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surf3d: A 3-D FINITE-ELEMENT PROGRAM FOR THE ANALYSIS OF SURFACE AND CORNER CRACKS IN SOLIDS SUBJECTED TO MODE-I LOADINGS

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Acknowledgements

The computer program *surf3d* was first written in 1976 and since then it has been continuously updated. This report describes the program as it stands today. (The authors realize that a computer program is never 'completed' nor is it totally bug proof!) This documentation was performed at the NASA Langley Research Center (contracts NAS 1-18599 and NAS 1-19317) and at the Center for Composite Materials Research in the Department of Mechanical Engineering at the North Carolina A&T State University, where the first author was a Research Professor. The program was implemented on the CRAY Y-MP (*flyer*), a UNIX supercomputer, at the North Carolina Supercomputing Center, Research Triangle Park, North Carolina. The first author takes this opportunity to express his gratitude to these organizations and the people who helped us in all these years.

ABSTRACT

A computer program, *surf3d*, that uses the 3D finite element method to calculate the stress-intensity factors for surface, corner and embedded cracks in finite-thickness plates with and without circular holes, was developed. The cracks are assumed to be either elliptic or part-elliptic in shape. The computer program uses eight-noded hexahedral elements to model the solid. The program uses a skyline storage and solver. The stress-intensity factors are evaluated using the force method, the crack-opening displacement method and the 3D virtual crack closure methods.

In the manual the input to and the output of the *surf3d* program are described. This manual also demonstrates the use of the program and describes the calculation of the stress-intensity factors. Several examples with sample data files are included with the manual. To facilitate modeling of the user's crack configuration and loading, a companion program (a preprocesssor program) that generates the data for the *surf3d* called *gensurf* was also developed. The *gensurf* program is a three dimensional mesh generator program that requires minimal input and that builds a complete data file for *surf3d*. The program *surf3d* is operational on Unix machines such as CRAY Y-MP, CRAY-2, and Convex C-220.

INTRODUCTION

Stress-intensity factors are fundamental quantities used to predict fatigue crack propagation rates and crack growth profiles. Surface and corner cracks usually initiate at imperfections and voids in metallic structures. These cracks usually grow into near or part-elliptical shapes [1]. Therefore stress-intensity factors for elliptical cracks are needed. A computer program, *surf3d*, that uses 3D finite elements was developed to calculate the stress-intensity factors for surface and corner cracks in finite plates and in plates with circular holes. This program was used extensively by the authors [2-15] and the results were compared to those obtained by others and other methods.

The purpose of this manual is to document this program, to describe the input to and the output of the program, and to demonstrate the program. Several examples are presented and several sample data files are included with this manual. To model crack configurations and loadings, a companion program (a preprocesssor program) that generates the data for the *surf3d* called *gensurf* [16] was also developed. The *gensurf* is a three dimensional mesh generator program that requires minimal input and that builds the complete data file for *surf3d*.

First, the crack configurations and loading that can be analyzed with *surf3d* are discussed. Next the program specifications and organization is presented. The procedure for the development of the models is explained. Then models for elliptic cracks are presented. Finally, several example problems and their output are presented. Appendix A lists names and functions of subroutines and major program variables. Appendix B describes the procedures to compile and execute *surf3d* both interactively and in the batch mode on UNIX supercomputers such as *CRAY Y-MP*.

CRACK CONFIGURATIONS AND LOADING

Several crack configurations can be analyzed with *surf3d*. A cross-section through the crack plane of each configuration is shown in Figure 1. The configurations are (see Figure 1)

(a) Surface crack
(b) Embedded crack
(c) Corner crack
(d) Corner crack from a circular hole
(e) Surface crack at a semicircular notch
(f) Surface crack from a circular hole

The cracks are assumed to be elliptic, semi-elliptic or quarter elliptic in shape and are defined by the semi-major axis, c, and the semi-minor axis, a. Any point on the crack front is defined by c, a, and ϕ, the parametric angle of the ellipse (see Figure 2).

The first three cases (a), (b), and (c), can be analyzed by imposing appropriate boundary conditions on the model shown in Fig. 2(b). Similarly, the next three cases, (d), (e), and (f) can be analyzed by imposing appropriate boundary conditions on the model shown in Figure 2(c). The boundary conditions for all the six cases are described below.

For all the six cases, $v = 0$ is prescribed for all nodes on the uncracked portion (shaded portion in Figure 2), including the nodes on the crack front, of the $y = 0$ plane.

(a) Surface crack in a plate:
  $u = 0$ for all nodes on the $x = 0$ plane
  $w = 0$ for the node at $(W, h, 0)$
(b) Embedded crack in a plate :
  $u = 0$ for all nodes on the $x = 0$ plane
  $w = 0$ for all nodes on the $z = 0$ plane
(c) Corner crack in a plate :
  $u = 0$ for nodes at $(0, h, 0)$ and $(0, h, -t)$
  $w = 0$ for the node at $(W, h, 0)$
(d) Corner crack from a circular hole :
  $u = 0$ for all nodes on the $x = -R$ plane
  $w = 0$ for the node at $(W, h, 0)$
(e) Surface crack from a semicircular hole :
  $u = 0$ for nodes at $(-R, h, 0)$ and $(-R, h, -t)$
  $w = 0$ for all nodes on the $z = 0$ plane
(f) Surface crack from a circular hole :
  $u = 0$ for all nodes on the $x = -R$ plane
  $w = 0$ for all nodes on the $z = 0$ plane

Loading

Several types of loading conditions can also be imposed on the cracked configurations. Four types of loading are commonly used:

(a) Remote tensile loading
(b) Remote bending loading about the x−axis
(c) Remote bending loading about the z−axis
(d) Uniform crack face pressure loading

The three loading conditions (a), (b), and (c) are illustrated in Figure 3. These loadings are usually applied on the $y = h$ plane as shown in Figure 3. The uniform pressure loading condition on the crack face is usually described by $\sigma_y = -1$ at all the nodes on the crack face.

The magnitudes of the tractions at the four nodes of a loaded face of an element are taken from the global vector of the nodal tractions. *surf3d* assumes that the tractions at the nodes are input in the global Cartesian coordinate directions. Using these nodal tractions, the tractions at any point on the loaded face of an element are obtained by interpolating with the element shape functions. From these tractions the consistent loads at the nodes are calculated following the standard FE principles [17].

Sometimes the user may require stress-intensity factors for loadings applied simultaneously on several faces of the model. In these situations *surf3d* may be executed with (a) several loadings in a single run or (b) once for each loading condition. In the first case, case (a), however, the boundary conditions should be the same for all the loading conditions and such that the loading produces only mode-I type deformations.

In addition to traction type loadings, *surf3d* accepts displacement type loadings. The same restrictions mentioned in the previous paragraph for combined loading conditions apply.

FINITE ELEMENT ANALYSIS

The cracked solid is modeled with 8-noded hexahedra (Hex-8), isoparametric elements. These isoparametric elements are serendipity elements based on a linear displacement field [17]. Near the crack front, singularity elements that are in the shape of pentahedrons are used. The formulation and the details of these elements can be found in references 2 through 4.

A typical model of a quarter of a plate with a surface crack (with $a/c = 1$ and $a/t = 0.5$) is shown in Figure 4. At any station on the crack front, eight singularity elements are used as shown in Figure 4. Around the crack front these singularity elements form a torus. The rest of the solid is modeled with the Hex-8 elements. Models such as this were developed using the mesh generator program *gensurf*; the details of the program are explained in the *gensurf* user's manual [16].

Figure 5 shows the Hex-8 element and the pentahedral singularity element and how the elements are defined by their nodal connectivity. This figure also shows several ways to define the Hex-8 and singularity elements. The $\sqrt{\xi}$ terms in the shape functions require that η is always along the crack front in the singularity element.

Improper definition of the element connectivity leads to the calculation of zero or negative volumes. For example, two inconsistently defined elements are shown in Figure 6. When an element's volume is computed as zero or negative, *surf3d* aborts execution and prints the element number, nodal connectivity information, coordinates of each of the nodes of the element, and the value of the diagonal terms of the element stiffness matrix. The user can then use this information to check his input for improper definition of the elements.

STRESS-INTENSITY FACTOR CALCULATIONS

The stress-intensity factors are calculated at each station on the crack front using three methods.

1. Force Method

2. Crack Opening Displacement (COD) Method

3. Three-Dimensional Virtual Crack Closure Technique (3D-VCCT)

The first two methods are used when singularity elements are used at the crack front, while the 3D-VCCT method is used when the solid is modeled completely with nonsingular elements.

Force Method

The force method was developed in references 2 and 3 and is briefly explained below. The method assumes that the 2D state stress is valid within every infinitesimal portion of the crack front, so that the stress normal to the crack plane, σ_y, can be written as

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} + A_1 \ + O(r^{1/2}) \tag{1}$$

where K_I is the stress-intensity factor and A_1 is a constant. Note that the distance r in Eq.(1) is measured *normal* to the crack front. The total force normal to crack plane and in a region bounded by $z_1 \leq z \leq z_2$ and $0 \leq r \leq r_D$ is

$$\begin{aligned} F_y &= \int_{z_1}^{z_2} [\int_0^{r_D} \sigma_y \, dr] \, dz \\ F_y &= \frac{K_I}{\sqrt{2\pi}} 2\sqrt{r_D}(z_2 - z_1) + A_1 r_D (z_2 - z_1) \ + \cdots \\ &\simeq \frac{K_I}{\sqrt{2\pi}} 2\sqrt{r_D}(z_2 - z_1) + A_1 r_D (z_2 - z_1) \end{aligned} \tag{2}$$

The forces F_y in the region $z_1 \leq z \leq z_2$ and $0 \leq r \leq r_D$ are known from the finite element analysis and can be used to evaluate the unknowns K_I and A_1 in Eq. (2). However, Eq. (2) is valid only for small distances r from the crack front. A procedure for evaluating the K-values using the forces F_y at various distances r is explained below.

Figure 7 shows a portion of the crack plane with two consecutive layers i and $(i+1)$ in the model. The nodes $j, k, ...p$ and the nodes $b, d, e, ...h$ define the model ahead of the crack front in the i^{th} layer. The FE solution calculates the forces F_y at all these nodes as shown in Figure 8. In this figure, F_{k_1} is the force in the $y-$direction computed at node k due to element I and F_{k_2} is the force in the y-direction computed at node k due to element J. The total force in the $y-$direction at node k in the i^{th}layer is

$$F_k = F_{k_1} + F_{k_2} \tag{3}$$

Thus the total force F_y for $r = r_2$ in the i^{th} layer (see Figure 8) is

$$(F_y)_{r=r_2} = F_j + F_k + F_{l_1} + F_b + F_d + F_{e_1} \tag{4}$$

Similarly, the total force F_y for $r = r_3$ in the i^{th} layer is

$$(F_y)_{r=r_3} = F_j + F_k + F_l + F_{m_1} + F_b + F_d + F_e + F_{f_1} \tag{5}$$

Thus total F_y forces are computed for five values of r and are used in Eq. (2) to obtain the following set of equations.

$$\begin{aligned}
(F_y)_{r=r_1} &= F_{y_1} = \frac{K_I}{\sqrt{2\pi}} t_i 2\sqrt{r_1} + A_1 r_1 t_i \\
(F_y)_{r=r_2} &= F_{y_2} = \frac{K_I}{\sqrt{2\pi}} t_i 2\sqrt{r_2} + A_1 r_2 t_i \\
&\vdots \\
(F_y)_{r=r_5} &= F_{y_5} = \frac{K_I}{\sqrt{2\pi}} t_i 2\sqrt{r_5} + A_1 r_5 t_i
\end{aligned} \tag{6}$$

where t_i is the thickness of the i^{th} layer. The unknowns in Eq. (6) are K_I and A_1. The five equations in Eq. (6) are used in a least square procedure to evaluate K_I and A_1. Through numerical experimentation it was found that consistent results are obtained when

(a) Five force equations are used (as in Eq. (6)), and

(b) the maximum value of $r(r_5)$ is less than or equal to $(a/10)$, where a is the depth of elliptical crack (semi-minor axis).

Equation (5) was modified slightly to calculate K_I at the two nodes (nodes j and b in Figures 7 and 8.) that define the ends of the crack in the i^{th} layer. The total force on the 'bottom' * half of the i^{th}layer, for example, for $r = r_2$ is

$$(F_y)_{r=r_2} = F_j + F_k + F_{l_1} \quad = (F_{y_2})_{bot} \tag{7}$$

Substituting Eq. (7) in Eq. (2) one obtains

$$(F_{y_2})_{bot} = \frac{(K_I)_{bot}}{\sqrt{2\pi}} \frac{t_i}{2} 2\sqrt{r_2} + (A_1)_{bot} r_2 (\frac{t_i}{2}) \tag{8}$$

* Note that 'bottom' and 'top' are relative and are used here for convenience in presentation. Each layer has a top and bottom. For example, node j in figure 7 is at the bottom of layer i and is also at the top of layer $i-1$. Similarly, node b is at the top of layer $i$and at the bottom of layer $i+1$. Also note that the nodes j and b in figure 7 are at stations i and $i+1$, respectively.

Note that this force is the force calculated using σ_y only in the bottom half of the i^{th} layer. Similarly one can write for the top part of the i^{th}layer as

$$(F_{y_2})_{top} = \frac{(K_I)_{top}}{\sqrt{2\pi}} \frac{t_i}{2} 2\sqrt{r_2} + (A_1)_{top} r_2 (\frac{t_i}{2}) \tag{9}$$

where

$$(F_{y_2})_{top} = F_b + F_d + F_{e_1} \tag{10}$$

Again the least square procedure is used independently for both top and bottom of the i^{th} layer individually to calculate the stress-intensity factors at both sides of this layer.

This procedure is repeated for all layers. The stress-intensity factors are then calculated as

$$(K_I)_{\text{station } (i)} = \frac{1}{2}[\{(K_I)_{\text{top}}\}_{\text{layer } (i-1)} + \{(K_I)_{\text{bot}}\}_{\text{layer } (i)}]$$

for all stations except the first and the last. (Note that node j in Figure 7 defines station i, node b defines station $(i+1)$, and so on.) For the first station, K_I is calculated from

$$(K_I)_{\text{station } (1)} = \{(K_I)_{\text{bot}}\}_{\text{layer } (1)}$$

and for the last station,

$$(K_I)_{\text{station } (Nlayer+1)} = \{(K_I)_{\text{top}}\}_{\text{layer}=(Nlayer)}$$

COD (Crack-Opening Displacement) Method

In the COD method, the crack opening displacement at the nodes just behind the crack front are used to calculate the stress-intensity factor from

$$COD = 2v = 2\frac{K_I}{2G}\sqrt{\frac{r}{2\pi}}\, 4(1-\nu^2) + O(r^{3/2}) + \ldots \tag{11}$$

where G is the shear modulus and ν is the Poisson's ratio of the material.

The crack opening displacements are known at all nodes behind the crack front. Therefore, as in the force method, equation (11) is evaluated at several values of r as follows

$$\frac{2G\,v}{\sqrt{\frac{r}{2\pi}}4(1-\nu^2)} = K_I + A' r + \cdots \simeq K_I + A' r \tag{12}$$

where K_I and A' are unknown constants. For various values of r the nodal opening displacement v is known; hence, Eq. (12) can be written as

$$\begin{aligned} \frac{c\cdot(v)_{r=r_1}}{\sqrt{r_1}} &= K_I + A' r_1 \\ \frac{c\cdot(v)_{r=r_2}}{\sqrt{r_2}} &= K_I + A' r_2 \\ &\vdots \\ \frac{c\cdot(v)_{r=r_5}}{\sqrt{r_5}} &= K_I + A' r_5 \end{aligned} \tag{13}$$

where c is a constant equal to $2G\sqrt{2\pi}\,/\,[4(1-\nu^2)]$.

Again the least square procedure is used to evaluate the constants K_I and A' in Eq. (13). As in the force method, five values of r are used in Eq. 13 and the nodes in the region $0 \le r \le (a/10)$ are used. The nodes that are used in the COD method $k', l', \cdots, p'$ for the i^{th} layer are shown in Figure 8.

3D VCCT

When the model does not contain singularity elements, the stress-intensity factors can be calculated from the strain energy release rates, G_I, assuming a state of plane strain [18,19].

For the i^{th} layer, reference 19 proposed that

$$(G_I)_i = -\frac{1}{2\Delta_i t_i}\left[F_j v_{k'} + F_b v_{d'}\right] \tag{14}$$

where Δ_i is the average of the radial distance between the set of nodes j and k and nodes b and d, and t_i is the thickness of the i^{th} layer . The nodes k' and d' are the COD-nodes on the crack front (see Figure 8). Note that the forces F_j and F_b are the force calculated using the elements in the i^{th} layer alone. The stress-intensity factor is then calculated assuming plane strain conditions as

$$K_{I_i} = \sqrt{\frac{EG_{I_i}}{(1-\nu^2)}} \tag{15}$$

The stress-intensity factor calculated from Eq. (15) is assumed to be the value at the center of the i^{th} layer.

A slightly different approach, suggested in reference 18, is used in *surf3d* and is described below. The node j on the crack front (see Figure 8) belongs to both layers $(i-1)$ and (i). The strain energy release rate at node j (i.e., station i) in Figure 8 is given by

$$(G_I)_j = -\frac{1}{2\Delta t_{\text{av}}}[F_j' v_{k'}] \tag{16}$$

where

$$\begin{aligned} F_j' &= (F_j)_{\text{layer}(i\text{-}1)} + (F_j)_{\text{layer}(i)} \\ \Delta &= \text{ radial distance between nodes } j \text{ and } k' \\ &= (\text{also radial distance between nodes } j \text{ and } k\ ) \\ t_{\text{av}} &= \frac{(t_{i-1}+t_i)}{2} \end{aligned} \tag{17}$$

In Eq. (16), F_j' is the total force in the y-direction at node j, t_{av} is the average thickness of layers $(i-1)$ and (i) computed as in the force method.

Eq. (16) is used for every station along the crack front. The stress-intensity factors are evaluated as before assuming plane strain as

$$(K_I)_j = \sqrt{\frac{E\cdot(G_I)_j}{(1-\nu^2)}} \tag{18}$$

or assuming plane stress as

$$(K_I)_j = \sqrt{E\cdot(G_I)_j} \tag{19}$$

Input Data for Stress-Intensity Factors

Figures 7 and 8 show the i^{th} and $(i+1)^{th}$ layers and the variables that are used to calculate the stress-intensity factors. Figure 7 shows the modeling with 4 singularity elements and shows the nodes, MNODE, and the elements, KELEM, used by the three methods for the i^{th} layer. Note that in this figure, for clarity, the modeling detail behind crack front and above the crack plane are not shown. Figure 8 shows the forces that are used in the force method and the COD nodes that are used in the COD and the VCCT methods. In this figure, the subscript IR in the arrays FCENT and FTIP denotes the IR^{th} loading condition (right hand side).

Figure 9 shows the details on the $z = 0$ plane for a typical surface crack model. Here eight singularity elements are used. As explained in the *gensurf* user's manual, first the $z = 0$ plane with a width of t units and height of h units is modeled. This is termed as the base model. Figure 9 shows only a part of the base model for $a/t = 0.8$ and $a/c = 1$. In this base model, 151 nodes and 128 elements were used. This figure also defines the input data for variables that are needed in the stress-intensity factor computations. Figure 10 shows details on the crack plane ($y = 0$ plane) and very close to the crack front for a 4-layer model. This figure also shows the complete data that is needed to define all the variables that are necessary in the stress-intensity factor computations. The tedious preparation of the input data can be avoided by using a mesh generator like *gensurf*.

The required input for calculating the stress-intensity factors using any of the three methods is described in the following section. By referring back to Figs. 7-9, the user's understanding of input parameters will be greatly enhanced.

INPUT TO *surf3d*

The required input data is described in this section. The data is created on a file, DFN, and is given an input to *surf3d*. A complete data file for *surf3d* could be generated using a mesh generator program like *gensurf*. Several sample input data files are attached. This section and the examples section together with Figures 6-8 will completely explain the input. (In this section, for convenience in presentation, the phrases cards, card sets, data sets, and lines are used interchangeably.)

Card set	*Format*	**Variable**	**Description**
1	20A4	TITLE	Title of the job
2	A6	POUT	Output Option. Input SHORT for short output and XLONG for long output option.
3	*†	EMOD,NU	Young's modulus and Poisson's ratio

† denotes free format

Card set	*Format*	**Variable**	**Description**
4	*	NPOIN,NELEM	Number of nodes and elements in the model
5	*	I, X(I,1), X(I,2),	Node number, $x-, y-$ and $z-$ coordinates of the model
	*	X(I,3)	

Warning: *surf3d assumes that Node 1 is at* $(c, 0, 0)$, *i.e., at the end of the major axis of the elliptic or part-elliptic crack.*

Card set	*Format*	**Variable**	**Description**
6	*	I, NOD(I,1), NOD(I,2), . . NOD(I,8), NINDX(I)	Element number, nodal connectivity and the index of each element (=1 for singularity elements and =0 for Hex-8 elements).

Repeat this line NELEM times until all the elements are specified.

7	*	NP,IU,IV,IW	Boundary Conditions - Node number, Restraint code for the u−displacement Restraint code for the v−displacement Restraint code for the w−displacement Restraint code: = 0 is free = 1 is fixed.

Repeat this line until all boundary conditions are prescribed.
Terminate this set of lines with four zeros.

