sin\ \delta\ \cos\ \gamma)\bar{i} + (\cos\ \delta\ \cos\ \gamma)\bar{j} + (\sin\ \gamma)\bar{k} \tag{19}$$

Resolution of the Total Velocity Vector

For either the conventional laser anemometer coordinate system or the radial flow coordinate system, the projection of the total velocity vector lies in a plane described by the bisecting line and the velocity projection for that system (fig. 5). Therefore the total velocity vector must lie along a line that is the intersection of the two planes (fig. 5). To determine the total velocity vector, the two planes must be defined and the intersection calculated.

Description of the velocity vector projection planes.—The velocity vector projection planes can be described by taking the crossproduct of two unit vectors. One of these is in the direction of the bisecting line, and the other is the appropriate velocity unit vector. For the radial flow coordinate system, the two unit vectors are the $\bar{k}'$ vector from equation (5):

$$\bar{k}' = (-\sin\ \Phi\ \sin\ \varepsilon)\bar{i} + (-\sin\ \Phi\ \cos\ \varepsilon)\bar{j} + (\cos\ \Phi)\bar{k} \tag{5}$$

and the $\bar{N}''$ vector from equation (14):

$$\bar{N}' = (\lambda_1\ \sin\ \kappa + \mu_1\ \cos\ \kappa)\bar{i} + (\lambda_2\ \sin\ \kappa + \mu_2\ \cos\ \kappa)\bar{j} + (\lambda_3\ \sin\ \kappa + \mu_3\ \cos\ \kappa)\bar{k} \tag{14}$$

The crossproduct of $\bar{k}'$ and $\bar{N}'$ yields an equation describing the plane:

$$P_R = A\bar{i} + B\bar{j} + C\bar{k} \tag{20}$$

where

$$\begin{aligned} A &= -\{(\sin\Phi\ \cos\varepsilon)(\lambda_3 \sin\kappa + \mu_3 \cos\kappa) + (\cos\Phi)(\lambda_2 \sin\kappa + \mu_2 \cos\kappa)\} \\ B &= +\{(\sin\Phi\ \sin\varepsilon)(\lambda_3 \sin\kappa + \mu_3 \cos\kappa) + (\cos\Phi)(\lambda_1 \sin\kappa + \mu_1 \cos\kappa)\} \\ C &= -\{(\sin\Phi\ \sin\varepsilon)(\lambda_2 \sin\kappa + \mu_2 \cos\kappa) - (\sin\Phi\ \cos\varepsilon)(\lambda_1 \sin\kappa + \mu_1 \cos\kappa)\} \end{aligned} \tag{21}$$

For this derivation, this plane is defined as passing through the origins of all three coordinate systems, which are collocated in space. The equation of the plane becomes

$$0 = AR + B\theta + CZ \tag{22}$$

Similarly, for the conventional velocity coordinate system, where the bisecting line unit vector from equation (16) is

$$-\bar{i}'' = (-\cos\delta)\bar{i} + (-\sin\delta)\bar{j} \tag{16}$$

and the unit vector in the direction of the velocity vector projection from equation (19) is

$$\bar{N}'' = (-\sin\delta\ \cos\gamma)\bar{i} + (\cos\delta\ \cos\gamma)\bar{j} + (\sin\gamma)\bar{k} \tag{19}$$

the crossproduct of $\bar{i}''$ and $\bar{N}''$ yields an equation describing the plane:

$$P_c = ER + F\theta + GZ \tag{23}$$

where

$$\begin{aligned} E &= \sin\delta\ \sin\gamma \\ F &= -\cos\delta\ \sin\gamma \\ G &= \cos\gamma \end{aligned} \tag{24}$$

Next, P_c is set to zero for reasons similar to those used for setting P_R to zero in equation (22).

Definition of the intersection line of the velocity vector projection planes.—The two planes in which the total velocity vector lies were described in the preceding section. To determine the line of intersection of the two planes, two points that lie in both planes must be identified. One of the two points is defined by the origins of both coordinate systems; it lies at the center of the probe volumes of both optical systems.

