N93-24701

TDRSS Orbit Determination Using Short Baseline Differenced Carrier Phase

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Abstract

This paper discusses a covariance study on the feasibility of using station-differenced carrier phase on short baselines to track the TDRSS satellites. Orbit accuracies for the TDRSS using station-differenced carrier phase data and range data collected from White Sands, NM are given for various configuarations of ground stations and range data precision. A one-sigma position position accuracy of 25 meters can be acheived using two orthogonal baselines of 100 km for the station-differenced phase data and range data with 1 m accuracy. Relevant configuration parameters for the tracking system and important sources of error are examined. The ability of these data to redetemine the position after a station keeping maneuver is addressed. The BRTS system, which is currently used for TDRSS orbit determination, is briefly described and its errors are given for comparison.

I. Introduction

The Tracking and Data Relay Satellite System (TDRSS) is a network of geosynchronous satellites used to communicate with low earth orbiters. Each satellite has two single access (SA) high gain antennae which operate at 2 GHz (S-band) and 14 GHz (Ku-band) and one multiple access (MA) array at 2 GHz that receive signals from users. Telemetry received from a user satellite is sent from a Tracking and Data Relay satellite (TDRS) to White Sands, NM on a dedicated link at 14 GHz. A user satellite may obtain 2-way range and doppler data from a TDRS in order to determine the user's position. The two TDRSS satellites currently in use are 67° of longitude west and east of White Sands. The TDRSS positions are required to be known to 50 m at one-sigma and their position must be redetermined to this accuracy within 2 hours of a station-keeping maneuver.[1] Currently, position determination for the TDRSS is done using range and doppler data at 2 GHz collected using the Bi-lateral ranging transponder system (BRTS) described below. In addition, rough position determination is done using angle and range radio metric data from three ground antennae at the White Sands complex also described below. The tracking technique presented in this paper is part of a study of alternative tracking strategies for supporting orbit determination of the Advanced Tracking and Data Relay Satellites (ATDRS). Other approaches, involving GPS tracking techniques, are also being explored.[2]

In this paper, the possibility of using station-differenced carrier phase data on short baselines to perform TDRSS orbit solutions is explored. Biased carrier phase data could be obtained passively by each of a few antennae near White Sands by tracking the 14 GHz carrier on which telemetry is sent from a TDRSS satellite to White Sands. The differenced phase between a pair of stations measures the plane-of-sky position of a transmitter in the direction of the baseline formed by the stations. The precision of the measurment is given by $\Delta\theta = \Delta\phi / B\sin\theta$, where B is the baseline length, $\Delta\phi$ is the precision of the differenced phase measurement and θ is the angle between the baseline and the transmitter. The differenced phase observable has a remaining bias due to the difference in instrumental signal path through the two stations; as a result, only the change in plane-of-sky position over a data arc is effectively measured. This technique differs from Connected Element Interferometry (CEI)[3] in which the phase bias between the stations and the integer cycle ambiguity is resolved. While station-differenced biased phase provides weaker position information than CEI, it is operationally simpler and does not require a capability to record quasar signals for calibration as required for CEI.

Table 1
Error budget for TDRSS orbit determination with BRTS

Error Source	Error model	Position Error (m)
Computed error:		
Range data noise†	10 m white noise, 10 m bias	
Doppler data noise†	0.003 Hz white noise	
Total computed error		69
Considered errors:		
Solar pressure	2% error in reflectivity	4
Troposphere	5 cm zenith error	2
Ionosphere	10 TEC error	18
BRTS station locations	5 m error per axis	13
RSS position error:		73

† The data have a 10 second integration time, and are scheduled for 4 minutes every four hours for a 34 hour arc.

In this paper, baselines small enough to fit within the few hundred kilometer footprint of the TDRSS space-to-ground link signal are considered. Because of the high precision with which phase delay can be measured, the station-differenced phase data type can provide good plane-of-sky velocity measurements with baselines of this modest size. For stations within about 100 km, it is possible to distribute a common frequency reference signal to the stations over a fiber optic link, reducing errors associated with drifts in separate station clocks.[4] Baselines of 1 km, 10 km and 100 km with stations sharing a frequency reference and baselines of 100 km and 500 km in which the stations have separate frequency references are discussed.

II. Current TDRSS Orbit Determination

a) BRTS system

The BRTS consists of several ground-based transponders at four near equatorial locations around the globe. Each TDRS can view two or three BRTS stations. A range code particular to the transponder is sent from White Sands through a TDRSS satellite to each BRTS transponder and back once every four hours. TDRSS orbit solutions are calculated for 34 hour passes of BRTS four-way range and doppler data. The BRTS geometry results in a robust orbit solution for the TDRSS; however, data-taking with BRTS occupies the SA or MA user antenna of each satellite for about four minutes every four hours.

Because it is the operational data type used for TDRSS orbit determination, many studies have been done to determine the accuracy of the BRTS solutions.[1] BRTS regularly achieves 50 - 100 m one-sigma position errors. For completeness, an error budget for BRTS is included here (Table 1). A data accuracy of 10 m for the BRTS range data and 0.003 Hz for the BRTS doppler data is used for 10 second integration times.[5] Four minute arcs of data are scheduled every four hours, and an epoch state solution is found for a 34 hour data arc. A systematic error in the range measurements was included by estimating a bias parameter with an *a priori* error of 10 m. Errors from propagation media, solar pressure mismodelling and station location mismodelling are considered at the levels shown in Table 1. The computed error dominates the TDRSS position error; however, ionosphere mismodelling and station location uncertainty contribute significant consider error.