8	*	NCASE	Number of loading conditions (number of right hand sides)
9	A6	CTYPE	Loading type. Input REMOTE for remote loading. Input CFACE for crack-face pressure loading.
10	*	NF, NL, NI, LIND, IFACE	Pressure loading definition - Number of first element, Number of last element, Increment, Load index-Input Unity, i.e LIND=1 Number of face where pressure loading is prescribed.

Repeat until all loaded elements are defined.
Terminate this set of cards with five zeros.

11	*	IP, PX, PY, PZ	Traction Definitions - Node number Magnitude of traction in the global x−direction at node IP Magnitude of traction in the global y−direction at node IP Magnitude of traction in the global z−direction at node IP

Repeat this line until all the tractions are specified for current loading condition.
Terminate each loading condition with zero for the integer and zeros for all three tractions.
Repeat Item 11 until all traction loading conditions (NCASE) have been defined.

12	*	NP,IU,IV,IW, U, V, W	Nodal Displacements - Node Number, Restraint code for the u-displacement, Restraint code for the v-displacement, Restraint code for the w-displacement, Prescribed U-displacement, Prescribed V-displacement, Prescribed W-displacement Restraint code: = 0 is free = 1 is fixed.

Repeat this line until all displacements are prescribed.
Terminate this set of lines with zeros for all four integers.

13	*	IRENUM	Renumbering option: = 0 if no renumbering given; = 1 if renumbering scheme is provided.

14	*	JNEW(NPOIN)	Renumbering scheme: Needed only if IRENUM is not equal to zero.

Use as many lines as needed to read NPOIN integers.

15	*	NLOAD	Number of concentrated loads

16	*	NA(1) NA(2) . . NA(NLOAD)	Degree of freedom at which the concentrated loads are applied.

Use as many lines as needed to read NLOAD integers.

17	*	XY(1) XY(2) . . XY(NLOAD)	Magnitude of concentrated loads corresponding to the degree of freedom directions defined in NA(I).

Use as many lines as needed to read NLOAD values.

18	*	NSINGU, NLAYER	Number of singularity elements in each layer and number of layers in the model.

Note that the number of singularity elements are assumed to be the same in each layer of the model.

19	*	ICOD(I,1), ICOD(I,2), ICOD(I,3), ICOD(I,4), ICOD(I,5)	COD Node Definitions - Node numbers of the nodes that are used in the COD method (see Fig. 8).

Read (NLAYER+1) lines to define all the COD nodes along the complete crack front.

20	*	MTIP(I,1), MTIP(I,2)	Node numbers defining the crack front for each layer of the model with singularity elements (see Fig. 9).
21	*	NTIP(I,1), NTIP(I,2), . . NTIP(I,NSINGU)	Element numbers for pentahedral (singularity) elements around the crack front for each layer of the model (see Fig. 9).
22	*	KELEM(I,1), KELEM(I,2), . . KELEM(I,5)	Element numbers of elements on the crack plane and ahead of the crack front for each layer of the model (see Figs. 7 and 9).

23	*	MNODE(I,1), MNODE(I,2), . . MNODE(I,10)	Node numbers of nodes on the crack plane and ahead of the crack front for each layer of the model (see Figs. 7 and 9).

24	*	HT, WIDTH, AOT, RPT, AOC	Height of the model, h Width of the model, W (a/t) ratio $(R+t)$ value for a plate with a hole (a/c) ratio.

As previously mentioned, Node 1 is assumed to be at $(c, 0, 0)$. Thus the value of c is automatically defined. Using this value of c and the ratios AOC, (a/c), and AOT, (a/t), the values of a and t are computed by the program.

OUTPUT OF *surf3d*

The output of *surf3d* is described in this section. Two output options, long and short, are available. If the user exercises the long output option (XLONG on the second card in columns 1-5), the following items will be printed. If the user specifies the short output option (SHORT on the second card),the items listed below with an asterisk will be omitted. Only a general description of major output sections is presented. Example output files are discussed and presented in the next section.

- Title
- Output Specification
- Description of the model
- Nodes and coordinates
- Element connectivity and index of the element.
- Boundary conditions
- Pressure or traction loading - elements, faces, nodes & traction magnitudes.
- Prescribed displacements
- Renumbering option and renumbering scheme
- Data for calculating stress-intensity factors
- Volume of the solid modeled
- For each loading condition:

  (a) Sum of x–forces before boundary conditions
  (b) Sum of y–forces before boundary conditions
  (c) Sum of z–forces before boundary conditions

- Projected surface areas: $x-$, $y-$, and $z-$ components
- Nodal displacements at each node in the model for the first loading condition.

- *Nodal displacements at each node in the model for each loading condition.
- *Nodal forces at each node in the model for each of the loading condition. (If the absolute value of the forces F_{x_i}, F_{y_i} and F_{z_i} at node i are less than 10^{-6}, then the forces are not listed.)
- *Average nodal stresses at each node for each loading condition.
- Equilibrium checks for each of the loading conditions ($\sum_i F_{x_i}, \sum_i F_{y_i}, \sum_i F_{z_i}, i = 1,$NPOIN in the model)

- Sum of applied loads and surface area components for the current loading condition.
- Nominal stresses for the current loading condition.
- *Stress-intensity factor calculations:-

  COD method:- K-value for each station along the crack front
  Force-method:- Apparent K-values at various distances from the crack front.
  These values are computed on 'top' and 'bottom' for each layer.

- Summary of stress-intensity factors calculated using

  (a) the Force method
  (b) the COD method when singularity elements are present
  (c) the 3D-VCCT method, when singularity elements are not present in the model. K-values using both plane stress and plane strain assumptions are output with this method.

- Element Equilibrium: If each element satisfies equilibrium, $\sum F_x = \sum F_y = \sum F_z \leq 1.0E-6$, then the program prints "All elements satisfy equilibrium". If some elements do not satisfy equilibrium, the program prints a warning message that N number of elements do not satisfy equilibrium and lists the element numbers.

EXAMPLES

In this section several examples illustrating the use of *surf3d* are presented. Only part of the input data files are shown because the data files are very long. The output files are also very long even when the short output option is exercised, and, therefore only the pertinent parts of the output file are presented here. However, the complete data and output files are available on the disk accompanying this manual.

In all examples the following assumptions are made: For remote loading, the applied stress S is assumed to be unity. The maximum bending stresses (the outer surface fiber bending stresses) S_{b_x} and S_{b_z} are also assumed to be unity (see Figure 2). The crack depth, a, is assumed to be unity. The square of the elliptic integral of the second kind, Q, is approximated in the program as

$$
\begin{aligned}
Q &= 1.0 + 1.464(a/c)^{1.65}, && \text{for } a/c \leq 1.0 \\
&= 1.0 + 1.464(a/c)^{1.65}, && \text{for } a/c > 1.0
\end{aligned}
$$

All the stress intensity factors are normalized by $S_n\sqrt{\frac{\pi a}{Q}}$ where

$$
\begin{aligned}
S_n &= S, && \text{for remote tension} \\
&= S_{b_x}, && \text{for bending about } x\text{-axis} \\
&= S_{b_z}, && \text{for bending about } z\text{-axis}
\end{aligned}
$$

The loadings in these examples in this section are selected so that the nominal stress S is unity. In general this will not be the case. Therefore, *surf3d* calculates the nominal stresses in the $x-, y-$, and $z-$directions for each of the loading conditions. The nominal stresses are calculated by evaluating the sum of the $x-, y-$, and $z-$components of the forces on the loaded faces of the model and dividing these components by the corresponding nonzero $x-, y-$, and $z-$components of the area of that face. The normalized stress-intensity factors computed by the program can be then be divided by S, where S is the correct value of the nominal stress. For bending loadings the nominal stress is zero. Therefore, for bending cases the outer-fiber stress is used as the nominal stress. This is assumed to be unity in the program.

For loading in the form of the prescribed displacements, the user needs to calculate the nominal stress. This is because the information on the loaded face areas are not available to *surf3d*. Fortunately, this calculation is very simple to perform. The sum of the $x-, y-$, and $z-$components of the forces on this face is available from the output of *surf3d*. These force components are divided by the corresponding nonzero components of the area to obtain the value of S. The normalized stress-intensity factors given by the program are then divided by S to obtain the correct normalized value. Note that this division needs to be performed by the use externally, i.e., after obtaining the output from *surf3d*.

The following table defines the input and output files used in each of the examples presented in this section.

Input and Output Files in the Examples

Example Number	*Input File*	*Output File*
1.	*dex1* (Table 1†)	*out12* (Table 2†)
2.		*outn12* (Table 3)
3(a).	*dex3a* (Table 4)	*outr22* (Table 5)
(b).	*dex3b*	*outc22* (Table 6)
4(a).	*dex4a*	*outr28* (Table 7)
(b).	*dex4b*	*outc28* (Table 8)
5.	*dex5*	*outcor28* (Table 9)
6.	*dex6*	*outem28* (Table 10)
7.	*dex7*	*occor15* (Table 11)
8.	*dex8*	*oscor15* (Table 12)
9.	*dex9*	*osmcor15* (Table 13)
10.	*dex10* (Table 14)	*outd12* (Table 15)
11.	*dex11*	*outdx12* (Table 16)

† Partial listing is shown in these tables

Traction-Type Loading

Surface, Corner and Embedded Cracks in Finite Plates

Example 1: Surface crack with $a/c = 1$ and $a/t = 0.2$ in a plate subjected to remote tension and bending about $x-$ and $z-$axes. ($W = 25; h = 125; t = 5.0$)

One quarter of the solid is modeled with 2161 nodes and 1664 elements. Eight layers are used to model the solid. In each layer 8 singularity elements are used at the crack front. The input data file is partially presented in Table 1. Part of the output file is shown in Table 2. The complete output file *out12* is available on the disk. The first part of the output is for remote tensile loading (loading number 1). The stress-intensity factors at each station along the crack front from $\phi = 0$ to $\pi/2$ calculated by the force and COD methods are given. In this part of the output, first the absolute value of K and then the normalized value of K from both methods are presented. Next the normalized values of K by the force and COD methods are tabulated for each station along the crack front, i.e., for each value of ϕ. The results for loadings 2 and 3, bending about x- and $z-$axes, respectively, are also listed in Table 2.

Example 2: Same configuration as in example 1 but without singularity elements.

This example uses non-singular elements through out the model. The program scans the NINDX array to determine if singularity elements are present in the model. When singularity elements are not present, force and COD methods are *not* used and instead the 3D-VCCT method is used to calculate the stress-intensity factors. This example presents the results obtained using the same model as in example 1, and setting all the indices INDX, in the NINDX array to zero. When 3D-VCCT method is used, one has to assume either plane stress or plane strain. The program calculates the normalized values of the stress-intensity factors using both assumptions. The results for example 2 are presented in Table 3. Comparison of these results with those in Table 2 shows that the 3D-VCCT assuming plane strain agrees well with the force method results for most of the crack front for all three loading conditions.

Example 3: Surface crack with $a/c = 0.2$ and $a/t = 0.2$ in a plate subjected to
(a) remote uniform tensile loading and
(b) uniform crack face pressure loading.

As in the previous examples, one quarter of the solid is modeled with 2441 nodes and 1872 elements. Again eight layers are used and in each of these layers 8 singularity elements are used at the crack front. Table 4 presents a partial listing of the data file.

From superposition principles, it can be shown that the stress-intensity factors for the two loading conditions are identical. This example demonstrates that *surf3d* nearly reproduces this result. Tables 5 and 6 present, respectively, the partial output files for remote tensile and crack face pressure loadings. Comparison of the results shows that the stress-intensity factors calculated by both methods are nearly but not exactly identical.

The slight differences between the two sets of stress-intensity factors are due to the non-exact nature of the loading on the crack face. Note that on the crack face, there is a semi-elliptic slit with a semi-major axis of $\sqrt{c^2 - (0.001a)^2}$ and semi-minor axis of $0.001a$. The loading for the two cases are not exactly identical to each other and, hence, the slight differences in the stress-intensity factors.

Example 4: Deep surface crack in a plate with $a/c = 0.2$ and $a/t = 0.8$; ($W = 50; h = 125; t = 1.25; a = 1; c = 5$)

One quarter of the solid is modeled with 2464 nodes and 1856 elements. As before eight layers are used to model the crack front and 8 singularity elements are used in each layer. The model is subjected to

(a) remote uniform tensile loading, and
(b) uniform crack face pressure loading of magnitude S.

The input files for these two cases *dex4(a)* and *(b)* are on the disk. Table 7 and 8 present the output files for loadings (a) and (b), respectively. Comparing the results obtained for this deep elliptic crack show that both loadings give nearly identical stress-intensity factors along most of the crack front.

Example 5: Corner crack with $a/c = 0.2$ and $a/t = 0.8$; ($W = 50; h = 125; t = 1.25; a = 1; c = 5$)

Consider a quarter-elliptical corner crack in a plate subjected to remote tensile loading. One half of the plate is modeled with 2464 nodes and 1856 elements. This model is identical to that used in example 4 but the boundary conditions are changed to the corner crack boundary conditions. The output is presented in Table 9.

Example 6: Embedded crack with $a/c = 0.2$ and $a/t = 0.8$; ($W = 50; h = 125; t = 1.25; a = 1; c = 5$)

One-eighth of the solid is modeled with 2464 nodes and 1856 elements. This model is identical to those used in examples 4 and 5 but the boundary conditions are changed to embedded crack-boundary conditions. The output is presented in Table 10.

Surface and Corner Cracks in a Plate with Holes

In this subsection, three examples of surface and corner cracks at a semi-circular or a circular hole are presented. In all these examples, the crack front is modeled with 8 layers, with 8 singularity elements at the crack front in each layer. The hole is modeled (on the $z = 0$ plane, see Figure 2) with 5 unequal thickness layers. The thickness of the five layers from the deepest point of the crack are 5,10, 25, 35, and 40 percent of the radius of the hole. (See *gensurf* users manual [16] for details on hole modeling.)

Example 7: Corner cracks at a circular hole with $a/c = 1, a/t = 0.5$, and $R/t = 1$; ($W = 25; h = 125; a = 1; c = 1; t = 2; R = 2$)

Because of symmetries, one quarter of the solid is modeled with 2863 nodes and 2260 elements. Loading conditions of remote uniform tension and bending about x−axis are considered. Table 11 presents a partial listing of the output.

Example 8: Surface crack at a circular hole with $a/c = 1, a/t = 0.5$, and $R/t = 1$; $(W = 25; h = 125; a = c = 1; R = t = 2)$

The same model as in example 7 is used for this example. The boundary conditions on $z = 0$ plane are prescribed to reflect the symmetric boundary conditions, i.e., all nodes on $z = 0$ plane are prescribed to have zero w−displacements. Remote uniform tensile loading was prescribed. Table 12 presents a partial listing of the output.

Example 9: Surface crack at a semi-circular hole with $a/c = 1, a/t = 0.5$, and $R/t = 1$; $(W = 25; h = 125; a = c = 1; R = t = 2)$

The same model as in examples 7 and 8 is used in this example. All the boundary conditions, except those on the $x = -R$ plane see Fig 2(c)), used in example 8 are used. Only one node on the $x = -R$ plane is prescribed to have a zero u−displacement. Remote uniform tensile loading was applied to this model. Table 13 presents a partial listing of the output.

Prescribed Displacement Loadings

Surface Crack in a Finite Plate

Example 10: Surface crack with $a/c = 1$ and $a/t = 0.2$ in a plate subjected to remote prescribed displacements, $v = 10^{-6}$in. on the $y = h$ face. $(W = 25; h = 125; t = 5.0)$

The configuration in this example is identical to example 1. Part of the input data file is presented in Table 14. Part of the output file is shown in Table 15. The complete output file *outd12* is available on the disk. The calculation of the nominal stress is illustrated in this example. The sum of the forces on the loaded face $y = h$ are computed as $F_x = 0.0$, $F_y = 29.99872$ lbs, and $F_z = 0.0$. The components of the area are $A_x = 0.0$, $A_y = 25.0 \text{x} 5.0 = 125.0$ sq. in., and $A_z = 0.0$. Therefore, the nominal stress is $S = 29.99872/125.0 = 0.24$ psi. All the output stress-intensity factors are divided by 0.24, the value of the nominal stress S. For example, the $K/S\sqrt{\pi\, a/Q}$ at $\phi = \pi/2$ using the force method will now be equal to $0.24519/0.24 = 1.0216$. This value agrees extremely well with that for the traction-type loading in example 1 for the same configuration (using the force method this value is 1.0221). This is expected because of the large plate used in both examples.

Example 11: Surface crack with $a/c = 1$ and $a/t = 0.2$ in a plate subjected to remote prescribed displacements, $u = -0.3 \cdot 10^{-7}$in. on the $x = b$ face. $(W = 25; h = 125; t = 5.0)$

The configuration in this example is identical to that used in example 1. The loading used in this example gives zero values for the stress-intensity factors. This example shows that the stress-intensity factors calculated by *surf3d* will be nearly zero but not identically zero. Note that the run was aborted because of the attempted square root of small negative stress-intensity factors. Part of the output file is shown in Table 16. The complete output file *outdx12* is available on the disk.

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APPENDIX A

PARAMETERS, SUBROUTINES, MAJOR PROGRAM VARIABLES, AND COMMON BLOCKS

This appendix presents the names and functions of the subroutines and major program variables and the common blocks with their elements and the subroutines which use the common blocks. A flow chart of *surf3d* is presented in Figure 11.

A-1: PARAMETER STATEMENT VARIABLES

NAME	DEFINITION
MAXBK	Maximum dimension of the assembled stiffness matrix - BIGK
MAXNOD	Maximum number of nodes
MAXEL	Maximum number of elements
MAXRHS	Maximum number number of right hand sides (loading conditions)
MAXBC	Maximum number of boundary conditions
MAXB	Maximum bandwidth of equations
NNODE	Number of nodes on the Hex-8 element = 8
NDOF=NFREE	Number of degrees of freedom per node = 3
NSIF	Number of stations along the crack front where the stress-intensity factors are evaluated.
MAXRUND	Maximum dimension of IRUND
MAXDIS	Maximum number of degrees of freedom = MAXNOD*NFREE
NSMK	Dimension of the stiffness matrix of the Hex-8 element. = NNODE * NFREE =24

A-2: SUBROUTINES

NAME	FUNCTION
1. ADJUST	Reorders the nodal coordinate array, the element connectivity array and the boundary condition array according to the renumbering scheme provided by the user. If no renumbering is given, no reordering of these arrays is performed.
2. ASEMB	Processes all elements, obtains element stiffness matrices, load vectors and assembles the global stiffness matrix.
3. ASTAR	Obtains the transformation relationship between the generalized coordinates and the nodal coordinates. This routine is called only once.

4. BLOCK DATA	Contains the Gaussian coordinates and weights up to an 8-point integration rule. Also contains the parent coordinates (ξ, η, ζ) of the Hex-8 elements.
5. BOUND	Prescribes the boundary conditions.
6. CCLOCK	Calculates the CPU and accumulated CPU times between successive calls.
7. CDER	Calculates the Cartesian derivatives at the Gaussian points and forms the BJ matrix.
8. CORDIN	Reads the coordinates of all the nodes in the model.
9. DERIVE	Obtains the derivatives of the shape functions at any point (ξ, η, ζ) in the Hex-8 and pentahedron singularity elements.
10. FORCES	Calculates the element forces and checks the element equilibrium. Calculates the nodal stresses and forces and checks global equilibrium.
11. GDERV	Calculates the parent derivatives, $\frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta}, \frac{\partial N_i}{\partial \zeta}, i = 1,8$, at each of the $(NGAUSS)^3$ Gaussian points, both for singularity and Hex 8 elements (If reduced integration is used (IRED=1), the parent derivatives are calculated at the center of the element).
12. LITTLE	Calculates the numerically smallest node number on each element
13. LOAD	Calculates the consistent loads on each of the loaded elements.
14. MATINV	Obtains the inverse of a square matrix.
15. MATMUL	Obtains the product of two matrices.
16. MODULUS	Computes the modulus matrix of an isotopic material.
17. PARENT	Calculates the parent derivatives, $\frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta}, i = 1,8$, at each of the $(NGAUSS)^2$ Gaussian points.
18. REND	Calculates the half-band width for each degree of freedom, sets up the row pointer array and calculates the total memory required to store the assembled stiffness matrix in profile form.
19. SETPP	Calculates the half-band widths required for each of the degree of freedom in the model.

20. *SHAPE*	Calculates the shape functions of the Hex-8 element.
21. *SFACTOR*	Calculates the stress-intensity factors using the force and the crack-opening displacement methods.
22. *SMALL*	Controlling subprogram which calls the stiffness matrix and load routines for the Hex 8 and singularity elements.
23. *SMALLK*	Calculates the element stiffness matrix of the Hex 8 and singularity elements. **Note:** *The element stiffness matrices are calculated using a procedure similar to that presented by Noor and Hartley [20] for vector computers like the STAR-100, CYBER 203, CYBER 205. The vector version was very efficient and was later devectorized for UNIX machines. Numerical experimentation showed that the current UNIX version is as efficient as the CYBER 205 version. The source code in this subprogram does not follow conventional procedures used for evaluating element stiffness matrices.*
24. *SOLV*	Calls the solver, SYMBAN, and prints the displacements of the all the nodes in the model.
25. *STRAN*	Calculates the stress-transformation matrices for the singularity elements.
26. *STRESS*	Calculates the stresses in each element by calculating the stresses at the 2x2x2 Gaussian points.
27. *SYMBAN*	Solves the system of equations using the Cholesky decomposition. The left hand side matrix is arranged in profile form. This is an in-core solver.
28. *TRANF*	Calculates the singularity element transformation matrices.
29. *TRANS*	Obtains the transpose of a matrix.
30. *ZEROLN*	Zeros out an integer array.
31. *ZEROLV*	Zeros out a real variable array.