For the second point, a value of $\theta = 1$ was selected for one of the coordinate unknowns. Substituting this value into equation (23), which describes the conventional plane, gives

$$0 = ER + F + GZ \tag{25}$$

or

$$R = -\left(\frac{F}{E} + \frac{GZ}{E}\right) \tag{26}$$

Substituting equation (26) into the radial flow plane equation (eq.(22)) yields

$$0 = -A\left(\frac{F}{E} + \frac{GZ}{E}\right) + B + CZ \tag{27}$$

Solving equation (27) for Z, and substituting Z into equation (26) and solving for R results in

$$\theta = 1 \tag{28}$$

$$Z = \frac{(BE - AF)}{(AG - CE)} \tag{29}$$

and

$$R = -\frac{1}{E}\left(F + \frac{G(BE - AF)}{(AG - CE)}\right) \tag{30}$$

A unit vector in the direction of the total velocity (the line of intersection of the planes) then is

$$\overline{N}_t = \frac{-[F(AG - CE) + G(BE - AF)]\bar{i} + E(AG - CE)\bar{j} + E(BE - AF)\bar{k}}{\sqrt{[F(AG - CE) + G(BE - AF)]^2 + E^2(AG - CE)^2 + E^2(BE - AF)^2}} \tag{31}$$

The magnitude of the total velocity vector is determined by taking the dot product of the axial velocity unit vector, $\bar{k}$, and $\overline{N}_t$. The $\bar{k}$ vector was selected because this velocity component is directly and completely resolved by the conventional laser anemometer system. Solving for the angle between these two vectors gives

$$\cos\omega = \frac{\bar{k}\cdot\overline{N}_t}{|\bar{k}||\overline{N}_t|} \tag{32}$$

so

$$\omega = \cos^{-1}\left(\frac{E(BE - AF)}{\sqrt{[F(AG - CE) + G(BE - AF)]^2 + E^2(AG - CE)^2 + E^2(BE - AF)^2}}\right) \tag{33}$$

The magnitude of the axial velocity is calculated with the following equation:

$$V_Z = V_c \sin \gamma \tag{34}$$

And the magnitude of the total velocity vector is

$$V_t = \frac{V_Z}{\cos \omega} = V_c\left(\frac{\sin \gamma}{\cos \omega}\right) \tag{35}$$

The dot product of the other axes with the appropriate unit vectors will yield the magnitudes of the other velocity components. To calculate the tangential velocity, the dot product of the tangential direction unit vector and the total velocity unit vector, equation (31), is used; this gives the angle between the tangential velocity and the total velocity as

$$\Delta = \cos^{-1}\left(\frac{E(AG - CE)}{\sqrt{[F(AG - CE) + G(BE - AF)]^2 + E^2(AG - CE)^2 + E^2(BE - AF)^2}}\right) \tag{36}$$

Thus the tangential velocity is

$$V_\theta = V_Z\left(\frac{AG - CE}{BE - AF}\right) \tag{37}$$

The dot product of N_t and $\overline{i}$ gives

$$\Lambda = \cos^{-1}\left(\frac{-[F(AG - CE) + G(BE - AF)]}{\sqrt{[F(AG - CE) + G(BE - AF)]^2 + E^2(AG - CE)^2 + E^2(BE - AF)^2}}\right) \tag{38}$$

and the radial velocity then is

$$V_R = V_Z\left(\frac{F(AG - CE)}{E(BE - AF)} + \frac{G}{E}\right) \tag{39}$$

Thus, the total average velocity vector can be resolved from information provided by observing the flow, from two different physical locations, and measuring it with a single component laser anemometer system.

A NUMERICAL EXAMPLE

In the research program to resolve the radial velocity vector in a compressor rotor, two independent sets of laser anemometer flow measurements were made. These two data sets measured the flow at seven axial locations in an average rotor passage, from −5 percent chord to 105 percent chord, and at a constant axial inlet location. At each chord location, measurements were taken at 10 span locations varying from 5 percent to 90 percent span from the hub. At each of these axial/radial locations, measurements were taken at 50 circumferential locations spanning an average blade passage. Additional measurements at axial/radial locations were taken with the radial velocity laser anemometer system. The final resolved velocity field will contain calculated total velocity vectors at over 3500 spatial locations. A detailed explanation of the data acquisition system, the data acquisition and reduction protocol, the conventional laser anemometer system, and the test compressor is presented in reference 5.