b) White Sands Ground Tracking System

The White Sands site has angle and range radio metric data available for rough TDRSS position determination. The angle data, consisting of azimuth and elevation measurements from antenna pointing,

Table 2

Differenced Troposphere Models for Several Baseline Lengths

Station separation (km) (baseline length)	Differenced troposphere sigma (cm) at 10° elevation	Differenced troposphere time constant (s)
1†	0.63	370
	0.11	2380
10	2.53	1333
100	5.86	7050
500	9.40	17,460
1000	11.2	30,000

† In the case of a 1 km baseline, the model used was the sum of two Gauss-Markov processes (see text).

have a precision of 0.1° and are biased. The current ranging system at White Sands is used to maintain an uncertainty in TDRSS range better than 10 km when the TDRSS position solution is propagated forward a few days.[6] The ranging signal has a 4 MHz bandwidth and the code is 244 μs long. The precision of the range measurements is about 20 m one-sigma for a 1 minute integration time with systematic error of about 30 m one-sigma. The system is not intended to be used for precision orbit determination and it may not be possible to calibrate the existing ground any antennae better. For the purposes of this study, it is assumed that a better ranging system could be put in place, although the current ranging system enhanced with station-differenced phase data is also studied.

III. Data Modelling and Filter Assumptions

a) Configuration

Figure 1 shows the configuration of antennae used in the covariance study. Three antennae, all near White Sands, form a north-south and an east-west baseline. For simplicity, north-south and east-west baselines of the same length are used. Baselines from 1 to 500 km are considered. In addition to the station-differenced phase data, range and doppler data collected at one of the stations are also included. The position solutions are found for 24 hour data arcs including station-differenced phase measurements on each of the two baselines and range data points every 10 minutes using the OASIS filter.[7] Position solutions calculated for TDRS-east and TDRS-west are very similar due to their symmetrical location with respect to White Sands. Hence, only the solutions for TDRS-east are shown.

Stochastic troposphere and station clock models are used to simulate the station-differenced phase data noise as discussed below. The station-differenced phase data are weighted at one-tenth of the line-of-sight troposphere sigma for each baseline, in order that it have negligible effect on the computed position error in the filter program. This results in data weights from about 0.025 cycle to about 0.5 cycle for the 14 GHz (Ku-band) phase data, depending on the baseline length. While carrier phase precision is much better than 1/40th of a cycle the data weight must be kept large enough to prevent unrealistic sensitivity to modelling errors. A constant phase bias is estimated at each of the three stations. The phase biases, which would result from integer cycle ambiguity and uncalibrated signal path delays at each station, are taken to be constant and initially unknown.

b) Noise modelling for station-differenced phase data

While carrier phase can be measured with high precision at each antenna, there would be noise in the station-differenced phase due to fluctuations in the difference between the propagation media along the two lines of sight and the difference between the station frequency references. If the stations have a common frequency reference, the noise in the station-differenced phase data should result mainly from fluctuations in the signal propagation media. For 14 GHz signals, the dominant media fluctuations are