A-3: MAJOR PROGRAM VARIABLES

The variables in various commons are listed alphabetically by the common block name and then the other major variables are listed in alphabetical order.

COMMON	VARIABLE	DEFINITION
ASTIF	*BIGK(MAXBK)*	Assembled stiffness matrix stored in profile form.
AVERAGE	*FSUMS(3,MAXRHS)*	Sum of the forces in the $x-, y-$ and $z-$directions on the loaded faces of the model.
	ASUMS(3)	Projected areas in the $x-, y-$ and $z-$ coordinate directions of the loaded faces,
BANDW	*P(MAXDIS)*	Work array used to store the reciprocal of the diagonal coefficients in the solver
	T(MAXB)	Work array needed to store the half-band widths in the solver
BNOD	*X(MAXNOD,NFREE)*	$x-, y-$ and $z-$ coordinates of all the nodes in the model. $X(I,1), X(I,2), X(I,3)$ represent the $x-, y-$ and $z-$coordinates of node I.
CLOCKS	*TSTART*	Temporary storage for CPU Time
COD	*ICOD(NSIF,MAXKE)*	Nodes that are used in the COD method to evaluate the stress-intensity factors at various stations along the crack front.
CLIST	*LIST(MAXBC)*	Boundary condition array. LIST(I) defines degrees of freedom (dof) I to be zero.
	NA(NLOAD)	Defines the dof where external loads are prescribed.
	XY(NLOAD)	Defines the magnitude of the external load corresponding to the dof defined in the the NA array.
	NLOAD	Number of external loads prescribed.
COMB	*NCASE*	Number of loading conditions.

	DEPTH	Thickness of the solid (t, see Figure 1)
	NSINGU	Number of singularity elements around the crack front
	RHOLE	Radius of the circular hole
DISP	*DIS(MAXDIS,MAXRHS)*	Array that contains the loads corresponding to each of the dof before solution and the displacements after the solution for each loading condition.
GAUSS	*CORD (8,8)*	Gaussian coordinates
	WEIGHT(8,8)	Gaussian weights. An N-point Gaussian coordinate and weight can be found in the Nth column of the arrays CORD and WEIGHT, respectively, ($1 \leq N \leq 8$).
IND	*NINDX(MAXEL)*	Array used to index the elements NINDX(I)=0 denotes that the element I is a Hex-8 element. NINDX(I)=1 denotes that the element I is a pentahedron singularity element.
	LINDX(MAXEL,2)	Array used to indicate elements with pressure loading. LINDX(I,1)=0 denotes that the element I does not have any pressure loading. LINDX(I,1)=1 denotes that the element I is subjected to pressure loading. LINDX(I,2) defines the face on which the pressure loading is prescribed. The 6 faces are defined using the parent coordinates ξ, η, ζ as follows: $\xi=0$ face number = 1 $\xi=1$ face number = 2 $\eta=0$ face number = 3 $\eta=1$ face number = 4 $\zeta=0$ face number = 5 $\zeta=1$ face number = 6

	LIND	LIND = LINDX (I,1)
	INDX	INDX = NINDX(I)
	IFACE	IFACE= LINDX(I,2)
INTGR	*NPOIN*	Number of nodes in the model
	NBOUN	Number of boundary conditions
	NELEM	Number of elements in the model
	NBAND	Maximum half-bandwidth of the equations
	NDIS	Number of dof in the model = NPOIN*NFREE
INTNST	*KELEM(NSIF,MAXKE)*	Elements on the crack plane and ahead of the crack plane used in stress-intensity factor calculation. (See Figure 7 for definitions).
	*MNODE(NSIF,2*MAXKE)*	Nodes on the crack plane and ahead of the crack plane used in the stress-intensity factor calculation. (See Figure 7 for definitions).
	*FCENT(NSIF, 2*MAXKE,2, MAXRHS)*	Forces in the y-direction at the nodes on the crack plane and ahead of the crack used in the stress-intensity factor calculation. (See Figure 8 for definition of this array.)
	*NKOUNT(NSIF,2*MAXKE)*	An integer array to keep the count of the occurrence of nodes in the MNODE array.
	MTIP(NSIF,2)	Nodes on the crack front of each layer of the model. (See Figures 8 through 10 for definitions and examples).
	NTIP(NSIF,MSINGU)	The elements closest to and around the crack front. These elements are used to evaluate the forces at the crack front nodes. (See Figures 8 through 10 for definitions and examples).
	FTIP(NSIF,2,MAXRHS)	Forces at the crack tip nodes. (See Figures 8 through 10 for definitions and examples).
	NLAYER	Number of layers (wedges) in the model. Stress-intensity factors are evaluated at (NLAYER+1) stations on the crack front.
MASTER		(See subprograms GDERV and PARENT for details)

	DMASTX(216,3), DMASTE(216,3), DMASTZ(216,3)	Derivatives $\frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta}, \frac{\partial N_i}{\partial \zeta}$ for a Hex-8 element, at the quadrature points for 2 or 3-point Gaussian quadrature. For a 2-point Gaussian, 8 derivatives, For a 3-point Gaussian, 27 derivatives.
	DMASSX(216,3), DMASSE(216,3), DMASSZ(216,3)	Derivatives $\frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta}, \frac{\partial N_i}{\partial \zeta}$ for pentahedron singularity elements at the (8 or 27) quadrature points with 2- or 3-point Gaussian quadrature.
	DSHX(216,3), DSHE(216,3), DSHZ(216,3)	Derivatives $\frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta}, \frac{\partial N_i}{\partial \zeta}$ for Hex-8 elements at the center of the element for reduced integration
	WT(27,3)	Gaussian weights at the quadrature points for 2- or 3-point Gaussian quadrature in each direction.
PDER	*DNX(216), DNY(216), DNZ(216)*	Parent derivatives of all shape functions (i=1,8) at all the quadrature points (8 or 27) for 2- and 3- point Gaussian quadrature (see subprogram GDERV for details)
POINTER	*NBW(MAXDIS)*	Bandwidth for each degree of freedom
	IRPNT(MAXDIS)	Row pointer array for each degree of freedom
	NDSTK(MAXNOD)	NDSTK(I) gives the lowest node number that is connected to node I. This array is used to compute the IRPNT array in subprogram REND.
RENUM	*JOLD(MAXNOD)*	Array which relates the old nodal numbers to the new nodal numbers in the renumbered scheme. JOLD(IN) gives the old number of new node IN. This array is complementary to the array JNEW.
	JNEW(MAXNOD)	Array which relates the new node numbers to the old node numbers. JNEW(IO) gives the new nodal number of the old node IO. This array is complementary to the array JOLD.

STIF	*NGAUSS*	Order of Gaussian integration used to integrate the elements
	IRED	Flag used to turn on and off reduced integration IRED =1 Reduced integration is ON = 0 Reduced integration is OFF
	DELVOL	Volume of the element being processed
SUMM	*STRESS-INTENSITY FACTORS*	
	SIFF(NSIF)	Stress-intensity factors calculated by the force method.
	SIFD(NSIF)	Stress-intensity factors calculated by the COD method assuming plane strain.
TYPE	*CTYPE*	Loading Type: REMOTE-for remote loading CFACE - for crack face pressure loading
ULOAD	*UL(NSMK,MAXRHS)*	Consistent loads calculated by subprogram LOAD for each of the loading conditions
	ALOAD(MAXNOD, NFREE,MAXRHS)	Magnitudes of tractions at each of the nodes in the model for all loading conditions
	PLOAD(NNODE, NFREE,MAXRHS)	Magnitudes of tractions at each of the nodes of the element being processed for all loading conditions.

MAJOR PROGRAM VARIABLES NOT IN ANY COMMON BLOCK(S)

VARIABLE	DEFINITION
ELDIS(NSMK,MAXRHS)	Nodal displacements of the element being processed.
FOR(NSMK,MAXRHS)	Nodal forces of the element being processed.
FORCE(MAXNOD,3,MAXRHS)	Nodal forces at each node of the model. The first subscript indicates the node number, the second subscript indicates the direction, the third subscript indicates the loading condition.

ND(MAXNOD)	Array containing the number of elements connected to a specific node.
SP(MAXNOD,6,MAXRHS)	Average nodal stresses at each node of the model. The first subscript indicates the node number. The second subscript indicates one of the six stresses, $\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$. The third subscript indicates the loading condition.
ND(MAXNOD)	Array containing the number of elements connected to a specific node.
XE(NNODE,3)	Nodal coordinates of the element being processed.

APPENDIX B
EXECUTION OF *surf3d*

This appendix describes the procedures to compile and execute *surf3d* both interactively and in the batch mode on a *CRAY Y-MP* computer. To distinguish among commands and responses in this appendix the following code is used. The **`typewriter font`** is used to denote a system response. (For example, **`flyer 25%`** denotes the 25th command to the *Cray Y-MP* named **`flyer`**.) The *italic font* is used to denote the user commands and the file names. The Roman font is used in the explanation of various commands and responses.

B-1: Interactive Compilation and Execution of *surf3d*

`flyer 25%`	*cft77 surf3d.f*	Invoke the FORTRAN compiler and create the object file, *surf3d.o*

	`FF0001 CFT77 VERSION 4.0.3.13`
	`(392409) 04/30/91 13:15:30`
	`FF0002 COMPILE TIME 14.168 SECONDS`
	`FF0006 MAXIMUM FIELD LENGTH 409116`
	`DECIMAL WORDS`
	`FF0003 4963 SOURCE LINES`
	`FF0004 0 ERRORS`
	FF0005 CODE: 9683 WORDS, DATA: 2338
	WORDS	

	(For *Convex* computers use *fc -cfc -72 -o s.e surf3d.f*)	Here *-cfc* flag emulates the *Cray* FORTRAN compiler and the *-o s.e* names the executable *s.e*. If *-o s.e* is omitted the executable name defaults to *a.out*.
`flyer 26%`	*segldr -f indef -o s.e surf3d.o*	link and create the executable file, *s.e*. The core is set to indefinite values with the *-f indef* flag.
`flyer 27%`	*s.e < dat12 >& out12&*	Execution with *dat12* as data file and *out12* as output file. The first ampersand directs that the execution to be carried out in the background. The second ampersand directs the errors to the output file, *out12*.
`flyer 28%`	*ps*	Determine process status

```
                PID TTY TIME COMMAND
                41299 p001 0:00 ps
                41253 p001 0:00 quotamon
                41292 p001 0:14 s.e
                41247 p001 0:00 csh
```

flyer 29%	*ps* PID TTY TIME COMMAND 41253 p001 0:00 quotamon 41315 p001 0:00 ps 41292 p001 0:30 s.e 41247 p001 0:00 csh	Process status
flyer 30%		
	[1] Done s.e < dat12 >& out12&	Job completed.

B-2: Batch Compilation and Execution of *surf3d*

The following commands describe how to compile and execute *surf3d* in the batch mode. A batch file, *surfer*, is used to perform the majority of this task. Because of variations in the batch processes in different systems, the users are encouraged to consult their system administrator to determine specifics of their system batch processes.

Here the source and the data files are assumed to be in a temporary (space) directory */tmp/raju*.

flyer 25%	*cat surfer* *#QSUB -me* *#QSUB -lT 100* *#QSUB -lM 16mw* *ja* *set verbose* *date* *cd /tmp/raju* *pwd* *cft77 surf3d.f* *segldr -f indef -o s.e surf3d.o* *./s.e < dat12 > out12* *date* *ja -csft*	Lists the script file, *surfer*.
flyer 26%	*qsub -eo -o slog surfer*	Submits the script *surfer* to the batch processor with a request to write the log, the errors and all the information to the file *slog*.

flyer 27% *qstat -a* — Tells the user the status of batch job.

flyer 28% *cat slog* — Lists the log file *slog*.

The table B-1 lists only the pertinent portions of the log file.

Table B-1: Listing of the file *slog*

```
Warning:  no access to tty; thus no job control in this shell...

Beginning Batch Execution

18:41:21

18:41:21 + exit

18:41:21 + /usr/spool/nqs/scripts/++02f+++++0+++

date

Sun Sep 29 18:41:21 EDT 1991

cd /tmp/raju

/tmp/raju

pwd

/tmp/raju

cft77 surf3d.f
.
.
.

segldr -f indef -o s.e surf3d.o

./s.e < dat12 > out12

STOP (called by $MAIN )

CP: 29.470s, Wallclock:  29.823s, 24.7.
.
.

HWM mem:  7730286, HWM stack:  310502, Stack overflows:  0

ja -csft

Job Accounting - Command Report
```

================================

Command Started Elapsed User CPU Sys CPU I/O Wait I/O Wait

Name At Seconds Seconds Seconds Sec Lck Sec Unlck SBU's

======== ======== =========== ========== ========== ======== ========== =======

ja 18:41:21 0.0042 0.0004 0.0037 0.0000 0.0000 0.00

date 18:41:21 0.0026 0.0005 0.0020 0.0000 0.0000 0.00

pwd 18:41:21 0.0049 0.0005 0.0043 0.0000 0.0000 0.00

pwd 18:41:21 0.0048 0.0005 0.0042 0.0000 0.0000 0.00

cft77 18:41:21 14.3477 14.1998 0.1284 0.0030 0.0147 7.16

segldr 18:41:35 0.6375 0.5865 0.0503 0.0000 0.0000 0.32

s.e 18:41:36 29.8828 29.4708 0.3972 0.0007 0.0112 14.93

date 18:42:06 0.0026 0.0005 0.0020 0.0000 0.0000 0.00

Job Accounting - Command Flow Report

=====================================

parent -> child ...

========================

ja

date

pwd

pwd

cft77

segldr

s.e

date

Job Accounting - Summary Report

================================

Job Accounting File Name : /tmp/nqs.+++++002f/.jacct5018

Operating System : sn1015 flyer 6.0 woo.19 CRAY Y-MP

User Name (ID) : raju (14006)

```
Group Name (ID) : ncedu (14000)

Account Name (ID) : raju (14006)

Job Name (ID) : surfer (5018)

Report Starts :  09/29/91 18:41:21

Report Ends :  09/29/91 18:42:06

Elapsed Time :  45 Seconds

User CPU Time :  44.2593 Seconds

System CPU Time :  0.5922 Seconds

I/O Wait Time (Locked) :  0.0037 Seconds

I/O Wait Time (Unlocked) :  0.0259 Seconds

CPU Time Memory Integral :  231.3400 Mword-seconds

SDS Time Memory Integral :  0.0000 Mword-seconds

I/O Wait Time Memory Integral :  0.0070 Mword-seconds

Data Transferred :  3.1413 MWords

Maximum memory used :  7918592 Words

Logical I/O Requests :  1796

Physical I/O Requests :  755

Number of Commands :  8

Billing Units :  22.4258

logout

18:42:06 + clear

18:42:06 + set lo=logout

18:42:06 + set bye=logout
```

B-3: Lower to Upper Case Conversions

After the input to *surf3d* is generated, change all the lower case characters in the file to the upper case characters using the script file called *trans*. The conversion is recommended because of string inputs. When alphanumeric strings with mixed upper and lower case characters are used, comparisons cause problems. To avoid these difficulties all upper case characters are recommended in the input files. The conversions file *trans* is provided with this manual. For example, to change all the characters in an input file called *tinp* to upper case characters, type

trans tinp

The system response will be

```
remove tinp.n?
```

Type *y* to remove temporary files. All the lower case characters in the file *tinp* are now changed to upper case characters. If the original file does not contain any lower case characters then the above command has no effect on the file. Therefore, the use of *trans* is recommended on the input file before the file is submitted for execution with *surf3d*.

The script file *trans* contains the following statements.

```
#

tr a-z A-Z <$1> $1.n

cp $1.n $1

rm $1.n
```

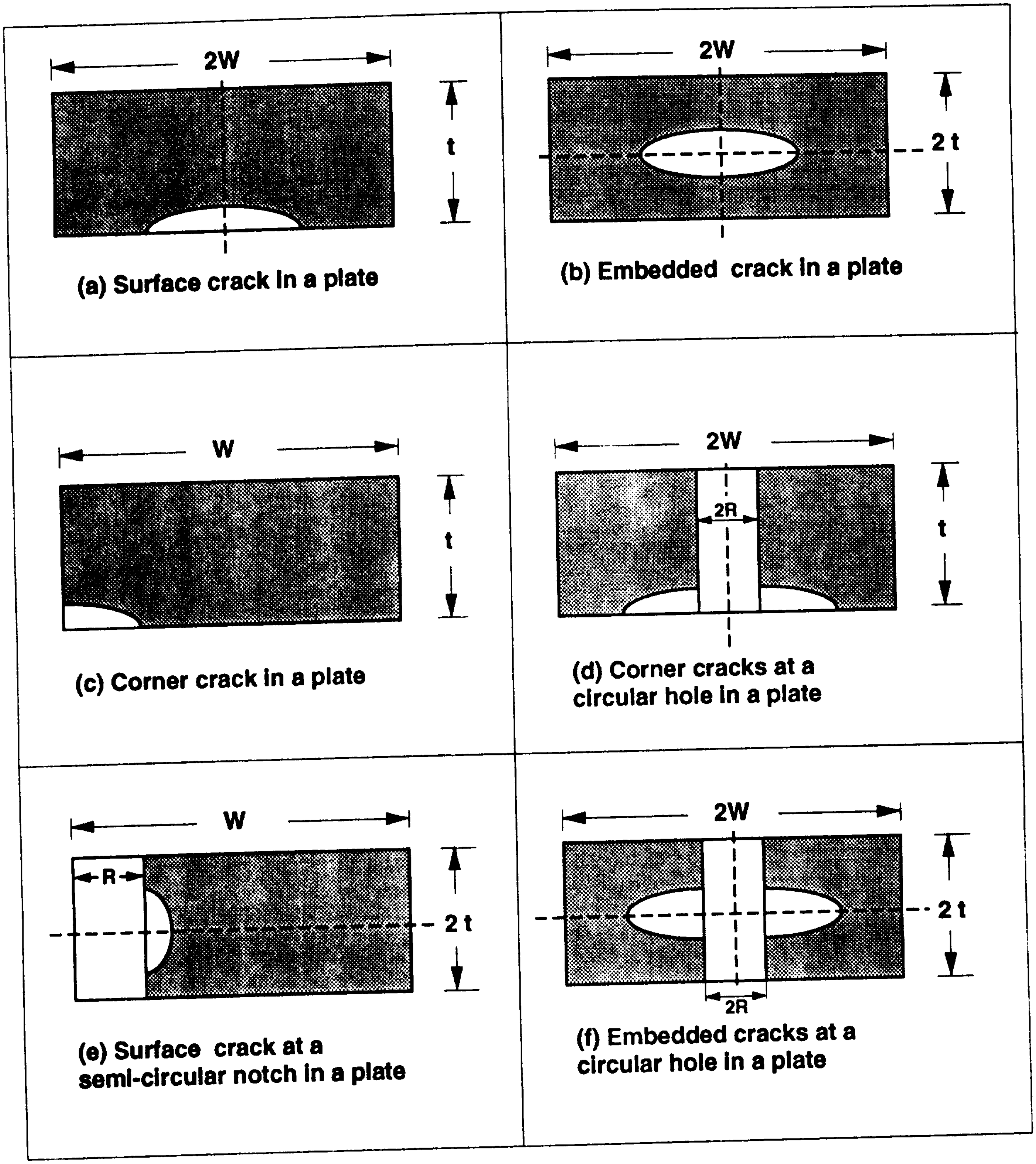
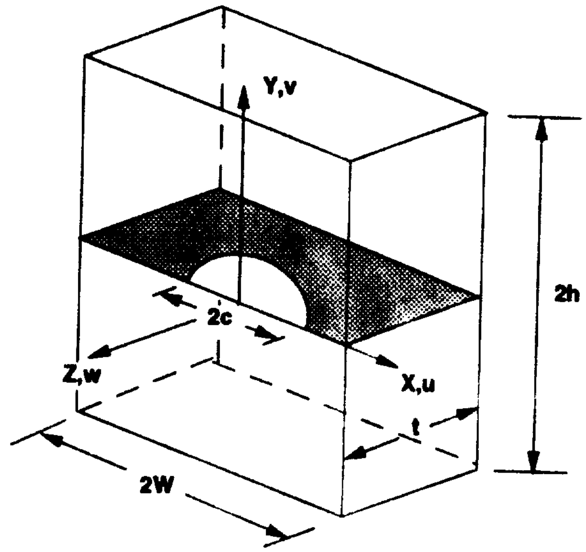
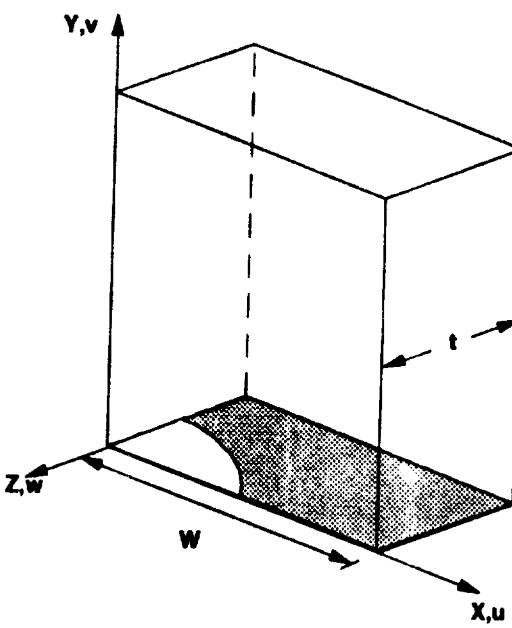


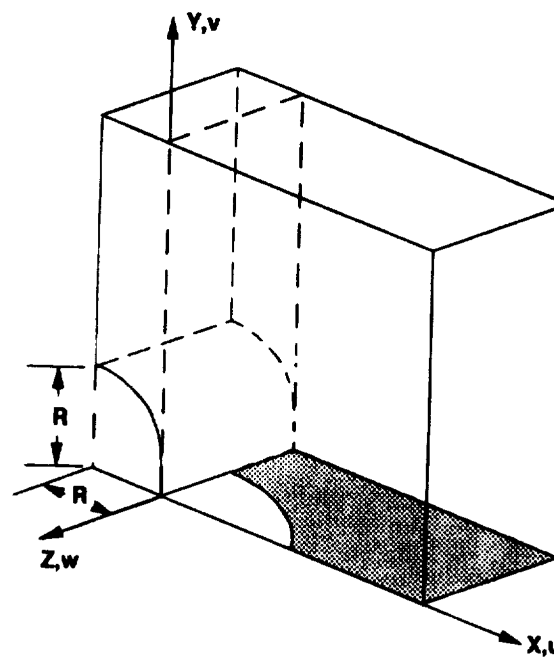
Figure 1: Crack Configurations.
(Elliptic crack: Semi-major axis= c ; Semi-minor axis= a)



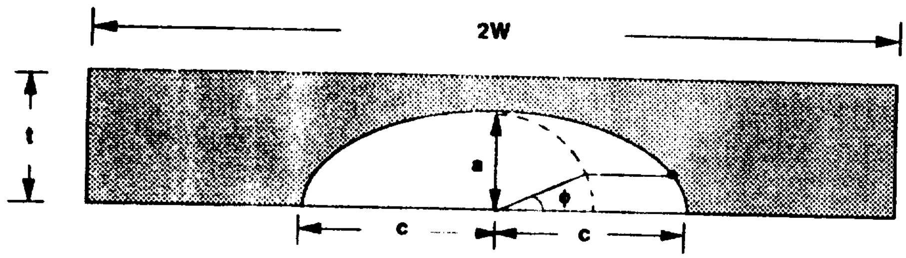
(a) Surface crack in a plate



(b) Region modeled for surface crack plate problems.