The following example uses preliminary data taken at 50 percent chord, 80 percent span from the hub, and approximately 10 percent pitch from the suction surface. The location was chosen because preliminary data processing indicated the measurements were of excellent quality.

This section demonstrates the calculation procedures for resolving the total velocity vector. The results given are preliminary and are provided principally to show the procedure. The order of solution is as follows:

(1) Solve equations (1) and (2) for ε

(2) Solve equations (11) for the radial system direction cosines

(3) Solve equations (23) and (28) for the constants that describe the radial and conventional plane

(4) Solve equation (37) for ω

(5) Solve equations (38) to (41) for the magnitudes of the velocity components.

The conventional laser anemometer system acquired data at $\delta = -3.00°$. The resolved velocity vector was 152.04 m/s, and γ was 63.01°. The measured radial laser anemometer geometry inputs were $|X| = 41.429$ cm and $\Phi = 58.10°$. This resolved radial velocity vector was 137.3 m/s and κ was −3.707°. The radial location at which data were acquired was 23.5660 cm, R_F was 35.1714 cm, and R_{op} was 37.3460 cm.

Using equation (1) to find ξ gives

$$\xi = \cos^{-1}\left(\frac{23.566^2 + 35.1714^2 - 37.346^2}{2(23.566)(35.1714)}\right)$$

or $\xi = 76.119°$. Therefore, $\varepsilon = -13.881°$. Using equations (11) to calculate the direction cosines gives

$$\lambda_1 = +\cos\,\varepsilon = 0.97080 \qquad \mu_1 = \cos\,\Phi\,\sin\,\varepsilon = -0.12678$$
$$\lambda_2 = -\sin\,\varepsilon = 0.23991 \qquad \mu_2 = \cos\,\Phi\,\cos\,\varepsilon = 0.51301$$
$$\lambda_3 = 0 \qquad \mu_3 = \sin\,\Phi = 0.84897$$

With this information and δ, the offset angle of the conventional system, coefficients for both laser anemometer system planes can be calculated by using equations (23) and (28). The radial plane coefficients are

$$A = -0.96057$$
$$B = -0.27257$$
$$C = 0.05489$$

Similarly, for the conventional system

$$E = -0.04664$$
$$F = -0.88983$$
$$G = 0.45391$$

Equation (37), which calculates the angle between the total velocity vector and the axial velocity component, gives $\omega = 29.231°$. The total and component velocity magnitudes from equations (38) to (41) are

$$V_t = 155.25 \text{ m/s}$$
$$V_Z = 135.48 \text{ m/s}$$
$$V_\theta = 70.55 \text{ m/s}$$
$$V_R = 27.76 \text{ m/s}$$

This radial velocity component is the correct order of magnitude for this location in the flow field. However, at this location, a negative radial velocity was anticipated. Preliminary calculations of the blade-to-blade radial velocity field across the passage at this axial/radial location indicate this outward velocity may be a tip clearance effect. Further work remains to be done to resolve this issue.

SUMMARY OF RESULTS

The total velocity vector field in a compressor rotor can be determined from information provided by a laser anemometer system that observes the same velocity field from two different locations. The equations to calculate the velocity components from the geometry and measured velocity projections have been developed in this paper.