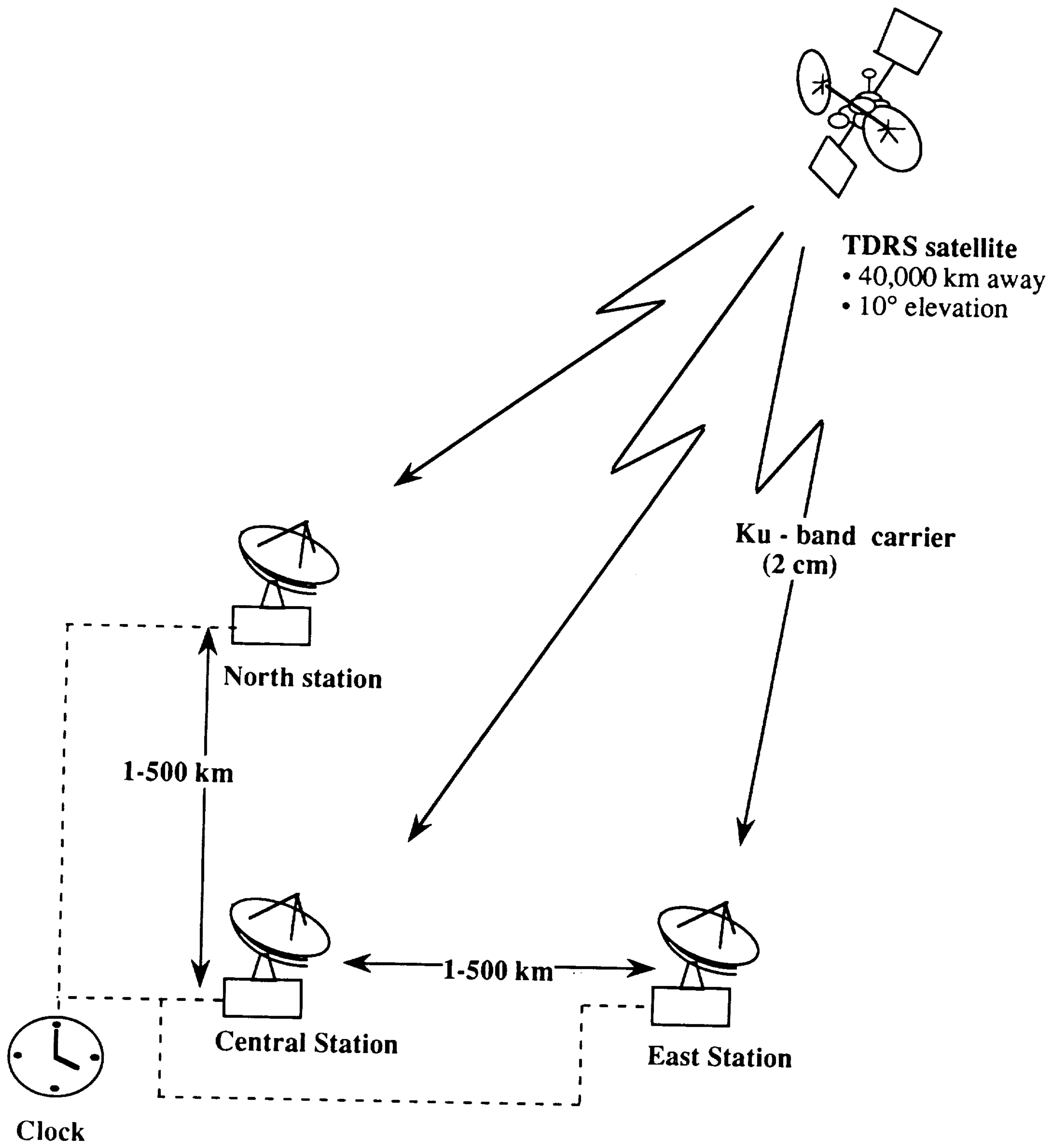

Figure 1. Configuration of antennae for carrier phase measurements used in covariance study.

Table 3
Station Clock Models Used

Clock type	Phase Random Walk growth (km/√s)	Frequency Random Walk growth ((km/s)/√s)	Phase growth in 24 hours (cm)
Rubidium	1.5×10^{-6}	4.7×10^{-10}	1200
Cesium	1.5×10^{-6}	1.5×10^{-11}	40
Hydrogen Maser	1.5×10^{-8}	4.7×10^{-12}	10

from troposphere, so charged particle media are ignored in this analysis. The data noise for differenced phase data from two stations sharing a frequency reference is simulated by estimating a stochastic troposphere model at each station. To simulate the noise in the differenced phase data from stations with separate references, a stochastic clock model at each station is also estimated.

i) Troposphere modelling

The sigma-squared and time constant of the Gauss-Markov stochastic troposphere model used for each baseline are determined by fitting the autocorrelation of the station-differenced troposphere to an exponential (table 2). In the case of a 1 km separation between stations, two exponentials, one accounting for the short term fluctuations and one for the longer term fluctuations, are required to fit the autocorrelation function well. For the longer baselines, a single exponential fits well. The autocorrelation function is calculated in a flat-earth frozen troposphere model using a wind velocity of 10 m/sec and a tropospheric height of 1 km.[8,9] The autocorrelation is calculated for north-south and east-west baselines of several sizes accounting for the 10° elevation of the TDRSS satellites from White Sands and the projection factor between the wind direction and baseline (fig 2). Since there is little dependence on baseline orientation, the autocorrelation with the largest value at t=0 for a given baseline length is used to model all the baselines of that length. For symmetry, a troposphere model is applied at each station, with a sigma-squared half the value of the differenced troposphere autocorrelation sigma. The differenced troposphere fluctuations grow with baseline; but they grow slower than the enhanced precision in angle measurement due to the longer baseline.

ii) Station Clock Modelling

Slightly longer baselines can be considered if frequency reference sharing between stations is not required. Baselines of 100 and 500 km in which the stations each have their own frequency reference are considered in this paper. If the stations forming a baseline have separate frequency references, drifts between the clocks at the stations increases the noise in the measurement of station-differenced phase.

Typical frequency standard stabilities have a short term behavior of a white frequency noise and long term behavior of a random walk in frequency. In the OASIS program, the phase of a station clock is modelled as a polynomial in time, $\phi = \phi_0 + \omega_0\tau + \alpha\tau^2$, where τ is time past some epoch, ϕ_0 is a bias parameter, ω_0 is a drift parameter and α is a drift rate parameter. In this analysis, the white frequency noise behavior of the station clocks is modelled by applying a random walk noise model to ϕ_0, the clock bias, and the random walk of frequency behavior is modelled by applying a random walk model to ω_0, the station clock drift. Clock models representing the performance of rubidium, cesium and hydrogen maser standards, shown in table 3, are used in this study.[10] The rubidium and cesium standards have comparable short term stability, but the cesium has better stability on a 24 hour or greater time scale. A hydrogen maser is stable enough over a 24 hour data arc that would result in position solutions comparable to the case in which the stations share a common clock.