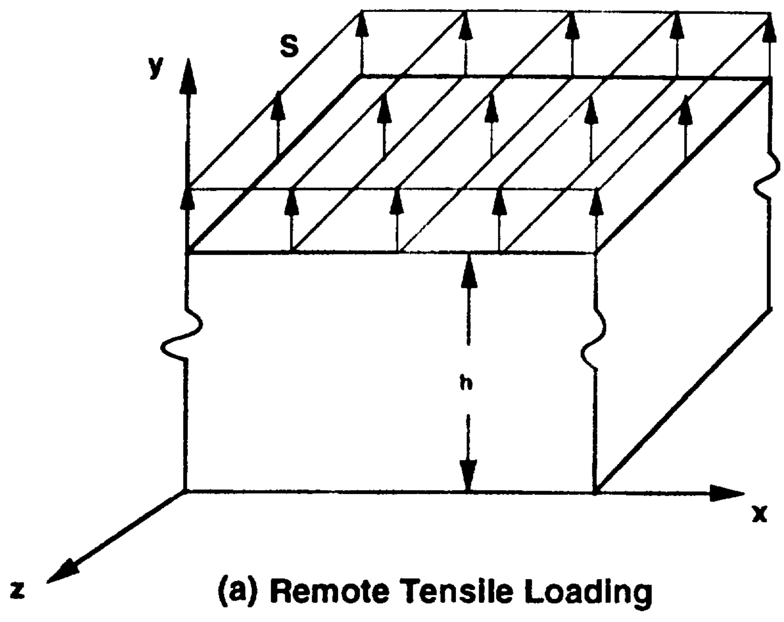


(c) Region modeled for surface crack in a plate with a circular hole problems.



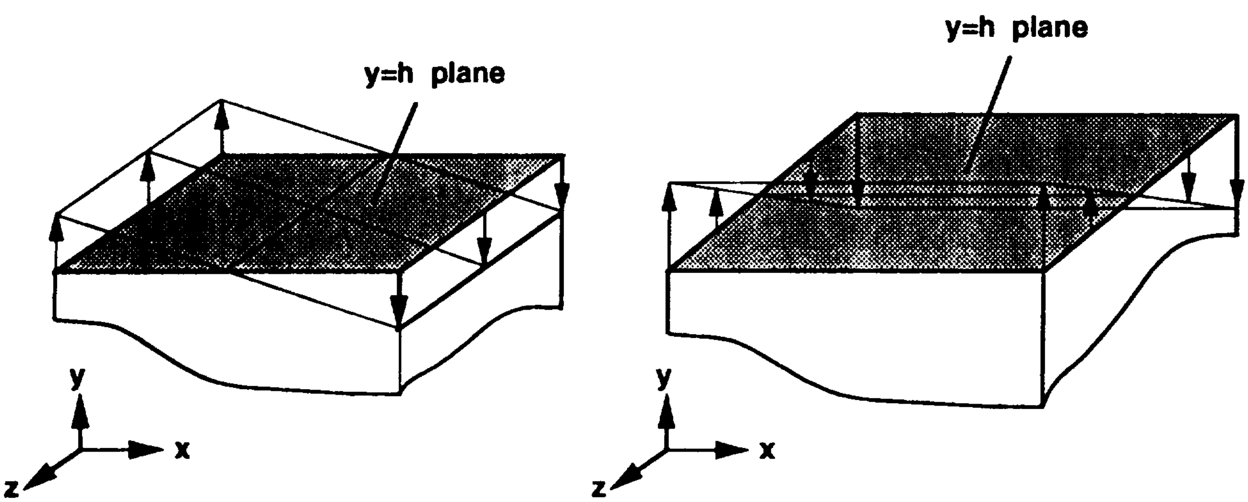
(d) Semi-elliptic surface crack and the crack plane.

Figure 2: Surface crack in a finite plate.



(a) Remote Tensile Loading

$$\sigma_y = S \text{ on } y=h$$
$$0 \leqslant z \leqslant -t \ ; \ 0 \leqslant x \leqslant W$$



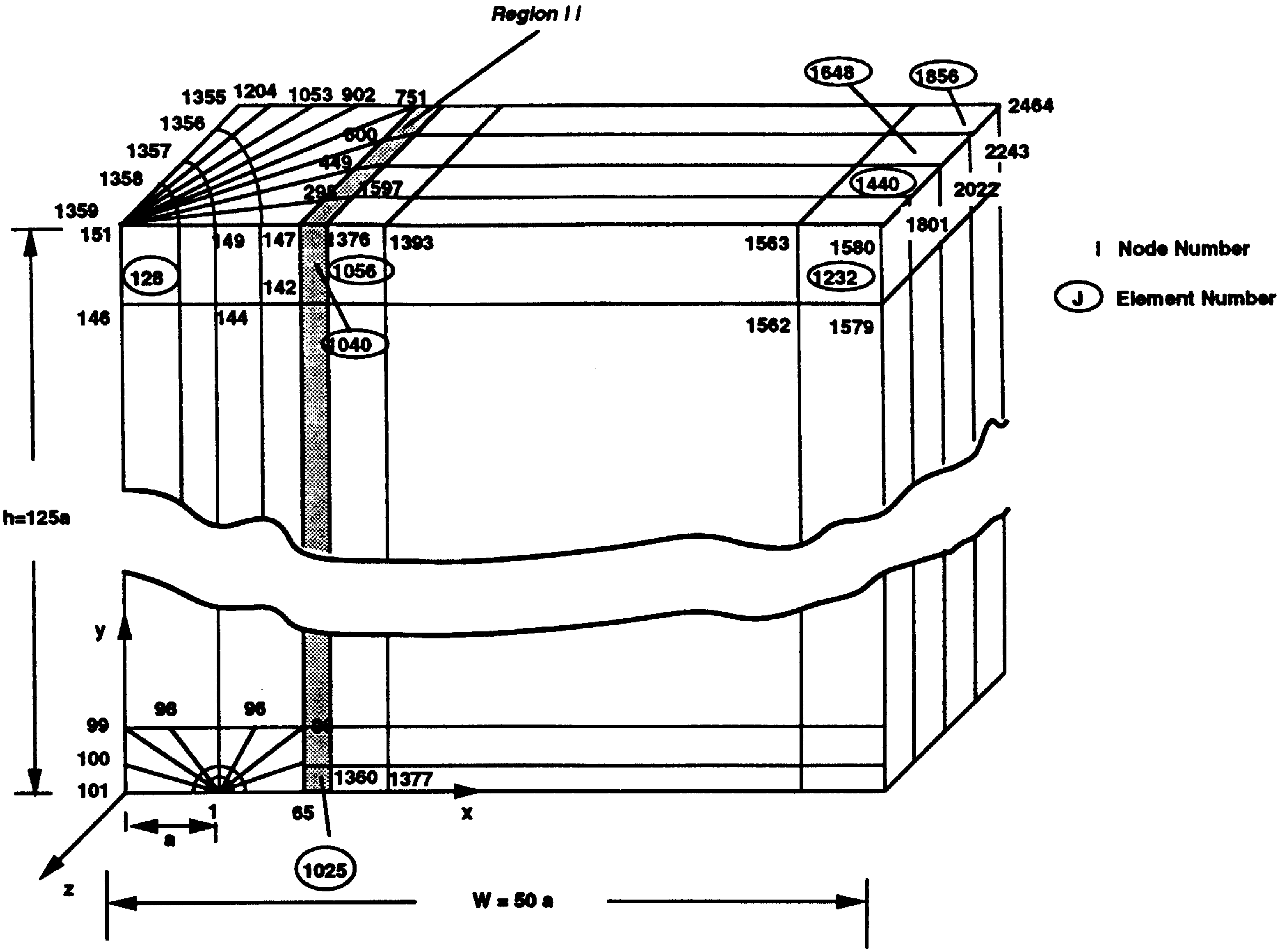
(b) Bending about z-axis

$$\sigma_y = 1 - 2x / W$$
$$0 \leqslant x \leqslant W$$

(c) Bending about x-axis

$$\sigma_y = 1 + 2 (z / t)$$
$$0 \leqslant z \leqslant -t$$

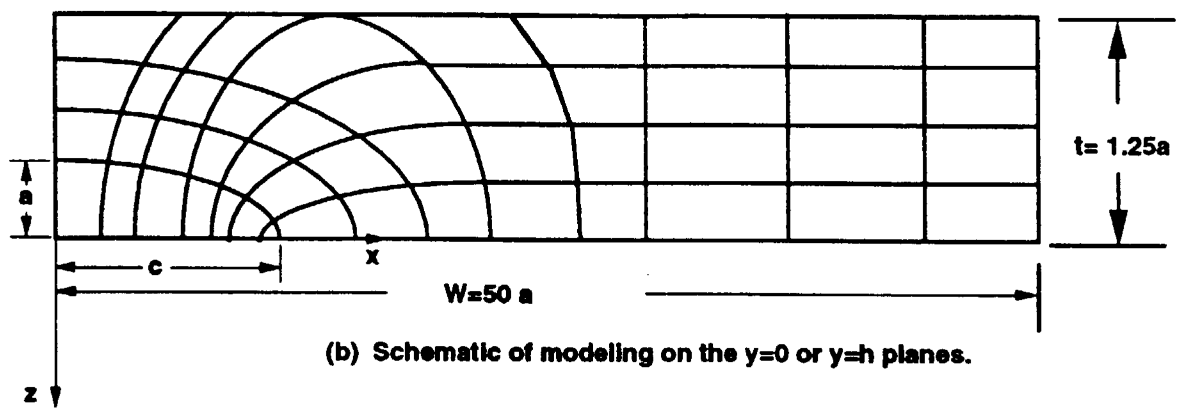
Figure 3: Remote loading applied to the models.



(a) Finite element model of a cracked solid with element and node numbers

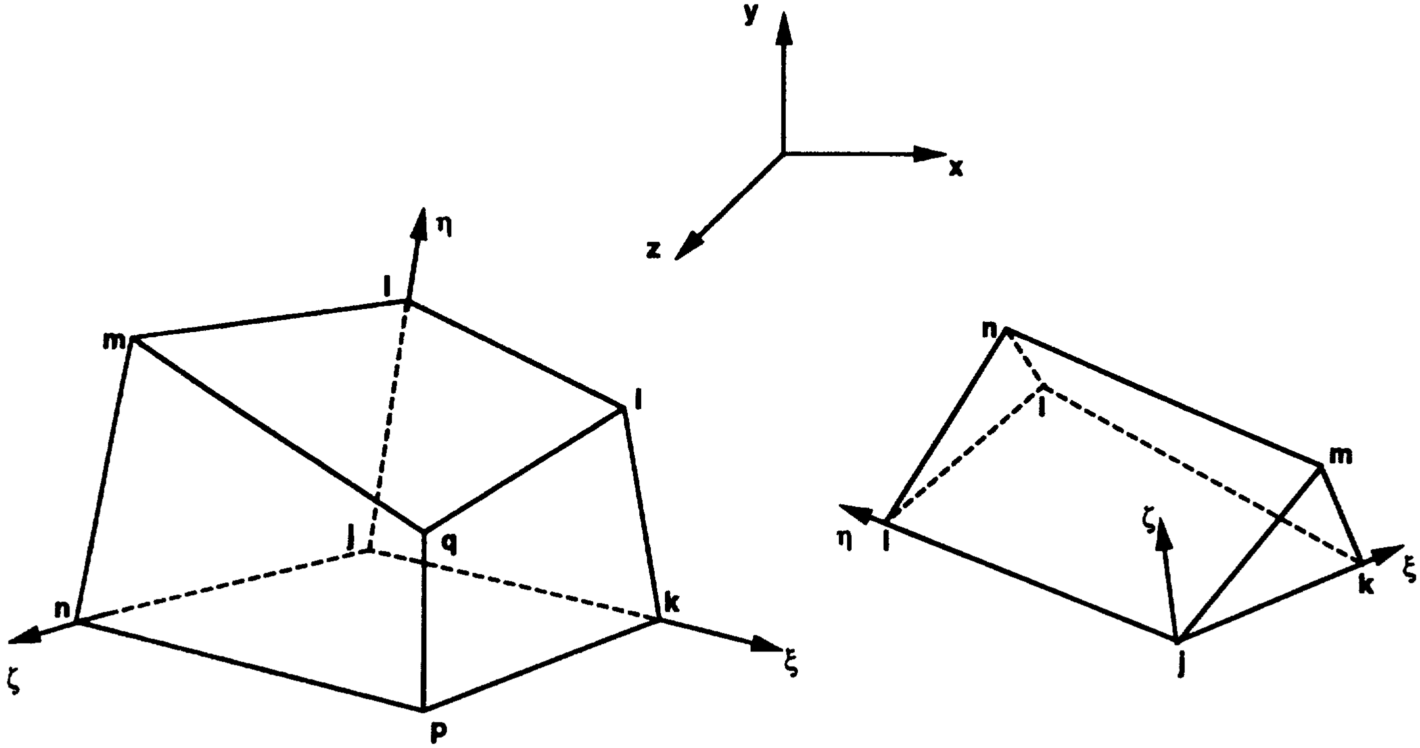
Figure 4 : 3D finite element model for a surface crack with a/c=1 and a/t=0.8.

(NLAYER=8, NSINGU=8)



(b) Schematic of modeling on the y=0 or y=h planes.

Figure 4 : (Concluded.)



(a) Regular hexahedral (Hex 8) element

(b) Pentahedral singularity element

Nodal Connectivity Definitions

Hex 8	i	j	k	l	m	n	p	q

Nodal Connectivity Definitions

Singularity Element	i	j	k	l	l	j	m	n

Other possible definitions for Hex-8 in (a)

Hex 8	j	k	l	i	n	p	q	m

or

Hex 8	k	l	i	j	p	q	m	n

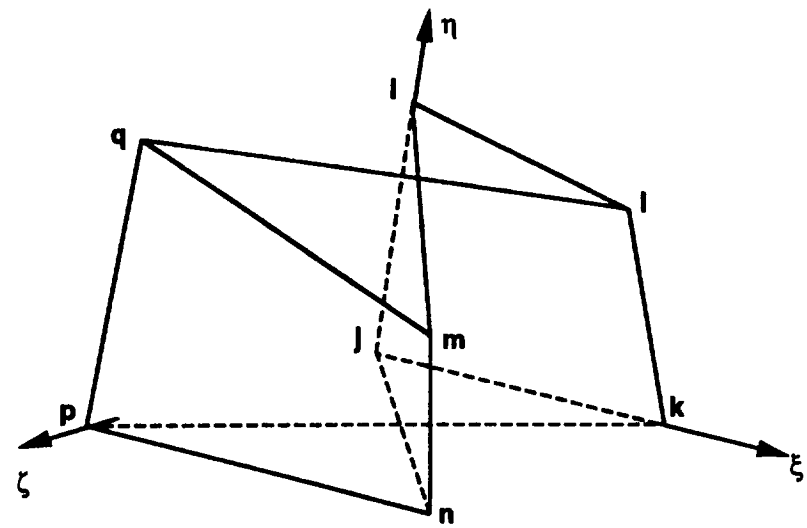
or

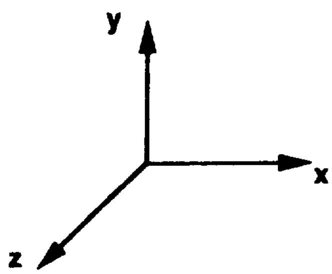
Hex 8	l	i	j	k	q	m	n	p

Definitions for the six faces of the Hex 8 element

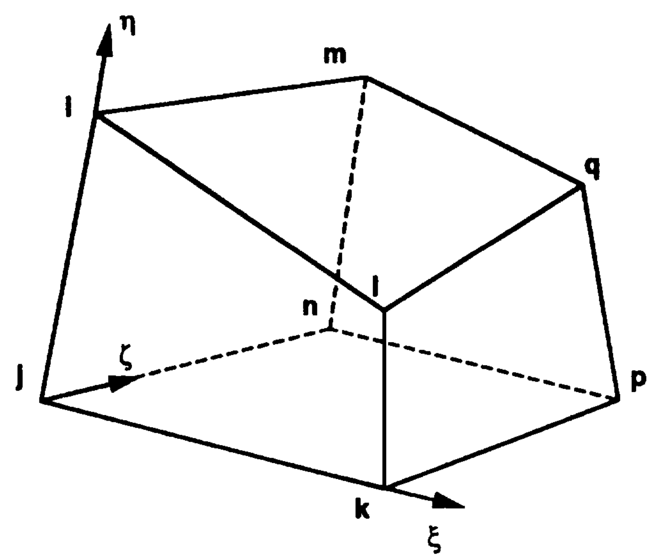
FACE	IFACE
i-j-m-n ($\xi=0$)	1
l-q-p-k ($\xi=1$)	2
k-p-n-j ($\eta=0$)	3
i-m-q-l ($\eta=1$)	4
i-l-k-j ($\zeta=0$)	5
p-q-m-n ($\zeta=1$)	6

Figure. 5: Definition of Hex-8 and singlularity elements.





(a) Twisted Element.



(b) Improperly defined element.
(i j k l m n p q)

Figure. 6 : Iconsistently defined elements.
(Zero or negative volumes will result for these elements.)