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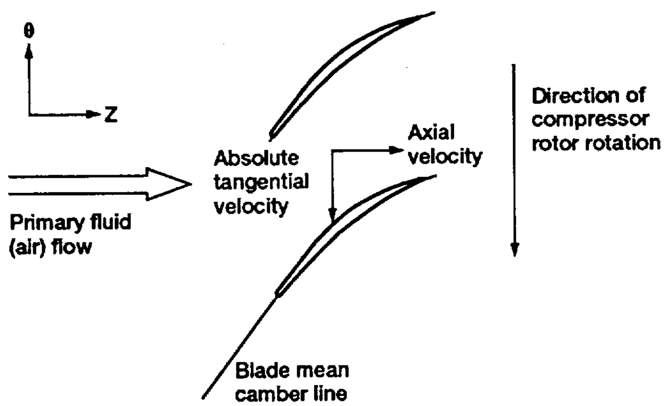
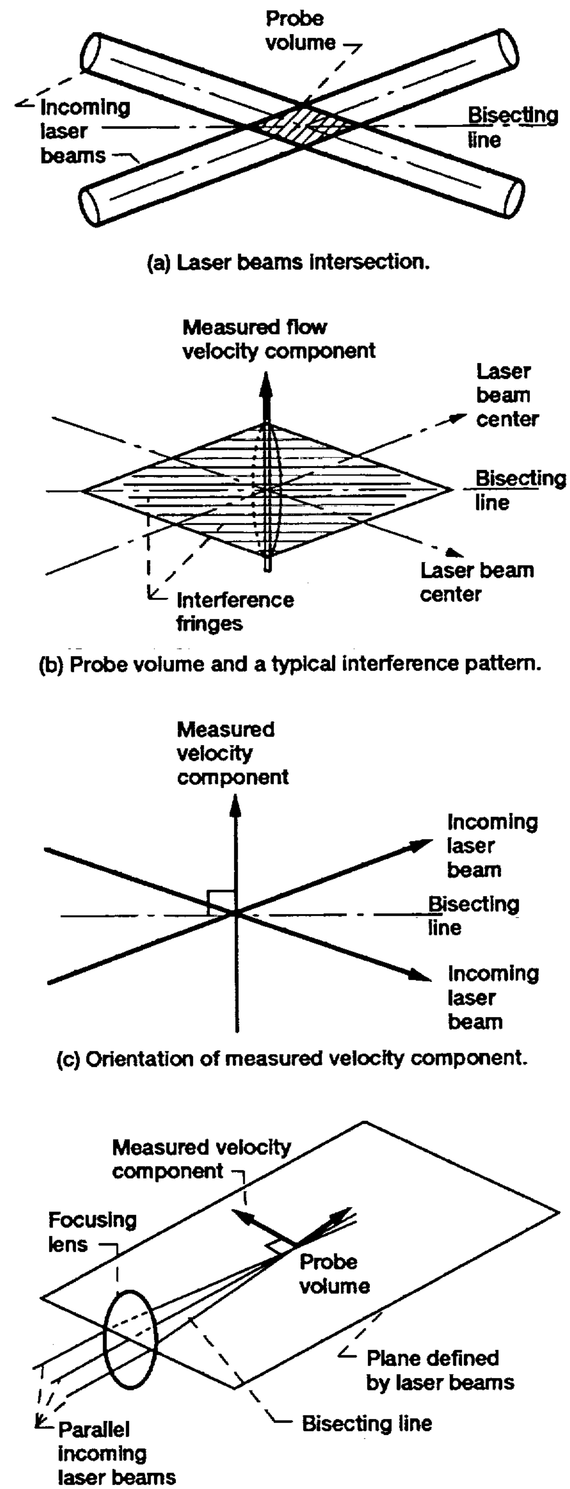


Figure 1.—Typical velocity components measured with laser anemometer.