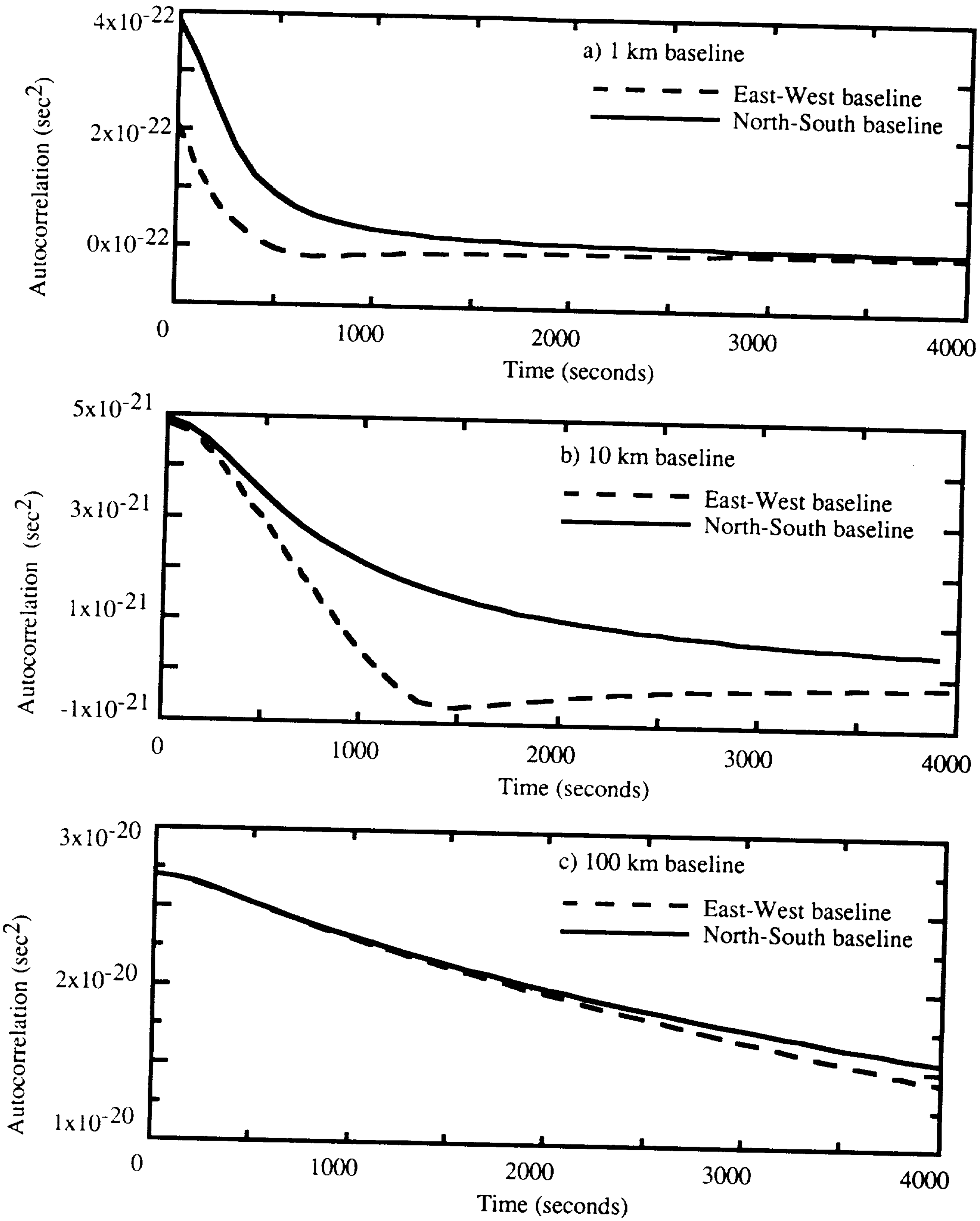

Figure 2. Autocorrelation function of the troposphere differenced between two lines of sight with elevation of 10° from stations forming an east-west or a north-south baseline of size: a) 1 km, b) 10 km or c) 100 km. These are based on the model developed in ref 9.

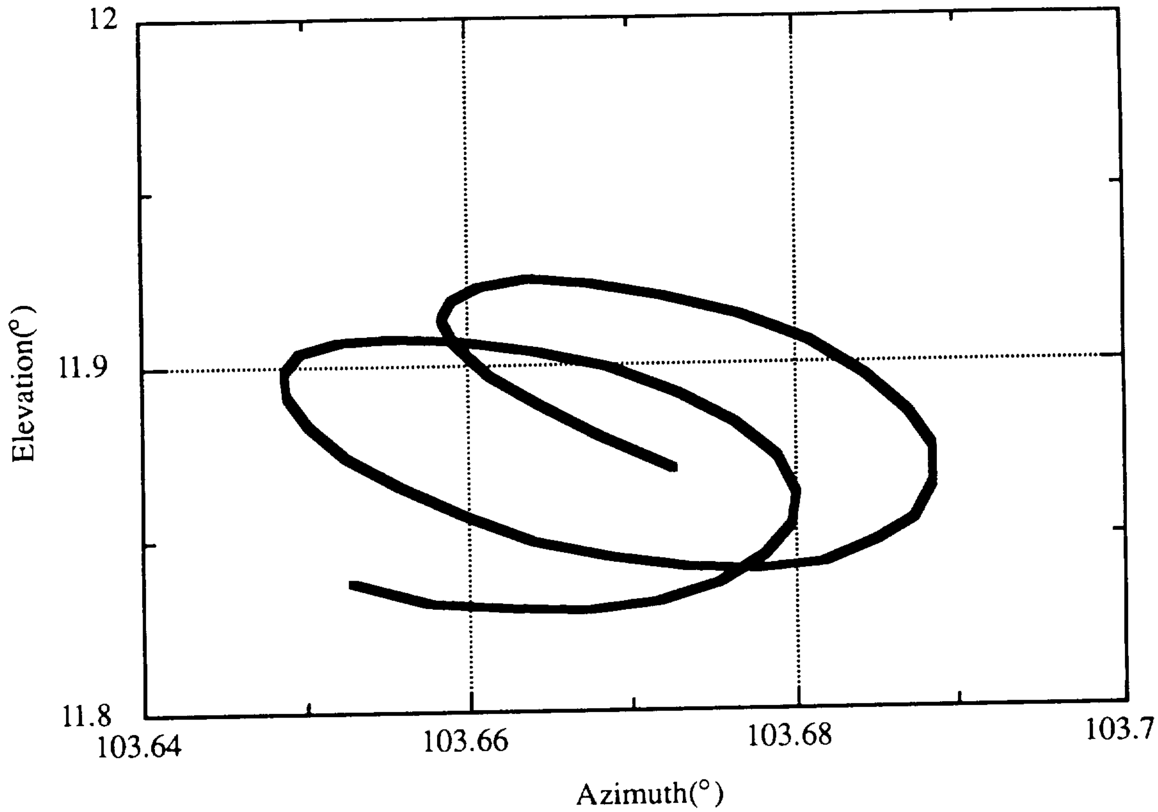

Figure 3. Signature on the plane of the sky of the TDRS-east as seen from White Sands, NM.