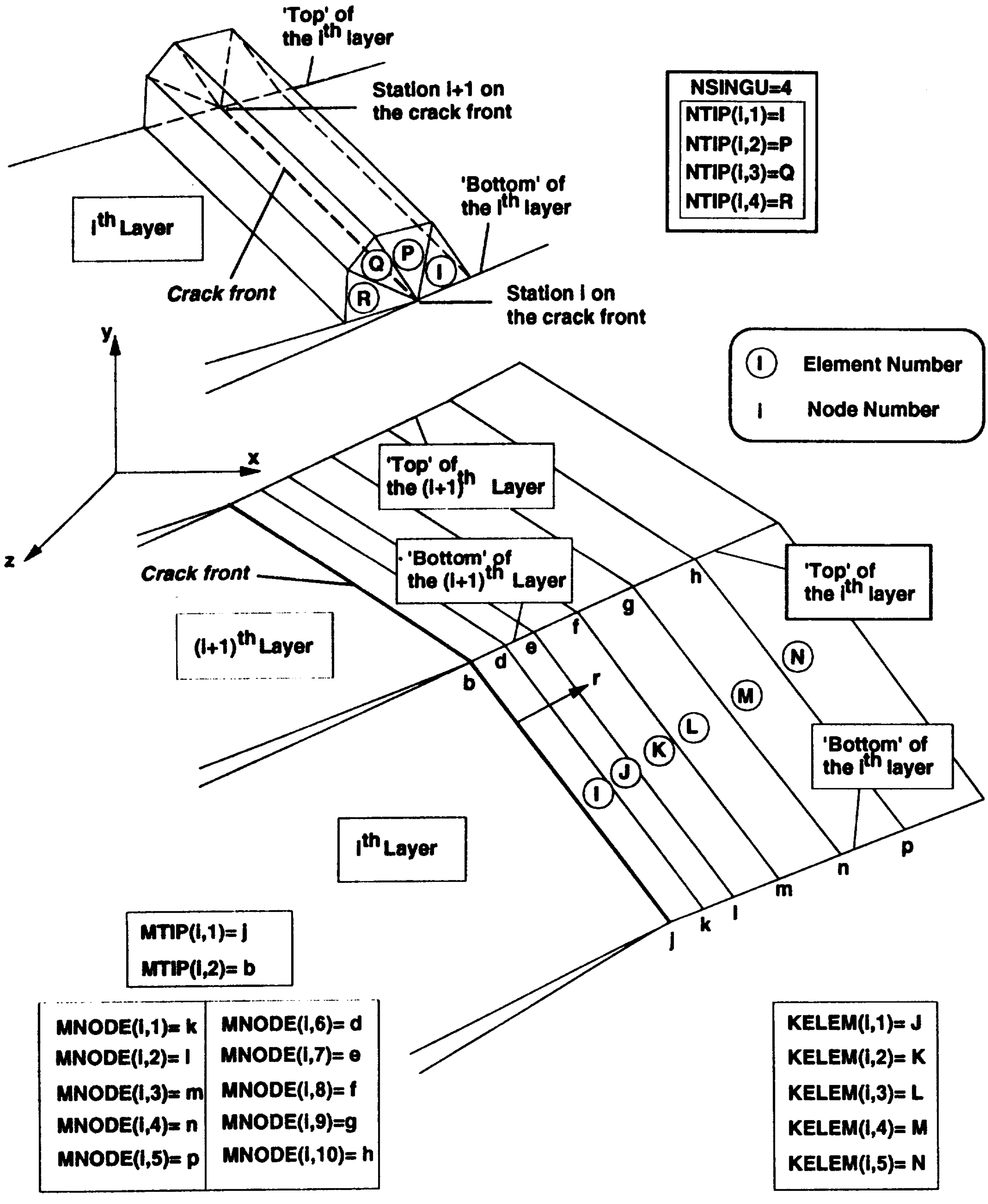
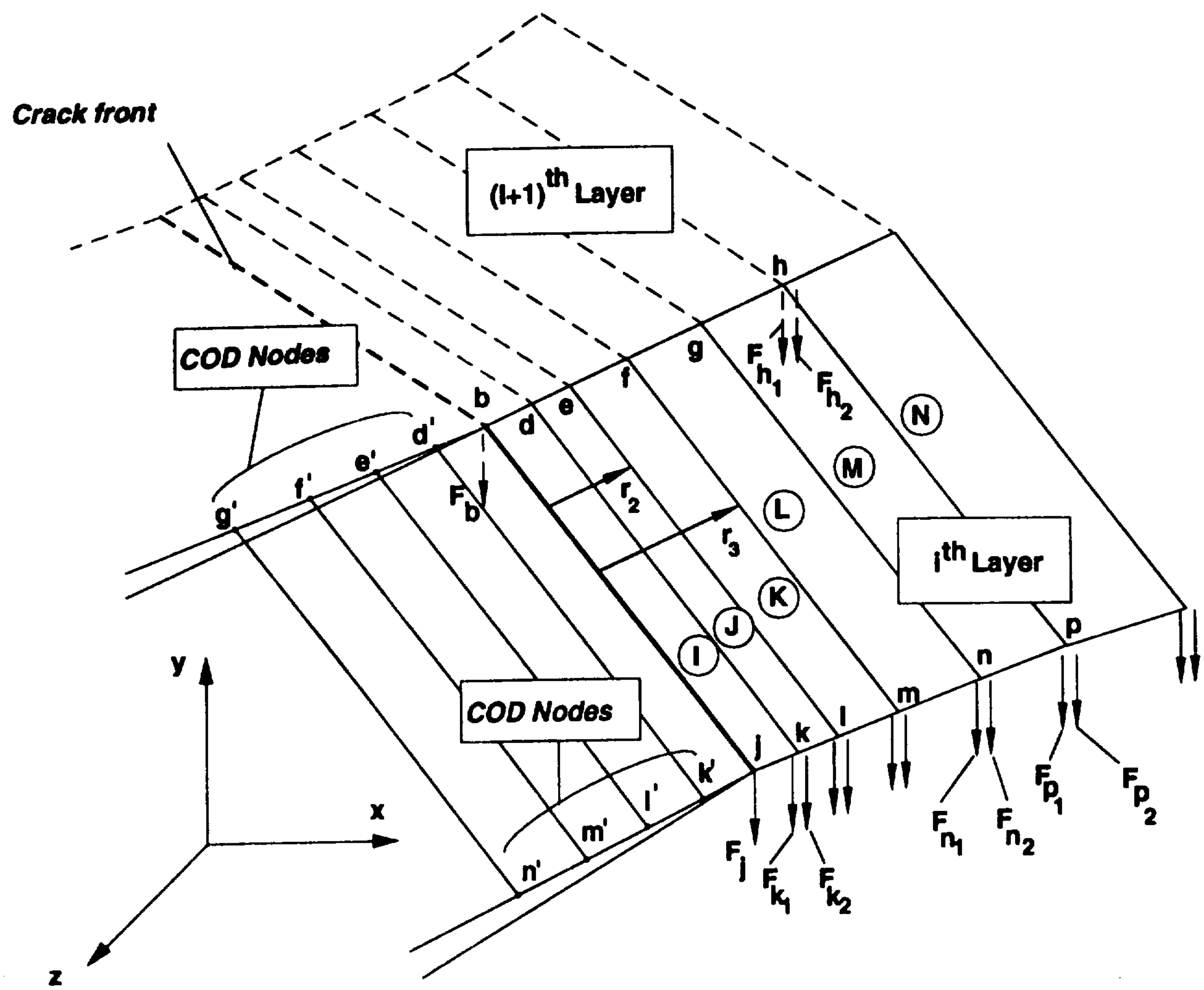


Figure 7: Definitions and various input parameters for the force method



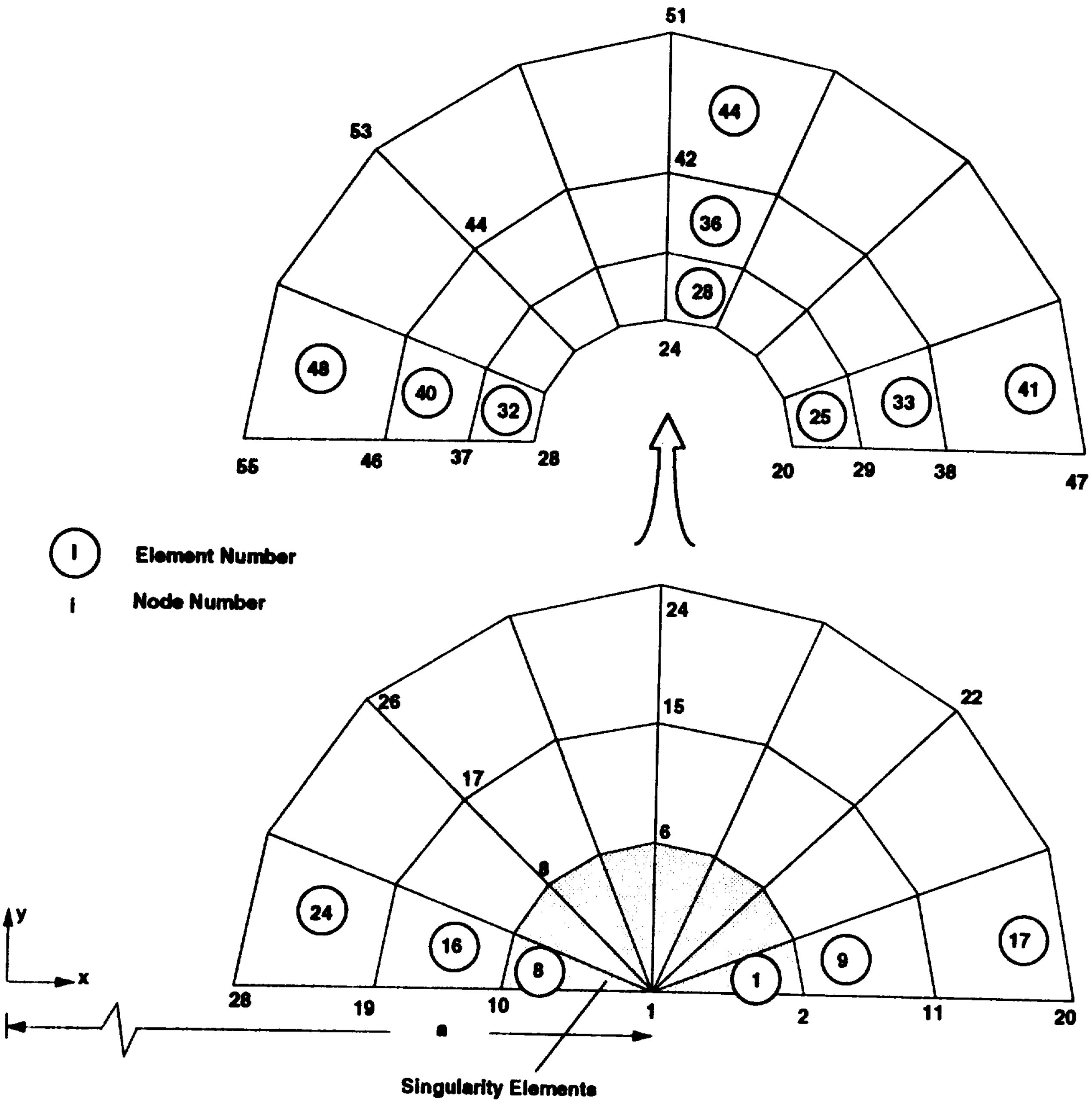
Forces Used in the Force Method

FCENT(I,1,1,IR) = F_{k_1}	FTIP(I,1,IR) = F_j
FCENT(I,1,2,IR) = F_{k_2}	FTIP(I,2,IR) = F_b
FCENT(I,2,1,IR) = F_{l_1}	
FCENT(I,2,2,IR) = F_{l_2}	
. .	I= Layer number
. .	IR= Current right hand side
. .	
FCENT(I,10,1,IR) = F_{h_1}	
FCENT(I,10,2,IR) = F_{h_2}	

COD Nodes

ICOD(I,1) = k'
ICOD(I,2) = l'
ICOD(I,3) = m'
ICOD(I,4) = n'
ICOD(I,5) = p'

Figure 8: Forces and displacements in the i^{th} layer



MAXKE=5, NSINGU=8

(KELEM(1,I), I=1,MAXKE) = 1 9 17 25 33

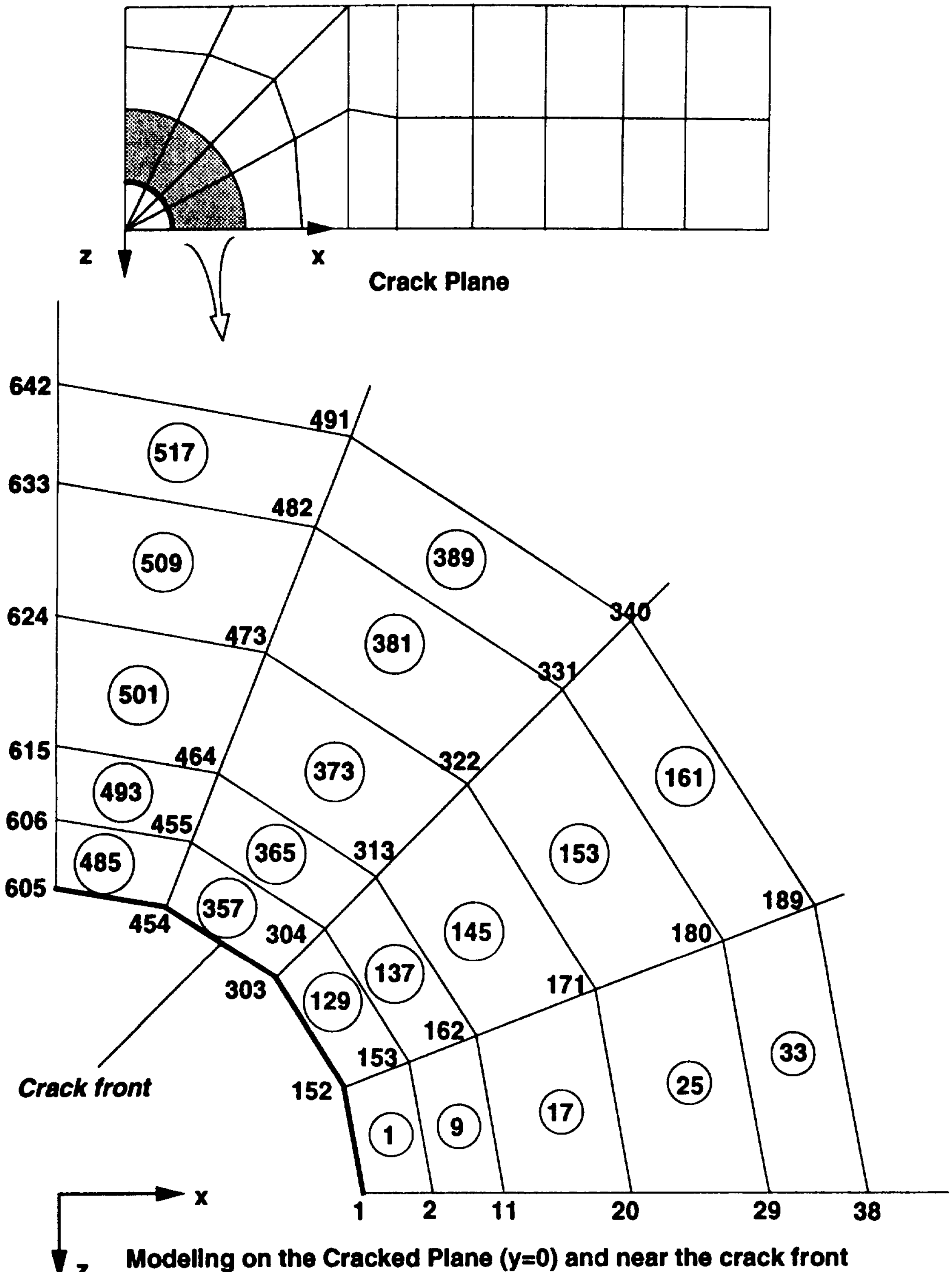
(MNODE(1, I), I=1, 2*MAXKE) = 2 11 20 29 38
153 162 171 180 189

(MTIP(1,I), I=1,2) = 1 152

(NTIP(1, I), I=1,NSINGU) = 1 2 3 4 5 6 7 8

(ICOD(1,I), I=1, MAXKE) = 10 19 28 37 46

Figure 9: Example of a base model with 151 nodes and 128 elements (details of data input for the first layer and only details near the crack front are shown)



Modeling on the Cracked Plane (y=0) and near the crack front
4-layer model (Not to scale)

Figure 10: Example of a model with 4 layers and 8-singularity elements (151 nodes and 128 elements in the base model)

NLAYER= 4   NSIF=NLAYER+1=5
MAXKE= 5   NSINGU=8

KELEM(NSIF,MAXKE) =

1	9	17	25	33
129	137	145	153	161
357	365	373	381	389
485	493	501	509	517

MNODE(NSIF, 2*MAXKE) =

2	11	20	29	38
153	162	171	180	189
153	162	171	180	189
304	313	322	331	340
304	313	322	331	340
455	464	473	482	491
455	464	473	482	491
606	615	624	633	642

MTIP (NSIF,2) =

1	152
152	303
303	454
454	605

NTI P(NSIF,NSINGU) =

1	2	3	4	5	6	7	8
129	130	131	132	133	134	135	136
357	358	359	360	361	362	363	364
485	486	487	488	489	490	491	492

ICOD(NLAYER+1,MAXKE) =

10	19	28	37	46
161	170	179	188	197
312	321	330	339	348
463	472	481	490	499
614	623	632	641	650

Figure 10: (Concluded.)

Table 1: Input file dexl for Example 1.

```
   SURFACE CRACK IN A PLATE TENSION AND BENDING  A/C=1.0 A/T=0.2
SHORT
   0.30000E+08     0.30000E+00
 2161 1664
    1          1.000000000         0.000000000          0.000000000
    2          1.013200000         0.000000000          0.000000000
    3          1.012200000         0.005200000          0.000000000
    4          1.009300000         0.009300000          0.000000000
    5          1.005200000         0.012200000          0.000000000
    6          1.000000000         0.013200000          0.000000000
    7          0.994800000         0.012200000          0.000000000
    8          0.990700000         0.009300000          0.000000000
    9          0.987800000         0.005200000          0.000000000
   10          0.986800000         0.000000000          0.000000000
   ...
   ...
   ...
 2161         25.000000000       125.000000000         -5.000000000
    1  210    1    2  211  210    1    3  212    1
    2  210    1    3  212  210    1    4  213    1
    3  210    1    4  213  210    1    5  214    1
    4  210    1    5  214  210    1    6  215    1
    ...
    ...
    ...
 1663 2146 2145 2159 2160 2090 2089 2103 2104    0
 1664 2147 2146 2160 2161 2091 2090 2104 2105    0
         1    0    1    0
         2    0    1    0
        11    0    1    0
        ...
        ...
        ...
      1878    1    0    0
      1879    1    0    0
      1880    1    0    0
      1881    1    0    0
      2161    0    0    1
         0    0    0    0
         3
REMOTE
       177  177    1    1    4
       178  178    1    1    4
       179  179    1    1    4
       ...
       ...
       ...
      1625 1625    1    1    4
      1638 1638    1    1    4
      1651 1651    1    1    4
      1664 1664    1    1    4
         0    0    0    0    0
```

```
   203   0.0000   1.0000   0.0000
   204   0.0000   1.0000   0.0000
   205   0.0000   1.0000   0.0000
   206   0.0000   1.0000   0.0000
   207   0.0000   1.0000   0.0000
   ...
   ...
   ...
  2147   0.0000   1.0000   0.0000
  2161   0.0000   1.0000   0.0000
     0   0.0000   0.0000   0.0000
   203   0.0000   1.0000   0.0000
   204   0.0000   1.0000   0.0000
   205   0.0000   1.0000   0.0000
   ...
   ...
   ...
  2105   0.0000  -0.5000   0.0000
  2119   0.0000  -1.0000   0.0000
  2133   0.0000  -1.0000   0.0000
  2147   0.0000  -1.0000   0.0000
  2161   0.0000  -1.0000   0.0000
     0   0.0000   0.0000   0.0000
   203   0.0000   0.6000   0.0000
   204   0.0000   0.6800   0.0000
   ...
   ...
   ...
  2091   0.0000  -0.3600   0.0000
  2105   0.0000  -1.0000   0.0000
  2119   0.0000   0.4400   0.0000
  2133   0.0000   0.1200   0.0000
  2147   0.0000  -0.3600   0.0000
  2161   0.0000  -1.0000   0.0000
     0   0.0000   0.0000   0.0000
     0   0    0    0    0.00000E+00   0.00000E+00   0.00000E+00
     1
     1       2       3       4       5
     6       7       8       9      10
    11      12      13      14      15
    16      17      18      19      20
   ...
   ...
   ...
  1310    1311    1312    1313    1314
  1315    1316    1317    1318    1319
  1320    1321    1322    1323    1324
  1325
     1
     1
0.0000
8    8
    10      19      28      37      46
   219     228     237     246     255
   428     437     446     455     464
   637     646     655     664     673
```

846	855	864	873	882
1055	1064	1073	1082	1091
1264	1273	1282	1291	1300
1473	1482	1491	1500	1509
1682	1691	1700	1709	1718
1	210	210	419	419
628	628	837	837	1046
1046	1255	1255	1464	1464
1673				
1	2	3	4	5
6	7	8	183	184
185	186	187	188	189
190	365	366	367	368
369	370	371	372	547
548	549	550	551	552
553	554	729	730	731
732	733	734	735	736
911	912	913	914	915
916	917	918	1093	1094
1095	1096	1097	1098	1099
1100	1275	1276	1277	1278
1279	1280	1281	1282	
1	9	17	25	33
183	191	199	207	215
365	373	381	389	397
547	555	563	571	579
729	737	745	753	761
911	919	927	935	943
1093	1101	1109	1117	1125
1275	1283	1291	1299	1307
2	11	20	29	38
211	220	229	238	247
211	220	229	238	247
420	429	438	447	456
420	429	438	447	456
629	638	647	656	665
629	638	647	656	665
838	847	856	865	874
838	847	856	865	874
1047	1056	1065	1074	1083
1047	1056	1065	1074	1083
1256	1265	1274	1283	1292
1256	1265	1274	1283	1292
1465	1474	1483	1492	1501
1465	1474	1483	1492	1501
1674	1683	1692	1701	1710
125.0000	25.0000	0.2000	5.0000	1.0000

Table 2: Output file out12 for Example 1.

```
*************************************************************
SURFACE CRACK IN A PLATE TENSION AND BENDING  A/C=1.0 A/T=0.2
*************************************************************

-------------------------------------------------------------
             DESCRIPTION OF THE MODEL
-------------------------------------------------------------
          OUTPUT OPTION                      =  SHORT
          YOUNG S MODULUS                    =   0.300000E+08
          POISSION S RATIO                   =          0.300
          NUMBER OF NODES IN THE MODEL       =  2161
          NUMBER OF ELEMENTS IN THE MODEL=  1664
-------------------------------------------------------------
                         NODAL COORDINATES
NODE               X-COORD              Y-COORD            Z-COORD
-------------------------------------------------------------

   1               1.00000              0.00000            0.00000
   2               1.01320              0.00000            0.00000
   3               1.01220              0.00520            0.00000
   4               1.00930              0.00930            0.00000
   ...
   ...
   ...

          IERR FROM SYMBN=     0

**********************************************************************
                  LOADING NUMBER    1
**********************************************************************

**********************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.1218192E-09
          SUM OF THE Y FORCE=       -0.2538115E-08
          SUM OF THE Z FORCE=        0.1493786E-09
----------------------------------------------------------------------

----------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
X-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
Y-COMPONENTS:      FORCE=     0.125000E+03     AREA=     0.125000E+03
Z-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
----------------------------------------------------------------------

**********************************************************************
                 N O M I N A L    S T R E S S E S
        NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
        NOMINAL STRESS IN THE Y- DIRECTION =        0.1000000E+01
        NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
----------------------------------------------------------------------
```