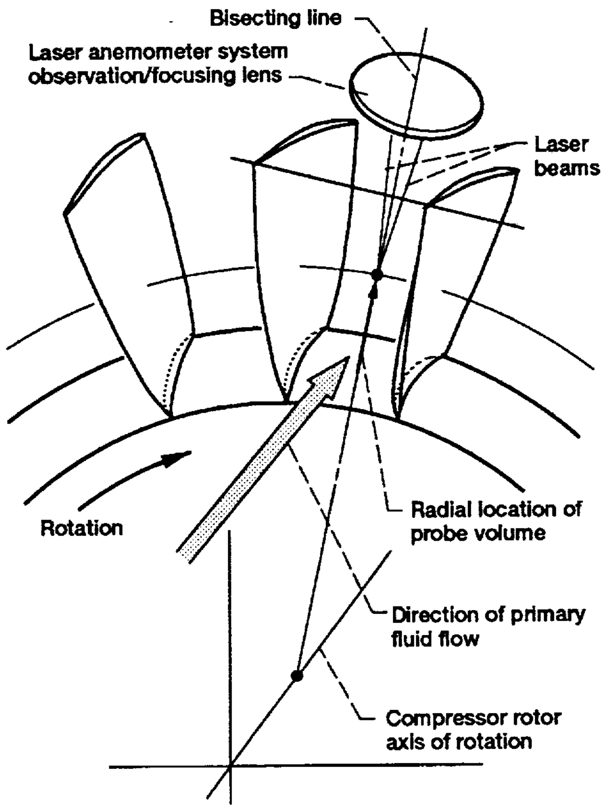
(a) Laser beams intersection.

(b) Probe volume and a typical interference pattern.

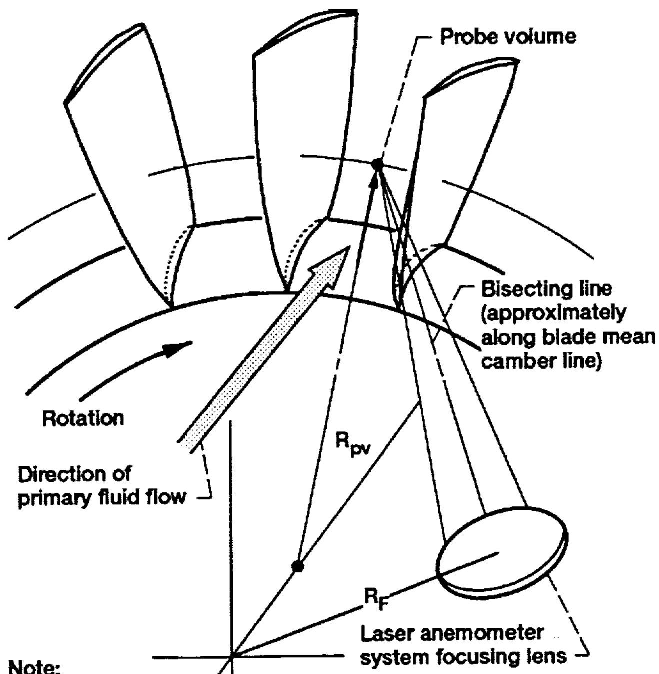
(c) Orientation of measured velocity component.

(d) Orientation of measured velocity projection.

Figure 2.—Typical laser anemometer probe volume geometry.



(a) Conventional laser anemometer system.



Note:
- $R_F > R_{pv}$
- Lens radius is greater than shroud radius
- Axial location of the lens is upstream of axial location of probe volume

(b) Laser anemometer system orientation for radial flow measurement.

Figure 3.—Laser anemometer system orientations with respect to test compressor rotor.

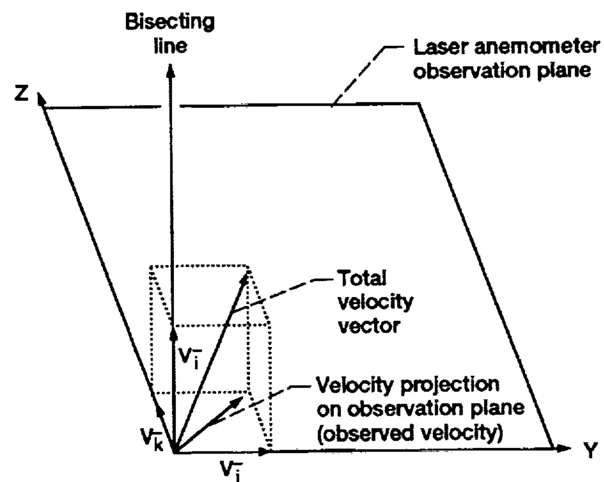


Figure 4.—Laser system observation plane.