c) Range data

Station-differenced phase data are able to trace the small signature in the plane-of-sky made by a geosynchronous satellite over a day (fig 3). In the geometry at hand, this determines 5 elements of the TDRSS orbits well, but leaves the longitude-of-node poorly determined. In addition, an orbit solution with this data type alone is very sensitive to mismodelling of forces such as solar pressure. A small mismodelling results in a several hundred meter error in the satellite position with most of the error in the satellite longitude. For these reasons, it is impractical to perform TDRSS orbit determination with station-differenced phase data alone.

In the analysis, range and doppler data from one station are included in the simulated data set. The range data are modeled with a white noise measurement error along with systematic bias parameter which is estimated, with an *a priori* constraint corresponding to the ranging system calibration accuracy. Doppler data are modeled simply with white measurement errors. We will consider weights and biases for the range measurements between 1 and 30 meters.

Range data can help determine the along-track position, which helps constrain the satellite longitude. Low precision range data (20-100m) is good enough to control the error due to solar pressure mismodelling. Better range data are required to reduce the computed along-track error to an acceptable level. The range partial with respect to the longitude of the TDRSS satellite orbit, $\delta\rho/\delta\phi$, for the geometry discussed here is 1/7, so a range measurement of better than 7m is required to get the longitude component of the TDRSS position error below 50m. The range data precision does not have to be quite this good, since it averages down over several measurments. However, the systematic error in the range measurment must be less than 1/7 of the required position error.

Table 4
TDRS-east satellite position errors using station-differenced phase data from connected-clock stations and range from White Sands.

	case 1	case 2	case 3	case 4	case 5
inputs					
Data weights:					
Station-differenced phase baseline (km)†	100	100	100	10	1
Dopper Noise (mm/s)‡	1.0	1.0	1.0	1.0	1.0
Range Noise (m)‡	10	10	1	1	1
Range Bias (m)	30	10	1	1	1
Consider Inputs:					
Range station location(m)	2	2	2	2	2
Station-differenced phase station locations(m)	.35	.35	.35	.35	.35
Solar pressure(% of reflectivity)	2%	2%	2%	2%	2%
results (epoch)					
Computed error (m)	247	84	14	25	63
Consider errors (m)					
Solar pressure	12.1	10.4	12.4	12.4	12.6
Range station location	14	14	14	14	14
Station-differenced phase station location	0.12	0.12	0.10	0.12	11.9
RSS position error (m)	**248**	**86**	**23**	**31**	**67**

† Two orthogonal baselines of this size are used.
‡ Data noise for 10 minute points.

IV. Covariance Results

a) Computed Errors

Table 4 shows the computed errors that result in several scenarios with station-differenced phase on two orthogonal baselines combined with range and doppler data all taken from White Sands for a 24 hour data arc. The first case corresponds to using the current WSGT ranging system data enhanced with station-differenced phase data. The others are examples of performance with various combinations of baseline length and improved range data.

i) Dependence on Range Bias

As is apparent from Table 4, the position accuracy is limited by the ranging system accuracy in several cases. Figure 4 shows the dependence of TDRS position error on range bias *a priori* uncertainty when the range data are combined with station-differenced phase data from various sized baselines. In all cases the range data noise was 1 m. In order that the position error be limited only by the station-differenced phase data, the range data bias must be better than approximately 1 m in the 100 km baseline connected frequency reference case. For smaller baselines, the station-differenced phase data are weaker and the TDRS position error becomes dominated by the station-differenced phase data at larger values of range bias error. In all cases, the range data largely determine the component of the TDRS position in the longitude direction, while the station-differenced phase data determine the other orbital elements. In order to obtain a range bias of 1 m it may be necessary to upgrade the current 2-way ranging from White Sands to TDRSS. Typically, a ranging system can be calibrated to about 90% of the inverse bandwidth of the system; thus a ranging system calibrated to 1 m would require a 30 MHz bandwidth.