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1297875E+01	0.1149418E+01
2	11.250	0.1261908E+01	0.1117566E+01
3	22.500	0.1215839E+01	0.1076766E+01
4	33.750	0.1188309E+01	0.1052385E+01
5	45.000	0.1171463E+01	0.1037467E+01
6	56.250	0.1161649E+01	0.1028774E+01
7	67.500	0.1156257E+01	0.1024000E+01
8	78.750	0.1153536E+01	0.1021590E+01
9	90.000	0.1152694E+01	0.1020844E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1335318E+01	0.1182579E+01
2	11.250	0.1243665E+01	0.1101410E+01
3	22.500	0.1202907E+01	0.1065313E+01
4	33.750	0.1176129E+01	0.1041599E+01
5	45.000	0.1159989E+01	0.1027305E+01
6	56.250	0.1150607E+01	0.1018996E+01
7	67.500	0.1145500E+01	0.1014473E+01
8	78.750	0.1142883E+01	0.1012155E+01
9	90.000	0.1142066E+01	0.1011432E+01

****

NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64

****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.11494E+01	0.11826E+01
2	11.250	0.11176E+01	0.11014E+01
3	22.500	0.10768E+01	0.10653E+01
4	33.750	0.10524E+01	0.10416E+01
5	45.000	0.10375E+01	0.10273E+01
6	56.250	0.10288E+01	0.10190E+01
7	67.500	0.10240E+01	0.10145E+01
8	78.750	0.10216E+01	0.10122E+01
9	90.000	0.10208E+01	0.10114E+01

****

LOADING NUMBER 2

****

****

E Q U I L I B R I U M   C H E C K S

SUM OF THE X FORCE= 0.8234746E-08
SUM OF THE Y FORCE= -0.4588969E-09
SUM OF THE Z FORCE= -0.1175395E-06

APPLIED LOAD AND THE SURFACE AREA COMPONENTS

X-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00
Y-COMPONENTS:	FORCE=	0.939978E-04	AREA=	0.125000E+03
Z-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION = 0.0000000E+00
NOMINAL STRESS IN THE Y- DIRECTION = 0.7519827E-06
NOMINAL STRESS IN THE Z- DIRECTION = 0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1160053E+01	0.1027362E+01
2	11.250	0.1097790E+01	0.9722205E+00
3	22.500	0.1012613E+01	0.8967860E+00
4	33.750	0.9453700E+00	0.8372347E+00
5	45.000	0.8916253E+00	0.7896376E+00
6	56.250	0.8502292E+00	0.7529765E+00
7	67.500	0.8206783E+00	0.7268058E+00
8	78.750	0.8028458E+00	0.7110130E+00
9	90.000	0.7968713E+00	0.7057219E+00

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1208218E+01	0.1070017E+01
2	11.250	0.1081892E+01	0.9581404E+00
3	22.500	0.1001718E+01	0.8871372E+00
4	33.750	0.9348640E+00	0.8279304E+00
5	45.000	0.8816859E+00	0.7808351E+00
6	56.250	0.8405767E+00	0.7444282E+00
7	67.500	0.8112083E+00	0.7184190E+00
8	78.750	0.7934419E+00	0.7026848E+00
9	90.000	0.7874846E+00	0.6974089E+00

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.10274E+01	0.10700E+01
2	11.250	0.97222E+00	0.95814E+00

3	22.500	0.89679E+00	0.88714E+00
4	33.750	0.83723E+00	0.82793E+00
5	45.000	0.78964E+00	0.78084E+00
6	56.250	0.75298E+00	0.74443E+00
7	67.500	0.72681E+00	0.71842E+00
8	78.750	0.71101E+00	0.70268E+00
9	90.000	0.70572E+00	0.69741E+00

****

LOADING NUMBER 3

****

****

E Q U I L I B R I U M   C H E C K S

SUM OF THE X FORCE= 0.1084288E-10

SUM OF THE Y FORCE= -0.3993250E-10

SUM OF THE Z FORCE= -0.4711880E-10

APPLIED LOAD AND THE SURFACE AREA COMPONENTS

X-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00
Y-COMPONENTS:	FORCE=	0.251484E-04	AREA=	0.125000E+03
Z-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION = 0.0000000E+00

NOMINAL STRESS IN THE Y- DIRECTION = 0.2011872E-06

NOMINAL STRESS IN THE Z- DIRECTION = 0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.2430848E-02	0.2152797E-02
2	11.250	0.2362947E-02	0.2092664E-02
3	22.500	0.2276044E-02	0.2015700E-02
4	33.750	0.2224167E-02	0.1969758E-02

5	45.000	0.2192557E-02	0.1941763E-02
6	56.250	0.2174276E-02	0.1925574E-02
7	67.500	0.2164344E-02	0.1916778E-02
8	78.750	0.2159391E-02	0.1912391E-02
9	90.000	0.2157872E-02	0.1911046E-02

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.2501464E-02	0.2215336E-02
2	11.250	0.2328975E-02	0.2062578E-02
3	22.500	0.2251980E-02	0.1994389E-02
4	33.750	0.2201486E-02	0.1949671E-02
5	45.000	0.2171161E-02	0.1922815E-02
6	56.250	0.2153651E-02	0.1907308E-02
7	67.500	0.2144214E-02	0.1898950E-02
8	78.750	0.2139433E-02	0.1894716E-02
9	90.000	0.2137952E-02	0.1893404E-02

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.21528E-02	0.22153E-02
2	11.250	0.20927E-02	0.20626E-02
3	22.500	0.20157E-02	0.19944E-02
4	33.750	0.19698E-02	0.19497E-02
5	45.000	0.19418E-02	0.19228E-02
6	56.250	0.19256E-02	0.19073E-02
7	67.500	0.19168E-02	0.18989E-02
8	78.750	0.19124E-02	0.18947E-02

```
                    9            90.000          0.19110E-02          0.18934E-02

                      ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 29.608s,  Wallclock: 84.966s,  8.7% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 3: Output file outnl2 for Example 2.

```
 ****************************************************************
    SURFACE CRACK IN A PLATE TENSION AND BENDING  A/C=1.0 A/T=0.2
 ****************************************************************

 ----------------------------------------------------------------
                   DESCRIPTION OF THE MODEL
 ----------------------------------------------------------------
            OUTPUT OPTION                         =  SHORT
            YOUNG S MODULUS                       =   0.300000E+08
            POISSION S RATIO                      =          0.300
            NUMBER OF NODES IN THE MODEL          =  2161
            NUMBER OF ELEMENTS IN THE MODEL=  1664
 ----------------------------------------------------------------
                              NODAL COORDINATES
 NODE               X-COORD               Y-COORD             Z-COORD
 ----------------------------------------------------------------

     1              1.00000               0.00000             0.00000
     2              1.01320               0.00000             0.00000
     3              1.01220               0.00520             0.00000
    ...
    ...
    ...

**********************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.1052262E-09
          SUM OF THE Y FORCE=       -0.2110426E-08
          SUM OF THE Z FORCE=        0.1165562E-09
----------------------------------------------------------------------

----------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
  X-COMPONENTS:      FORCE=     0.000000E+00     AREA=    0.000000E+00
  Y-COMPONENTS:      FORCE=     0.125000E+03     AREA=    0.125000E+03
  Z-COMPONENTS:      FORCE=     0.000000E+00     AREA=    0.000000E+00
----------------------------------------------------------------------
```

```
*************************************************************************
                 N O M I N A L    S T R E S S E S
     NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
     NOMINAL STRESS IN THE Y- DIRECTION =        0.1000000E+01
     NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
-------------------------------------------------------------------------

     **************************************************************
     NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=     0
     **************************************************************

     **************************************************************
                                K/( S*SQRT(PI A/Q) )
                                FROM VCCT          FROM VCCT
     STATION         PHI        P-STRAIN           P-STRESS
     **************************************************************
```

STATION	PHI	K/(S*SQRT(PI A/Q)) FROM VCCT P-STRAIN	FROM VCCT P-STRESS
1	0.000	0.11774E+01	0.11232E+01
2	11.250	0.11215E+01	0.10699E+01
3	22.500	0.10819E+01	0.10320E+01
4	33.750	0.10576E+01	0.10089E+01
5	45.000	0.10428E+01	0.99475E+00
6	56.250	0.10341E+01	0.98650E+00
7	67.500	0.10294E+01	0.98196E+00
8	78.750	0.10269E+01	0.97964E+00
9	90.000	0.10262E+01	0.97892E+00

```
*************************************************************************
                    LOADING NUMBER     2
*************************************************************************

*************************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.2532065E-08
          SUM OF THE Y FORCE=       -0.6705818E-09
          SUM OF THE Z FORCE=       -0.9852940E-07
-------------------------------------------------------------------------

-------------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
  X-COMPONENTS:       FORCE=     0.000000E+00    AREA=     0.000000E+00
  Y-COMPONENTS:       FORCE=     0.939978E-04    AREA=     0.125000E+03
  Z-COMPONENTS:       FORCE=     0.000000E+00    AREA=     0.000000E+00
-------------------------------------------------------------------------
```

```
******************************************************************
              N O M I N A L     S T R E S S E S
      NOMINAL STRESS IN THE X- DIRECTION =         0.0000000E+00
      NOMINAL STRESS IN THE Y- DIRECTION =         0.7519827E-06
      NOMINAL STRESS IN THE Z- DIRECTION =         0.0000000E+00
-------------------------------------------------------------------

     **********************************************************
      NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=    0
     **********************************************************

     **********************************************************
     STATION           PHI            K/( S*SQRT(PI A/Q) )
                                      FROM VCCT          FROM VCCT
                                      P-STRAIN           P-STRESS
     **********************************************************

        1             0.000          0.10588E+01        0.10100E+01

        2            11.250          0.97573E+00        0.93078E+00

        3            22.500          0.90118E+00        0.85967E+00

        4            33.750          0.84131E+00        0.80256E+00

        5            45.000          0.79357E+00        0.75701E+00

        6            56.250          0.75670E+00        0.72185E+00

        7            67.500          0.73037E+00        0.69673E+00

        8            78.750          0.71446E+00        0.68155E+00

        9            90.000          0.70912E+00        0.67646E+00

******************************************************************
                 LOADING NUMBER    3
******************************************************************

******************************************************************
          E Q U I L I B R I U M    C H E C K S
          SUM OF THE X FORCE=         0.1055867E-10
          SUM OF THE Y FORCE=        -0.2489742E-10
          SUM OF THE Z FORCE=        -0.4182784E-10
-------------------------------------------------------------------

-------------------------------------------------------------------
         APPLIED LOAD AND THE SURFACE AREA COMPONENTS
 X-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
 Y-COMPONENTS:      FORCE=     0.251484E-04     AREA=     0.125000E+03
 Z-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
-------------------------------------------------------------------
```

```
*************************************************************************
                 N O M I N A L    S T R E S S E S
          NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
          NOMINAL STRESS IN THE Y- DIRECTION =        0.2011872E-06
          NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
-------------------------------------------------------------------------

          ***************************************************************
           NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=     0
          ***************************************************************

          ***************************************************************
                                       K/( S*SQRT(PI A/Q) )
          STATION          PHI         FROM VCCT           FROM VCCT
                                       P-STRAIN            P-STRESS
          ***************************************************************

             1             0.000       0.22053E-02         0.21038E-02

             2            11.250       0.21001E-02         0.20033E-02

             3            22.500       0.20253E-02         0.19320E-02

             4            33.750       0.19795E-02         0.18883E-02

             5            45.000       0.19517E-02         0.18618E-02

             6            56.250       0.19356E-02         0.18465E-02

             7            67.500       0.19269E-02         0.18382E-02

             8            78.750       0.19225E-02         0.18339E-02

             9            90.000       0.19211E-02         0.18327E-02

               ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 30.631s,  Wallclock: 37.531s,  20.4% of 4-CPU Machine
HWM mem: 7730259, HWM stack: 310502, Stack overflows: 0
```

Table 4: Input file dex3a for Example 3.

```
  SURFACE CRACK IN A PLATE        A/C=0.2 A/T=0.2
SHORT
  0.30000E+08     0.30000E+00
 2441 1872
   1        5.000000000     0.000000000     0.000000000
   2        5.002656718     0.000000000     0.000000000
   3        5.002454282     0.005200000     0.000000000
   4        5.001868300     0.009300000     0.000000000
  ...
```

```
...
...
2437        25.000000000        15.000000000        -5.000000000
2438        25.000000000        25.000000000        -5.000000000
2439        25.000000000        45.000000000        -5.000000000
2440        25.000000000        85.000000000        -5.000000000
2441        25.000000000       125.000000000        -5.000000000
   1  210    1    2  211  210    1    3  212    1
   2  210    1    3  212  210    1    4  213    1
   3  210    1    4  213  210    1    5  214    1
   4  210    1    5  214  210    1    6  215    1
   5  210    1    6  215  210    1    7  216    1
   6  210    1    7  216  210    1    8  217    1
   ...
   ...
   ...
1867 2422 2421 2435 2436 2310 2309 2323 2324    0
1868 2423 2422 2436 2437 2311 2310 2324 2325    0
1869 2424 2423 2437 2438 2312 2311 2325 2326    0
1870 2425 2424 2438 2439 2313 2312 2326 2327    0
1871 2426 2425 2439 2440 2314 2313 2327 2328    0
1872 2427 2426 2440 2441 2315 2314 2328 2329    0
        1    0    1    0
        2    0    1    0
       11    0    1    0
      ...
      ...
      ...
     1878    1    0    0
     1879    1    0    0
     1880    1    0    0
     1881    1    0    0
     2441    0    0    1
        0    0    0    0
        1
REMOTE
      177  177    1    1    4
      178  178    1    1    4
      179  179    1    1    4
      ...
      ...
      ...
     1859 1859    1    1    4
     1872 1872    1    1    4
        0    0    0    0    0
      203    0.0000    1.0000    0.0000
      204    0.0000    1.0000    0.0000
      205    0.0000    1.0000    0.0000
      206    0.0000    1.0000    0.0000
      ...
      ...
      ...
     2399    0.0000    1.0000    0.0000
     2413    0.0000    1.0000    0.0000
     2427    0.0000    1.0000    0.0000
     2441    0.0000    1.0000    0.0000
```

```
   0   0.0000   0.0000   0.0000
   0   0    0   0   0.00000E+00   0.00000E+00   0.00000E+00
   1
   1      2      3      4      5
   6      7      8      9     10
  11     12     13     14     15
  16     17     18     19     20
 ...
 ...
 ...
1590   1591   1592   1593   1594
1595   1596   1597   1598   1599
1600   1601   1602   1603   1604
1605
   1
   1
0.0000
8   8
  10     19     28     37     46
 219    228    237    246    255
 428    437    446    455    464
 637    646    655    664    673
 846    855    864    873    882
1055   1064   1073   1082   1091
1264   1273   1282   1291   1300
1473   1482   1491   1500   1509
1682   1691   1700   1709   1718
   1    210    210    419    419
 628    628    837    837   1046
1046   1255   1255   1464   1464
1673
   1      2      3      4      5
   6      7      8    183    184
 185    186    187    188    189
 190    365    366    367    368
 369    370    371    372    547
 548    549    550    551    552
 553    554    729    730    731
 732    733    734    735    736
 911    912    913    914    915
 916    917    918   1093   1094
1095   1096   1097   1098   1099
1100   1275   1276   1277   1278
1279   1280   1281   1282
   1      9     17     25     33
 183    191    199    207    215
 365    373    381    389    397
 547    555    563    571    579
 729    737    745    753    761
 911    919    927    935    943
1093   1101   1109   1117   1125
1275   1283   1291   1299   1307
   2     11     20     29     38
 211    220    229    238    247
 211    220    229    238    247
 420    429    438    447    456
```

420	429	438	447	456
629	638	647	656	665
629	638	647	656	665
838	847	856	865	874
838	847	856	865	874
1047	1056	1065	1074	1083
1047	1056	1065	1074	1083
1256	1265	1274	1283	1292
1256	1265	1274	1283	1292
1465	1474	1483	1492	1501
1465	1474	1483	1492	1501
1674	1683	1692	1701	1710
125.0000	25.0000	0.2000	5.0000	0.2000

Table 5: Output file outr22 for Example 3(a).

```
************************************************************
   SURFACE CRACK IN A PLATE       A/C=0.2 A/T=0.2
************************************************************

------------------------------------------------------------
               DESCRIPTION OF THE MODEL
------------------------------------------------------------
           OUTPUT OPTION                     =  SHORT
           YOUNG S MODULUS                   =   0.300000E+08
           POISSION S RATIO                  =           0.300
           NUMBER OF NODES IN THE MODEL      =  2441
           NUMBER OF ELEMENTS IN THE MODEL=  1872
------------------------------------------------------------
                         NODAL COORDINATES
NODE              X-COORD              Y-COORD            Z-COORD
------------------------------------------------------------

   1              5.00000              0.00000            0.00000
   2              5.00266              0.00000            0.00000
   3              5.00245              0.00520            0.00000
   4              5.00187              0.00930            0.00000
  ...
  ...
  ...

           IERR FROM SYMBN=     0

**********************************************************************
                     LOADING NUMBER     1
**********************************************************************

**********************************************************************
        E Q U I L I B R I U M   C H E C K S
        SUM OF THE X FORCE=          0.4193217E-09
        SUM OF THE Y FORCE=         -0.3798505E-08
        SUM OF THE Z FORCE=         -0.4241372E-09
----------------------------------------------------------------------
```

APPLIED LOAD AND THE SURFACE AREA COMPONENTS

X-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00
Y-COMPONENTS:	FORCE=	0.124996E+03	AREA=	0.124996E+03
Z-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION =	0.0000000E+00
NOMINAL STRESS IN THE Y- DIRECTION =	0.1000000E+01
NOMINAL STRESS IN THE Z- DIRECTION =	0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1042638E+01	0.6177579E+00
2	11.250	0.1097014E+01	0.6499757E+00
3	22.500	0.1265662E+01	0.7498990E+00
4	33.750	0.1461145E+01	0.8657214E+00
5	45.000	0.1629536E+01	0.9654924E+00
6	56.250	0.1761401E+01	0.1043622E+01
7	67.500	0.1856268E+01	0.1099831E+01
8	78.750	0.1913422E+01	0.1133694E+01
9	90.000	0.1932516E+01	0.1145007E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.8606799E+00	0.5099489E+00
2	11.250	0.1028380E+01	0.6093103E+00
3	22.500	0.1265637E+01	0.7498842E+00
4	33.750	0.1475477E+01	0.8742133E+00
5	45.000	0.1642917E+01	0.9734206E+00
6	56.250	0.1769638E+01	0.1048502E+01
7	67.500	0.1859042E+01	0.1101474E+01

8	78.750	0.1912002E+01	0.1132852E+01
9	90.000	0.1929571E+01	0.1143262E+01

```
****************************************************************
 NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=   64
****************************************************************
```

```
****************************************************************
                                    K/( S*SQRT(PI A/Q) )
STATION                 PHI       FORCE-METHOD          COD METHOD
****************************************************************
```

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.61776E+00	0.50995E+00
2	11.250	0.64998E+00	0.60931E+00
3	22.500	0.74990E+00	0.74988E+00
4	33.750	0.86572E+00	0.87421E+00
5	45.000	0.96549E+00	0.97342E+00
6	56.250	0.10436E+01	0.10485E+01
7	67.500	0.10998E+01	0.11015E+01
8	78.750	0.11337E+01	0.11329E+01
9	90.000	0.11450E+01	0.11433E+01

ALL ELEMENTS SATISFY EQUILIBRIUM

```
STOP  (called by $MAIN )
CP: 38.705s,  Wallclock: 92.835s,  10.4% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 6: Output file outc22 for Example 3(b).

```
****************************************************************
   SURFACE CRACK -CRACK FACE PRESSURE LOADING  A/C=0.2 A/T=0.2
****************************************************************

----------------------------------------------------------------
                 DESCRIPTION OF THE MODEL
----------------------------------------------------------------
          OUTPUT OPTION                     =  SHORT
          YOUNG S MODULUS                   =   0.300000E+08
          POISSION S RATIO                  =          0.300
          NUMBER OF NODES IN THE MODEL      =  2441
          NUMBER OF ELEMENTS IN THE MODEL=  1872
----------------------------------------------------------------
```

NODAL COORDINATES

NODE	X-COORD	Y-COORD	Z-COORD
1	5.00000	0.00000	0.00000
2	5.00266	0.00000	0.00000
3	5.00245	0.00520	0.00000
4	5.00187	0.00930	0.00000
...			
...			
...			