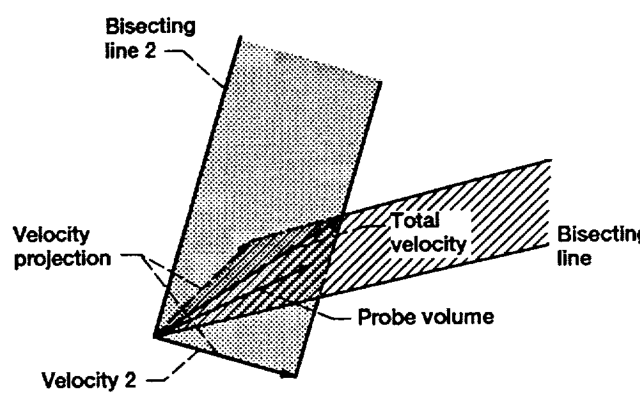
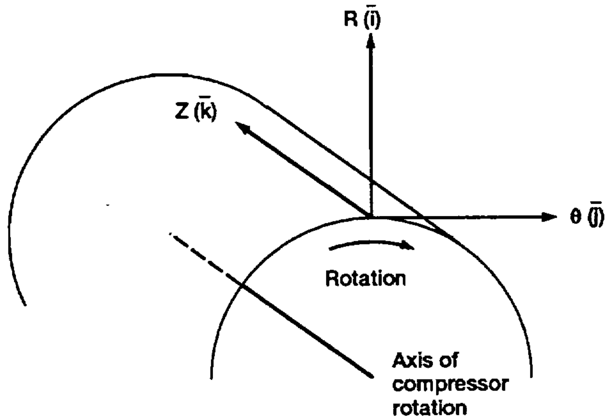
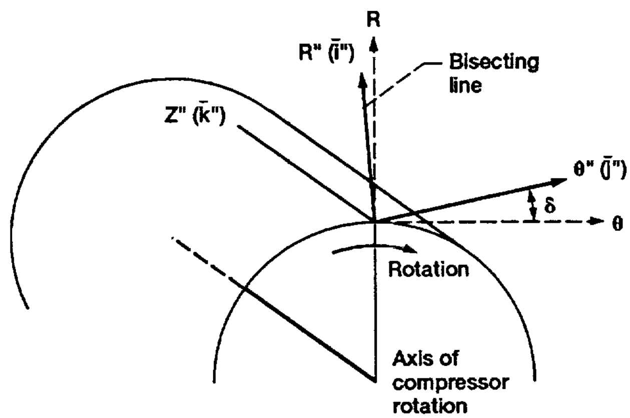


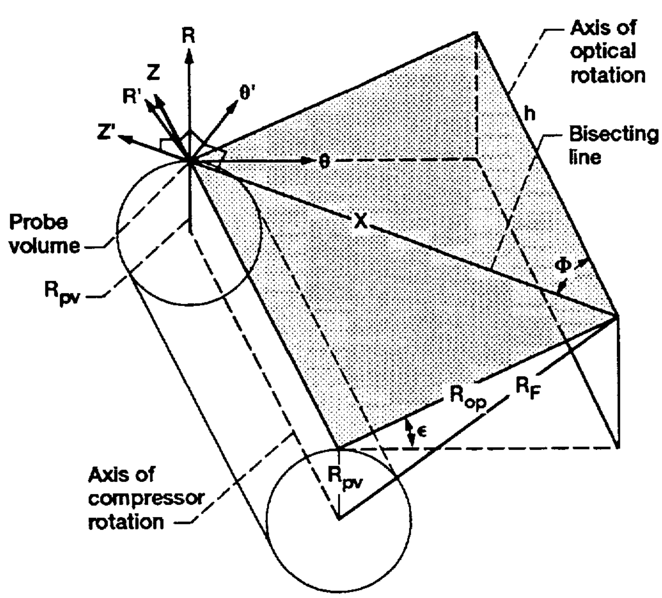
Figure 5.—Intersection line for two bisecting line velocity vector planes.



(a) Primary coordinate system.



(b) Conventional coordinate system and its relationship to primary cordinate system.



(c) Radial coordinate system and overall system construction.

Figure 6.—Coordinate systems.