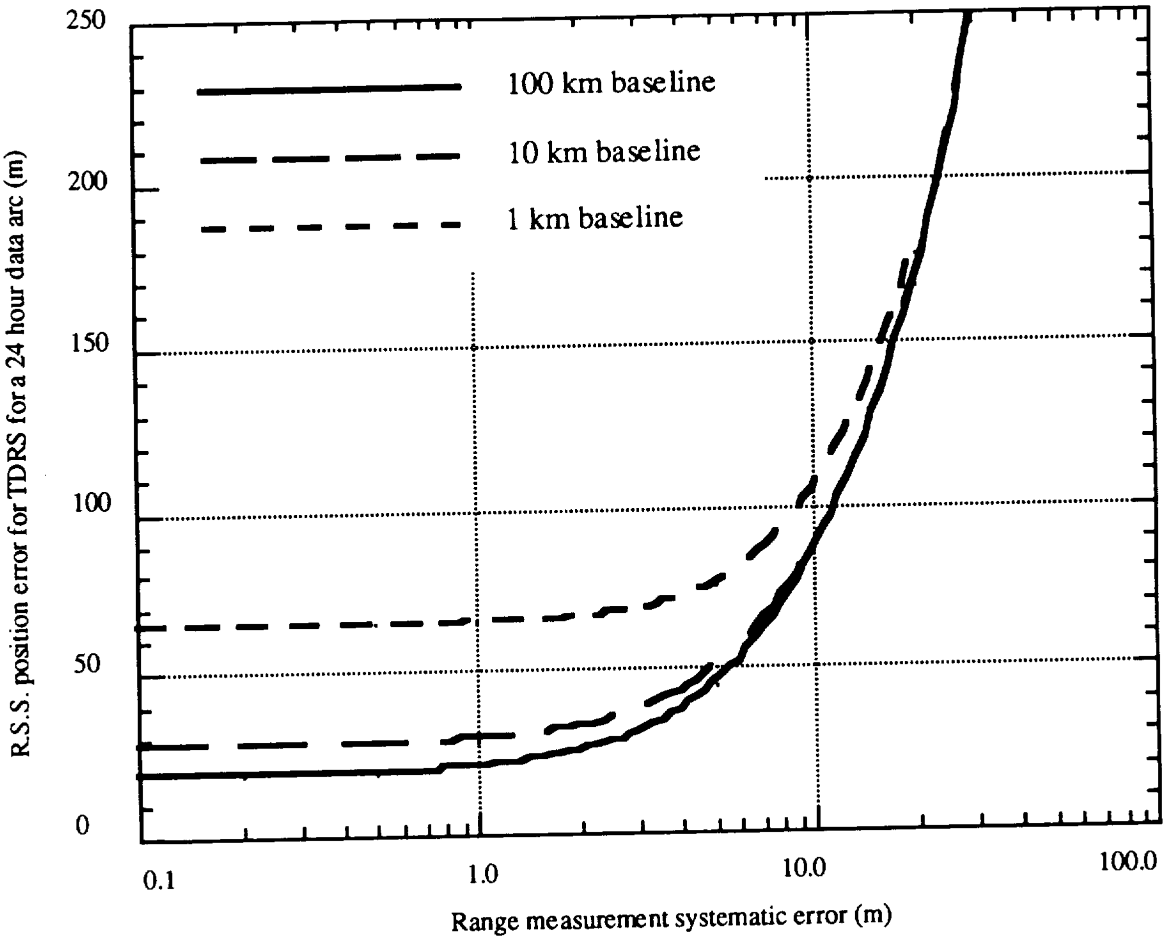

Figure 4. R.S.S. position error for a TDRSS satellite using a one day arc of station-differenced phase and range data from White Sands as a function of systematic error in the range measurement. The range noise is held fixed at 1 m for a 10 minute point.

ii) Dependence on Troposphere model

Since the troposphere model is the important source of data noise for the station-differenced phase data in the case where the stations share a common clock, it is useful to know how the troposphere model parameters affect the TDRS position error. The two parameters that describe the station-differenced troposphere, as modelled in this paper, are the steady-state sigma and the correlation time constant. Figure 5 shows how TDRS position accuracy varies with these model parameters. In Fig. 5a, The magnitude of the troposphere sigma is varied by a factor of two from the nominal values shown in Table 2, while the time constant is held fixed at its nominal value. In Fig. 5b, the time constant for the stochastic station-differenced troposphere is varied, while the steady-state sigma is held fixed. In both cases the position error changes slowly with the variation in troposphere parameters.

iii) Effect of Separate Station Clocks

Because a configuration in which the stations have independent freqeuncy references may be simpler to build, we consider it as well, despite the cost in position accuracy. Position determination with station-differenced phase depends on tracing the signature in the plane-of-sky from the station made by the satellite. In the case of a geosynchronous satellite like TDRS, there is a 24-hour period to the signature (Fig 3), so that is the time scale during which the difference in the station clocks must be stable. The expected plane-of-sky error from clock phase error growth is approximately $\rho\Delta\phi_C/B$, where ρ is the