IERR FROM SYMBN= 0

```
**********************************************************************
                    LOADING NUMBER    1
**********************************************************************

**********************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.1027622E-11
          SUM OF THE Y FORCE=       -0.9257040E-11
          SUM OF THE Z FORCE=       -0.5503000E-09
----------------------------------------------------------------------

----------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
 X-COMPONENTS:    FORCE=    0.000000E+00    AREA=    0.175788E-16
 Y-COMPONENTS:    FORCE=    0.389798E+01    AREA=    0.389798E+01
 Z-COMPONENTS:    FORCE=    0.000000E+00    AREA=    0.278156E-16
----------------------------------------------------------------------

**********************************************************************
                 N O M I N A L    S T R E S S E S
     NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
     NOMINAL STRESS IN THE Y- DIRECTION =        0.1000000E+01
     NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
----------------------------------------------------------------------

     ************************************************************
               STRESS INTENSITY FACTORS ARE AS FOLLOWS
     ************************************************************
```

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1038225E+01	0.6151432E+00
2	11.250	0.1095227E+01	0.6489169E+00
3	22.500	0.1265707E+01	0.7499253E+00
4	33.750	0.1462723E+01	0.8666567E+00
5	45.000	0.1632234E+01	0.9670910E+00

6	56.250	0.1764889E+01	0.1045689E+01
7	67.500	0.1860285E+01	0.1102210E+01
8	78.750	0.1917742E+01	0.1136254E+01
9	90.000	0.1936936E+01	0.1147626E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.8465032E+00	0.5015492E+00
2	11.250	0.1021579E+01	0.6052811E+00
3	22.500	0.1263493E+01	0.7486140E+00
4	33.750	0.1474976E+01	0.8739167E+00
5	45.000	0.1643138E+01	0.9735514E+00
6	56.250	0.1770183E+01	0.1048826E+01
7	67.500	0.1859740E+01	0.1101887E+01
8	78.750	0.1912768E+01	0.1133306E+01
9	90.000	0.1930357E+01	0.1143727E+01

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.61514E+00	0.50155E+00
2	11.250	0.64892E+00	0.60528E+00
3	22.500	0.74993E+00	0.74861E+00
4	33.750	0.86666E+00	0.87392E+00
5	45.000	0.96709E+00	0.97355E+00
6	56.250	0.10457E+01	0.10488E+01
7	67.500	0.11022E+01	0.11019E+01
8	78.750	0.11363E+01	0.11333E+01
9	90.000	0.11476E+01	0.11437E+01

```
ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 38.803s,  Wallclock: 80.092s,  12.1% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 7: Output file outr28 for Example 4(a).

```
*****************************************************************
   SURFACE CRACK-REMOTE TENSION   A/C=0.2 ,A/T=0.8
*****************************************************************

-----------------------------------------------------------------
                 DESCRIPTION OF THE MODEL
-----------------------------------------------------------------
           OUTPUT OPTION                     =  SHORT
           YOUNG S MODULUS                   =   0.300000E+08
           POISSION S RATIO                  =          0.300
           NUMBER OF NODES IN THE MODEL      =  2464
           NUMBER OF ELEMENTS IN THE MODEL=  1856
-----------------------------------------------------------------
                         NODAL COORDINATES
NODE              X-COORD           Y-COORD            Z-COORD
-----------------------------------------------------------------

   1              5.00000           0.00000            0.00000
   2              5.00266           0.00000            0.00000
   3              5.00245           0.00510            0.00000
   4              5.00187           0.00930            0.00000
   5              5.00102           0.01220            0.00000
  ...
  ...
  ...

           IERR FROM SYMBN=     0

*********************************************************************
                    LOADING NUMBER    1
*********************************************************************

*********************************************************************
           E Q U I L I B R I U M   C H E C K S
           SUM OF THE X FORCE=        0.8482506E-09
           SUM OF THE Y FORCE=       -0.5669275E-08
           SUM OF THE Z FORCE=       -0.1710956E-07
---------------------------------------------------------------------
```

APPLIED LOAD AND THE SURFACE AREA COMPONENTS

X-COMPONENTS:	FORCE= 0.000000E+00		AREA=	0.000000E+00
Y-COMPONENTS:	FORCE= 0.624962E+02		AREA=	0.624962E+02
Z-COMPONENTS:	FORCE= 0.000000E+00		AREA=	0.000000E+00

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION = 0.0000000E+00
NOMINAL STRESS IN THE Y- DIRECTION = 0.1000000E+01
NOMINAL STRESS IN THE Z- DIRECTION = 0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1942565E+01	0.1150961E+01
2	11.250	0.1945317E+01	0.1152591E+01
3	22.500	0.2148890E+01	0.1273207E+01
4	33.750	0.2397951E+01	0.1420775E+01
5	45.000	0.2637838E+01	0.1562907E+01
6	56.250	0.2815154E+01	0.1667966E+01
7	67.500	0.2934686E+01	0.1738788E+01
8	78.750	0.2980024E+01	0.1765651E+01
9	90.000	0.2990430E+01	0.1771816E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1667430E+01	0.9879444E+00
2	11.250	0.1837670E+01	0.1088811E+01
3	22.500	0.2173179E+01	0.1287598E+01
4	33.750	0.2450039E+01	0.1451637E+01
5	45.000	0.2625023E+01	0.1555314E+01
6	56.250	0.2841981E+01	0.1683861E+01
7	67.500	0.2935053E+01	0.1739006E+01

8	78.750	0.2960373E+01	0.1754007E+01
9	90.000	0.2963058E+01	0.1755598E+01

```
*************************************************************
 NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=   64
*************************************************************

*************************************************************
                                 K/( S*SQRT(PI A/Q) )
STATION              PHI        FORCE-METHOD        COD METHOD
*************************************************************

      1             0.000        0.11510E+01        0.98794E+00

      2            11.250        0.11526E+01        0.10888E+01

      3            22.500        0.12732E+01        0.12876E+01

      4            33.750        0.14208E+01        0.14516E+01

      5            45.000        0.15629E+01        0.15553E+01

      6            56.250        0.16680E+01        0.16839E+01

      7            67.500        0.17388E+01        0.17390E+01

      8            78.750        0.17657E+01        0.17540E+01

      9            90.000        0.17718E+01        0.17556E+01

          ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 45.682s,  Wallclock: 85.788s,  13.3% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 8: Output file outc28 for Example 4(b).

```
*************************************************************
  SURFACE CRACK-CRACK FACE PRESSURE LOADING A/C=0.2 ,A/T=0.8
*************************************************************

-------------------------------------------------------------
                DESCRIPTION OF THE MODEL
-------------------------------------------------------------
          OUTPUT OPTION                    =  SHORT
          YOUNG S MODULUS                  =   0.300000E+08
          POISSION S RATIO                 =          0.300
          NUMBER OF NODES IN THE MODEL     =  2464
          NUMBER OF ELEMENTS IN THE MODEL=  1856
-------------------------------------------------------------
```

```
                     NODAL COORDINATES
NODE            X-COORD            Y-COORD            Z-COORD
-------------------------------------------------------------

  1             5.00000            0.00000            0.00000
  2             5.00266            0.00000            0.00000
  3             5.00245            0.00510            0.00000
  4             5.00187            0.00930            0.00000
 ...
 ...
 ...

          IERR FROM SYMBN=    0

************************************************************************
                    LOADING NUMBER    1
************************************************************************

************************************************************************
           E Q U I L I B R I U M   C H E C K S
           SUM OF THE X FORCE=      -0.4298795E-10
           SUM OF THE Y FORCE=      -0.7681145E-10
           SUM OF THE Z FORCE=      -0.1726783E-07
------------------------------------------------------------------------

------------------------------------------------------------------------
           APPLIED LOAD AND THE SURFACE AREA COMPONENTS
  X-COMPONENTS:     FORCE=    0.000000E+00    AREA=    0.184609E-16
  Y-COMPONENTS:     FORCE=    0.389798E+01    AREA=    0.389798E+01
  Z-COMPONENTS:     FORCE=    0.000000E+00    AREA=    0.390507E-16
------------------------------------------------------------------------

************************************************************************
                    N O M I N A L    S T R E S S E S
           NOMINAL STRESS IN THE X- DIRECTION =      0.0000000E+00
           NOMINAL STRESS IN THE Y- DIRECTION =      0.1000000E+01
           NOMINAL STRESS IN THE Z- DIRECTION =      0.0000000E+00
------------------------------------------------------------------------

     *************************************************************
              STRESS INTENSITY FACTORS ARE AS FOLLOWS
     *************************************************************

                       FROM THE FORCE METHOD

  STATION      PHI          ABSOLUTE-K        K/(S*SQRT(PI*A/Q))

     1         0.000      0.1939965E+01        0.1149420E+01

     2        11.250      0.1946502E+01        0.1153294E+01

     3        22.500      0.2152937E+01        0.1275605E+01

     4        33.750      0.2403408E+01        0.1424008E+01

     5        45.000      0.2629350E+01        0.1557878E+01
```

6	56.250	0.2818348E+01	0.1669858E+01
7	67.500	0.2939771E+01	0.1741801E+01
8	78.750	0.2986851E+01	0.1769696E+01
9	90.000	0.2997954E+01	0.1776274E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1654822E+01	0.9804745E+00
2	11.250	0.1831812E+01	0.1085340E+01
3	22.500	0.2171463E+01	0.1286582E+01
4	33.750	0.2449731E+01	0.1451454E+01
5	45.000	0.2625300E+01	0.1555478E+01
6	56.250	0.2842561E+01	0.1684204E+01
7	67.500	0.2935786E+01	0.1739440E+01
8	78.750	0.2961178E+01	0.1754485E+01
9	90.000	0.2963886E+01	0.1756089E+01

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.11494E+01	0.98047E+00
2	11.250	0.11533E+01	0.10853E+01
3	22.500	0.12756E+01	0.12866E+01
4	33.750	0.14240E+01	0.14515E+01
5	45.000	0.15579E+01	0.15555E+01
6	56.250	0.16699E+01	0.16842E+01
7	67.500	0.17418E+01	0.17394E+01
8	78.750	0.17697E+01	0.17545E+01
9	90.000	0.17763E+01	0.17561E+01

```
                ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 45.254s,  Wallclock: 115.863s,  9.8% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

```
----------------------------------------------------------
    Table 9: Output file outcor28 for Example 5.
----------------------------------------------------------
```

```
 ************************************************************
   CORNER  CRACK-REMOTE TENSION   A/C=0.2 ,A/T=0.8
 ************************************************************

 ------------------------------------------------------------
                DESCRIPTION OF THE MODEL
 ------------------------------------------------------------
           OUTPUT OPTION                   =  SHORT
           YOUNG S MODULUS                 =   0.300000E+08
           POISSION S RATIO                =          0.300
           NUMBER OF NODES IN THE MODEL    =  2464
           NUMBER OF ELEMENTS IN THE MODEL=  1856
 ------------------------------------------------------------
                          NODAL COORDINATES
 NODE              X-COORD              Y-COORD              Z-COORD
 ------------------------------------------------------------

    1            5.00000              0.00000              0.00000
    2            5.00266              0.00000              0.00000
    3            5.00245              0.00510              0.00000
    4            5.00187              0.00930              0.00000
   ...
   ...
   ...
           IERR FROM SYMBN=    0

**********************************************************************
                    LOADING NUMBER    1
**********************************************************************

**********************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.1055787E-08
          SUM OF THE Y FORCE=       -0.4741175E-08
          SUM OF THE Z FORCE=       -0.2597931E-06
----------------------------------------------------------------------

----------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
   X-COMPONENTS:     FORCE=     0.000000E+00     AREA=    0.000000E+00
   Y-COMPONENTS:     FORCE=     0.624962E+02     AREA=    0.624962E+02
   Z-COMPONENTS:     FORCE=     0.000000E+00     AREA=    0.000000E+00
----------------------------------------------------------------------
```

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION = 0.0000000E+00
NOMINAL STRESS IN THE Y- DIRECTION = 0.1000000E+01
NOMINAL STRESS IN THE Z- DIRECTION = 0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.2016223E+01	0.1194603E+01
2	11.250	0.2019591E+01	0.1196598E+01
3	22.500	0.2228682E+01	0.1320484E+01
4	33.750	0.2490525E+01	0.1475624E+01
5	45.000	0.2750801E+01	0.1629837E+01
6	56.250	0.2961335E+01	0.1754578E+01
7	67.500	0.3142617E+01	0.1861986E+01
8	78.750	0.3339150E+01	0.1978431E+01
9	90.000	0.3552545E+01	0.2104866E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1740380E+01	0.1031167E+01
2	11.250	0.1905171E+01	0.1128805E+01
3	22.500	0.2252796E+01	0.1334771E+01
4	33.750	0.2542841E+01	0.1506621E+01
5	45.000	0.2734719E+01	0.1620308E+01
6	56.250	0.2983844E+01	0.1767914E+01
7	67.500	0.3129717E+01	0.1854343E+01
8	78.750	0.3271988E+01	0.1938638E+01
9	90.000	0.3648129E+01	0.2161499E+01

```
************************************************************
 NUMBER OF SINGULARITY ELEMENTS IN THE MODEL=   64
************************************************************

************************************************************
                                   K/( S*SQRT(PI A/Q) )
STATION            PHI          FORCE-METHOD         COD METHOD
************************************************************

   1              0.000         0.11946E+01         0.10312E+01

   2             11.250         0.11966E+01         0.11288E+01

   3             22.500         0.13205E+01         0.13348E+01

   4             33.750         0.14756E+01         0.15066E+01

   5             45.000         0.16298E+01         0.16203E+01

   6             56.250         0.17546E+01         0.17679E+01

   7             67.500         0.18620E+01         0.18543E+01

   8             78.750         0.19784E+01         0.19386E+01

   9             90.000         0.21049E+01         0.21615E+01

       ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 45.308s,  Wallclock: 116.253s,  9.7% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 10: Output file outem28 for Example 6.

```
************************************************************
  EMBEDDED CRACK-REMOTE TENSION    A/C=0.2 ,A/T=0.8
************************************************************

------------------------------------------------------------
               DESCRIPTION OF THE MODEL
------------------------------------------------------------
           OUTPUT OPTION                  =  SHORT
           YOUNG S MODULUS                =   0.300000E+08
           POISSION S RATIO               =          0.300
           NUMBER OF NODES IN THE MODEL   =  2464
           NUMBER OF ELEMENTS IN THE MODEL=  1856
------------------------------------------------------------
                       NODAL COORDINATES
NODE              X-COORD             Y-COORD             Z-COORD
------------------------------------------------------------

   1              5.00000             0.00000             0.00000
```

```
2          5.00266          0.00000          0.00000
3          5.00245          0.00510          0.00000
4          5.00187          0.00930          0.00000
...
...
...       IERR FROM SYMBN=     0

**********************************************************************
                LOADING NUMBER    1
**********************************************************************

**********************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.8550499E-09
          SUM OF THE Y FORCE=       -0.4030978E-08
          SUM OF THE Z FORCE=       -0.1866076E-09
----------------------------------------------------------------------

----------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
  X-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
  Y-COMPONENTS:      FORCE=     0.624962E+02     AREA=     0.624962E+02
  Z-COMPONENTS:      FORCE=     0.000000E+00     AREA=     0.000000E+00
----------------------------------------------------------------------

**********************************************************************
                 N O M I N A L    S T R E S S E S
          NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
          NOMINAL STRESS IN THE Y- DIRECTION =        0.1000000E+01
          NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
----------------------------------------------------------------------

     ************************************************************
             STRESS INTENSITY FACTORS ARE AS FOLLOWS
     ************************************************************

                    FROM THE FORCE METHOD

  STATION        PHI          ABSOLUTE-K          K/(S*SQRT(PI*A/Q))

     1          0.000        0.9477525E+00         0.5615390E+00

     2         11.250        0.1026610E+01         0.6082616E+00

     3         22.500        0.1222969E+01         0.7246035E+00

     4         33.750        0.1440403E+01         0.8534321E+00

     5         45.000        0.1682060E+01         0.9966126E+00

     6         56.250        0.1921766E+01         0.1138638E+01

     7         67.500        0.2169597E+01         0.1285476E+01

     8         78.750        0.2345304E+01         0.1389582E+01
```

9	90.000	0.2409995E+01	0.1427911E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.7604610E+00	0.4505696E+00
2	11.250	0.9681786E+00	0.5736414E+00
3	22.500	0.1228963E+01	0.7281549E+00
4	33.750	0.1465673E+01	0.8684046E+00
5	45.000	0.1658270E+01	0.9825174E+00
6	56.250	0.1928974E+01	0.1142908E+01
7	67.500	0.2174445E+01	0.1288349E+01
8	78.750	0.2351977E+01	0.1393536E+01
9	90.000	0.2418940E+01	0.1433211E+01

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.56154E+00	0.45057E+00
2	11.250	0.60826E+00	0.57364E+00
3	22.500	0.72460E+00	0.72815E+00
4	33.750	0.85343E+00	0.86840E+00
5	45.000	0.99661E+00	0.98252E+00
6	56.250	0.11386E+01	0.11429E+01
7	67.500	0.12855E+01	0.12883E+01
8	78.750	0.13896E+01	0.13935E+01
9	90.000	0.14279E+01	0.14332E+01

ALL ELEMENTS SATISFY EQUILIBRIUM

STOP (called by $MAIN)
CP: 45.622s, Wallclock: 88.919s, 12.8% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0

Table 11: Output file occor15 for Example 7.

```
**********************************************************
 CORNER CRACK AT A CIRCULAR HOLE  A/C=1 ,A/T=0.5 , R/T=1.0
**********************************************************

---------------------------------------------------------
               DESCRIPTION OF THE MODEL
---------------------------------------------------------
          OUTPUT OPTION                   =  SHORT
          YOUNG S MODULUS                 =   0.300000E+08
          POISSION S RATIO                =          0.300
          NUMBER OF NODES IN THE MODEL    =  2863
          NUMBER OF ELEMENTS IN THE MODEL=  2260
---------------------------------------------------------
                         NODAL COORDINATES
 NODE            X-COORD              Y-COORD              Z-COORD
---------------------------------------------------------

    1            1.00000              0.00000              0.00000
    2            1.01320              0.00000              0.00000
    3            1.01220              0.00520              0.00000
    4            1.00930              0.00930              0.00000
    ...
    ...
    ...

          IERR FROM SYMBN=     0
*****************************************************************
                  LOADING NUMBER     1
*****************************************************************

*****************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=        0.3811911E-09
          SUM OF THE Y FORCE=       -0.6030213E-08
          SUM OF THE Z FORCE=       -0.2428789E-08
-----------------------------------------------------------------

-----------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
 X-COMPONENTS:     FORCE=     0.000000E+00     AREA=     0.000000E+00
 Y-COMPONENTS:     FORCE=     0.539980E+02     AREA=     0.539980E+02
 Z-COMPONENTS:     FORCE=     0.000000E+00     AREA=     0.000000E+00
-----------------------------------------------------------------

*****************************************************************
                 N O M I N A L    S T R E S S E S
          NOMINAL STRESS IN THE X- DIRECTION =       0.0000000E+00
          NOMINAL STRESS IN THE Y- DIRECTION =       0.1000000E+01
          NOMINAL STRESS IN THE Z- DIRECTION =       0.0000000E+00
-----------------------------------------------------------------
```

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.2326628E+01	0.2060499E+01
2	11.250	0.2276628E+01	0.2016218E+01
3	22.500	0.2221275E+01	0.1967197E+01
4	33.750	0.2218767E+01	0.1964975E+01
5	45.000	0.2265604E+01	0.2006455E+01
6	56.250	0.2362867E+01	0.2092592E+01
7	67.500	0.2530806E+01	0.2241322E+01
8	78.750	0.2691461E+01	0.2383601E+01
9	90.000	0.2359771E+01	0.2089851E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.2387579E+01	0.2114478E+01
2	11.250	0.2243456E+01	0.1986840E+01
3	22.500	0.2194859E+01	0.1943802E+01
4	33.750	0.2191650E+01	0.1940961E+01
5	45.000	0.2237883E+01	0.1981904E+01
6	56.250	0.2333527E+01	0.2066608E+01
7	67.500	0.2482424E+01	0.2198474E+01
8	78.750	0.2787567E+01	0.2468713E+01
9	90.000	0.2379925E+01	0.2107699E+01

****

NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64

****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.20605E+01	0.21145E+01
2	11.250	0.20162E+01	0.19868E+01
3	22.500	0.19672E+01	0.19438E+01
4	33.750	0.19650E+01	0.19410E+01
5	45.000	0.20065E+01	0.19819E+01
6	56.250	0.20926E+01	0.20666E+01
7	67.500	0.22413E+01	0.21985E+01
8	78.750	0.23836E+01	0.24687E+01
9	90.000	0.20899E+01	0.21077E+01

****

LOADING NUMBER 2

****

****

E Q U I L I B R I U M   C H E C K S

SUM OF THE X FORCE= 0.7870824E-08
SUM OF THE Y FORCE= -0.1061700E-08
SUM OF THE Z FORCE= -0.2249274E-06

APPLIED LOAD AND THE SURFACE AREA COMPONENTS

X-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00
Y-COMPONENTS:	FORCE=	-0.201877E-02	AREA=	0.539980E+02
Z-COMPONENTS:	FORCE=	0.000000E+00	AREA=	0.000000E+00

****

N O M I N A L   S T R E S S E S

NOMINAL STRESS IN THE X- DIRECTION = 0.0000000E+00
NOMINAL STRESS IN THE Y- DIRECTION = -0.3738595E-04
NOMINAL STRESS IN THE Z- DIRECTION = 0.0000000E+00

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1709044E+01	0.1513557E+01
2	11.250	0.1542926E+01	0.1366440E+01
3	22.500	0.1310887E+01	0.1160942E+01
4	33.750	0.1113279E+01	0.9859379E+00
5	45.000	0.9482749E+00	0.8398074E+00
6	56.250	0.8170198E+00	0.7235657E+00
7	67.500	0.7235576E+00	0.6407942E+00
8	78.750	0.6573300E+00	0.5821419E+00
9	90.000	0.5045022E+00	0.4467952E+00

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1814866E+01	0.1607274E+01
2	11.250	0.1519294E+01	0.1345511E+01
3	22.500	0.1294912E+01	0.1146795E+01
4	33.750	0.1097889E+01	0.9723082E+00
5	45.000	0.9341272E+00	0.8272779E+00
6	56.250	0.8033078E+00	0.7114222E+00
7	67.500	0.7082324E+00	0.6272219E+00
8	78.750	0.6602082E+00	0.5846909E+00
9	90.000	0.5569405E+00	0.4932353E+00

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.15136E+01	0.16073E+01
2	11.250	0.13664E+01	0.13455E+01

3	22.500	0.11609E+01	0.11468E+01
4	33.750	0.98594E+00	0.97231E+00
5	45.000	0.83981E+00	0.82728E+00
6	56.250	0.72357E+00	0.71142E+00
7	67.500	0.64079E+00	0.62722E+00
8	78.750	0.58214E+00	0.58469E+00
9	90.000	0.44680E+00	0.49324E+00

```
        ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 34.501s,  Wallclock: 64.907s,  13.3% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 12: Output file oscorl5 for Example 8.