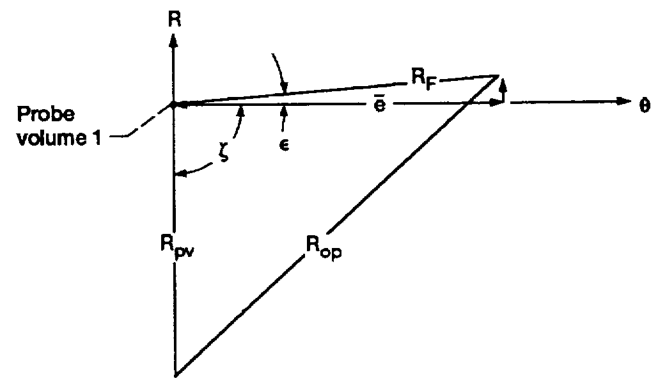


Figure 7.—Focusing lens position in primary coordination system.

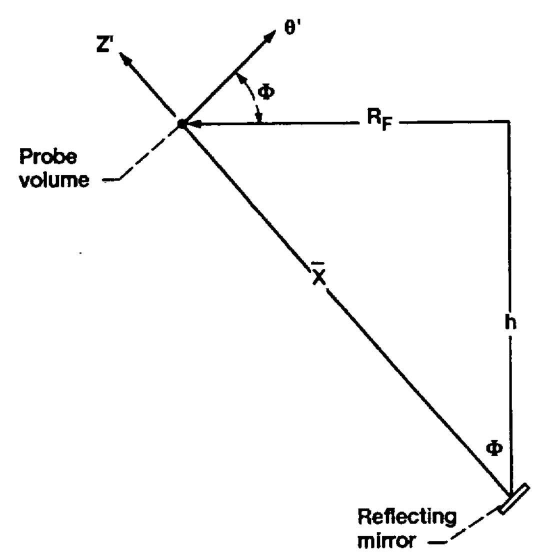


Figure 8.—Position of probe volume with respect to axis of rotation of the radial laser anemometer system.

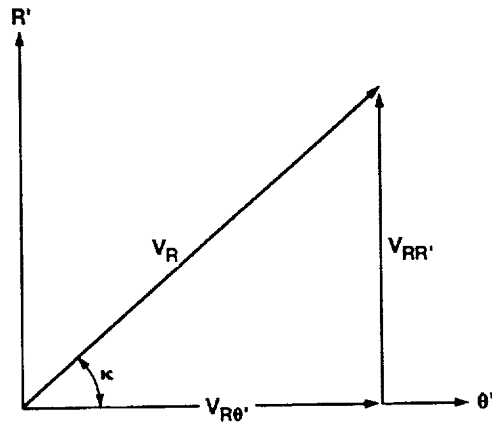


Figure 9.—Defintion of κ.

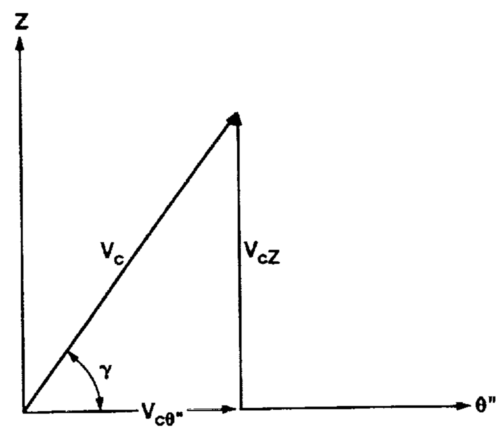


Figure 10.—Defintion of γ.

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13. ABSTRACT ***(Maximum 200 words)***
The mathematical relations between the measured velocity fields for the same compressor rotor flow field resolved by two fringe type laser anemometers at different observational locations are developed in this report. The relations allow the two sets of velocity measurements to be combined to produce a total velocity vector field for the compressor rotor. This report presents the derivation of the mathematical relations, beginning with the specification of the coordinate systems and the velocity projections in those coordinate systems. The vector projections are then transformed into a common coordinate system. The transformed vector coordinates are then combined to determine the total velocity vector. A numerical example showing the solution procedure is included.

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