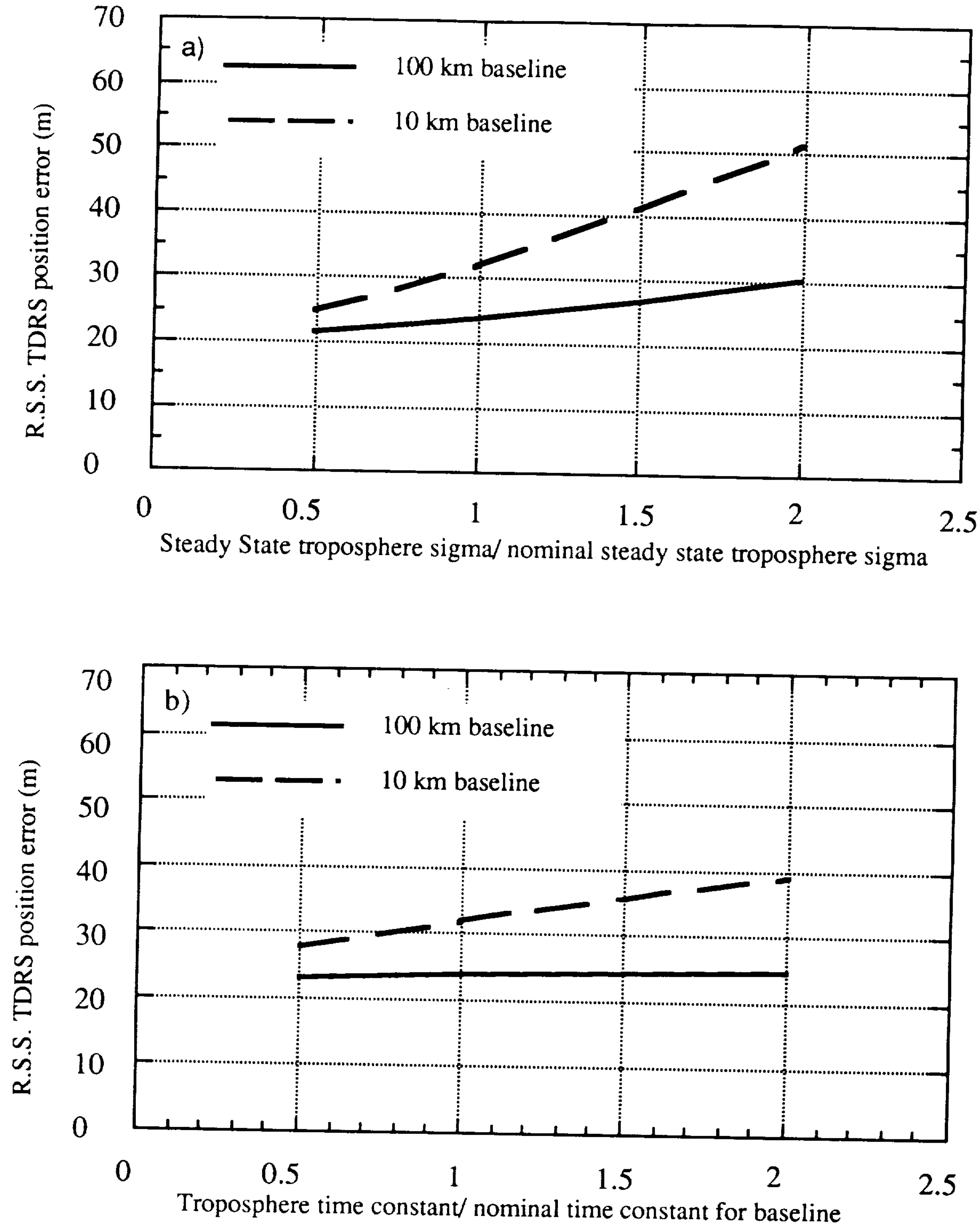

Figure 5 a.) R.S.S. position error for a TDRSS satellite using a one-day arc of differenced carrier-phase and range data from White Sands as a function of sigma of station-differenced zenith troposphere. b.) R.S.S.position error for a TDRSS satellite using a one day arc of differenced carrier-phase and range data from White Sands as a function of station-differenced zenith troposphere autocorrelation time constant. A range data noise of 1 m and range bias of 1 m were used in both figures.

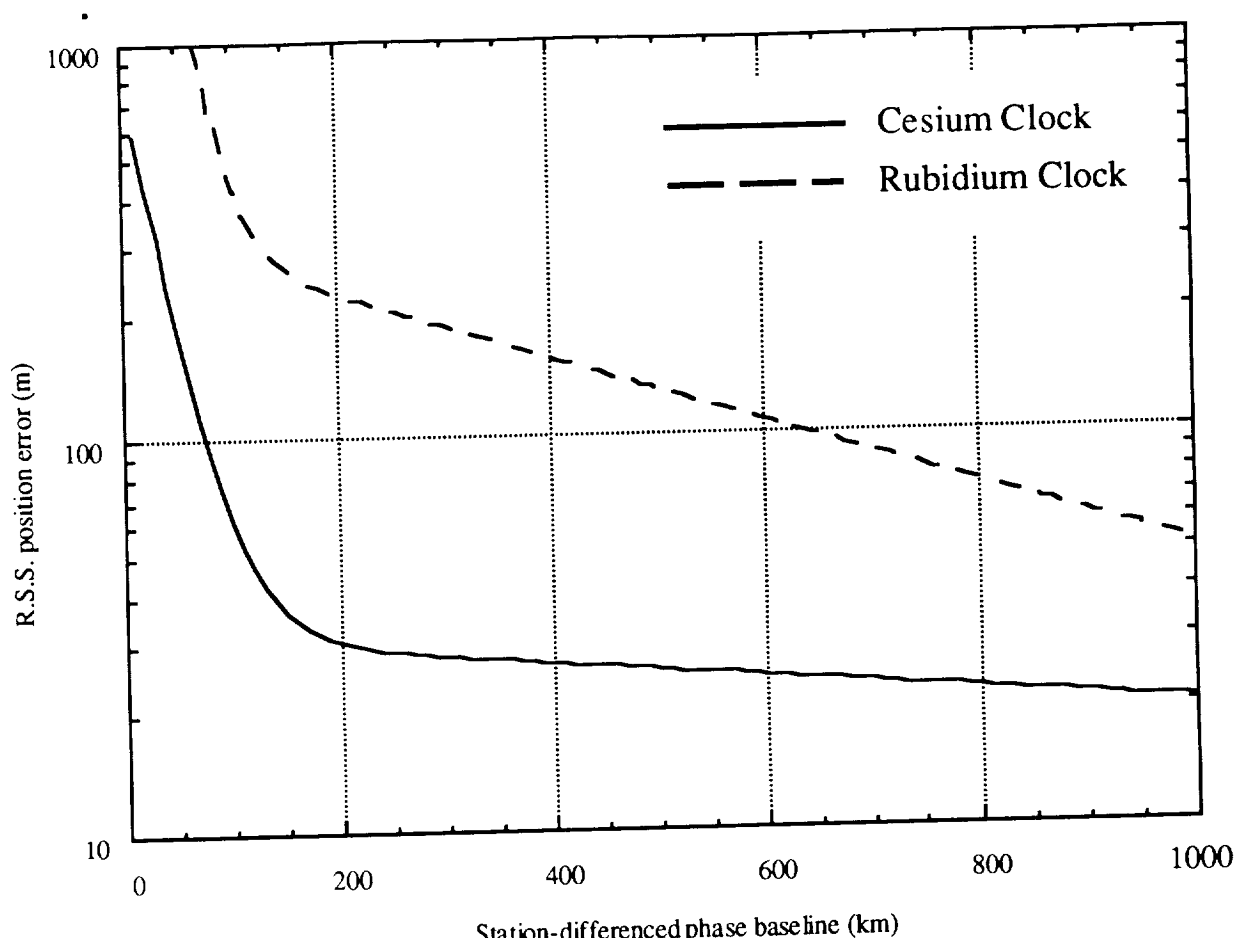

Figure 6 The position error for TDRS-east using station-differenced phase data from two orthogonal baselines plus range data (1 m error) as a function of the baseline size, if the stations have separate frequency references of the type indicated.