```
************************************************************
  SURFACE CRACK AT A CIRCULAR HOLE  A/C=1 ,A/T=0.5 , R/T=1.0
************************************************************

------------------------------------------------------------
              DESCRIPTION OF THE MODEL
------------------------------------------------------------
          OUTPUT OPTION                  =  SHORT
          YOUNG S MODULUS                =   0.300000E+08
          POISSION S RATIO               =          0.300
          NUMBER OF NODES IN THE MODEL   =  2863
          NUMBER OF ELEMENTS IN THE MODEL=  2260
------------------------------------------------------------
                        NODAL COORDINATES
NODE              X-COORD             Y-COORD           Z-COORD
------------------------------------------------------------

   1              1.00000             0.00000           0.00000
   2              1.01320             0.00000           0.00000
   3              1.01220             0.00520           0.00000
   4              1.00930             0.00930           0.00000
   ...
   ...
   ...

          IERR FROM SYMBN=     0

**********************************************************************
                      LOADING NUMBER     1
**********************************************************************
```

```
***********************************************************************
              E Q U I L I B R I U M   C H E C K S
              SUM OF THE X FORCE=        0.3116301E-09
              SUM OF THE Y FORCE=       -0.5856798E-08
              SUM OF THE Z FORCE=       -0.1453687E-09
-----------------------------------------------------------------------

-----------------------------------------------------------------------
             APPLIED LOAD AND THE SURFACE AREA COMPONENTS
  X-COMPONENTS:      FORCE=     0.000000E+00     AREA=    0.000000E+00
  Y-COMPONENTS:      FORCE=     0.539980E+02     AREA=    0.539980E+02
  Z-COMPONENTS:      FORCE=     0.000000E+00     AREA=    0.000000E+00
-----------------------------------------------------------------------

***********************************************************************
                   N O M I N A L   S T R E S S E S
             NOMINAL STRESS IN THE X- DIRECTION =      0.0000000E+00
             NOMINAL STRESS IN THE Y- DIRECTION =      0.1000000E+01
             NOMINAL STRESS IN THE Z- DIRECTION =      0.0000000E+00
-----------------------------------------------------------------------

    ***************************************************************
              STRESS INTENSITY FACTORS ARE AS FOLLOWS
    ***************************************************************
```

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.1972444E+01	0.1746828E+01
2	11.250	0.1981288E+01	0.1754661E+01
3	22.500	0.2008866E+01	0.1779084E+01
4	33.750	0.2058871E+01	0.1823369E+01
5	45.000	0.2139874E+01	0.1895107E+01
6	56.250	0.2259833E+01	0.2001344E+01
7	67.500	0.2442181E+01	0.2162834E+01
8	78.750	0.2614491E+01	0.2315435E+01
9	90.000	0.2308667E+01	0.2044592E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.1951981E+01	0.1728706E+01
2	11.250	0.1960570E+01	0.1736312E+01
3	22.500	0.1987496E+01	0.1760158E+01

4	33.750	0.2035504E+01	0.1802675E+01
5	45.000	0.2114568E+01	0.1872695E+01
6	56.250	0.2232217E+01	0.1976887E+01
7	67.500	0.2396266E+01	0.2122171E+01
8	78.750	0.2704875E+01	0.2395481E+01
9	90.000	0.2341106E+01	0.2073321E+01

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.17468E+01	0.17287E+01
2	11.250	0.17547E+01	0.17363E+01
3	22.500	0.17791E+01	0.17602E+01
4	33.750	0.18234E+01	0.18027E+01
5	45.000	0.18951E+01	0.18727E+01
6	56.250	0.20013E+01	0.19769E+01
7	67.500	0.21628E+01	0.21222E+01
8	78.750	0.23154E+01	0.23955E+01
9	90.000	0.20446E+01	0.20733E+01

ALL ELEMENTS SATISFY EQUILIBRIUM

```
STOP  (called by $MAIN )
CP: 33.958s,  Wallclock: 64.160s,  13.2% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 13: Output file osmcor15 for Example 9.

```
*************************************************************
SURFACE CRACK AT A SEMI-CIRCULAR HOLE  A/C=1 ,A/T=0.5 , R/T=1.0
*************************************************************

----------------------------------------------------------------
                DESCRIPTION OF THE MODEL
----------------------------------------------------------------
          OUTPUT OPTION                     =  SHORT
          YOUNG S MODULUS                   =   0.300000E+08
          POISSION S RATIO                  =          0.300
          NUMBER OF NODES IN THE MODEL      =  2863
          NUMBER OF ELEMENTS IN THE MODEL=  2260
----------------------------------------------------------------
                        NODAL COORDINATES
NODE             X-COORD              Y-COORD              Z-COORD
----------------------------------------------------------------

   1             1.00000              0.00000              0.00000
   2             1.01320              0.00000              0.00000
   3             1.01220              0.00520              0.00000
   4             1.00930              0.00930              0.00000
 ...
 ...
 ...

          IERR FROM SYMBN=      0
*************************************************************************
                     LOADING NUMBER     1
*************************************************************************

*************************************************************************
          E Q U I L I B R I U M   C H E C K S
          SUM OF THE X FORCE=          0.1603649E-08
          SUM OF THE Y FORCE=         -0.6038583E-08
          SUM OF THE Z FORCE=         -0.2318881E-09
-------------------------------------------------------------------------

-------------------------------------------------------------------------
          APPLIED LOAD AND THE SURFACE AREA COMPONENTS
X-COMPONENTS:       FORCE=      0.000000E+00      AREA=      0.000000E+00
Y-COMPONENTS:       FORCE=      0.539980E+02      AREA=      0.539980E+02
Z-COMPONENTS:       FORCE=      0.000000E+00      AREA=      0.000000E+00
-------------------------------------------------------------------------

*************************************************************************
                  N O M I N A L    S T R E S S E S
          NOMINAL STRESS IN THE X- DIRECTION =        0.0000000E+00
          NOMINAL STRESS IN THE Y- DIRECTION =        0.1000000E+01
          NOMINAL STRESS IN THE Z- DIRECTION =        0.0000000E+00
-------------------------------------------------------------------------
```

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.2132622E+01	0.1888684E+01
2	11.250	0.2142476E+01	0.1897411E+01
3	22.500	0.2173118E+01	0.1924548E+01
4	33.750	0.2228355E+01	0.1973466E+01
5	45.000	0.2317019E+01	0.2051989E+01
6	56.250	0.2446855E+01	0.2166974E+01
7	67.500	0.2641224E+01	0.2339110E+01
8	78.750	0.2817368E+01	0.2495106E+01
9	90.000	0.2474677E+01	0.2191613E+01

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.2110524E+01	0.1869114E+01
2	11.250	0.2120098E+01	0.1877593E+01
3	22.500	0.2150030E+01	0.1904100E+01
4	33.750	0.2203130E+01	0.1951127E+01
5	45.000	0.2289751E+01	0.2027840E+01
6	56.250	0.2417397E+01	0.2140886E+01
7	67.500	0.2592295E+01	0.2295777E+01
8	78.750	0.2918831E+01	0.2584963E+01
9	90.000	0.2493143E+01	0.2207967E+01

****

NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64

****

```
************************************************************
                                      K/( S*SQRT(PI A/Q) )
STATION            PHI             FORCE-METHOD          COD METHOD
************************************************************

    1             0.000            0.18887E+01          0.18691E+01

    2            11.250            0.18974E+01          0.18776E+01

    3            22.500            0.19245E+01          0.19041E+01

    4            33.750            0.19735E+01          0.19511E+01

    5            45.000            0.20520E+01          0.20278E+01

    6            56.250            0.21670E+01          0.21409E+01

    7            67.500            0.23391E+01          0.22958E+01

    8            78.750            0.24951E+01          0.25850E+01

    9            90.000            0.21916E+01          0.22080E+01

          ALL ELEMENTS SATISFY EQUILIBRIUM

STOP  (called by $MAIN )
CP: 33.899s,  Wallclock: 60.821s,  13.9% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 14: Input file dat12d for Example 10.

```
          SURFACE CRACK -Prescibed displacements-  A/C=1.0 A/T=0.2
SHORT
   0.30000E+08    0.30000E+00
 2161 1664
    1        1.000000000       0.000000000       0.000000000
    2        1.013200000       0.000000000       0.000000000
    3        1.012200000       0.005200000       0.000000000
    4        1.009300000       0.009300000       0.000000000
    5        1.005200000       0.012200000       0.000000000
    ...
    ...
    ...
 2156       25.000000000      10.000000000      -5.000000000
 2157       25.000000000      15.000000000      -5.000000000
 2158       25.000000000      25.000000000      -5.000000000
 2159       25.000000000      45.000000000      -5.000000000
 2160       25.000000000      85.000000000      -5.000000000
 2161       25.000000000     125.000000000      -5.000000000
    1  210    1    2  211  210    1    3  212    1
    2  210    1    3  212  210    1    4  213    1
    ...
    ...
```

```
     ...
1658 2141 2140 2154 2155 2085 2084 2098 2099    0
1659 2142 2141 2155 2156 2086 2085 2099 2100    0
1660 2143 2142 2156 2157 2087 2086 2100 2101    0
1661 2144 2143 2157 2158 2088 2087 2101 2102    0
1662 2145 2144 2158 2159 2089 2088 2102 2103    0
1663 2146 2145 2159 2160 2090 2089 2103 2104    0
1664 2147 2146 2160 2161 2091 2090 2104 2105    0
        1    0    1    0
        2    0    1    0
      ...
      ...
      ...
     1877    1    0    0
     1878    1    0    0
     1879    1    0    0
     1880    1    0    0
     1881    1    0    0
     2161    0    0    1
        0    0    0    0
        1
REMOTE
        0    0    0    0    0
        0    0.0000    0.0000    0.0000
      203    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      204    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      205    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      206    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      207    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      208    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      209    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      412    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      413    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      414    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      415    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      416    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      417    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      418    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      621    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      622    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      623    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      624    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      625    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      626    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      627    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      830    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      831    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      832    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      833    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      834    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      835    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
      836    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
     1039    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
     1040    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
     1041    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
     1042    0    1    0    0.00000E+00    0.10000E-05    0.00000E+00
```

```
1043   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1044   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1045   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1248   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1249   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1250   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1251   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1252   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1253   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1254   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1457   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1458   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1459   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1460   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1461   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1462   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1463   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1666   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1667   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1668   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1669   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1670   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1671   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1672   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1875   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1876   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1877   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1878   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1879   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1880   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1881   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1923   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1937   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1951   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1965   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1979   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
1993   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2007   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2021   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2035   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2049   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2063   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2091   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2105   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2119   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2133   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2147   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
2161   0   1   0   0.00000E+00   0.10000E-05   0.00000E+00
   0   0   0   0   0.00000E+00   0.00000E+00   0.00000E+00
   1
   1          2          3          4          5
   6          7          8          9          10
 ...
 ...
 ...
125.0000   25.0000   0.2000   5.0000   1.0000
```

Table 15: Output file outd12 for Example 10.

```
 ************************************************************
SURFACE CRACK IN A PLATE-DISPLACEMENTS V=1.0E-6 ON Y=H - A/C=1.0 A/T=0.2
 ************************************************************

 ------------------------------------------------------------
                DESCRIPTION OF THE MODEL
 ------------------------------------------------------------
           OUTPUT OPTION                    =  SHORT
           YOUNG S MODULUS                  =   0.300000E+08
           POISSION S RATIO                 =          0.300
           NUMBER OF NODES IN THE MODEL     =  2161
           NUMBER OF ELEMENTS IN THE MODEL=    1664
 ------------------------------------------------------------
                        NODAL COORDINATES
 NODE              X-COORD            Y-COORD            Z-COORD
 ------------------------------------------------------------

    1              1.00000            0.00000            0.00000
    2              1.01320            0.00000            0.00000
    3              1.01220            0.00520            0.00000
    4              1.00930            0.00930            0.00000
   ...
   ...
   ...

         IERR FROM SYMBN=     0

 ********************************************************************
                  LOADING NUMBER     1
 ********************************************************************

 --------------------------------------------------------------------
         REACTION FORCES AT PRESCRIBED DISPLACEMENT NODES

         SUM OF THE X FORCE=         0.3833682E-04
         SUM OF THE Y FORCE=         0.2999872E+02
         SUM OF THE Z FORCE=        -0.1479277E-09
 --------------------------------------------------------------------

 ********************************************************************
         E Q U I L I B R I U M   C H E C K S
         SUM OF THE X FORCE=         0.2801653E-10
         SUM OF THE Y FORCE=        -0.7433272E-09
         SUM OF THE Z FORCE=         0.5562037E-10
 --------------------------------------------------------------------

     ************************************************************
             STRESS INTENSITY FACTORS ARE AS FOLLOWS
     ************************************************************
```

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	0.3113099E+00	0.2757010E+00
2	11.250	0.3026873E+00	0.2680647E+00
3	22.500	0.2916434E+00	0.2582840E+00
4	33.750	0.2850461E+00	0.2524414E+00
5	45.000	0.2810111E+00	0.2488679E+00
6	56.250	0.2786616E+00	0.2467871E+00
7	67.500	0.2773719E+00	0.2456450E+00
8	78.750	0.2767214E+00	0.2450689E+00
9	90.000	0.2765203E+00	0.2448908E+00

FROM THE CRACK OPENING DISPLACEMENT METHOD

1	0.000	0.3202891E+00	0.2836531E+00
2	11.250	0.2983115E+00	0.2641894E+00
3	22.500	0.2885413E+00	0.2555368E+00
4	33.750	0.2821246E+00	0.2498540E+00
5	45.000	0.2782588E+00	0.2464304E+00
6	56.250	0.2760132E+00	0.2444417E+00
7	67.500	0.2747916E+00	0.2433598E+00
8	78.750	0.2741661E+00	0.2428059E+00
9	90.000	0.2739710E+00	0.2426330E+00

****
NUMBER OF SINGULARITY ELEMENTS IN THE MODEL= 64
****

****

STATION	PHI	K/(S*SQRT(PI A/Q)) FORCE-METHOD	COD METHOD
1	0.000	0.27570E+00	0.28365E+00
2	11.250	0.26806E+00	0.26419E+00

3	22.500	0.25828E+00	0.25554E+00
4	33.750	0.25244E+00	0.24985E+00
5	45.000	0.24887E+00	0.24643E+00
6	56.250	0.24679E+00	0.24444E+00
7	67.500	0.24565E+00	0.24336E+00
8	78.750	0.24507E+00	0.24281E+00
9	90.000	0.24489E+00	0.24263E+00

ALL ELEMENTS SATISFY EQUILIBRIUM

```
STOP  (called by $MAIN )
CP: 28.635s,  Wallclock: 55.219s,  13.0% of 4-CPU Machine
HWM mem: 7733236, HWM stack: 310499, Stack overflows: 0
```

Table 16: Output file outdx12 for Example 11.

```
**************************************************************
SURFACE CRACK IN A PLATE-PRESCRIBED DISPLACEMENTS - U= -0.3E-7 on x=b
A/C=1.0  A/T=0.2
**************************************************************

-------------------------------------------------------------
              DESCRIPTION OF THE MODEL
-------------------------------------------------------------
          OUTPUT OPTION                      =  SHORT
          YOUNG S MODULUS                    =   0.300000E+08
          POISSION S RATIO                   =          0.300
          NUMBER OF NODES IN THE MODEL       =  2161
          NUMBER OF ELEMENTS IN THE MODEL=  1664
-------------------------------------------------------------
                       NODAL COORDINATES
NODE              X-COORD              Y-COORD              Z-COORD
-------------------------------------------------------------

    1             1.00000              0.00000              0.00000
    2             1.01320              0.00000              0.00000
    3             1.01220              0.00520              0.00000
    4             1.00930              0.00930              0.00000
   ...
   ...
   ...

 HEIGHT OF THE MODEL             =      125.00
 WIDTH OF THE MODEL              =       25.00
 SURFACE LENGTH OF THE CRACK =        1.00
 DEPTH OF THE CRACK              =        1.00
 THICKNESS OF THE PLATE          =        5.00
```

```
A/C RATIO                    =        1.00
A/T RATIO                    =        0.20
RADIUS OF THE CIRCULAR HOLE  =        0.00
*************************************************************

-------------------------------------------------------------
     MAXIMUM BANDWIDTH                    =          1002
     TOTAL CORE REQUIREMENT OF BIGK=             4449930
-------------------------------------------------------------

*************************************************************
 SUM OF THE FORCES BEFORE BOUNDARY CONDITIONS
 FOR LOADING CONDITION    =    1
*************************************************************

 SUM OF THE X-FORCES ARE  =          0.000000E+00
 SUM OF THE Y-FORCES ARE  =          0.000000E+00
 SUM OF THE Z-FORCES ARE  =          0.000000E+00

PROJECTED SURFACE AREAS IN EACH OF THE COORDINATE DIRECTIONS
 SURFACE AREA X-COMPONENT =          0.000000E+00
 SURFACE AREA Y-COMPONENT =          0.000000E+00
 SURFACE AREA Z-COMPONENT =          0.000000E+00

     VOLUME OF THE SOLID MODELED =          0.156250E+05

AT SUBPROGRAM SOLVE-B    CPU TIME=   0.453569E+01
 ACCUMULATED CPU=   0.453569E+01

AT SUBPROGRAM SOLVE-E    CPU TIME=   0.231621E+02
 ACCUMULATED CPU=   0.276978E+02

           IERR FROM SYMBN=    0

**********************************************************************
                    LOADING NUMBER    1
**********************************************************************

----------------------------------------------------------------------
        REACTION FORCES AT PRESCRIBED DISPLACEMENT NODES

        SUM OF THE X FORCE=        -0.2250000E+02
        SUM OF THE Y FORCE=        -0.6472670E-08
        SUM OF THE Z FORCE=         0.1818062E-10
----------------------------------------------------------------------

**********************************************************************
        E Q U I L I B R I U M   C H E C K S
        SUM OF THE X FORCE=         0.1828226E-10
        SUM OF THE Y FORCE=        -0.2951272E-10
        SUM OF THE Z FORCE=        -0.8830405E-11
----------------------------------------------------------------------
```

****

STRESS INTENSITY FACTORS ARE AS FOLLOWS

****

FROM THE FORCE METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	-0.1037153E-04	-0.9185194E-05
2	11.250	-0.2597900E-05	-0.2300741E-05
3	22.500	0.9410255E-06	0.8333872E-06
4	33.750	0.5019746E-05	0.4445567E-05
5	45.000	0.9920509E-05	0.8785760E-05
6	56.250	0.1478098E-04	0.1309027E-04
7	67.500	0.1890774E-04	0.1674499E-04
8	78.750	0.2166702E-04	0.1918866E-04
9	90.000	0.2263688E-04	0.2004758E-04

FROM THE CRACK OPENING DISPLACEMENT METHOD

STATION	PHI	ABSOLUTE-K	K/(S*SQRT(PI*A/Q))
1	0.000	-0.9245535E-04	-0.8187993E-04
2	11.250	-0.7967842E-04	-0.7056447E-04
3	22.500	-0.6818843E-04	-0.6038876E-04
4	33.750	-0.4885792E-04	-0.4326935E-04
5	45.000	-0.2653945E-04	-0.2350376E-04
6	56.250	-0.4131448E-05	-0.3658876E-05
7	67.500	0.1484371E-04	0.1314583E-04
8	78.750	0.2753143E-04	0.2438228E-04
9	90.000	0.3198674E-04	0.2832797E-04

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13. ABSTRACT *(Maximum 200 words)*

A computer program, surf3d, that uses the 3D finite-element method to calculate the stress-intensity factors for surface, corner, and embedded cracks in finite-thickness plates with and without circular holes, was developed. The cracks are assumed to be either elliptic or part-elliptic in shape. The computer program uses eight-noded hexahedral elements to model the solid. The program uses a skyline storage and solver. The stress-intensity factors are evaluated using the force method, the crack-opening displacement method, and the 3-D virtual crack closure methods.

In the manual the input to and the output of the surf3d program are described. This manual also demonstrates the use of the program and describes the calculation of the stress-intensity factors. Several examples with sample data files are included with the manual. To facilitate modeling of the user's crack configuration and loading, a companion program (a preprocessor program) that generates the data for the surf3d called gensurf was also developed. the gensurf program is a three dimensional mesh generator program that requires minimal input and that builds a complete data file for surf3d. The program surf3d is operational on Unix machines such as CRAY Y-MP, CRAY-2, and Convex C-220.

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