distance between the satellite and the tracking antennae, B is the projected baseline length, and $\Delta\phi_C$ is the path delay due to the phase error from the two clocks drifting. Clearly, somewhat larger baselines are required to offset the loss.

Figure 6 shows the expected position error as a function of baseline length if each of the three stations measuring phase (see Fig 1) have their own frequency reference. Only cesium and rubiduim standards are shown, since consider and troposphere errors dominate the position error if each station has a hydrogen maser standard. In these examples, the range data have a 1 m data noise and 1 m bias error and are unaffected by the clock model.

b) Consider Errors

A consider analysis was performed to assess the sensitivity of the solution to mismodelling due to solar pressure and station location uncertainty. Errors due to polar motion uncertainty and mismodelling of gravity harmonics are found to be small, and not included in Table 4. The same consider errors are applied to all the cases described in this paper.

In this covariance analyis, solar pressure is modelled as a force due to specular reflection from the satellite surface: $F = CA(1+\eta)/r^2$, where C is the solar flux, r is the distance between the sun and the satellite, A is the reflecting area, and η is its reflectivity. The reflecting area is assumed to be constant with a value of 40 m^2. Solar pressure mismodelling is simulated as a 2% of the solar reflectivity

Table 5

Position error (m) achieved after a station-keeping maneuver

	case 1	case 2	case 3	case 4	case 5
inputs					
carrier phase baselines (km)	100	100	100	10	1
range weight (m)	10	10	1	1	1
range bias (m)	30	10	1	1	1
doppler weight (mm/s)	1	1	1	1	1
results					
error after 30 mins†	379	301	287	124	319
error after 60 mins†	249	94	49	97	242
error after 90 mins†	248	91	43	85	206
error after 120 mins†	247	88	40	76	182

† The solution includes 22 hours of data from before the maneuver and data after the maneuver up to the times listed in column one of table (see text). The errors include consider errors at the same levels as table 2.

parameter, $(1+\eta)$, with a nominal value of 1.42. It has been shown that solar pressure effects on the TDRSS satellites can be modelled to 2%.[11] Operationally, the solar reflectivity might be estimated, enlarging the computed error somewhat, and a much smaller mismodelling error could be considered.

The error due to uncertainty in the antenna locations is also determined with consider analysis. A 2 m uncertainty in each component of the ranging station location is used. A 35 cm uncertainty in each component of the stations measuring carrier phase is used. This corresponds to allowing about 0.5 m uncertainty in the baselines formed by pairs of stations. The sensitivity to the baseline length error is small except in the case of the smallest baselines studied.

c) Recovering Position After a Maneuver

The TDRSS satellites occasionally make corrective maneuvers in order to stay in their desired orbits to the required tolerance. The requirement for TDRSS is 50 m position error one-sigma within two hours after the maneuver. Because good position determination with station-differenced phase data requires about a 1-day arc, it is impossible to redetermine all the state parameters to this accuracy within two hours. Instead of redetermining the whole state, we may take advantage of the good instantaneous plane-of-sky velocity information in this data type by estimating the velocity change in the satellite state associated with the maneuver. Line-of-sight doppler data collected along with the range data at one station, provides the third component of the satellite velocity.

It is assumed that there is nearly a day long arc of station-differenced phase, range and doppler data prior to the maneuver. The corrective maneuver is modelled as a velocity impulse with well known time of burn. No *a priori* knowledge of the error in the impulses is assumed. Table 5 shows position error achievable after such a maneuver, if the state and the maneuver are both estimated using a day long arc of data ending two hours after the maneuver. In each case, the position error is about twice as large as after a full day data arc uninterupted by a maneuver.

V. Conclusions

We have examined the possibility of using station-differenced carrier phase with stations forming small baselines for orbit determination for the TDRS satellites. We find that two orthogonal baselines between 10 and 100 km, located near White Sands, are sufficient to obtain position accuracies of 25 to 50 meters. However, for the geometry considered, station-differenced phase data alone poorly determine the longitude of the spacecraft, and thus range data with systematic error of about 1 m must be included to obtain these accuracies. The accuracy of the station-differenced carrier phase data from such moderate baselines results from the use of a common frequency reference at the stations. As can be seen in figure 6, acceptable position accuracies can also be acheived with separate station clocks; however, much larger baselines are required.

The station-differenced phase data along with two-way doppler can be used to estimate the velocity change associated with a station-keeping maneuver well enough to determine TDRSS position to 50-100 meters within two hours of a maneuver. Nearly a full day is required to recover the best possible position accuracy with this method. No *a priori* knowledge of the size of the impulse is required; though the time of the impulse is assumed to be known.

Range data with accuracy of about 1 m is crucial for this technique to be viable independent of the baseline size used and whether or not the stations share a frequency reference. With a 1 m systematic error in the ranging system, the position accuracy is only slowly dependent on the exact value of the bias.

Acknowledgment

The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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